Special Theory of Relativity
Prof. Shiva Prasad
Department of Physics
Indian Institute of Technology, Bombay

Lecture - 14
Velocity Four Scalar

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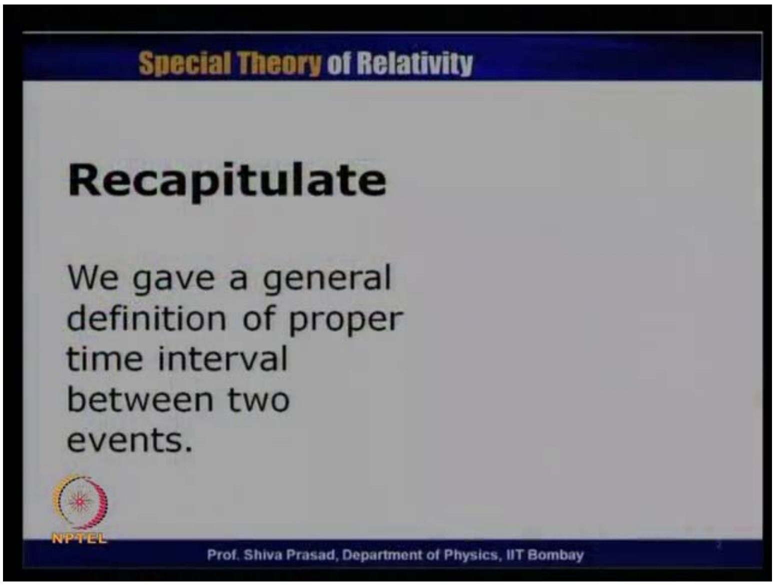

Hello. Let us recapitulate what we did in our last lecture. We had given a general definition of proper time interval between two events. When we had first introduced the concept of proper time interval, it was in the context of the use of time dilution formula. At that time we told there is two events occur in a given frame in the same position, then the time interval between these two events as measured in that frame of reference will be called proper time interval.

Of course, this particular definitions sort of assumed, that it is possible to find out two events, which occur in it is possible to find out a frame of reference in which the two events occur at the same position. Later we have seen when we are defining space like and time like events that it need not always be so. If we are restricting our speeds to up to the speed of light, then it is not necessary that if we take any two arbitrary events. I will be able to find a frame of reference, in which these two arbitrary events would occur at the same position.

But then we discussed in our last lecture, that we can still give a more general definition of proper time interval. Even in such cases where it is not possible to find a frame, in which the two events occur at the same position. Only thing in this particular case, the proper time interval would turn out to be imaginary and of course, that we have we have now defined the proper time interval it turns out to be a four scalar. It means it does not change when I change the frame of reference.

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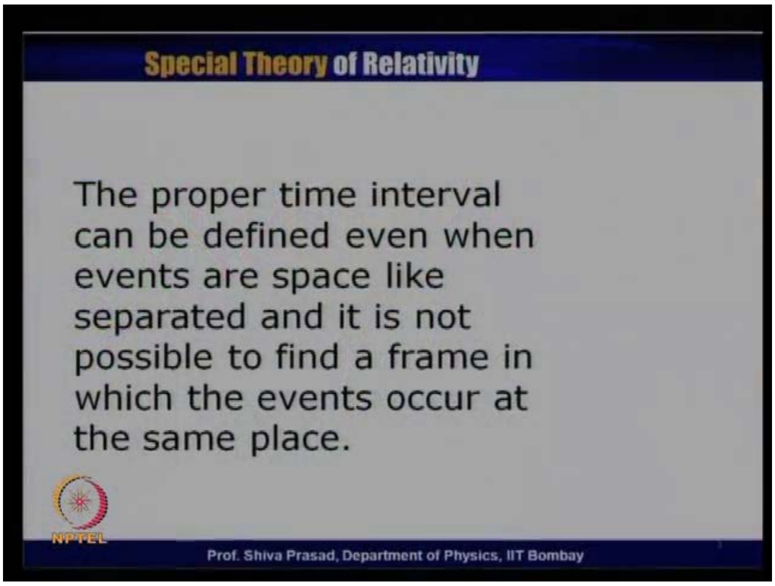

So, what we have said that this particular time of interval, proper time interval can be defined even when the events are space like separated and remember when the events are space like separated, it is not possible to find a frame in which the events occurs at the same place. So, even in such cases a proper time interval can be found out by finding out delta x delta y delta z and delta t.

Only thing for space like intervals, space like separated events the proper time interval will turned out to be imaginary. While for time like separated events, it will turn out to be real because in if the events are time like separated, then it always possible to find a frame of difference in which the two events occur in the same position. So, it is possible to make a measurement in that particular frame of reference, of time difference between two events which will turn out to be real time interval and that is what is the proper time interval. That of course, we have given a certain examples, to mention about hmm the how the time proper time interval turns to be same just to convince our self that whatever we are trying to say is correct.

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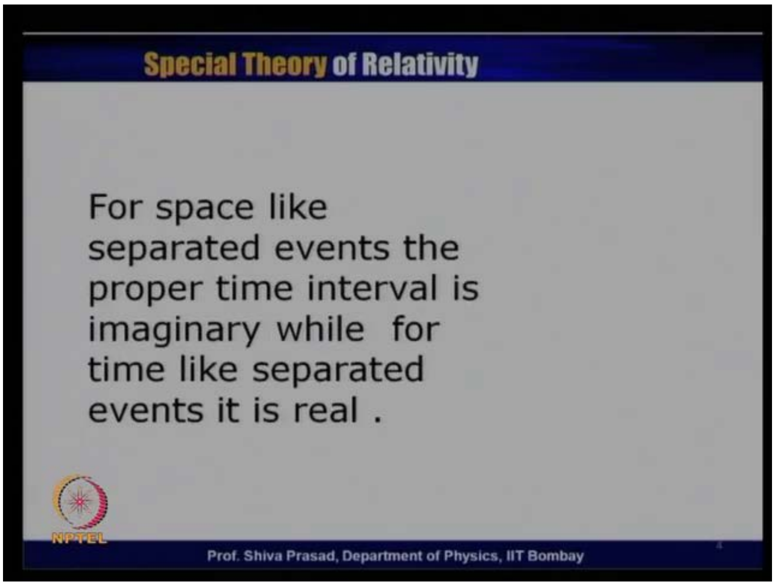

Now, let us go a step further. We have defined a position four vector. Then we had define a displacement four vector. Now, let us try to define a velocity four vector. The question is that why we are at all talking about four vectors. One thing is of course, I have mention that convenience any new quantity that we define is because it makes further mathematics often the physics simple. Also here we are doing with a specific aim of looking at the new definition of momentum. So, that I assure that conservation of momentum in a frame of reference implies the conservation of momentum is same, unchanged in any other frame of reference. It becomes an universal quantity and advantage of dealing with four vectors is that certain amount of universality is maintained in this things.

So, now let us go the velocity four vector. Now, we imagine that two events that we had talked about earlier, those two events are not an arbitrary events, but their specific events related to displacement of a particle near a space. So, that is one particular particle which is found at a given position. This I called as one event. Then the same particles get displaced and at a different time is found at a different position, in the same frame of reference. Then I call this as the second event. So, the two events that I am talking or not really now arbitrary event, but the events that are related to the displacement of a single particle of a particular particle in a given frame of reference.

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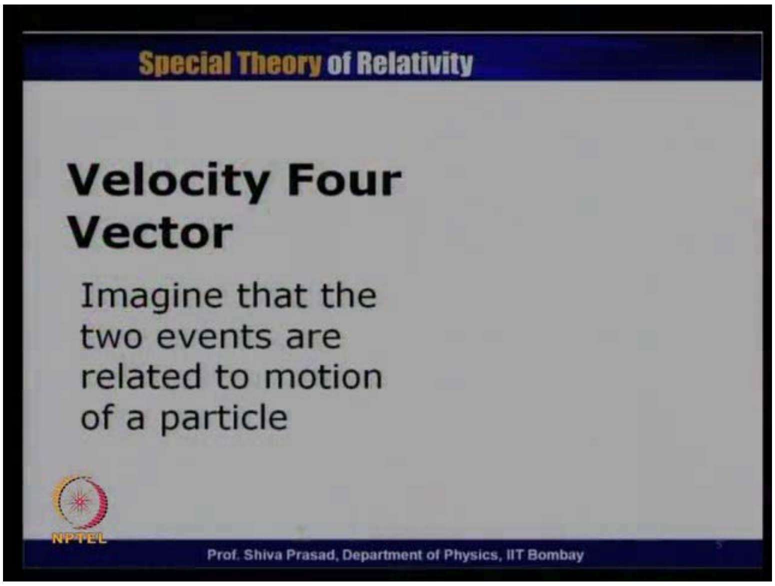

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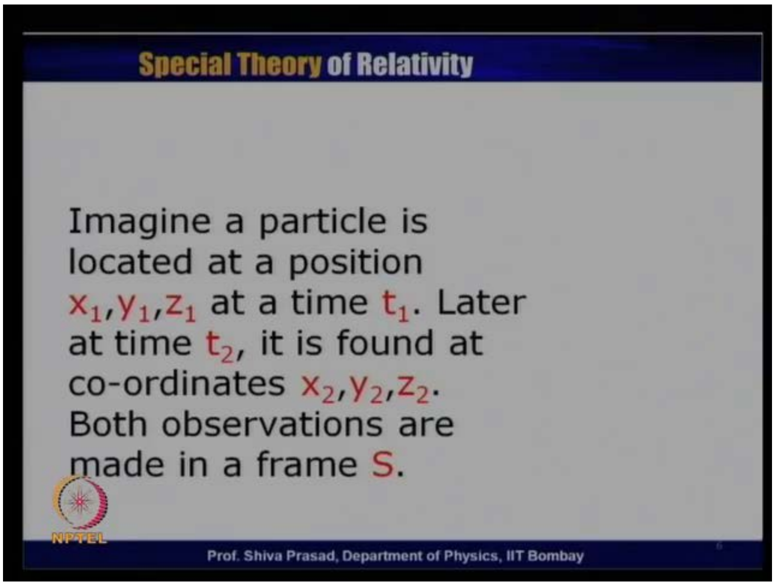

So, let us imagine that a particle is located at a position x 1 y 1 z 1 at time t 1, so in a given frame S. We are, we have x is and we have our watch. We find that at a particular time t 1 the particle is located at the position x 1 y 1 z 1 and at a later time t 2 the same particle is now found at a co-ordinate x 2, y 2, z 2. So, these become the co-ordinates or position co-ordinate of the two events which we are talking about. Relating to the movement of the particular particle, and of course we assume that both this observation are being made in the frame in a given frame S.

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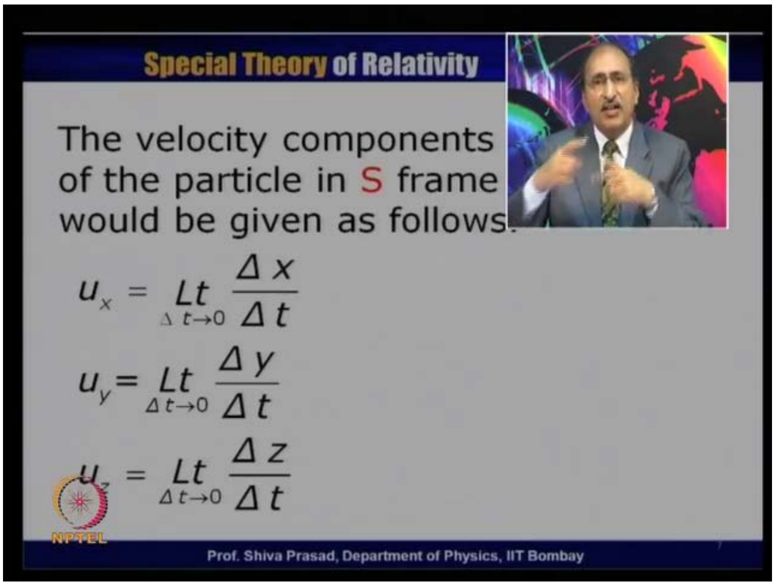

Now, I take the difference between this two and if I called delta x is x 2 minus x 1 as we have been calling earlier.

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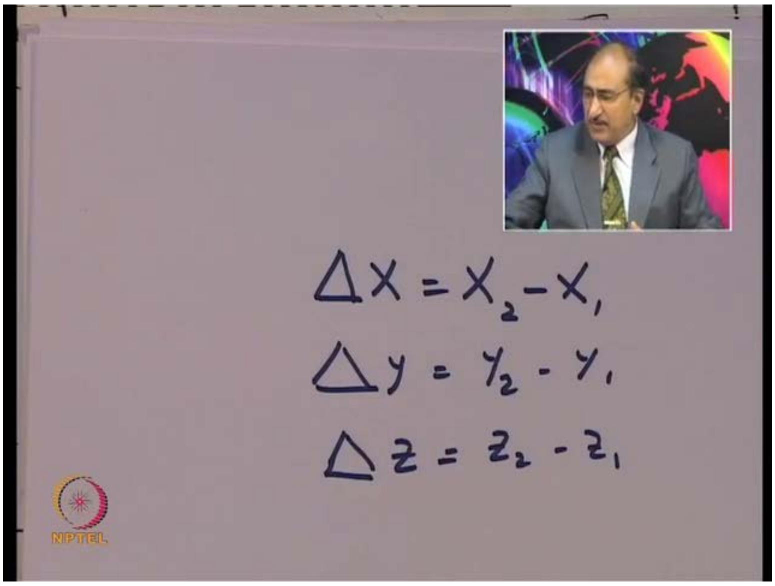

Similarly, we call delta y has y 2 minus y 1 and delta z has z 2 minus z 1 and assume that the time that has elapsed between this particular this particular events is very small. Then in the limit delta t tend into 0, if I divide these quantities by delta t. I will get the instantaneous velocity of the particle. Let me again remind you that this particular particle need not be moving with the constant velocity. The constant velocity instruction we had put only on the frames side, within the relative velocity between the frames because that is why we are talking of the initial frames. Now, both this initial observers could be observing one particular particle, which may or may not be have an acceleration.

So, on the particle which is being observed by the two observers, in the two frames of references that particle need not be moving with the constant velocity. So, I have to talk about the instantaneous velocity and this aspect we have discussed in the earlier that instantaneous velocity will be given by u x in the limit delta t tend into 0 delta x by delta t.

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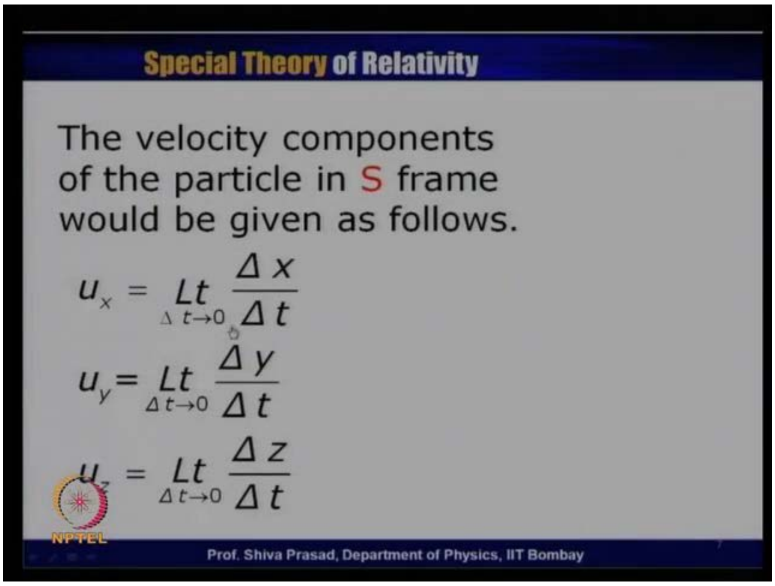

Similarly, u y in the limit delta t tend into 0 delta y by delta t in the limit u z in the limit delta t tend into 0 delta z divided by delta t. So, this is how an observer in the frame of references. S would measure the components of the velocity. Once he knows the time differences between these two events and the position difference between these two events, then by dividing by appropriately by the time as measure by his, by him in this own frame of reference. Remember we have always being emphasis in these aspects, that a observer has to be consistence all the measurement have to be made in his or her own frame of reference. So, if I am talking of delta x this delta x is in his or her frame of reference. Similarly, delta t is also in his or her frame of reference.

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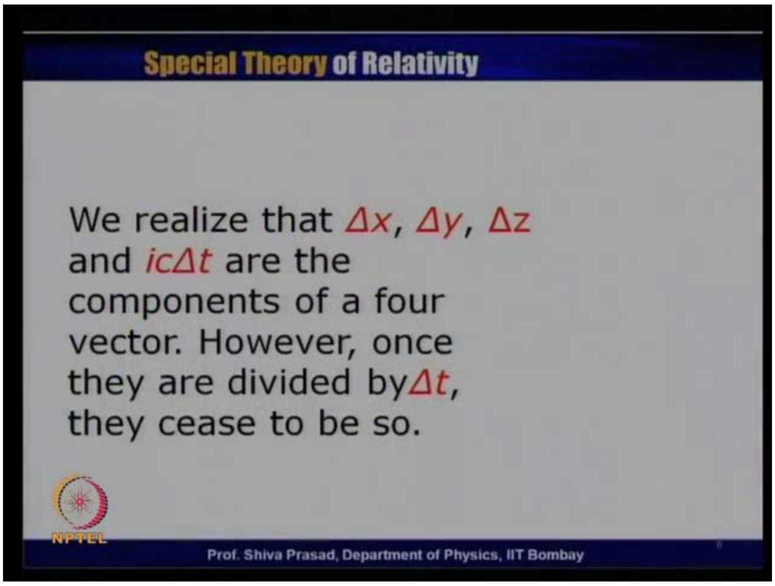

Now, let us tried to talk about the four vector language. We have earlier seen that delta x delta y delta z and i c delta t, these are the components of the four vector. What it means that if I go to a different frame of reference, delta x would be different from delta x measured in x prime, which I call now as delta x prime. Delta x prime will be different from delta x. Similarly, delta t prime will be different from delta t. But they will transform using the, same transformation matrix that we have written earlier. Then only they qualified to be called as the components of the four vector.

So, we realize the delta x delta y delta z and i c delta t are the components of the four vector. Now, if I divide these things or multiply this things by something which is a constant, which is not a frame dependent quantity, then that quantity will also remains or will also turn out to be a components of four vectors.

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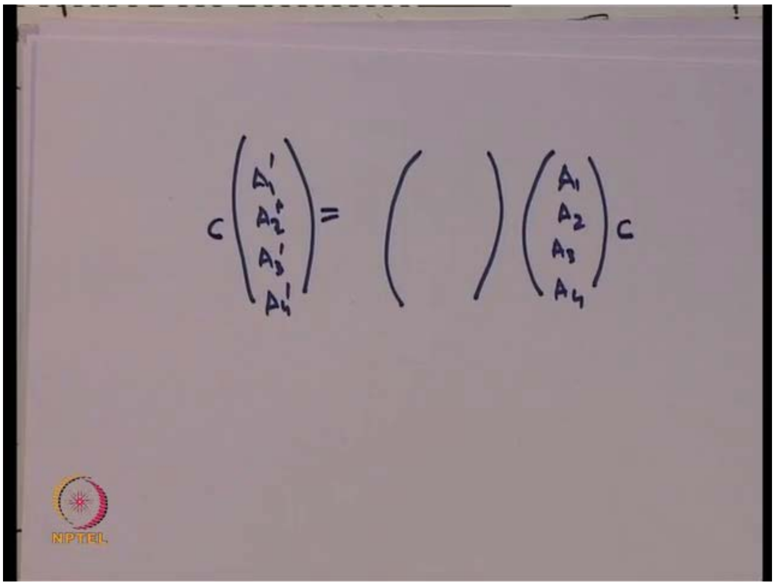

So, say for example, if we have matrix here and I am multiplying two matrix matrices to get something here. Here we have A 1 A 2 A 3 A 4, here we have A 1 prime A 2 prime A 3 prime A 4 prime and this is the transformation matrix, which is used to transform these to this. If these are the components of four vector. If I multiply this by a constant C here and this also I have multiplied by a constant C. C times A 1 C times A 2 C times A 3 C times A 4, will automatically give me C times A 1 prime C times A 2 prime C times A 3 prime C times A four prime, provided this C and this C are same. It means once I change my frame of reference, this C should not change.

So, if it is a four scalar, then if multiply the components of four vector by a four scalar multiply or divide by a four scalar, then that quantity remains the component. Still I, even after the multiplication will remain the components of four vector, but if we, I look at the velocity definition what I have done, have divided in how to achieve this particular velocity. Have divided this delta x and delta t I have divided this delta y and delta t I have divide this delta z by delta t now, delta t is not a frame independent quantity.

We have seen from Lorentz transformation that if I change my frame of reference delta t would become different the time interval between this two events will turned out to be different. Hence to I agree that delta x delta y delta z and i c delta t are the components of the four vector, but once I divided by delta t. Delta t not being a four scalar being something which depends on the frame. Once I divide these things, they I will not get what I will get will not be the components of four vectors. Hence, division by delta t to these equations will not will not let them remain a four vector and that is how we had define our velocities.

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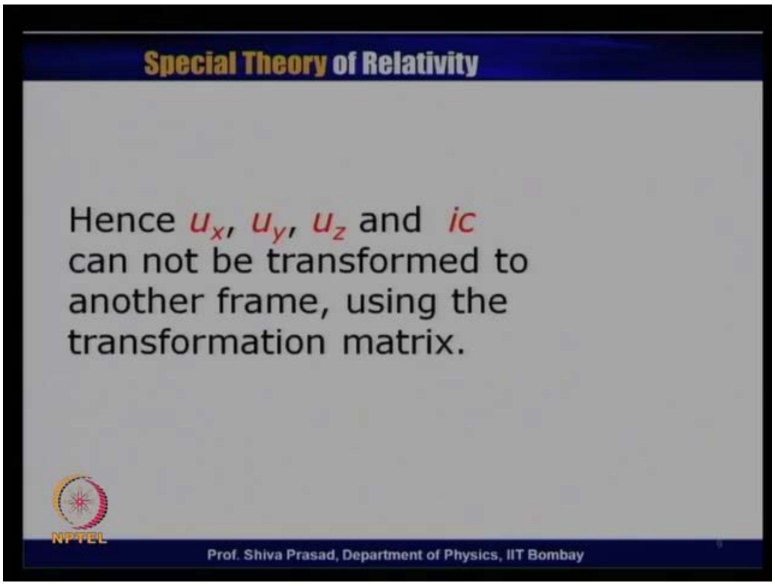

Effectively means that u x u y u z and i c cannot be transformed to another frame using the transformation matrix. Hence, u x u y u z and i c are not the component of a four vectors. They will not transform using this similar equation. This you might have given notice earlier that when I transform the velocity. The velocity transformation equation that I was getting they were very different from the normal Lorentz transformation equation. They will appear very very different. It is basically because u x u y u z and i c they do not transform by the same transformation matrix because they are not the component of a four vector. Now, because I want certain amount of universality to be built in all my definition, because eventually from velocity I have to go to the momentum.

Let us see how can I make this particular quantity and are how can I define the velocity four vector which will transform the way I want all the transformation of four vector. To be and if you realize the answer is very simple. If you instead of dividing delta x by delta t, I would have divided delta x by delta tau. I would have realize the delta tau is the proper time interval and this would not change when I change my frame of reference, so instead of dividing delta x by delta y by delta t delta z by delta t. If I would have define, if I would have divided delta x by delta tau delta y by delta tau delta z by delta tau, then I would be getting them to be the components of four vector. So, what I would should see, I should take delta x by delta tau.

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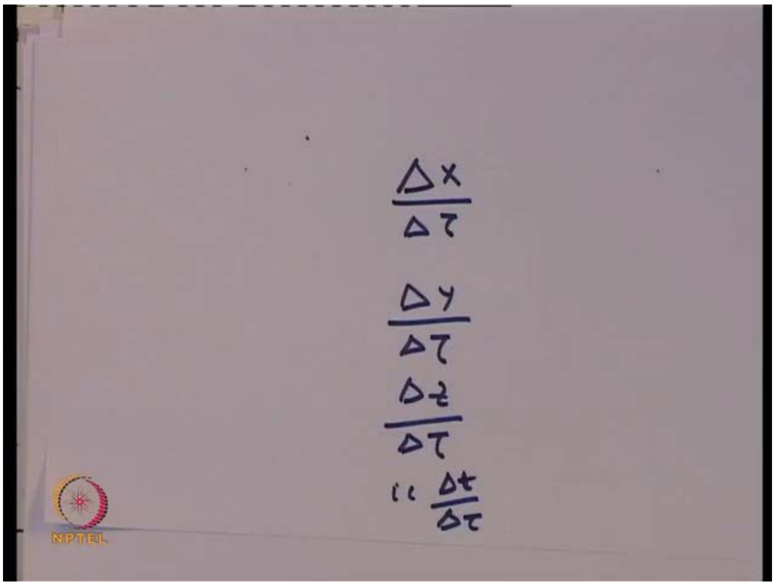

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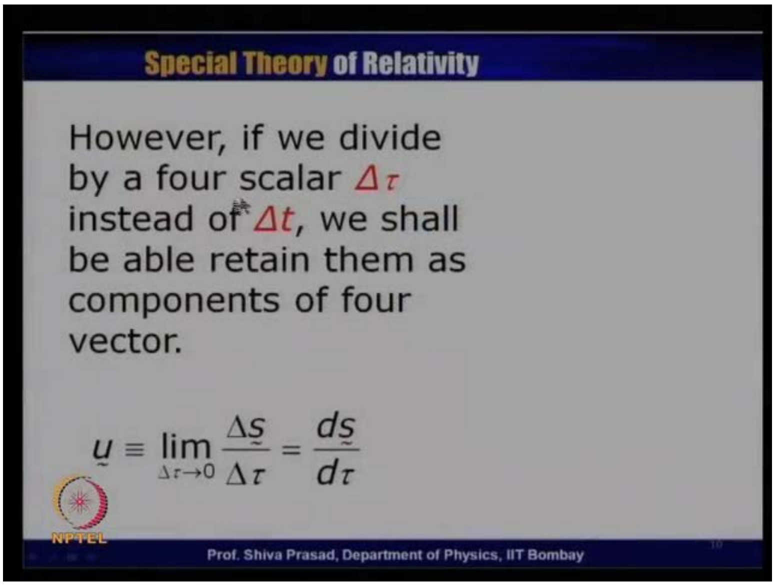

We have delta tau is the proper time interval. Similarly, delta y by delta tau delta z by delta tau and of course, I see delta t by delta tau. If I have, would have define them in this particular fashion. Then resultant of four number or four variables will really be the components of four vector. Hence I define a velocity of four vector using delta tau and not using delta t. Therefore, that is what have written here.

If we divide by a four scalar which is delta tau instead of delta t, we shall be able to retain them as components of four vector and eventually have to take the limit of delta t tend into 0 with effectively means delta tau also tend into 0, then the velocity four vector is defined has limit of delta t tend into 0, delta S this is what we have used for four vectors divided by delta tau which I can write as d s d tau using exactly the similar type of terminology that we use in differential calculus. So, we agreed that the velocity four vector is defined as d s d tau rather than d s d t.

Now, my questions is that if I am in a given frame of reference I directly measure only delta t. If I have to find out how or if I have to find out the components of the velocity four vector of a given particle, I have to go to and find out the delta tau. So, let us look at the way I can find it out essentially very simple, but let formulize it.

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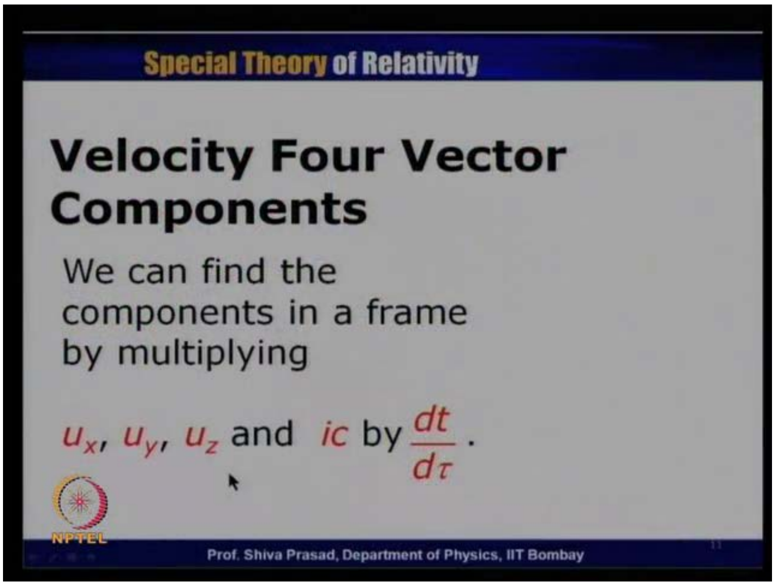

So, what I should do if you remember in u x I have divided delta x by delta t. What I want it is that it should have been divided by tau. So, whatever velocity component or a instantaneous velocity that I measure from the particle in a given frame of reference.

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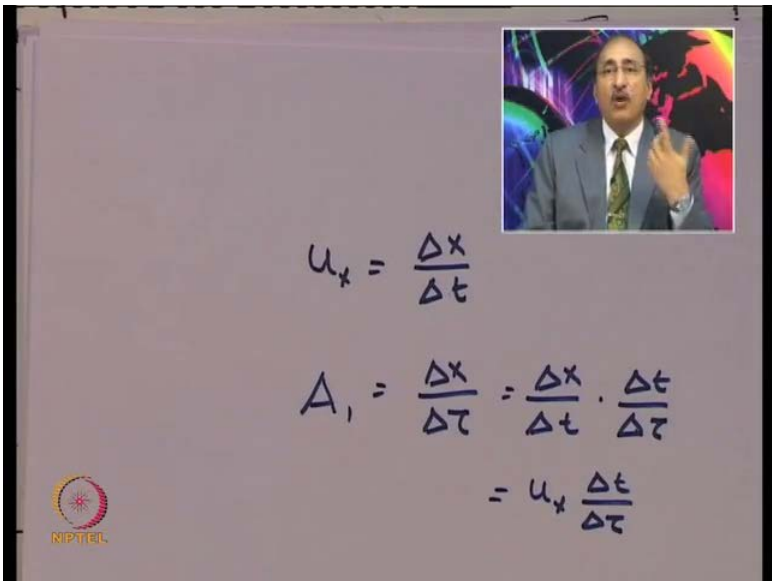

With if, this u x I have multiplied by d t by d tau, then I should be able to get the velocity component the component of the velocity four vector. Just to make it clear by u x by delta x by delta t in the limit of course, delta t tend into 0. Now, I want to take first component velocity four vector. So, let us see A 1. I want this to be divided by delta tau so A 1 is defined as delta x by delta tau. What I can do, I can write this as delta x divided by delta t into delta t divided by delta tau. This quantity is u x. So, I can write this as u x delta t by delta tau. So, it means all I have to do is take my velocity u x u y u z and multiplied them by delta t by delta tau.

Which of course, in the limit differential limit means I multiplied by d t and d tau. So, if each the component which we have said u x u y u z and i c if I multiplied by d t by d tau, then I would be able to convert them into components of velocity four vector. Now, let us evaluate this quantity delta t by delta tau. Once I evaluate the component, the quantity I will be able to eventually find out the method of how to find out in the given frame of reference the components of the velocity four vectors.

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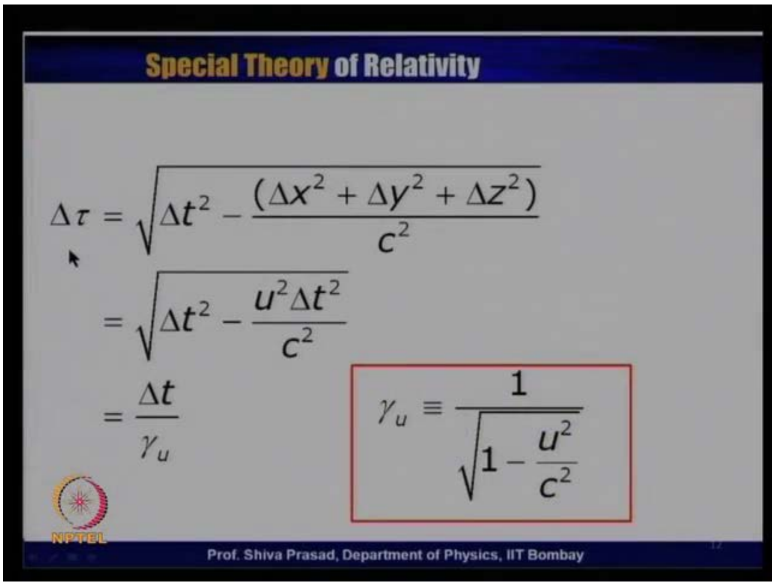

Now, we have seen by the definition of delta tau, a proper time interval between this two events. This is the general definition, which is under root of delta t square minus delta x square plus delta y square plus delta z square divided by C square. Now, we realize that delta x in given frame of references S is actually the motion of the particle. We have said earlier that the two events that I am talking are related to the motion of a single particle. Therefore, this delta x essentially is the displacement of the particle along the x direction. Similarly, delta y is the displacement of the particle along y direction. Delta z is the displacement of the particle along the z direction.

If I would have divided this by delta t I would have bought this as u x or rather u x square. If I would have divided by this delta t square rather I would have got as u y square . So, I can write this delta x square as u x delta t square u y as delta y as u y delta t square and delta z as u z delta t square. I can take delta t square common out of this, then I will get in this bracket u x square plus y square plus z square which eventually means u square. So, this whole quantity can be written as minus u square delta t square divided by C square.

If I take this delta t is common rather delta t square common because of this particular bracket under root, this become delta t and in the bracket what will be the remaining is 1 minus u square by C square under root. So, what will I beginning is delta t tau is equal to delta t 1 minus u square by C square.

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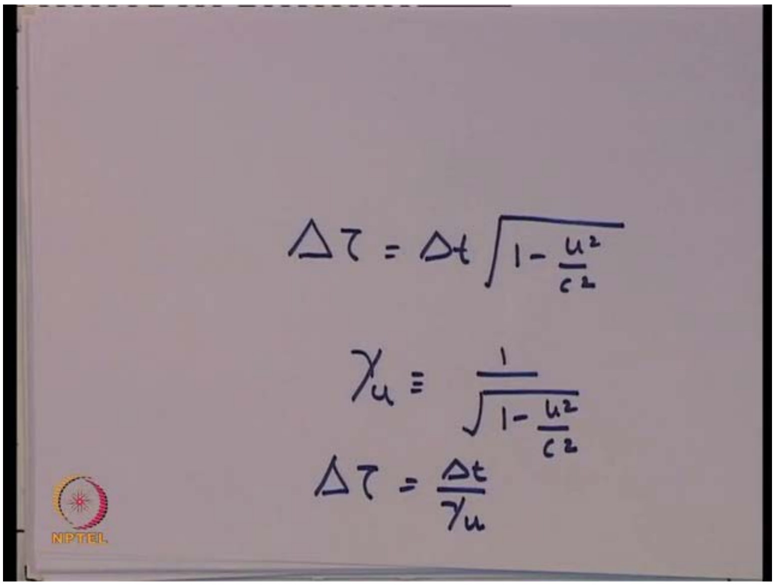

Now, I define a quantity. Gamma u has, let us put three lines just to say that this is a definition 1 minus u square by C square therefore, delta tau becomes delta t by gamma u.

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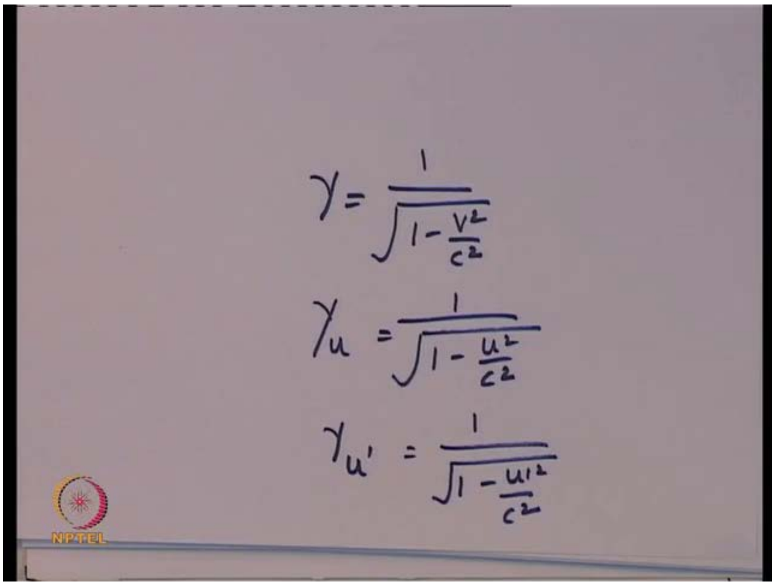

There is a little chance of confusion here. So, let us be clear. Let us revice our ideas because as you will be seeing that we have so many u v's and gamma gamma u's etcetera, are all appearing. Remember earlier we have talked of only one single gamma. This gamma was equal to 1 upon under root 1 minus v square by C square. So, our gamma was 1 upon under root 1 minus v square by C square. This v was the relative velocity between the frames and in fact this is the only gamma that we have in using in the present context. In fact if you look at the transformation matrix that we have used, for transforming a four vector there also use gamma. This gamma is something which depends on v and v is relative velocity between the frames.

This v is constant because the two frames that I am talking where initial. Therefore, this v has to be constant. So, this is the gamma that we have been talking so far, but when use velocity transformation and that time we have said that we might have to handle with three different speeds, three different velocities. So, you have one particular particle which is being observed by two observers. One observer setting in S frame another observer setting in S prime frame of reference. Both this observers are observing one particular particle and that particular particle need not be moving with constant velocity. Now, the relative velocity between S and S prime the frames in which are our observers are setting for that we have reserved the symbol v and using that v I defined gamma.

Now, an observer setting in S frame, observes this particular particle and find its instantaneous speed as u. Of course, you in fact in in not be even constant. As we have said now whatever is the value of u using that particular value of u that observers uses exactly a similar expression and finds a different gamma that we will call as gamma u. So, this gamma u depends on u which is the speed of the particle mind it as being observed by the frame S and this is the instantaneous velocity. Therefore, gamma u need not be the constant as the function of time, while gamma is constant as the function of time. Now, the same particle is also being observed by the another observer in S prime, which is moving with the relative speed v.

Now, then that particular particle observes that particular particle and at a given instant finds its speed as u prime then using that particular u prime that observer at that instant of time calculates gamma using that particular u in his or her own frame of reference, that I will cal as gamma u prime. So, gamma u prime is 1 upon under root 1 minus u prime square divided by C square, as we had three different speeds that we have talking. Speed v relative velocity between the frame, speed u the speed of a particle being observed in a frame, S and a speed u prime this speed of a particle has being observed in frame S prime.

Similarly, using these three speeds I can define three different gammas, one gamma relating to v for which I am not putting any subscript. Another is gamma u which depends on u and the gamma u prime which depends on u prime. Again as I have mention u and u prime need not be constant therefore, gamma u and gamma u prime need not be constant as a function of time.

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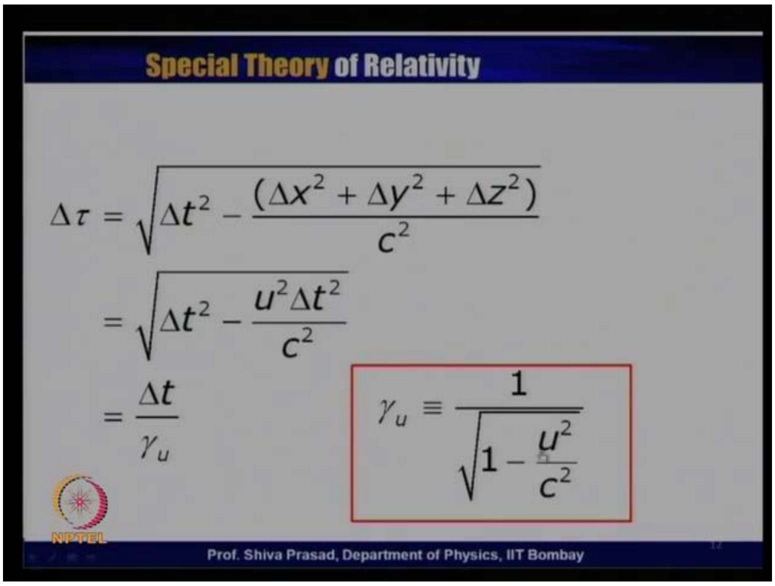

So, I realize that if I am sitting in frame S, then if I have to calculate delta tau, then if I notice instantaneous speed of the particle. This u I can evaluate using that particular instantaneous speed of the particle gamma u, which is a very similar expression as gamma except that v is replace by u. Then this t tau will turn out to be t tau d t sorry delta t divided by gamma u. Delta tau will be delta t divided by gamma u and all that I have to do is in velocity expression I have to multiply by an appropriate constant, so that this particular thing gets converted into the component of four vector.

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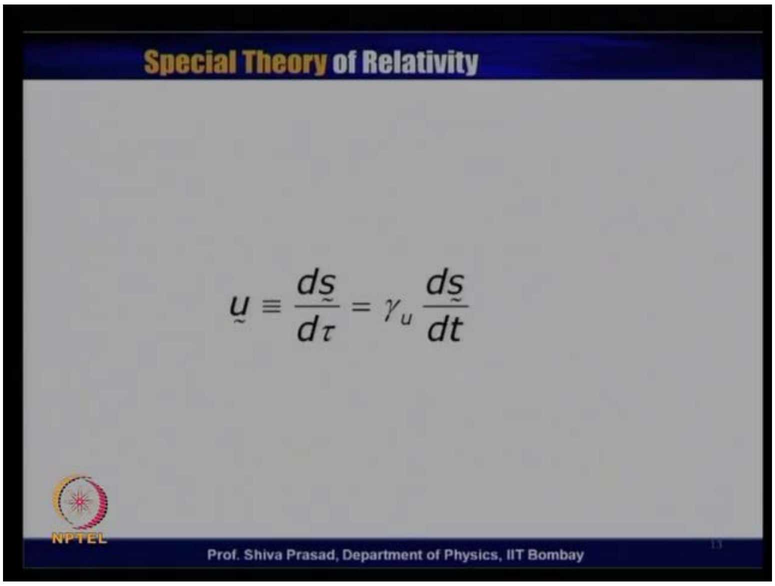

So, this is the way I would write the components of the velocity four vector. This is the definition that we had given as d s d tau. This I have expressed in terms of d t because the advantage of expressing this in terms of d t is that the first three component will directly turn out to be the instantaneous speed of the particle in S frame of reference. But in all to make this delta d t here, I will have to multiply by gamma u. Therefore, the components of velocity four vector have to be derived first by taking the velocity, standard velocity components and that multiplying them by gamma u, and of course the fourth component i c will also get multiplied by gamma u.

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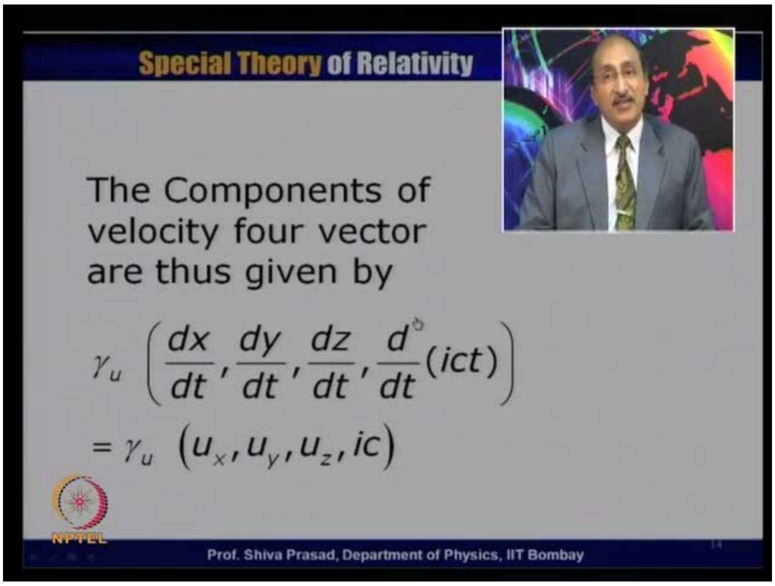

I have just expanded this particular thing. The components of the velocity four vector will thus be given by I have multiplied by gamma u, d x d t d y d t d z d t d d t of i c t. d d t of i c t if I take it just becomes i c. So, the components is gamma u, u x u y u z i c. It means this I have to multiplied by gamma u, inorder to covert them to a velocity four vector.

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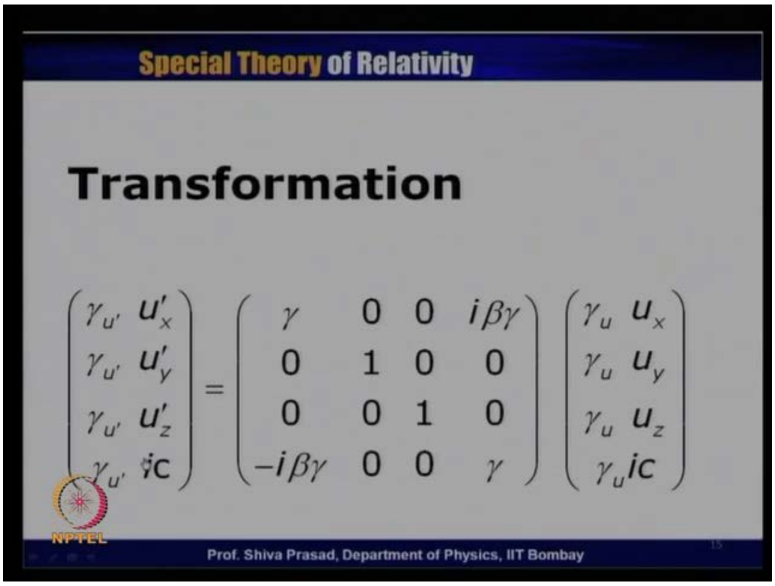

Now, is these are velocity four vectors. They must transform with this particular transformation equation. That is the way we have always be saying. That the components of any four vector, if I know in a given frame of reference if I multiplied this by instated four by four matrix, then the resultant then I would be getting will turn out to be the component of the four vector in S prime frame of reference. Here I have to use gamma which is dependent on the relative velocity between the frames. As for velocity four vectors is concerned, the components of gamma u u x gamma u u y gamma u u z gamma u i c as we have just now mention. Now, if an observers tries to measure the speed of the same particle in his or her own frame of reference.

The way he or she would calculate the components of four vectors would be, first calculate the velocities components u x of course, the velocity in the that particular frame would be u x prime. u x prime u y prime u z prime for c does not change so the fourth component is i c. Then he or she has to multiplies by gamma u prime because the speed has change in his frame. So, using his speed will have to calculate his gamma u which will be different from the gamma u in S so that is why I have call this as gamma u prime. Therefore, what according to him would be the components of the four vector will be gamma u prime u x prime gamma u prime u i prime gamma u prime u z prime gamma u prime i c.

These gammas depend on u prime, the instantaneous speed of the particle. These gamma is depend on again u which is in instantaneous speed of the particle in S frame of reference transformation matrix depends on gamma, which is the relative velocity between S and S prime frame of reference. So, there are three gammas which one has to be clear, that we are using the appropriate gamma in order to to work out this particular equation. Now, let us try to see whether, if I multiplied this particular matrices I get back my velocity transformation, because that is what it should happen. I know that I have by totally different methods I have found out velocity transformation in principle. If I expand this particular matrix I should be able to get back my velocity transformation.

So, let us try to do this particular thing. Small amount of mathematics so let us just try to see it. Let us look at this particular equation here. First I, look at this particular component here this component the first component will be given by gamma multiplied by this first component plus 0 multiplied by this component plus 0 multiplied by this component plus i beta gamma multiplied by this component. Eventually these two component do not leave me to anything because they get multiplied by 0.

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$$\gamma_{u'} u_x' = \gamma \gamma_u u_x + i\beta\gamma \gamma_u ic$$
$$\gamma_{u'} u_y' = \gamma_u u_y$$
$$\gamma_{u'} u_z' = \gamma_u u_z$$
$$\gamma_{u'} ic = -i\beta\gamma \gamma_u u_x + \gamma\gamma_u ic$$

So, this should be the gamma times this i beta gamma times this. This must be equal to the first component. So, let me first write the first component. Say gamma gamma u u x plus i beta gamma times the fourth component. The fourth component is gamma u i c. So, this becomes my first equation. Let us look into the second thing here. As for second equation is considered as simple because this is equal to this multiplied by the first component, one multiplied by the second component third multiplied by the third component here fourth multiplied by the fourth component.

So, this is just gamma u prime u i prime is equal to gamma u u z. So, let us write a again here. Similarly, we can write from the same component because it is exactly similar. Now, let us like try to write from the fourth component. For fourth component we have gamma u prime i c should be this multiplied by the first component 0 multiplied by second component 0 multiplied by third component.

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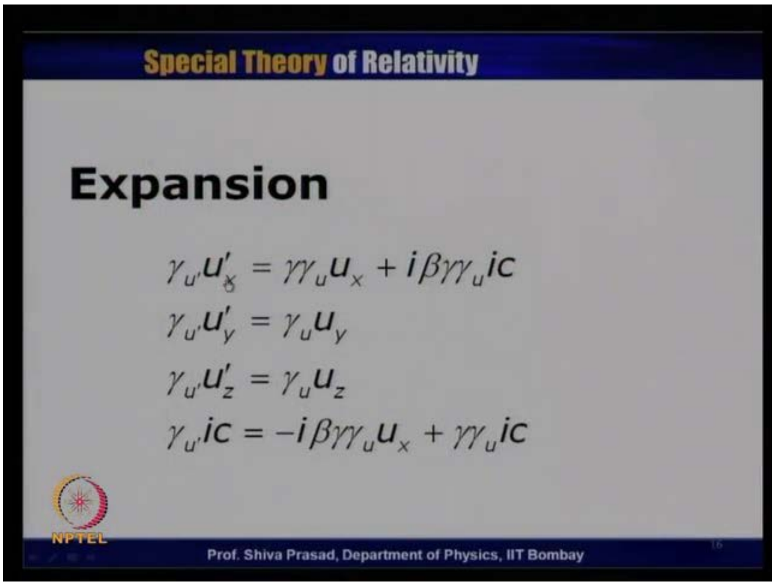

So, that does not give me anything. Plus gamma multiplied by gamma u i c. So, let us write this equation as the fourth equation minus i beta gamma times u u x plus gamma gamma u i c. So, this this is what I get after opening up this particular matrix. Let us see with the this particular matrix deals me the velocity transformation. What I have done just written the same equation which I have written here gamma u prime is equal to u x prime multiplied by u x prime is equal to gamma gamma u u x plus i beta gamma gamma u i c this two are just equality gamma u prime i c is equal to minus i beta gamma gamma u u x plus gamma gamma u i c.

The equations look complicated, but let us this try to work out. Let us start with the fourth equation. This is the equation. Let us first look this particular equation and then whatever I get I will substitute this here. In fact from this equation will get gamma u prime in terms of gamma u. Let us realize that this i c there is a i here there is a i here so all this i's will cancel i will, let the c also cancel. There is a c here so that c also will get cancel. There is no c here, but there is a beta which is v by c. So, once I cancel c this will become v by c square. So, what I will get is gamma u prime is equal to gamma gamma u. Gamma gamma u in fact I can take common. So, I will get 1 minus u x. There is a u x here and this will become v by c square.

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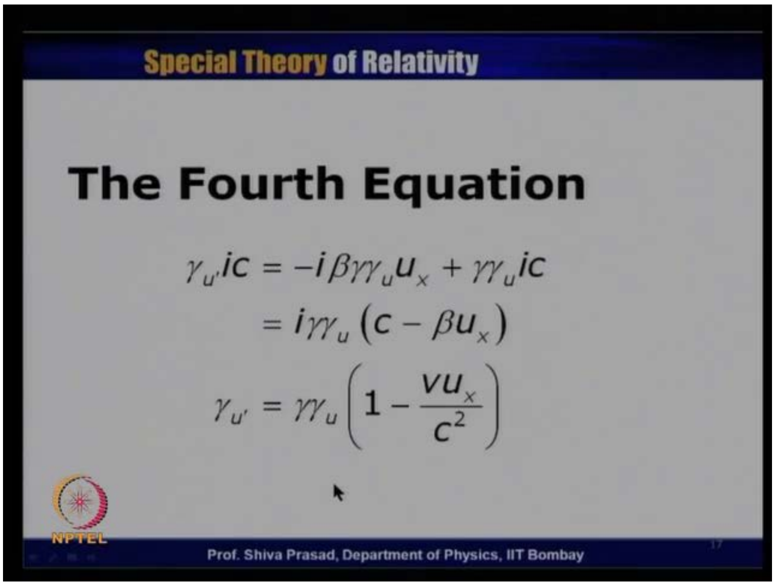

So, this is what happens in the fourth equation. So, what I have written here to be more careful. Gamma u prime i c is just the repetition of the same equation. I have taken i gamma gamma common. I get there is a c here so this gets c. This time I have I am writing first. This time I am writing the second. So, this becomes I have already taken, i gamma gamma u common. So, this may comes beta times u x and I realize that beta is equal to u by sorry v by c. So, once I write in terms of once, I try to cancel this particular c and i then I get gamma u prime is equal to gamma gamma u one minus v by c square u x. So, this is an equation which essentially solve out of relates gamma u prime in terms of gamma u. This is the equation which I will now put into the other equation and try to work it out.

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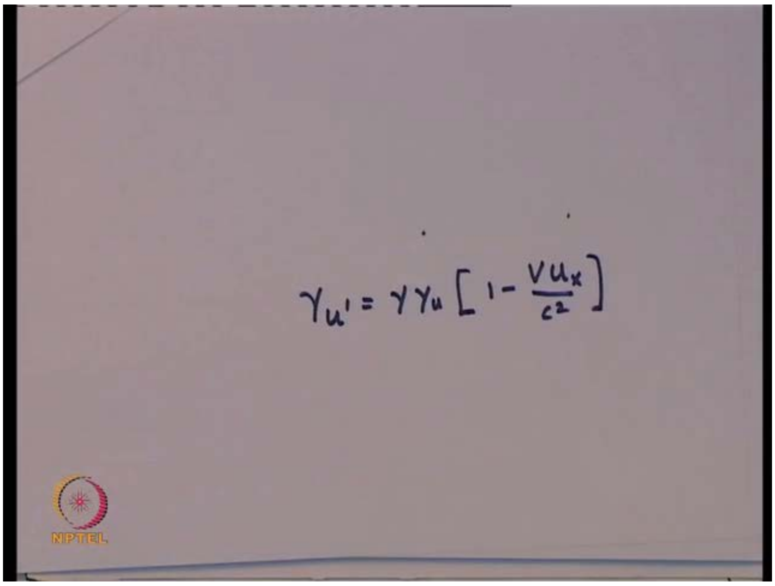

So, let me first write this equation here. Gamma u prime is equal to gamma gamma u 1 minus v u x by c square. Let we put it back into the first equation.

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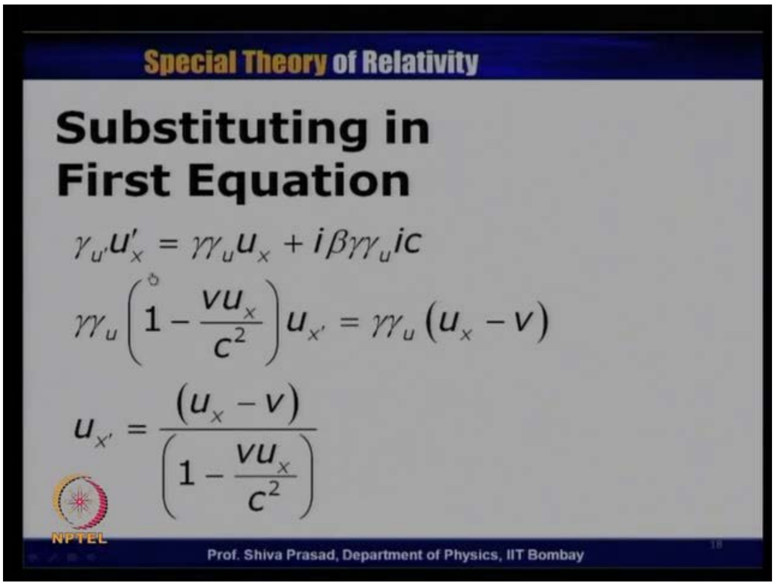

This was my equation. All I am doing is putting for gamma u prime gamma gamma u 1 minus v u x by c square, which we have just, now seen from the fourth equation. So, this gamma u prime has been replaced by this quantity multiplied by u x prime should be equal to gamma gamma u x u x minus v just stop. Repeat this transparency. So, let us substitute this value of gamma u prime. This particular first equation, which have written here. This is the same equation which I had obtained after expanding the matrix transformation matrix. Now, this for this gamma u prime I substitute gamma gamma u multiplied by 1 minus v u x upon c square as we have just now seen from the fourth equation and this u x prime remains u x prime.

On the right hand side I take gamma gamma u out. So, this become gamma gamma u and in bracket what is remaining is this u x. Then this i square become minus 1. So, this become there is a v by c here there is a c here. So, this c cancels with this c what remains here is just v, if you get here as gamma gamma u x u x minus v. This gamma gamma u cancels from this side I can find out for u x prime. u x prime will be this divided by this so will get u x prime as u x minus v divided by 1 minus v u x upon c square, which has we know with we had derived earlier directly by the use of Lorentz transformation, the first equation of the velocity transformation the transformation of the x component of the velocity of the particle.

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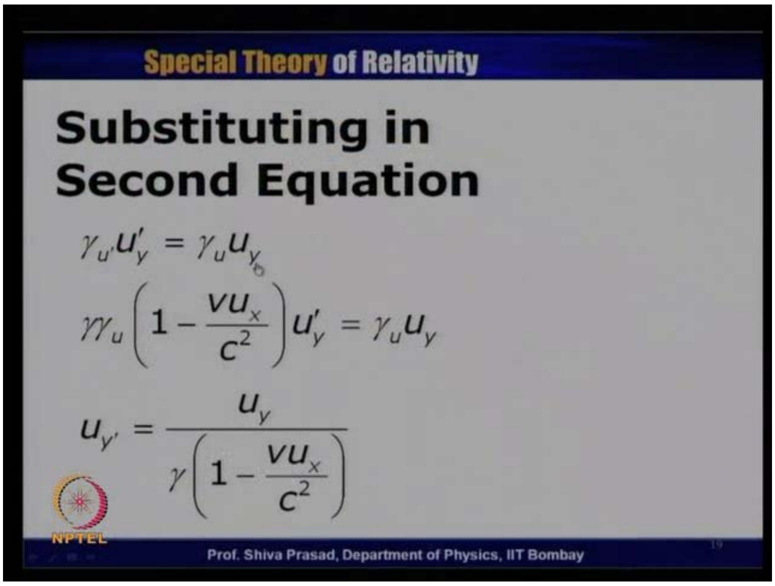

Now, let us look at the second equation. This was my second equation which was gamma u prime u y prime is equal to gamma u u y. I do exactly the same thing for gamma u prime I substitute the same expression, which I obtain from the fourth equation, which is gamma gamma u multiplied by 1 minus v u x divided by c square. Of course, u y prime by means here gamma u into u y. gamma u here cancels with this gamma u. I can write u y prime is equal to u y divided by this quantity and of course, one of the gamma remains here because this gamma does not cancel here. So, I get u y prime is equal to u y divided by gamma one minus v u x upon c square which as you would have realize is the standard transformation of the y component of the velocity.

Similarly, I could have substituted this into the third equation and shown that this turns out to be the transformation for the z component of the velocity. So, I can derive the velocity transformation from the transformation of velocity of four vector.

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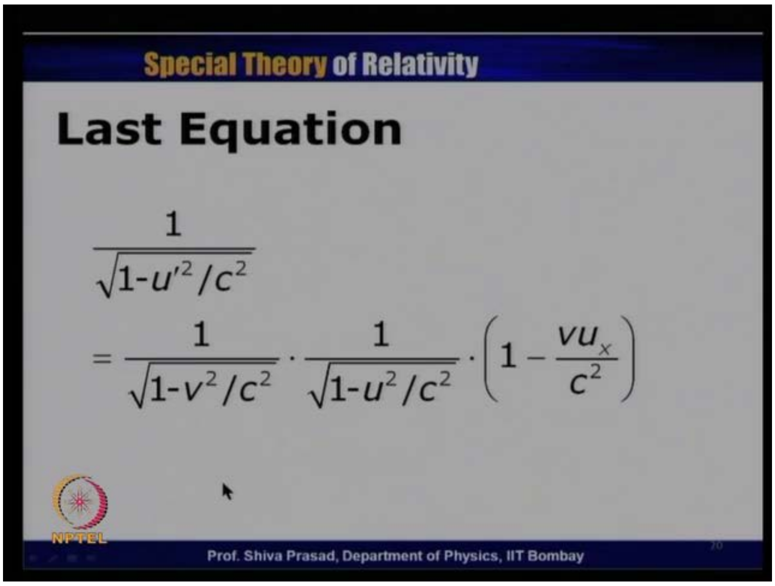

Let us re look at the fourth equation and let us just see what it gives. I have expanded it for gamma u prime. I have written the actual expression of gamma u prime which is 1 upon under root 1 minus u prime square divided by c square here. There was a gamma so for this gamma I have written as 1 upon under root 1 minus u square divided by c square, the was a gamma u here from this gamma u I have written this as 1 minus u square, but c square and this gets multiplied by 1 minus v u x by c square. As we have just now seen. So, all I have to do is to take write this particular equation in terms of c square readjust this terms and you will get comparatively a simplified expression which I can interpret slightly in a different fashion. That is why I am discussing little more about this last equation.

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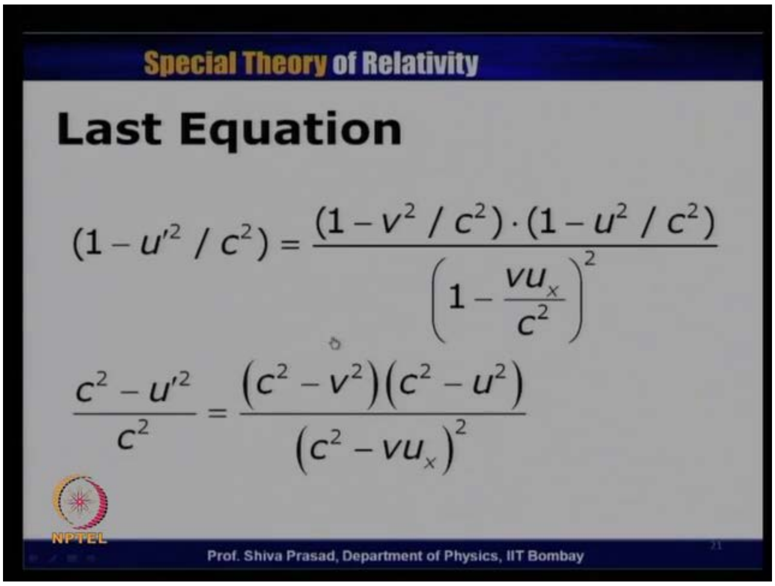

So, this equation can be very easily shown to be written in terms of 1 minus u prime square by c square and writing in this particular fashion by taking the inverses. Do a small amount of simplification by a multiplying by c square of c 2 bar 4 or whatever it is depending upon this cases and eventually you would lined into this particular equation. From this equation I find out something which is interesting which is I would like to point out. One is that if I assumed for example, that we have already said that I would not expect the relative velocity between the frames co exceed c. From this quantity for any value of v less than c, is always going to be positive. In the denominator we have something which is square, which is always going to be positive.

There is no imaginary number involved here. Here you have c square minus u square. Here you have c square minus u prime square. So, if u is less than c which is what we expect, then this quantity is positive. So, this quantity will also turn out to be positive, which it means shows that if in any frame u is less than c, in any other frame u prime will also turn out to be less than c because this quantity has to be positive. Nothing surprising, but let us little little extend the little bit the argument. If you happens to be see, if there was a particle which was at all found to be moving with the same speed as c about this we will discuss about little later, then this particular quantity will be 0, this particular quantity will also be 0. Means if a particle is found to be travelling with the speed of light, then in any other frame of reference also that particular particle will found to be travelling with the speed of light and remember this is something which we have earlier said.

This is nothing special about light as far as transformation equations are constant relating to the particle or light. If light velocity is same in all frame of reference. If at all we find the particle which also travels with a same speed as speed of light, it is possible then every other frame of reference that particular particle will also be found to be moving with the speed of light. Now, let us estipulate in detail further. If at all if at all we able, we are able to find the particle which travels with the speed greater than the speed of light. Then this quantity in this particular bracket will be negative. Then here also should be negative. t means in any other frame of reference also that particular particle will turn out to be moving with speed greater than speed of light.

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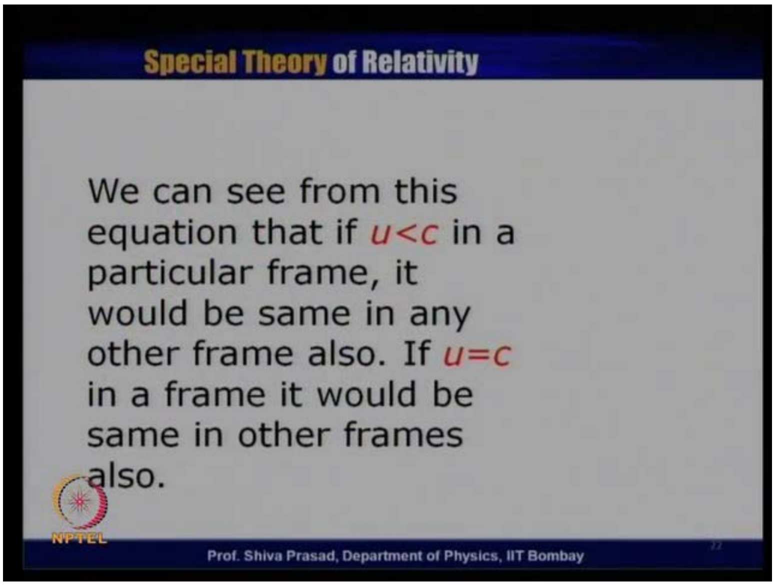

So, speed of light is something which is in between say any particle, if at all we are able to find a particle which is travelling with the speed later in the speed of light in every frame this will move with the speed greater than the speed of light. If it will be found to be travelling with speed of light in every other frame of reference it will be found to be travelling with the speed of light. If a frame it is found to be travelling with the speed less than the speed of light in any other frame it will be found to be travelling with the speed less than the speed of light. Thiswhat I have summarize in next few transparencies.

We can see from this equation that if u is less than c in a particular frame, it would be same in any other frame also. If u is equal to c in a frame it would be same in any other frames also.

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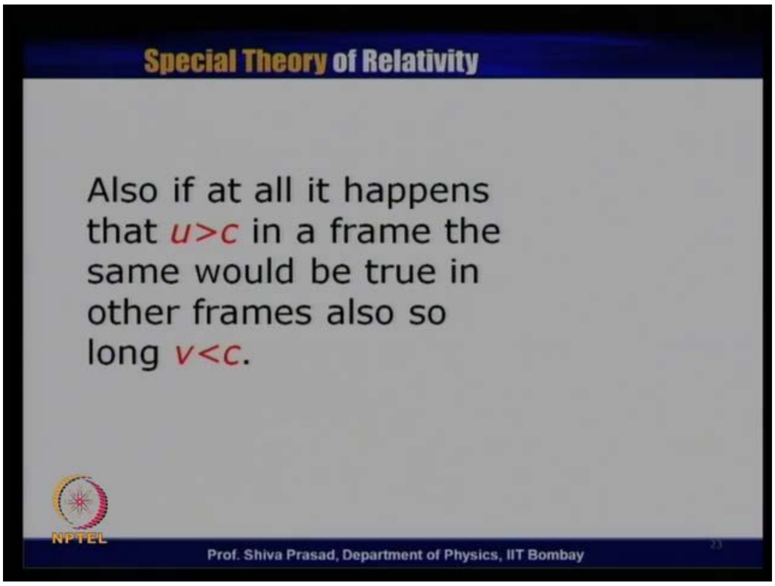

Also if it happens, that u is greater than c in a frame that same would be true in other frame also. So long as v is less than c that is what we have said. When curiosity we may have we have always said that the length of four vector is always same in all the frame of references. Let us just evaluated the length of the velocity four vector and see whether the real trends are to be same in frame of reference because see like from the displacement four vector we have taken that particular four vector and look at its length and set that is a proper time interval is same in all the frame of reference. Similarly, the length of the velocity four vector should also be same in all the frame of reference. So, let us evaluate the length of the velocity four vector.

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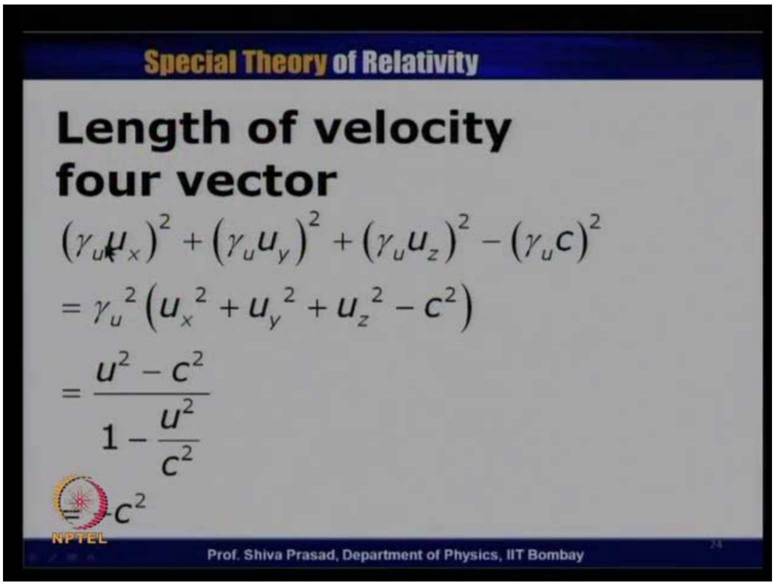

This not I have done in this particular transparency. This is the first component. This is second component. This the third component, fourth component was gamma i u c. I have taken the length, I was squared and because of this I, this sign becomes negative so the length of velocity four vector will be given by gamma u u x whole square gamma u u y whole square gamma u u z whole square minus gamma u c whole square. I can take gamma u square common. If I take gamma u square common, I will get u x square plus u y square plus u z square minus c square. u x square plus u y square plus u z square is u square. So, this whole thing I can write as u square minus c square. So, this is what I have written here u square minus c square.

I have expanded gamma u square which is 1 divided by 1 minus u square by c square. To look at this particular denominator, this you can write as c square minus u square divided by c square. So, this c square will come into the numerator. Here will have u square minus c square divided by as c square minus u square. So, this will become minus 1. So, the whole quantity will become minus c square and indeed this is going to be save in all the frame of reference references because I know c id independent of frame of reference it is same in all the frame of reference. So, obviously the length of the four velocity, four vector is really a four scalar on the frame of reference, that you have to use to describe this particular four vector.

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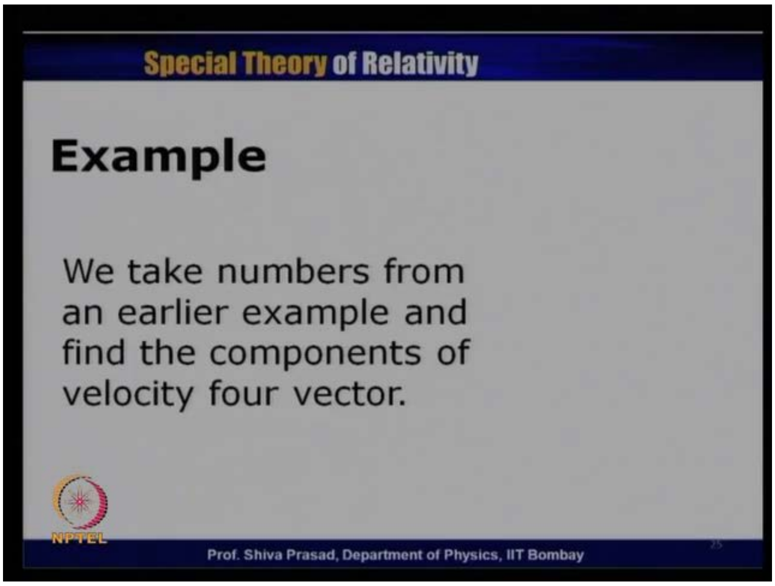

Now, let us take some example. Let us take one particular example and this example is essentially similar to the ever we have done in our last class. Essentially the numbers had been picked up from the same things so you do not have to describe another different problem. So, we just take number from an earlier examples, and let us try to work out and find out the velocity four vectors. So, we know how we are doing it.

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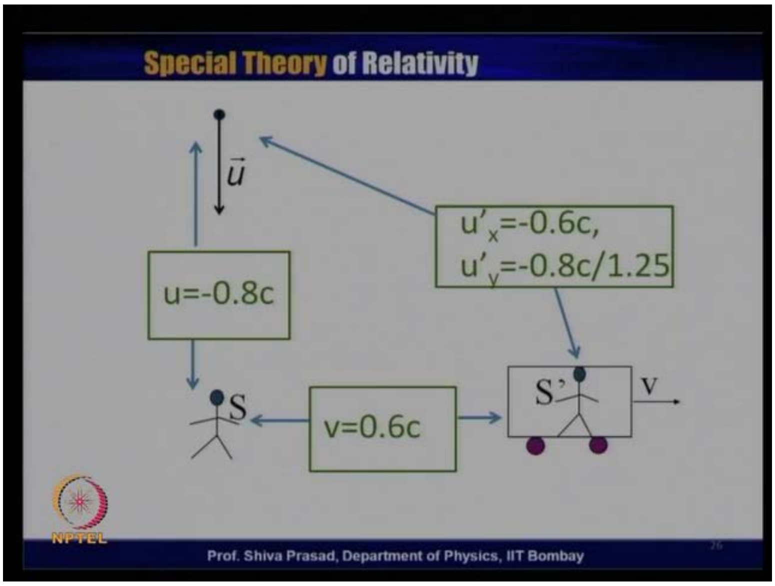

So, this is the one which we have done it earlier. That there is a particular particle which is falling on the along the tower etc., All those things but that is not all that important as for as we are concern. Let us treat that this is the particle, which is falling vertically downwards. So, it has the velocity along the minus y direction and the velocity that we have evaluated at that time was minus 0.8 c. Let us try to be little more careful about our symbols, which we have not always been earlier because we have not described this things so much in detail. So, let us call this as u because this is the velocity this is the particles speed has been observed in S frame of reference for that we have reserved the symbol u.

We have another frame of reference S prime which is moving a relative to S. This for this relative velocity I have reserved the symbol v. So, v turns out to be equal to 0.6 c. The same particle is being observed by this particular observer and we had done a velocity transformation last time and found out that it has a x components of velocity, which is minus 0.6 six c and the y component of the velocity which is minus 0.8 c divided by gamma, which is gamma which turns out to be equal to 1.25 in this particular case because this is the particle speed has been observed in S prime frame of reference. I have reserved the symbols primes u is u’s primes. The particle I have to that being the particle speed I have put u not v and because this is being observed in S prime frame of reference I have putting u prime.

So, you have u x prime u y prime. So, these are the two frame of reference. This is the relative velocity between the frames. This is particle this is particle velocity S being observed in S. This is the particle S being observed in the S prime frame of reference the number. Of course, in this particular example this u we had assumed to be constant because we want (( )) transform in u frame of reference, but now now we have only talking between S and S prime. This speed has not have been constant. So, this could then be instantaneously u, this could instantaneously u prime at a given instance of time, but of course, in this particular example we have taken the case when this particular particle velocity is also constant.

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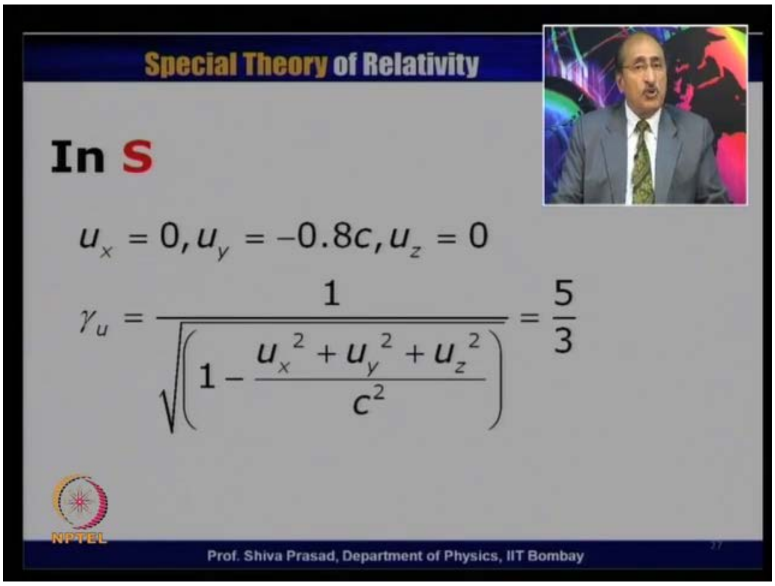

Now, let us first look at the frame S and try to calculate the component of velocity four vector. I just put the same thing here. u x is equal to 0 u y is equal to minus 0.8 c and u z is equal to 0. First thing that I have to used to calculate gamma u because just u x u y u z is not enough, if I calculate gamma u using these velocity component, the velocity of the particle. So, gamma u is 1 upon under root 1 minus u x square plus u y square plus u z square divided by c square. Essentially means because all 0ther component are 0 only 0.8 c we have use this particular gamma value for 0.8 this turns out to be equal to 5 by 3.

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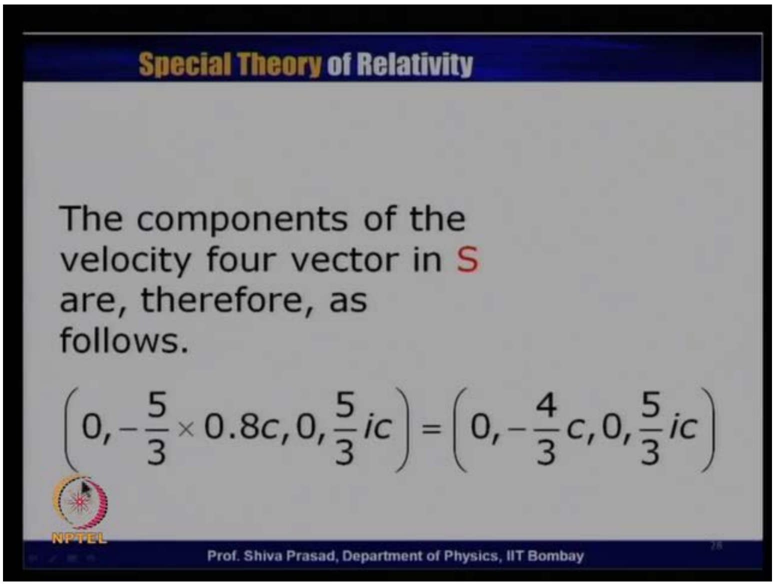

So, once I know gamma u I can very easily calculate the components of the velocity four vector. Remember by velocity components were 0 minus 0.8 c and 0. So, I have to multiply them by by gamma u. Of course, 0 multiplied by gamma u still remains 0. So, point, minus 0.8 c I have to multiplied by 5 by 3 which is the gamma u which I just have calculated. 0 nothing happens. i c multiplied by 5 by 3. If I just simplified I get the component of the velocity four vectors of that particular particle has 0 minus 4 by 3 c 0 5 by 3 i c. These are the components of the velocity four vector of the particle has seen in S frame of reference. Let we try to find out for S prime frame of reference.

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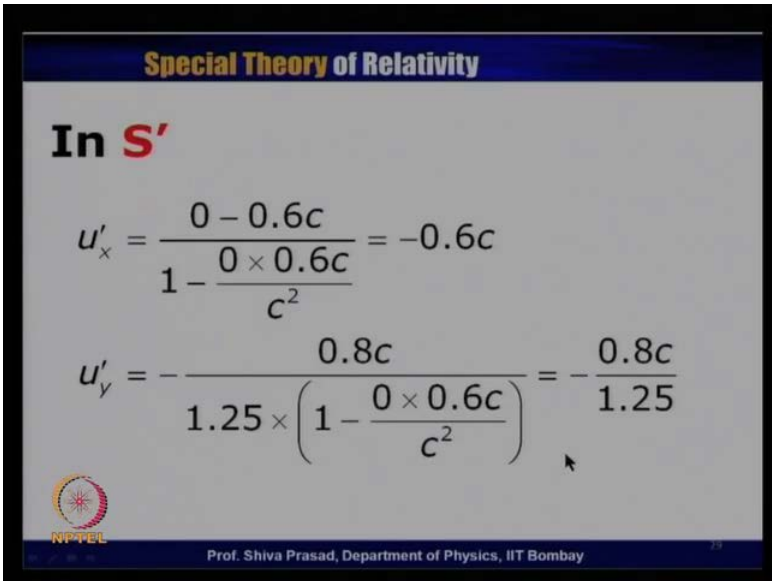

So, this this particular transparency is super (( )), because this we have done earlier. Calculate the x component of the velocity and y component of the velocity of the particle in S prime frame of reference. This in fact this even, I have even written in the trans, figure that u x prime is minus 0.6 c u y prime is minus 0.8 c divided by 1.25

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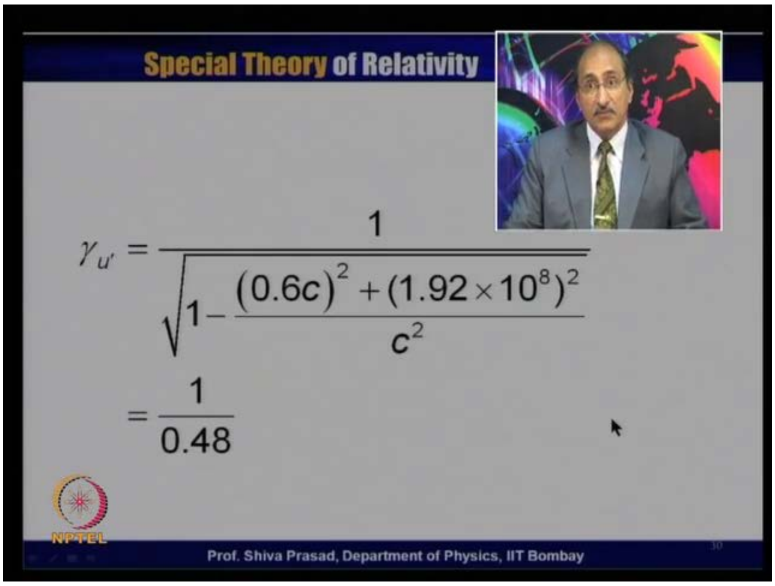

I calculate gamma u prime. So, the observers sitting in the S prime finds u x to be this much and u y in fact I put the number here, this much. So, I can calculate that particular observers will calculate gamma u prime in his or her frame of reference. This also we have done in the last lecture. This gamma u prime will turn out to be 1 divided by 0.48

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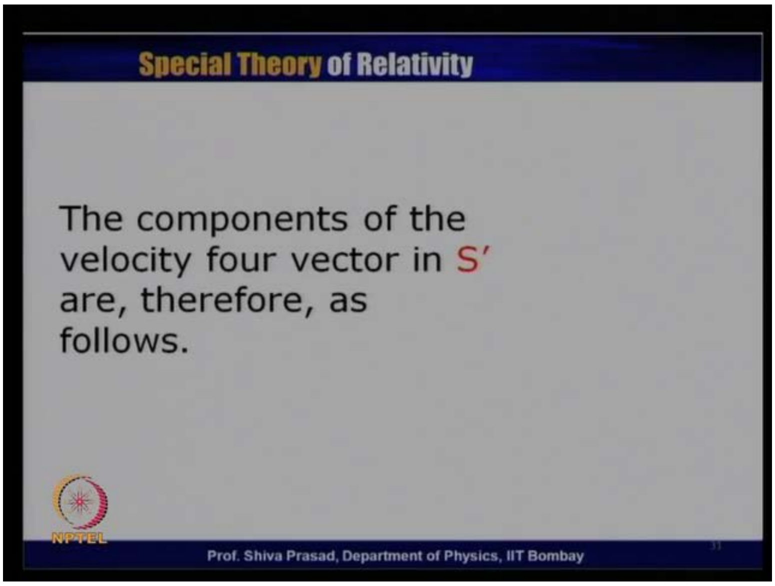

So, the components of the velocity four vectors in S prime frame of reference, he has to take the velocity components. After getting the velocity component, multiplied by gamma u prime. If I have to convert them into the components of the velocity four vector. So, this was u x I have multiplied by gamma u, which is 1 divided by a 0.48. In fact I have put it now 0.8 c divided by instead of 1.25 I have put 5 by 4 here. Divided by gamma u which is multiplied by gamma u sorry gamma u prime which is 1 divided by 0.48 third has any ways 0 fourth is i c multiplied by gamma u prime which is 1 divided by 0.48 If you solved or simplify this. This becomes minus 5 by 4 c minus 4 by 3 c 0 1 divided by 0.48 i c.

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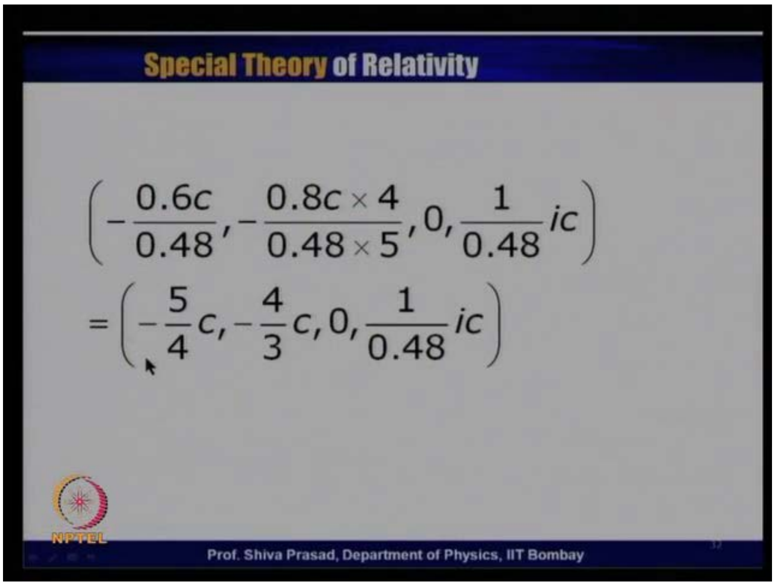

So, according to S prime observer the components of the velocity four vector are given here. Now, if whatever I am saying is consistent, then if I use the transformation matrix for transforming the velocity four vector. These four components must be obtain by from this component change S frame of reference. Let us just try to see it. We thus see that the components are different in S prime frame of reference. However they should transform as per the four vector transformation equation. This is the four vector transformation equation.

These are the components in S prime frame of reference. This equation is must be true if I, whatever I am saying is correct. Remember here what is appearing is gamma and what is appearing here is beta. This gamma depends on the relative velocity between the frames which is 0.6 c for 0.6 c I get 1.25 here. So, this is 1.25 this I get as 0.6. So, let me explain this particular thing that writing this number.

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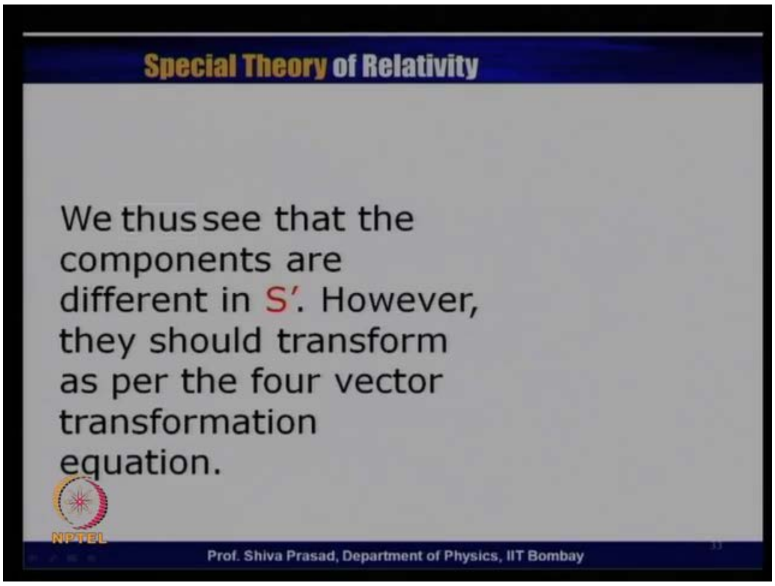

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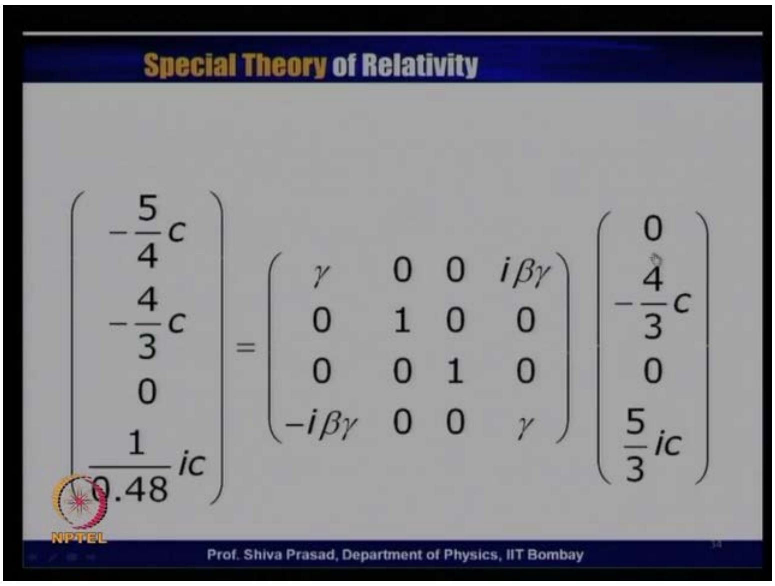

So, all the I have done is expanded this particular matrix by writing the value of gamma which turns out to be 5 by 4. Here we had 0.6 multiplied by 5 by 4 which gives me 3 by 4. Here I had minus i beta gamma. So, that I get minus i 3 by 4. This is the same gamma 5 by 4. You can very easily see that, this equation is satisfied. For example, if I take the first component 5 by 4 multiplied by 0, 0 multiplied by 0. 0 multiplied by 0 here, sorry 0 multiplied 0 multiplied by minus 4 by 3 c 0 multiplied by 0 Then i 3 by 4 multiplied by 5 by 3 i c. If I multiplied these things what I will be getting 3 will be cancelling. I will be getting 5 by 4 i square will become minus 1.

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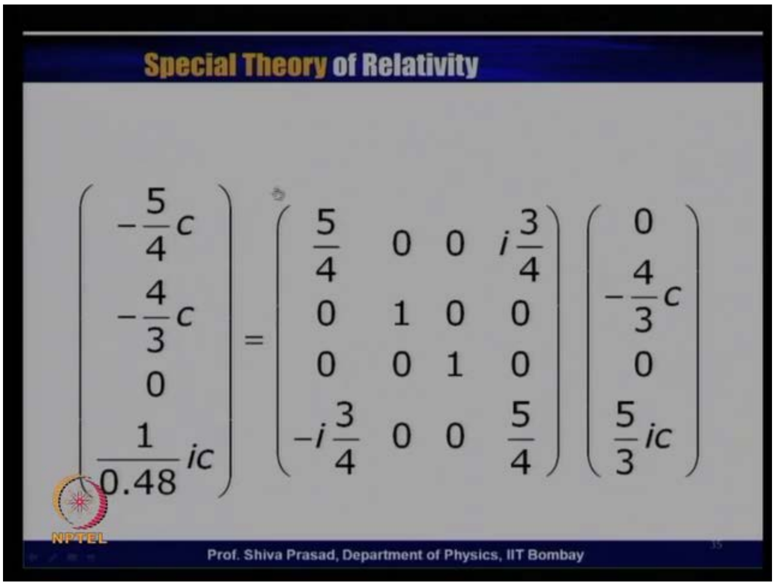

So, I will get minus 5 by 4 c. this is same as this, this is same as this as this expected. Similarly, minus i 3 by 4 multiplied by 0, 0 multiplied by minus 4 by 3 c 0 multiplied by 0. It means the fourth component is just 5 by 4 multiplied by 5 by 3 i c, you can see very easily that this will give you 1 divided by 0.48. So, this equation is satisfied these are the components of the velocity four vector in S frame. These are the components components of the velocity four vector in S prime frame of reference and this equation does follow the same y transformation equation that I expected into follow.

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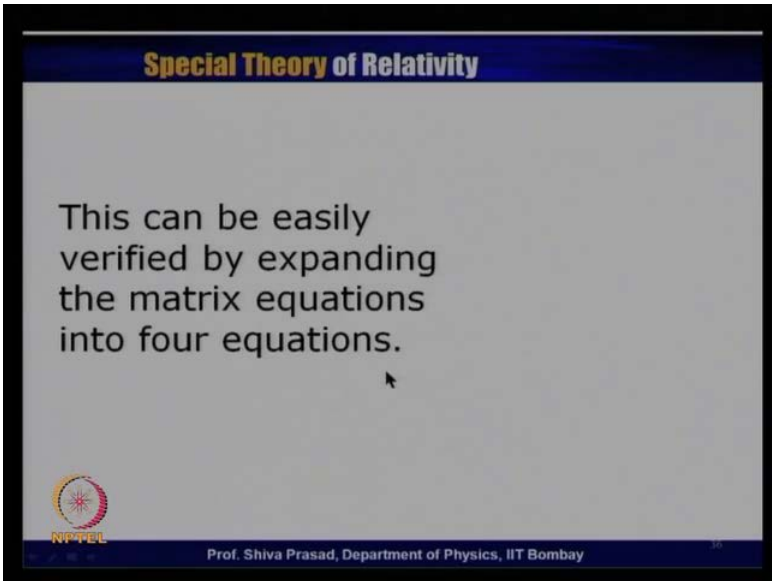

So, this is what I have written this can be easily seen that this equation is true by expanding the matrix equation into four equations.

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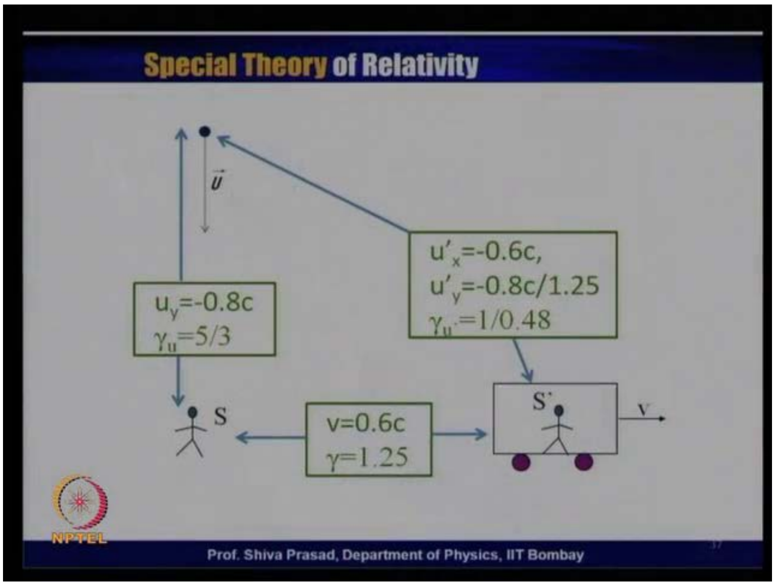

So, then I was, I will just show this particular figure to make the things about the gamma is very, very clear. So, remember you have put here u y here, we have put u x prime u y prime. If I use this particular u to calculate gamma I will get gamma u. If I calculate this particular values of u to calculate gamma I will get gamma u prime and if I use this particular value of v to calculate gamma I will get gamma.

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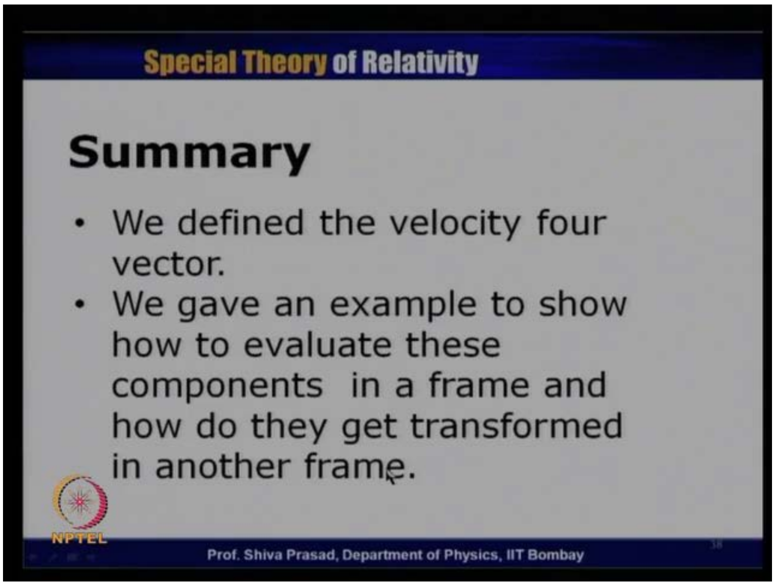

I will just summarize whatever we have said today. We have defined the velocity four vector. Then we gave an example to show how to evaluate these components in a frame, and how do they get transformed in another frame.

Thank you.