**Special Theory of Relativity**
**Prof. Shiva Prasad**
**Department of Physics**
**Indian Institute of Technology, Bombay**

**Lecture - 13**
**Proper Time a Four Scalar**

So, let us start with recapitulating what we had done in our last lecture. We introduced a sort of a abstract concept of a four vector. What we call as Minkowski space.

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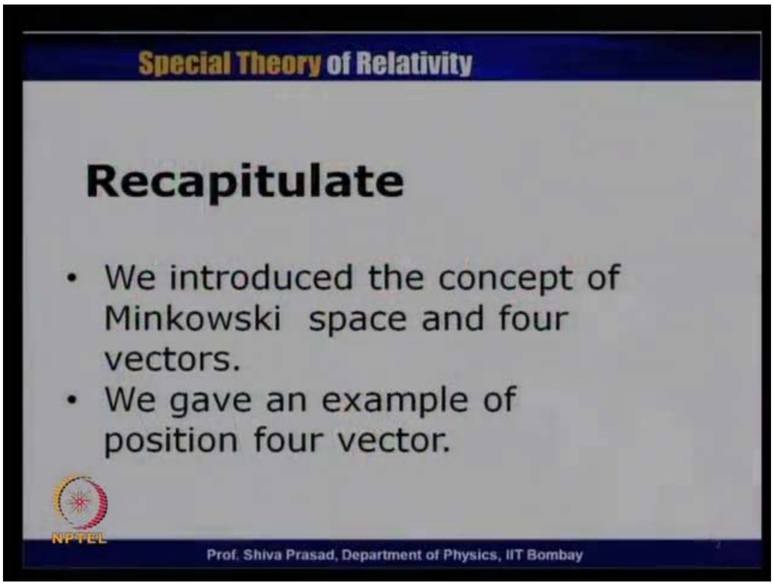


We discussed like we can represent a standard traditional vector with which we are reasonably familiar in terms of its components along a set of axis. So, we can call a traditional vector as just set of three quantities. Similarly, we can extend this particular idea to so called force dimension and we can construct what we called as a four vector, which has which is set of four variables. These four variables are measured in a given frame of reference and if these variables satisfies a certain transformation equation, when we go from one frame to another frame because we are always talking of inertial frame.

Then we will call this set of four variables as forming or as the component of if or components of a four vector. We visually gives an example of a position four vector, where we has said that x y z which are the three co-ordinates of an event and i c t where t is the time of that event in a given frame. These four form a component of a four vector.

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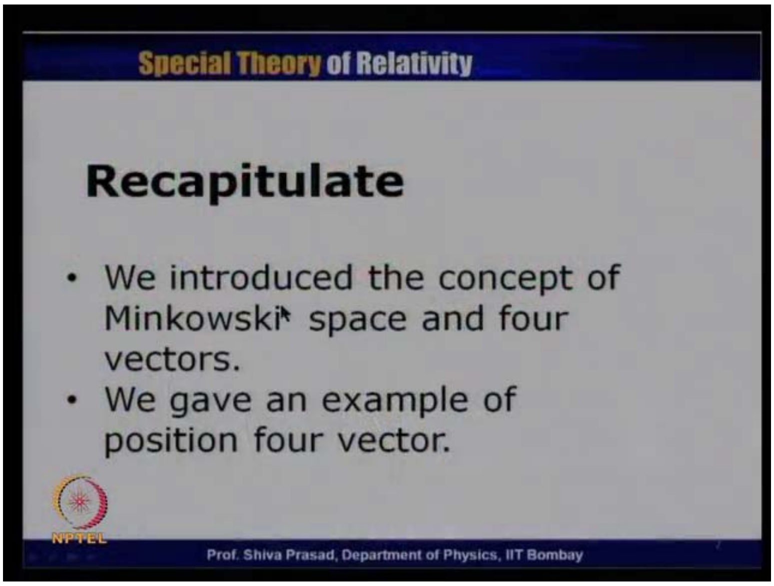


So, this is what we have written. We introduced the concept of Minkowski space and four vectors and gave an example of position four vector. I would just like to mention before we go ahead in this particular thing that the way we represented the four vectors in terms of transformation equation, is not always very standard. Their alternate ways also of defining this four vector. The physics does not get change. Only thing for the convenience sake many times we would like to use a slightly different set. The basic philosophy is exactly identical. The basic final equations that will get is completely identical. Is just a matter of following the path that we are choosing in terms of the four vectors.

So, one of the modification that we make often in this particular definition that we avoids all those imaginary numbers i's because no there every time i i coming see there is an i c t then the transformation equation also there i's this cumbersome would keep track of all these i's. So, what one one can do is to remove all those i's and then use a different slightly different type of transformation equation which does not involve any imaginary number. The only differences here in that particular case would be that the dot product that we will just now define today that will slightly change. The advantage of the way I have written this particular transformation equation is that the definition are very easily extendable. Definition of dot product for example, is very easily extended extendible from the standard dot product definition of traditional three vectors.

In this case, we have to slightly modify by changing moral of the science. (( )) always say there after what is so special about dot product and what is so special for about for that matter about multiplication or division. This is the way we have defined about things and we have define them this particular fashion because our convenience. Why we say 2 multiplied by 2 is equal to 4 because that is we have defined it. So, all this things are essentially the way of def, defining things, which make our further study of further advance studies comparatively easier. So, if I can define it in different fashion and get rid of those imaginary numbers, why not do it?

So, this one if the thing that one finds the many of the textbooks and especially those persons who are working in this particular area, who remove those i's to make things know less (( )).

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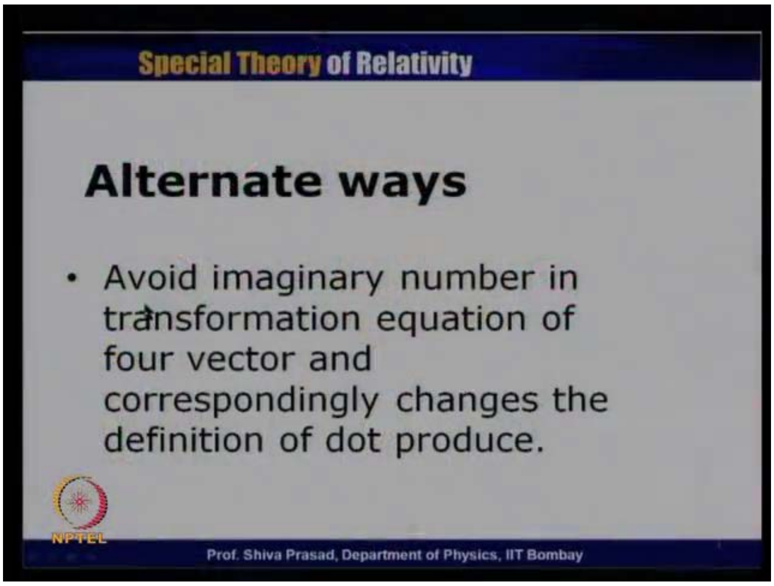


So, these what one of the ways is to avoid imaginary number in transformation equation and definition of the four vectors and correspondingly change the definition of dot product. We are not yet define the dot product but, we will define today. There is another way of choosing the four vectors in which the fourth component that we have discussed is taken as the first component. In terms of the position four vector, it means that the time component becomes the first component and x y z becomes second third and fourth component. If we have to do that, the transformation equations will slightly become different and this is what they will show.

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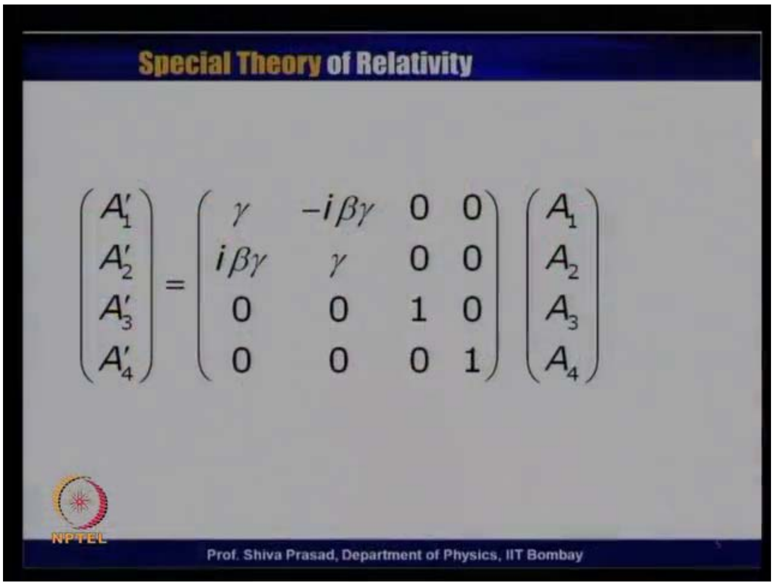


So, the fourth component has become the first component and correspondingly second, first second and third components have becomes second third and fourth. The basic advantage here is, that if you take this particularly transformation matrix and if we take this square plus this square, this will turn out to be equal to one, which is very similar to the rotational thing that we have discussed in the case of the traditional vectors. Where also there is a cos theta and sin theta appearing it and there cos square theta plus sin square theta was giving equal to 1. So one can get little bit more abstract idea in this particular case and say that this Lorentz transformation appears to be are a somewhat similar to a rotation in the Minkowski space. So, this is the another way in which many of the textbooks would define the four vector.

So, as I have said that, if you see gamma square plus of course, with bracket minus i beta gamma square. This will give you gamma square whose minus will become plus when you squared and i square will give you minus 1. So, this becomes gamma’s square multiplied by one minus beta square. As we know that gamma square is equal to one divided by one minus beta square. That is the definition of gamma.

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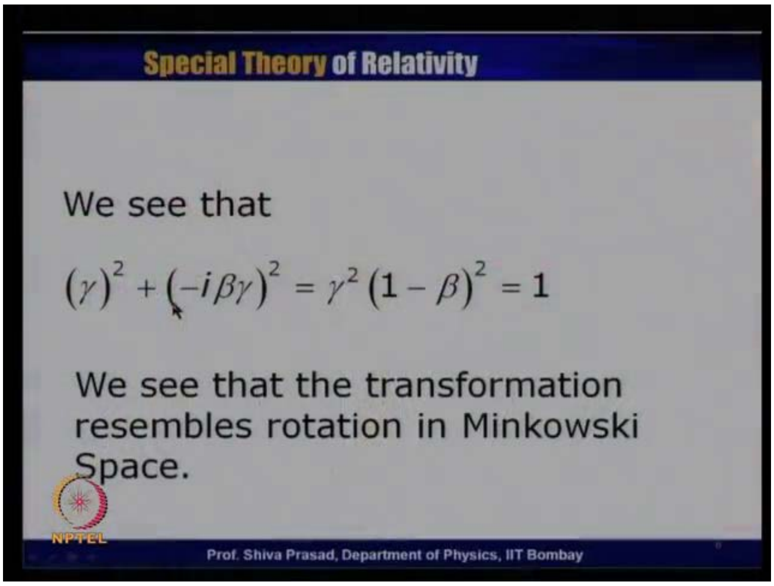


So, this whole quantity becomes equal to 1. So, it is in that way similar to cos square theta plus sin square theta giving you to 1. So, it is as we have said that the transformation, the Lorentz transformation resembles rotation in Minkowski space. So, these are the alternate ways of using the concept of the four vectors. Now, let us go little ahead. We had define a position four vector. Now, let us define a displacement four vector. (( )) if you go through should to the traditional mechanics, when we are dealing with standard three vectors then in that particular case, if you choose a set of axis and if this is a point, then we draw origin to this particular point.

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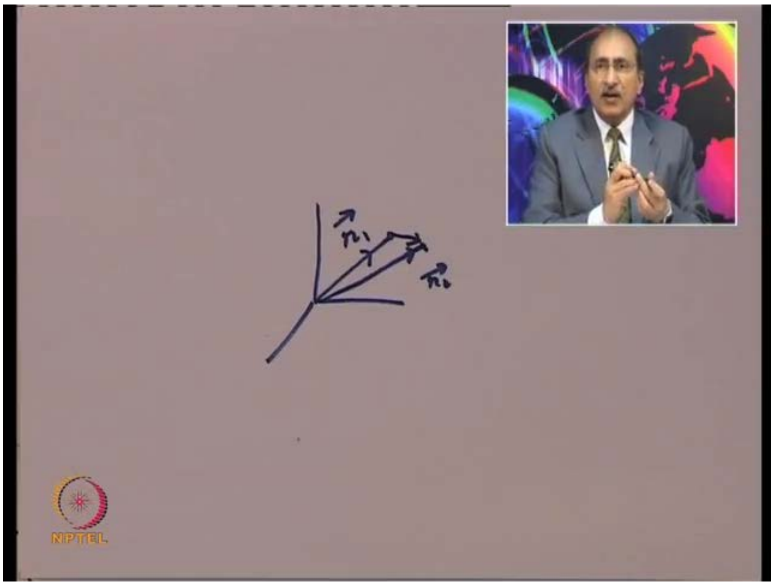

This is what we call as a position vector, but if there are two points then, this particular point for example, is displaced to this particular point, the position vector now becomes different. Let us call this is an r 1 let us call this is an r 2; you can see that r 1 plus this vector becomes r 2. This vector is what is called as displacement vector and this displacement vector in the traditional mechanics is independent of the origin that we have chosen. Now, taking somewhat parallel from this particular concept, what we have define when we are talking of a absolute values of x y z and i c t. We call that as a position four vector. Now, let us talk on terms of two different events and correspondingly take the differences like in this case, of this vector displacement traditional vector displacement. We can take the differences and call that as a displacement four vector.

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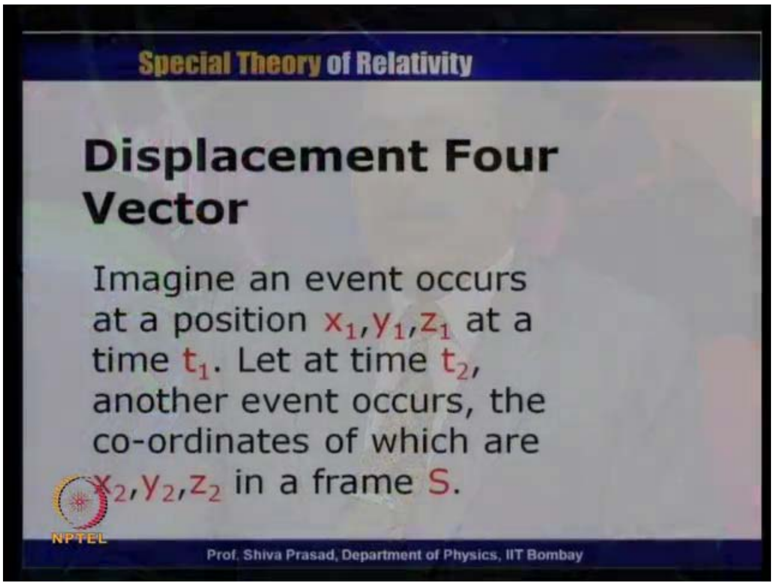


So, let us imagine that an event occurs in a given frame of reference S. Everything is given in terms of a particular frame of the reference S and this event occurs at a position x 1 y 1 z 1 and at a time t 1. Now, let us assume that at a time t 2 another event occurs, everything again give a in terms of S frame of reference; they co-ordinates of which are x 2 y 2 and z 2.

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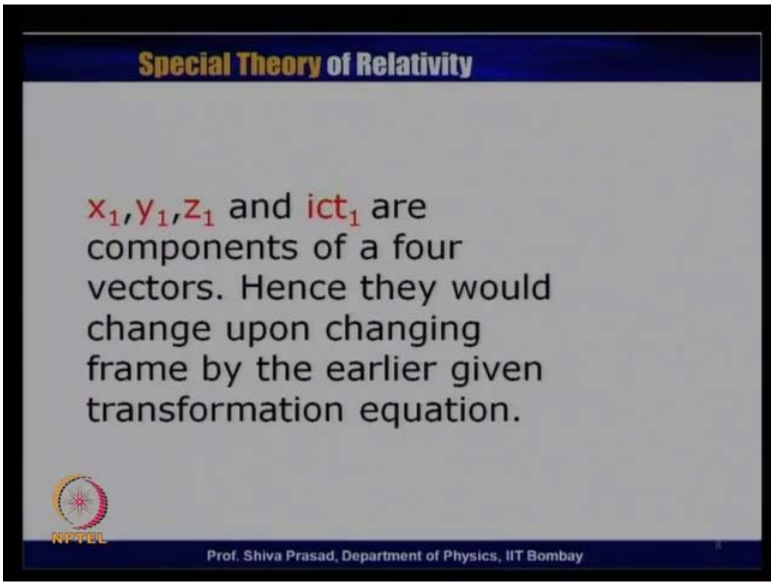


Now, we know that x 1 y 1 z 1 and i c t 1 are components of a four vectors. This we have discussed already earlier. Hence if I change my frame of reference and go for S frame of reference to some additional S prime frame of reference, these quantities will change and they would change by the transformation matrix that we have discussed in our earlier lecture. So, by changing the frame of reference these quantities will change. Similarly, x 2 y 2 z 2 and i c 2, which are also a components of four vector they would also change.

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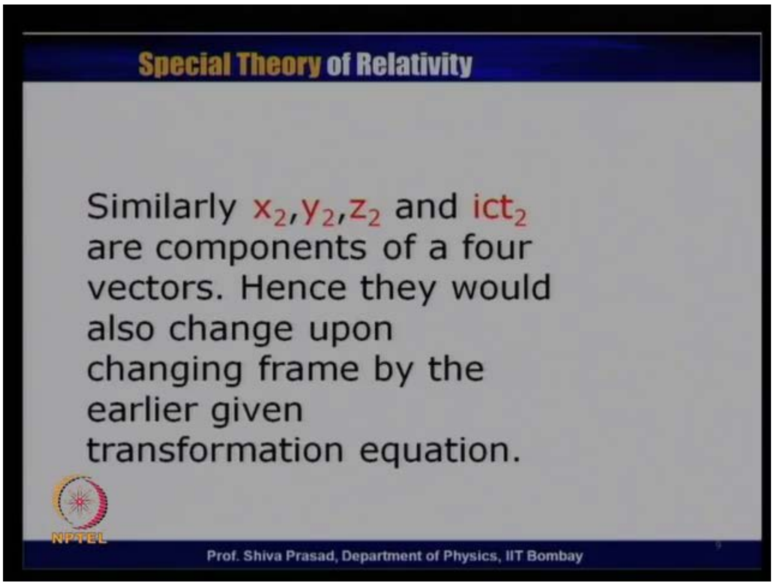

If I go to a different frame of reference and they will also change exactly by using this same transformation equation. So, now if there is one particular set of 4 numbers, which follow one particular transformation equation, an another set of four vector four components which also form which also transform exactly using this same transformation matrix. Therefore, their differences will also transform exactly using this same transformation matrix. This is a simple, this come for simple matrix algebra.

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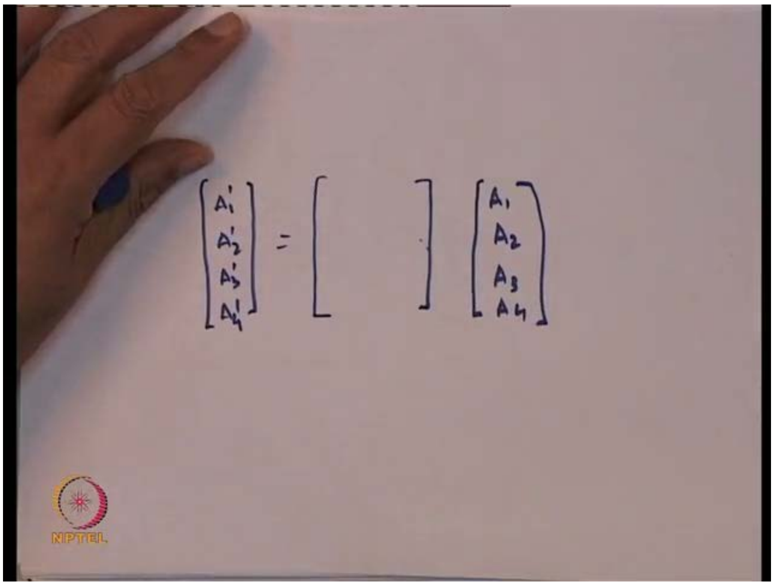


So, if for example, we have let us say components A 1 A 2 A 3 A 4 and we have one particular matrix. We changes them to A 1 prime A 2 prime A 3 prime and A 4 prime. We have another one with B 1 B 2 B 3 B 4. We changes to B 1 prime B 2 prime B 3 prime B 4 prime, using exactly the same matrix. Then, if I take the difference of these two equations, then A 1 minus B 1, A 2 minus B 2, A 3 minus B 3, A 4 minus B 4 will also exactly transform using the same transformation matrix. Hence, the differences are also the component of four vector. That is what I wanted to say.

Therefore, I expect the delta x which is equal to x 2 minus x 1, delta y which is equal to y 2 minus y 1 and delta z which is equal to z 2 minus z 1 and i c delta t which means i c multiplied by t 2 minus t 1. Of course, see same all the frames are also the components of the four vector. So, this is what I will call as the displacement four vector. See like traditional displacement which does not depend on the origin of the frame, here also it does not depend on the origin of x y z because we are only talking of the differences.

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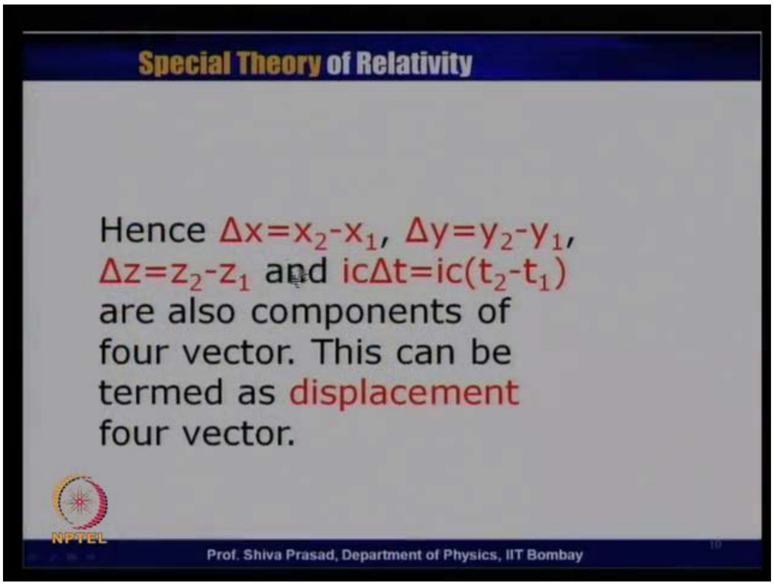


So, all we are looking how much x has change, how much y has change, how much z has change, how much time has change. So, what is the origin to define this particular thing is that so much so important. So, we are looking only at the differences. So, this is what we call as in displacement four vector. Now, we agreed the this displacement four vector would also obey exactly the same transformation equation. If it has that way then obviously delta S. If I take it dot product of delta S with the same delta S. This will obviously be equal to the length of the four vector. Other length square of the four vector that is what we have already define.

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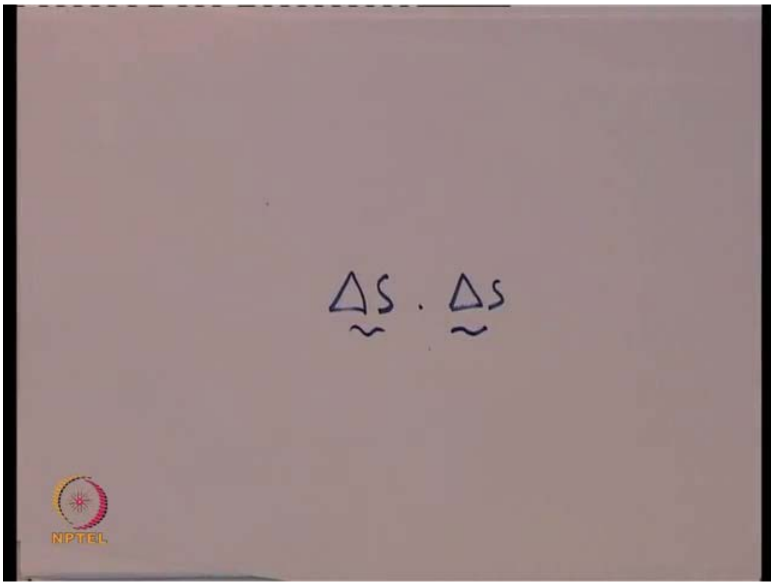

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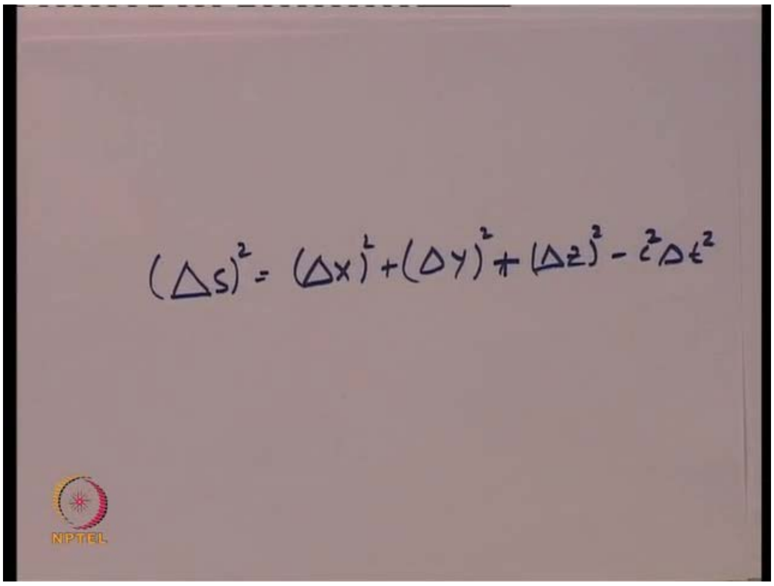


Therefore, if I take the length length square that will give me by the definition, delta S square is equal to delta x square plus delta y square plus delta z square minus c square delta t square and if I take that under root of that that will called the magnitude of the displacement. That is call magnitude of the displacement and this obviously by the definition of dot product would be same in all the inertial frames of a reference because that quantity is a four scalar.

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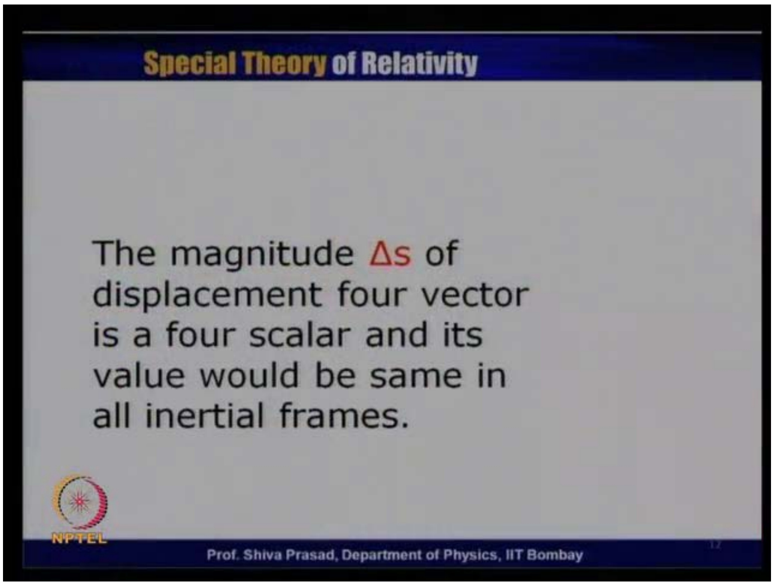

This does not changed when we change the frame of reference. Delta x would change delta y will change delta z will change in general of course, the transformation matrix delta y is transfer to be same as delta y prime delta z is transfer to be same as delta z prime, but all these quantities am in general would change, but if you take the length of the four vector that will not change. The magnitude delta S of the four vector displacement four vector therefore, is a four scalar and its value would be same in all inertial frames.

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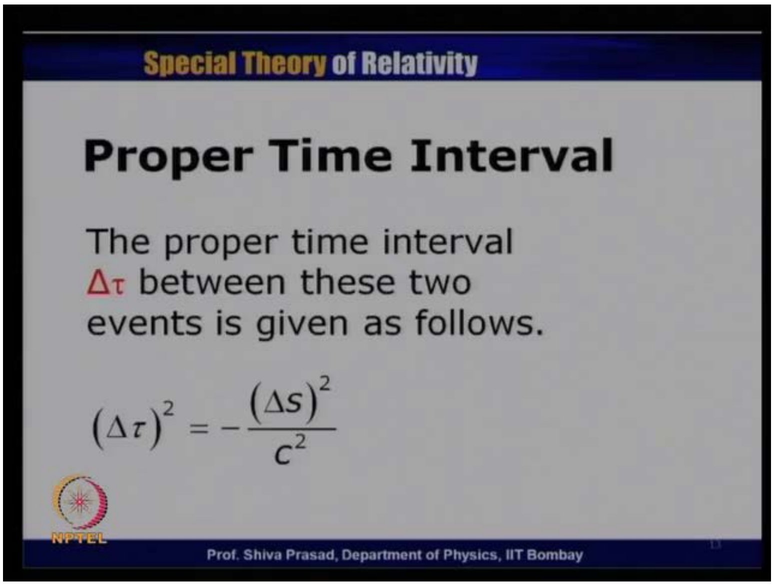


Now, let us define a quantity which we have already define, but talk in a little more general fashion. I define a proper time interval between these two events. Earlier if you remember we had discuss in some of earlier our earlier lectures, that for of defining the proper time interval you must find a frame of reference in which these two events occur exactly at the same position.

If they occur exactly at the same position then the time difference measure between these two events in that particular frame of reference will be a proper time interval. But recently we have also seen that there events in which it may not may possible for anybody to find a frame of reference in which these two events occur exactly at the same position. We have discussed earlier time like separated and space like separated events. In that case cannot we defined a proper time interval.

The thing is that we can still define a proper time interval, but that will turn out to be imaginary. That is what I will see. That will turn out to be imaginary. So, I am now giving a more general definition of proper time interval, which is many times not only easy to execute or to find out because this does not depend on finding out a frame of reference in which these two events occurs exactly at the same position and also it can be generalized to such case where it is not possible to find a frame of reference, in which these two events occur at the same position. So, this proper time interval is defined in terms of delta S square the proper time interval delta tau between these two events is given as follows which is delta tau square is equal to we put a minus sign delta S square divided by c square. So, if we look at this particular definition of deltas S square here, we put a negative sign.

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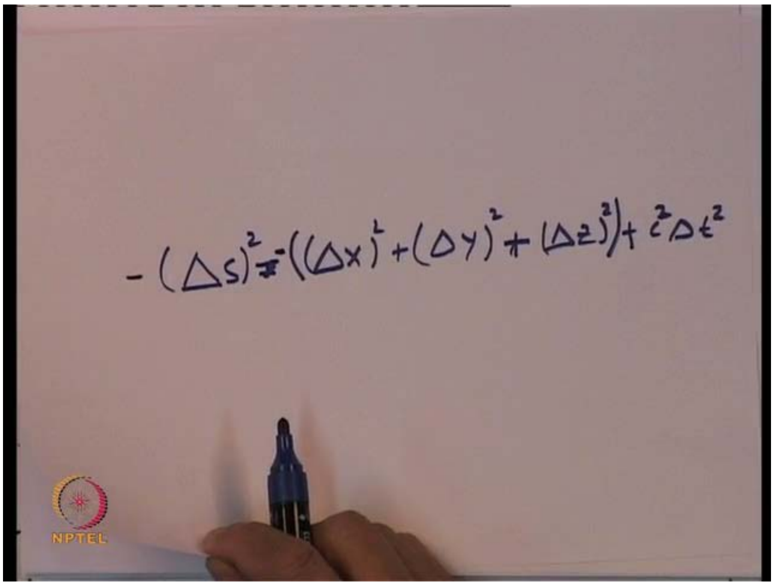


If I put a negative sign here, this quantity becomes plus. All these quantities become negative. I divide by c square. If I divide by c square this will become just delta t square an all these quantities will get divided by c square. This leads to a following definition of delta tau square.

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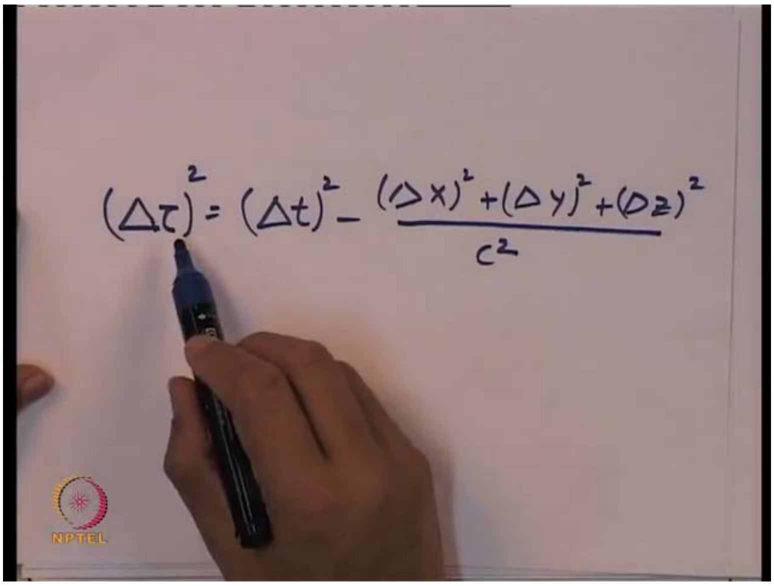


Delta tau square will be turning out to be equal to delta t square minus delta x square plus delta y square plus delta z square divided by c square. So, my definition of delta tau is now given in terms of these quantities.

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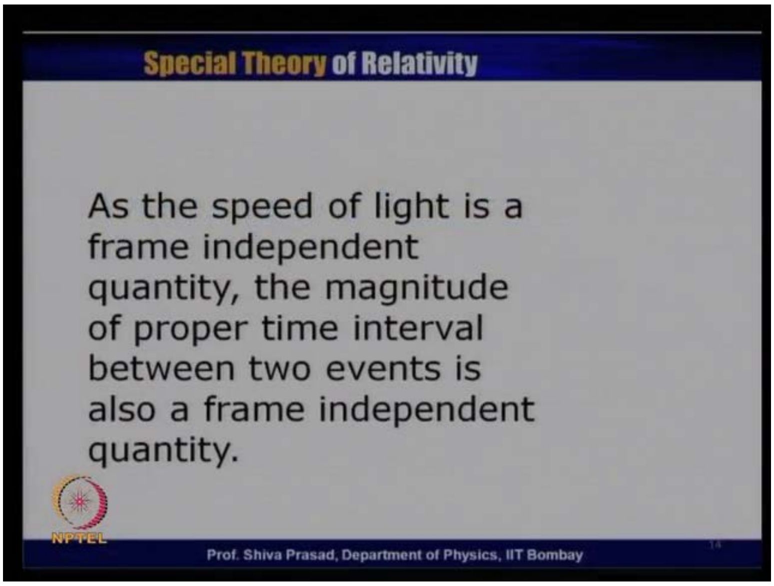


As the speed of light is a frame independent quantity, the magnitude of proper time interval between these two events will also be a frame independent quantity.

It means the proper time interval between these two events would not change, if I use this particular definition to find out the proper time interval. That because speed of light is does not depend on on the frame of reference. So, what I should, I must do. If I have any two events, I do not have to bother about finding out the frame of reference in which these two events occur at the same place.

I choose delta x that I have obtained in a frame of reference delta y that I have obtained in a frame of reference delta z that I have obtained in my my frame of reference and delta t that I have obtained in my frame of reference substitute in this particular equation which have written on this particular paper, that is delta tau square is equal to delta t square minus delta x square plus delta y square plus delta z square divided by c square and I can calculate what is delta tau square.

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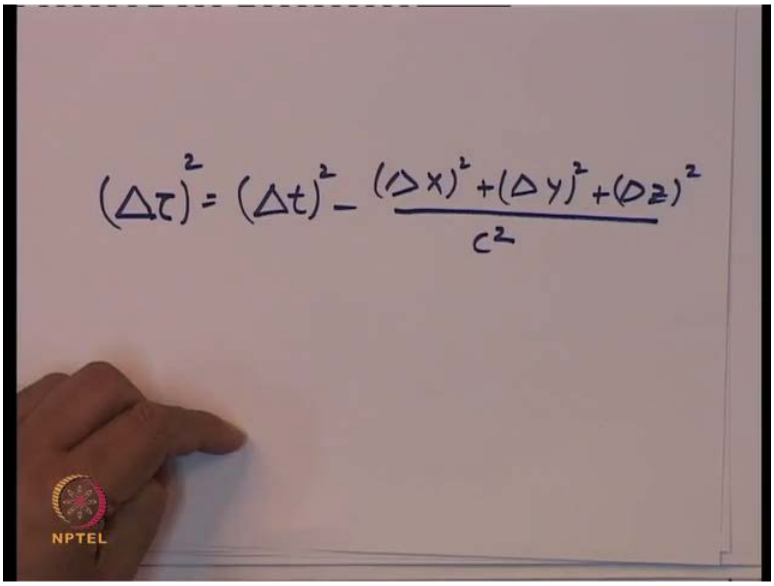


Now, obvious question is that is it consistent with whatever we have defined earlier? I want to show that this is consistent because if the two events in a frame of reference occur exactly at the same position. It means delta x is 0 because in a given particular frame of reference if delta x is 0 they obviously occur at the same value of x. If delta x is 0 delta y is 0 and delta z is also 0. Then it means they occur exactly at the same place there is no difference between the x values of the two events. There is no difference in the y values of the two events. There is no difference in the z values of the two events.

Therefore, obviously the two events occur have occurred that is in same position and they have occur at a same position this is 0 delta x is 0 delta y is 0 delta z is 0 and therefore, delta t will definitely equal to delta tau. That is what we have said earlier that in a frame of reference if this is 0 this is 0 this is 0 then delta t that we measure is equal to the proper time interval.

So, this definition is consistent with the definition that we have used earlier and also we notice from this particular equation, that if it so happens that this particular quantity delta x square plus delta y square plus delta z square divided by c square turns out to be larger than the delta t square then this whole quantity will turn out to be negative and delta tau will turn out to be imaginary.

This is the case in which it is not possible to find the frame of reference in which these two events occur at the same position and in that case, delta tau will turn out to be imaginary. So, we can still define a proper time interval irrespective to the fact whether we have is space like separated events or a time like separated events, is still we should able to find out delta tau to all the thing it will turn out to be real or imaginary.

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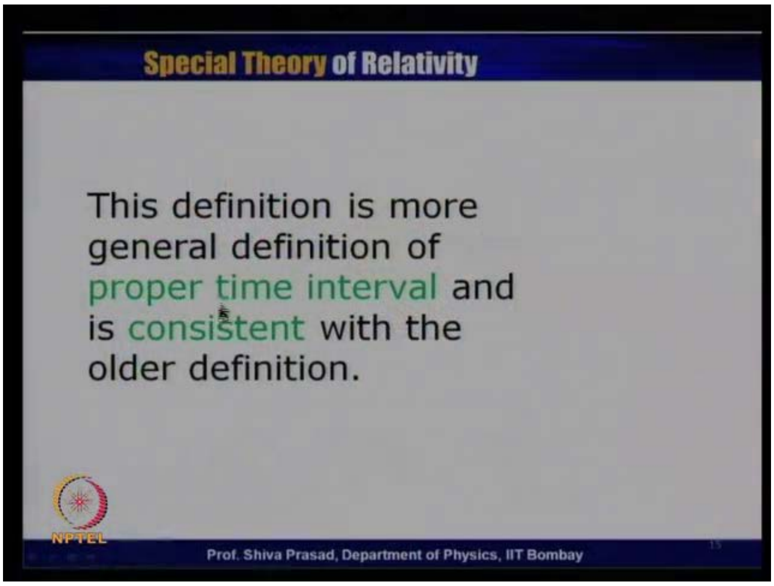


So, let us summarize what we have said. The definition of proper time interval that we have given is a more general information of a proper time interval and is consistent with the older definition.

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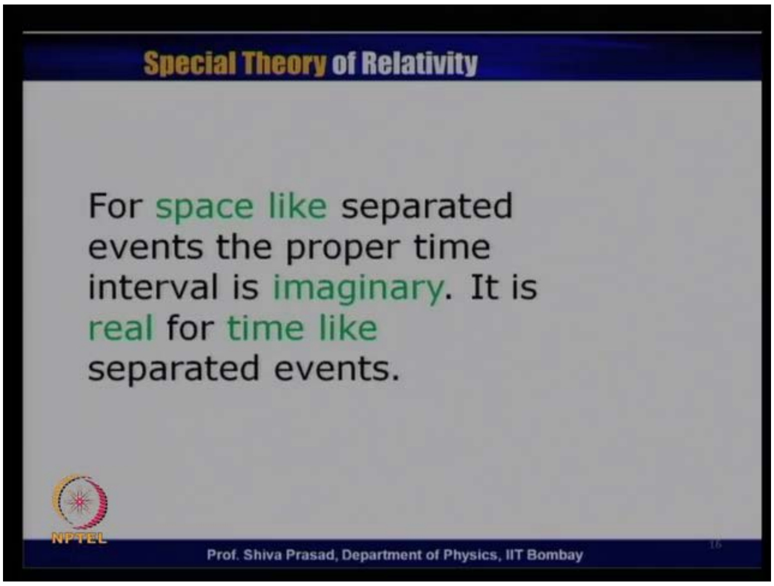


For space like separated events, the proper time interval is imaginary because for space like separated events it is not possible to find a frame in which these two events occurs exactly at the same position, but still we can find out a proper time interval which turns out to be imaginary. While it is going to be a real for time like separated events.

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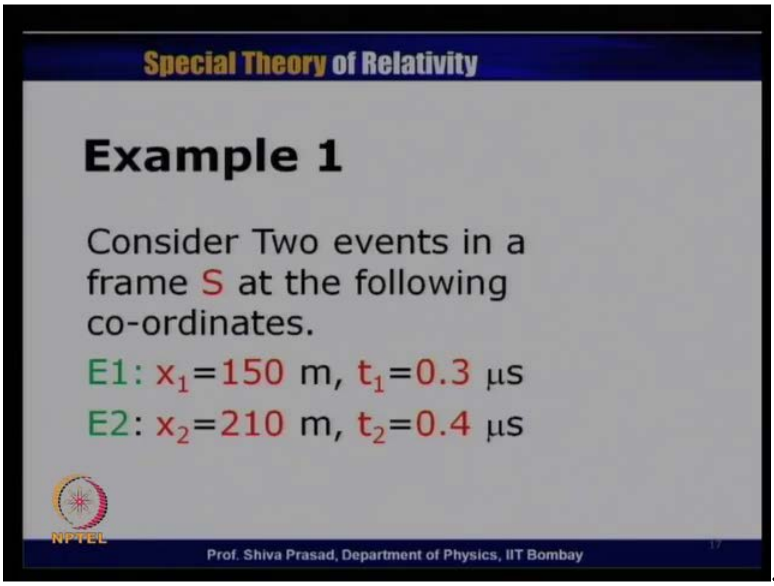


Let us take example. Let us take a very very simple example. Let us take two any arbitrary events. Let us imagine, that the first event let us take y and z equal to 0 y prime z prime be also become equal to 0. As I said that is a simple example. Second example will make some more differences. So, first event occurs at x is equal to 150 meters and time is equal to point 3 micro second. 0.3 micro seconds.

In the same frame of reference and event two, whatever might be that event appears to be occurring at a value of x is equal to 210 meters and at a time 0.4 micro seconds. So, these are the two events. Our question is that find out, we have to find out what is the proper time interval between these two events.

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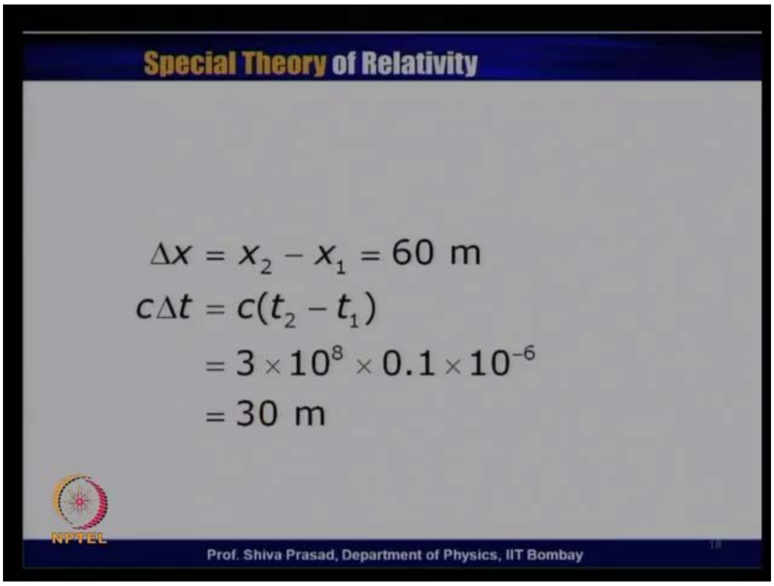


So, let us first find out delta x and let us first find out delta t. So, what I will do, I will find out x two minus x one which will give me delta x. I will find out t two minus t one which gives me delta t because y and z co-ordinates are same for both the events. Delta y is 0 delta z is 0. So, these two values have a substitute in the general definition of proper time interval and eventually obtain the proper time interval.

So, let us go ahead delta x is x 2 minus x 1 which is clearly 60 meters. c delta t, is c multiplied by t 2 minus t 1, we have taken c as 3 into 10 to the power of 8 meters per second and time interval was 0.1 into 10 to the power of minus 6 because there is a micro seconds. So, micro is a 10 to the power of minus 6. So, that is what, I have substituted here. Remember I am talking only of the differences now.

If I multiply this 10 to power 8 into 10 power minus 6 gives me a factor of 100. 3 multiplied by 0.1 gives me 0.3. If I multiplied by 100 I get 30 meters. So, x 2 minus x 1 is equal to 60 meters and c delta t is 30 meters. Obviously delta x is greater than c delta t. Therefore, these are space like separated events and therefore, it is not possible to find the frame of reference in which these two events occur exactly at the same position unless of course, that particular frame of reference moves with the speed greater than a speed of light which we have discounted because a many other reasons.

But as I have said still using the definition that we have just now evolved I will be able to find out a proper time interval which will turn out to be imaginary.

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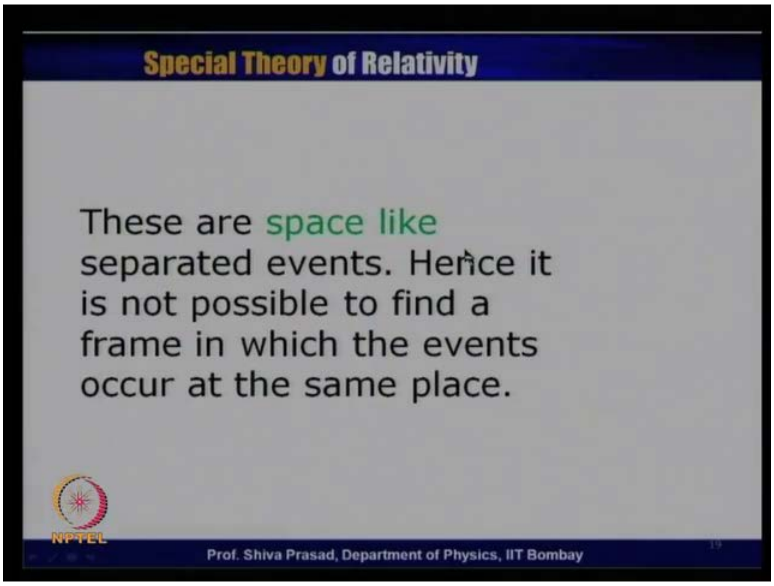


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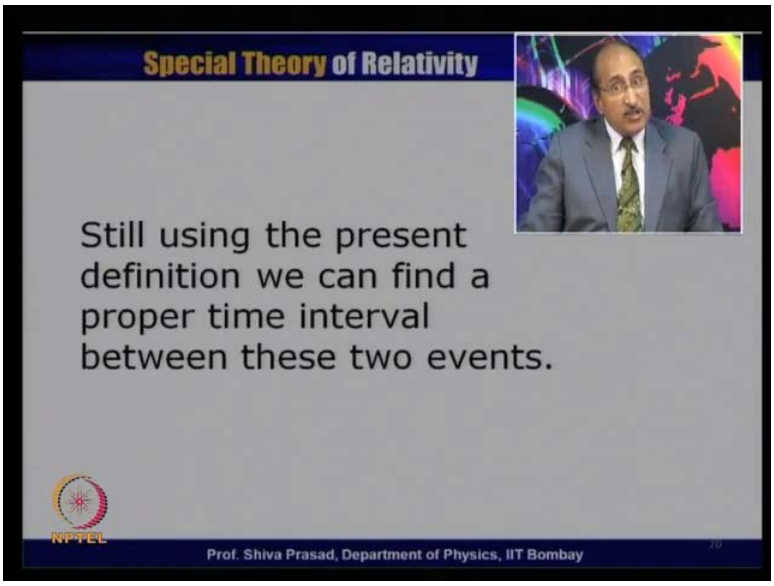

So, this is what I have said, these are space like separated events. Hence, it is not possible to find a frame in which the events occur at the same place. Still using the present definition we can find a proper time interval between these two events. Let us use this particular definition.

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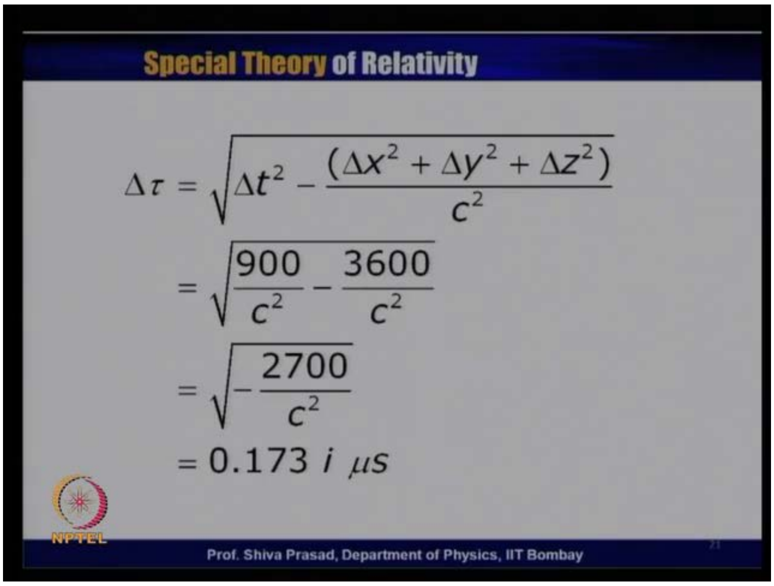


Delta x square plus delta y square plus delta z square. Remember our delta x values. Delta x was 210 minus 50 which was giving me 60 meters. c delta t was 30 meters.

I put here 60 that gives me 3600 delta y is 0 delta z is equal to 0 divided by c square. Delta t was 30. c delta t was 30 rather. So, delta t will be 30 divided by c. So, I have squared it. So, this becomes 930 square is 900 divided by c square. As you can see 3600 is greater than 900.

So, you get under root of minus 2700 divided by c square and therefore, delta tau fighting the under root of that substituting the value c, I get this as 0.173 multiplied by i which is imaginary number under root minus 1 micro second. So, proper time interval between these two frames of reference is 0.173 i micro second, is a imaginary number.

Now, the next thing I would like to show, that if I go to a different frame these delta x is delta t will change but, I will still get delta tau to the same. Remember in none of these two frame of reference, these two events occur at exactly at the same position that is not possible in fact in this case that is not we have seen, but still I can find out delta tau.

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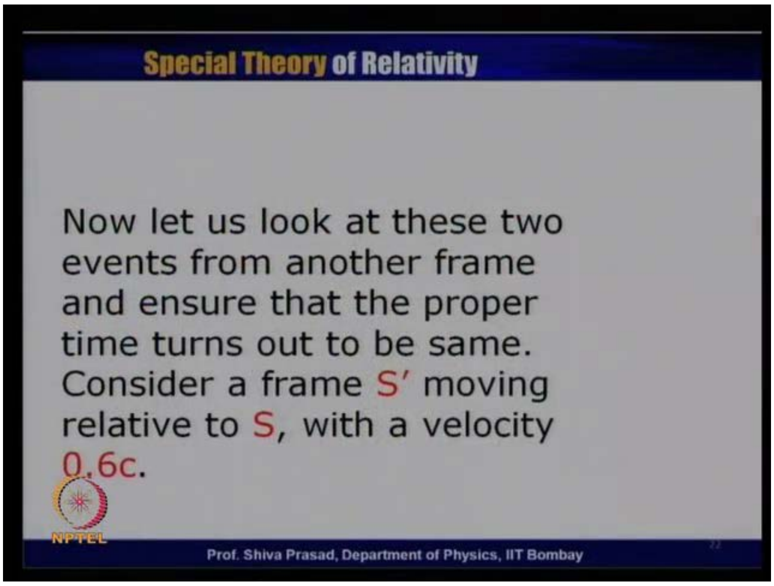


So, now let us look at these two events from another frame and ensure that the proper time turns out to be same. Now, consider a frame S prime moving relative to S along x direction, we should we have not been mentioning it every time with a speed of 0.6 c. We have always been taking 0.6, 0.8 because this gives me a clean clear gamma number and we are trying to sort of illustrate so, that is why let us take mathematical things which are comparatively simple.

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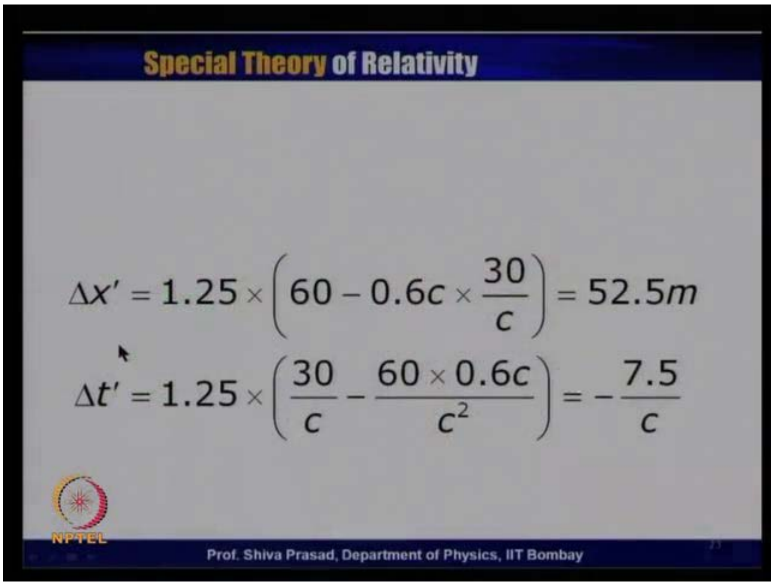

So, if I look at these two events I can find out what are the delta x values what are the delta y of course, delta y is same delta t values. Once I change my frame from x to x prime. This what I have done here. We has shown number of times that a value of v is equal to 0.6 c gives me a value of gamma is equal to 1.25. You can substitute it back, the value of gamma and you can see that this relative velocity of 0.6 c gives me a gamma equal to 1.25.

So, delta x prime is equal to gamma which is 1.25 multiplied by delta x which is 60 meters minus v which is 0.6 c into t. t is 30 divided by c, that is what we have seen. Put this numbers you get 52.5 meters. So, according to x prime observer these two events did not occur at a separation of 30 meters along the x direction sorry 60 meters along the x direction but, they occurred at difference of 52.5 meters

Similarly, time interval between these two events would also be different according to the observer in S prime frame of reference and that time interval will be given by delta t prime is equal to gamma which is 1.25 multiplied by t which is 30 by c minus v which is 0.6 c into x which is 60 divided by c square. If you put all these numbers you get minus 7.5 c.

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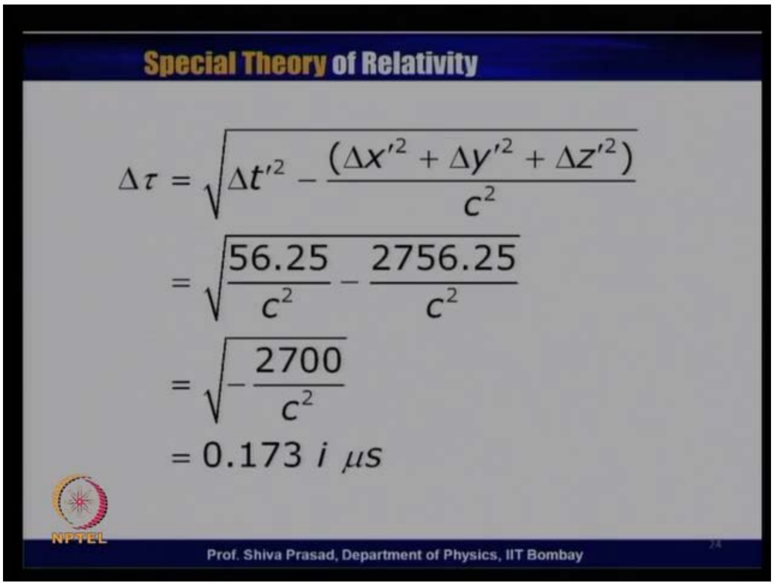


So, we see the de, delta t also has change which is now instead of being 30 by c has become minus 7.5 c. In fact even though order has change because it has become from positive to s negative number. Time order of course, can change in these two events because these are space like separated events.

I substituted back in my equation, which is delta tau is equal to now delta t prime square minus delta x prime square plus delta y prime square plus delta z prime square divided by c square. Put the value of delta t prime that we have obtain put the value of delta x prime that we have obtain. Take a small calculator work out these things. You will get exactly as under root minus 2700 divided by c square which is 0.173 i micro second. Exactly similar to the earlier case.

So, we see that this has changed this has changed but, not this. This is a four scalar. Once I change my frame reference I will not change the proper time interval which will turn out to be same that is what I am saying because it is a four scalar quantity.

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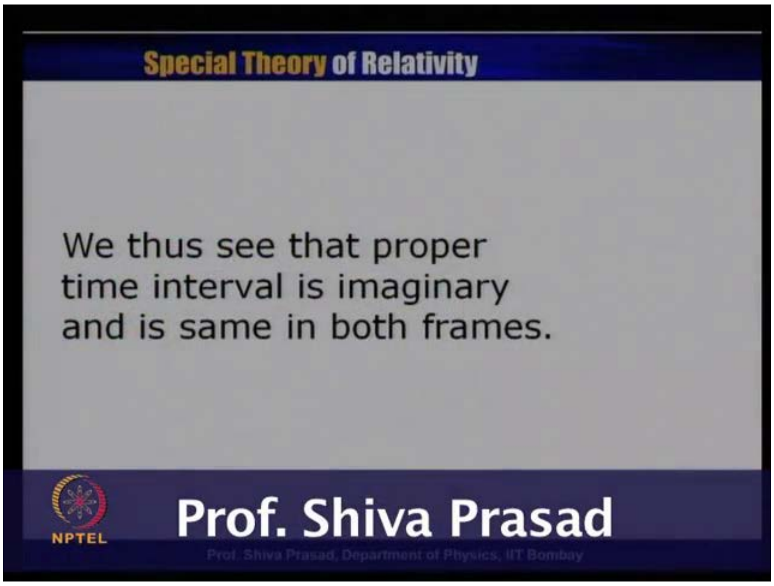


We thus see that the proper time interval is imaginary and is same for both the frames. Let us say one more example which is slightly more difficult because you know you have a motion involving both in y direction as well as in x direction.

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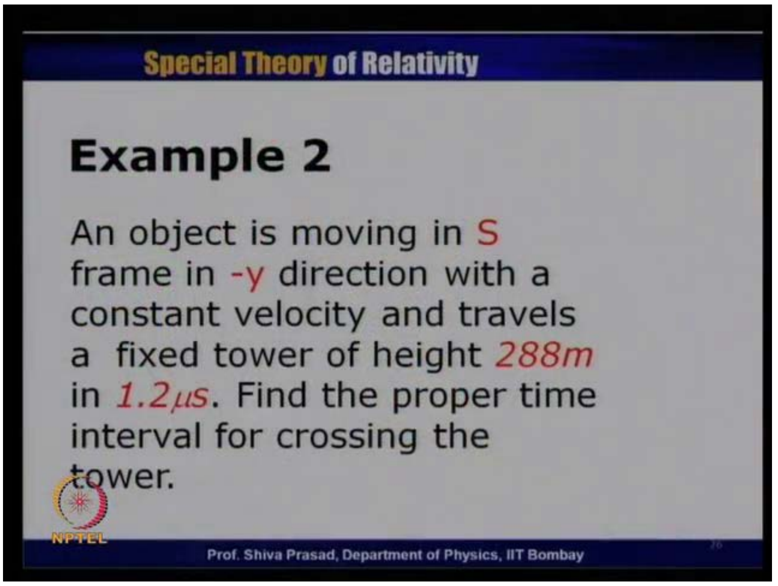


So, let us imagine that there is an object which is moving in S frame in minus y direction and is moving with a constant velocity. Let us imagine that you know y direction is vertically a fixed direction and there is a tower which is of a height 288 meters and a particle is falling with a constant velocity. Of course, in a traditional gravity the particle will not fall with a constant velocity. It will always have acceleration, but you can always imagine that probably this space of gravity free. Or you can imagine that this particular particle has acquired stable velocity whatever it is.

Let us imagine for the sake of illustration that this particular particle is actually moving with a constant velocity. So, now in observer finds that this particular particle which is coming from the top of the tower to the bottom to the ground. Covers the total distance of the tower which is 288 meters in a time of 1.2 micro second. What we have to do is to find the proper time interval for crossing the tower.

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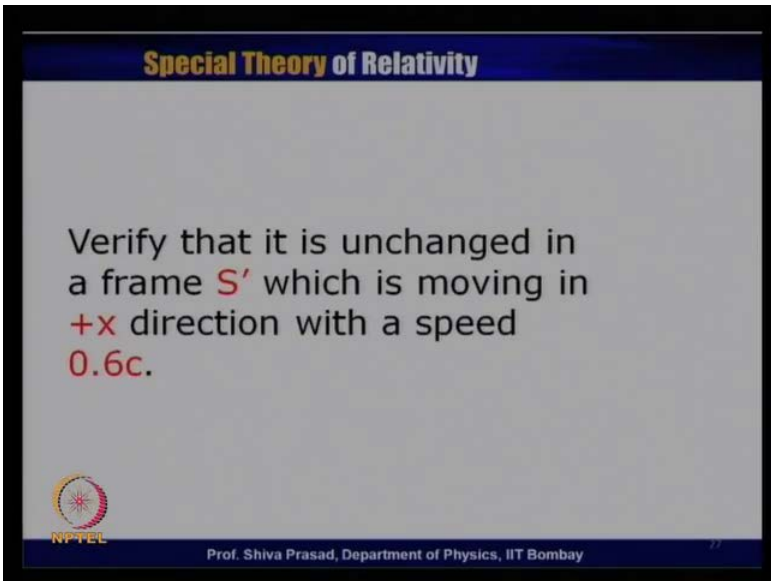


And of course, that we will show that is same in a different frame of reference. Verify that it is unchanged in a frame S prime which is moving in plus x direction with a speed of a 0.6 c. So, second part which is identical to the first part, where the same event now where the displacement is along the y direction and not along the x direction and to find out that in this particular case also the time interval turns out to be same.

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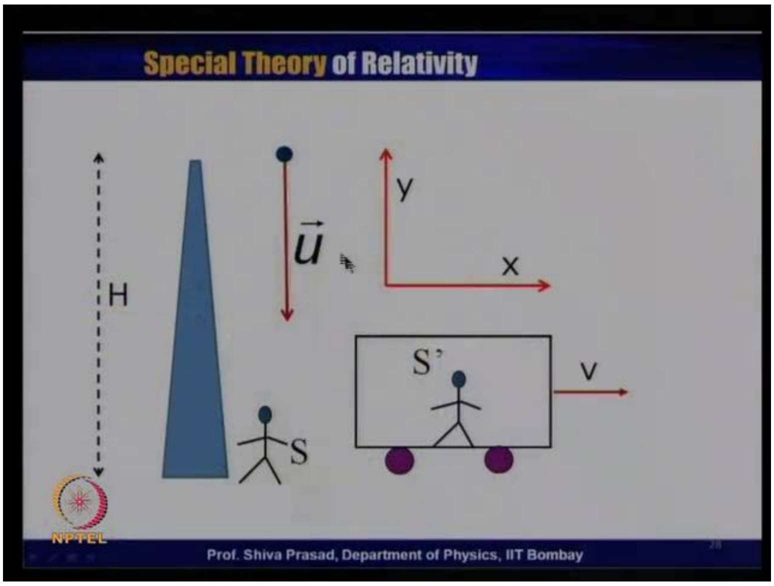


If we calculate the proper time interval not time interval as seen by an observer, but if we calculate the proper time interval that we turn out to be same. So, this what I have sort of picturized, sort of roughly picturized that there is a towers. Everything is viewed with respect to an observer of the ground, which I call as S observer this is my ground observer S observer. There is a tower which has a height H which I have said as 288 meters.

There is the particle which is coming down, vertically down. We assume that this is coming down with a constant velocity. Let us call this particular velocity as u. This is my y y direction which is vertically upward direction. This is my x direction. So, this particular vector u is pointing out in minus y direction because this is coming down while the y is pointing up. (( )) another observer which is moving along the plus x direction it means it moving like this.

This observer I am calling as S prime observer, which is moving with a speed of 0.6 c this particular observer also observes this particular particle being falling from this particular point and coming down to this particular time, coming out from the height of the tower and coming back to the ground. Now, question is that. We have to find out what is the proper time interval between the events. It means when it starts with the particle starts here and the particle reaches here.

So, fall the particle was particle is falling, but there will be a point where it comes just here. Let us call that time t is equal to 0 and let us assume at that this particular time this particular origin was also coinciding with this particular origin, but that is not all that important because we talking in terms of deltas. But when we are writing event let us that assume that particular thing that there origin was coinciding. So, this is my event number one that this particular particle being at the top, are aligning with the top of the tower and event number two that this particular particle reach in the ground. These are my event one and event two and between these two events, I have to find out the proper time interval. Between in S frame and S prime frame of reference and show that this time interval will be same in more the cases.

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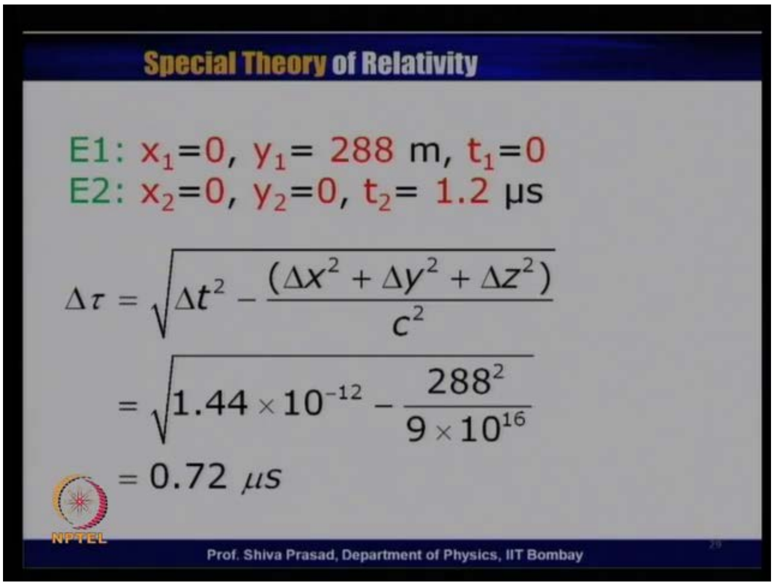


So, these are the co-ordinates of my event one and event two. I have assumed that this particular falling as seen by an S observer is at the origin as I just now mention. So, first event occurred at the origin as for as the x values concerned. sorry I am sorry It does not occur at the origin, it occurs at x is equal to 0, but it occurs at y is equal to 288 meters because the tower was 288 meters height. So, the y value of that particular event was 288 meters. Essentially, what I am trying to say that if this is my tower and this particle falls here so, this is my origin. So, event number one occurs at a y value of 288 meters and the event number two occurs y is equal to 0.

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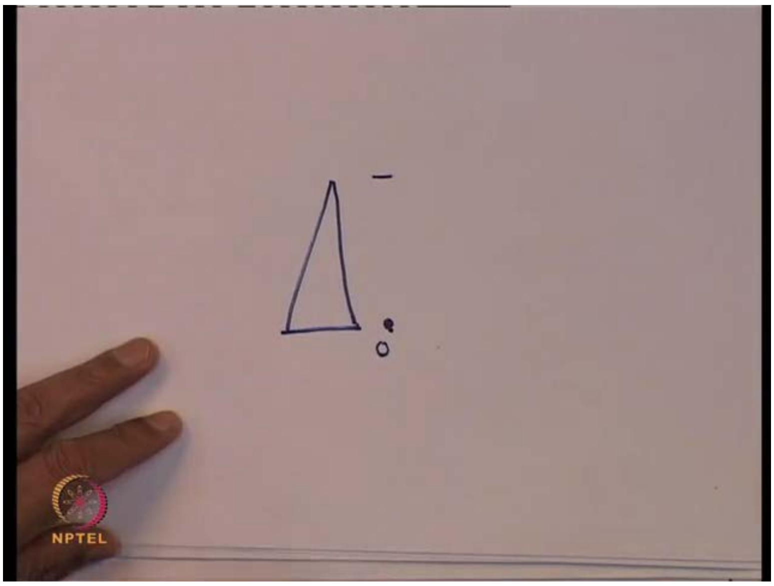

Of course for both these two events the value of x is 0.

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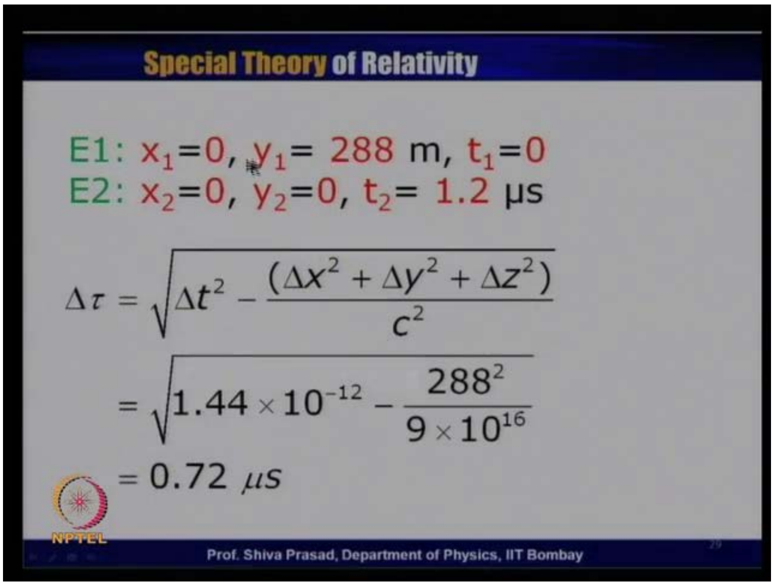


So, what I have written here x 1 is equal to 0 y 1 is equal to 288 meters and let us assume that at the time with this particular particle starts from the top or it aligns itself to the top of the tower, time is 0. For the second event of course, it falls to the origin so x 2 was 0 y 2 was equal to 0 and because S given in the problem this particular particle to 1.2 micro second to reach the ground therefore, time according to S observer must be equal to 1.2 micro second.

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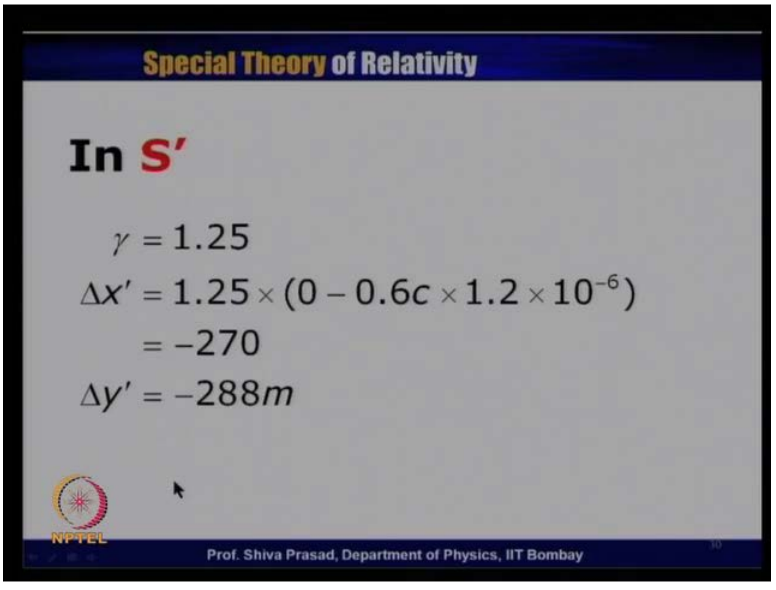

I substitute in this particular equation. Delta t which is the difference between these two which is 1.2 micro second. Delta x now 0 delta y is 288. I substituted here, delta z is equal to 0 c .I substitute the value of c I get this proper time interval to be equal to 0.72 micro second. So, proper time interval between these two events is 0.72 micro second.

Now, let us go to S prime observer, which is that particular train or car whatever you want to say which is moving on the plus x direction. As we have said gamma is equal to 1.25 because 0 a relative velocity 0.6 c. So, I write gamma is equal to 1.25. Just like before I do a Lorentz transformation find out delta x prime. Delta x prime turns out to be equal to gamma which is 1.25. Delta x which is equal to 0 minus v delta t. v is 0.6 c delta t is 1.2 into 10 power minus 6 second because which is 1.2 micro second.

This will turn to be minus 270. Of course, as for as delta y prime is concerned, there is no change because delta y prime is equal to delta y. Which is equal to minus 288 meters because remember y 1 was 0 and y 2 was 288. So, if I take I am sorry. y 1 was 288 and y 2 was equal to 0. So, if I take y 2 minus y 1, you will get minus 288 meters. So that is why it is minus 288 meters.

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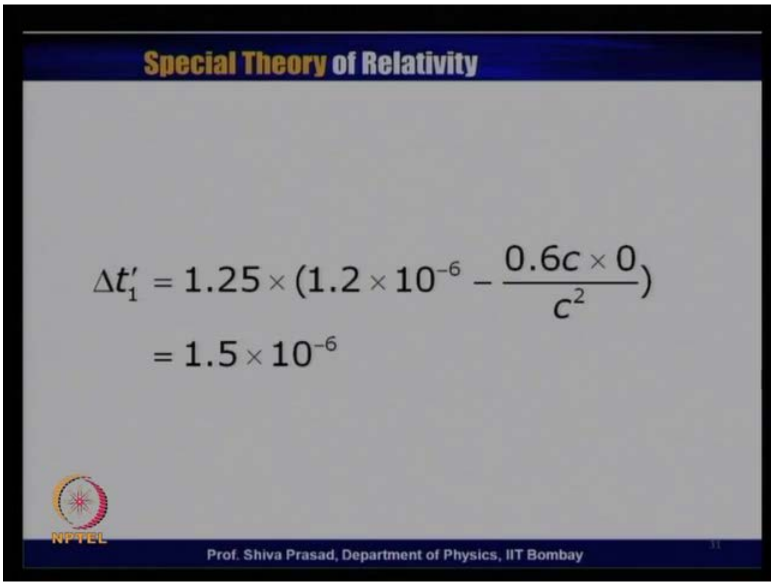


I can calculate delta t prime. Which is 1.25, which is gamma. Delta t which is 1.2 into 10 power minus 6 minus v which is 0.6 c into delta x which happens to be 0 divided by c square I get delta t prime is equal to 1.5 into 10 power minus 6 second.

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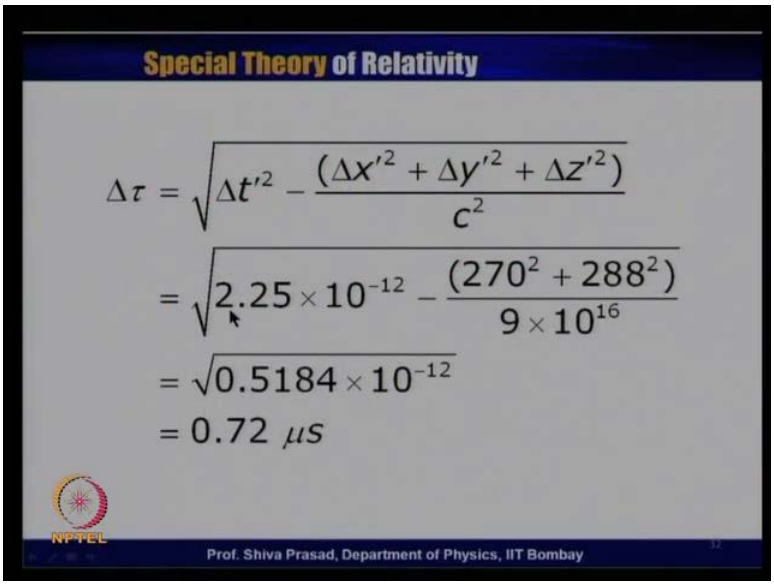


So, now we have calculated delta x prime and delta t prime. So let us substitute in this equation standard equation for finding out the proper time interval. We have delta t prime which we have calculated as 1.5 into 10 power minus 6. So, we take square of that. This becomes 2.5 into 10 power minus 12. Now, delta x prime is note not 0. Which happened to be 270 meters. So, I take 270 meter square. Delta y prime was of course, same as earlier which was minus 288. I square it so minus of course, becomes plus. This is 288 square. Delta z prime of course, is 0 divided by c square which is 9 into 10 power 16. I calculate this number this gives me this which gives me exactly equal to 0.72 micro second which we have obtained earlier.

So, proper time interval turns out to be same in both S and S prime frame of reference to strictly speaking the events. The two events neither in S nor in S occur exactly at the same position. How this one, very interesting thing which you might have noticed. When I calculated it delta t prime, it is so happen the delta t prime turn out to be just equal to gamma times delta t.

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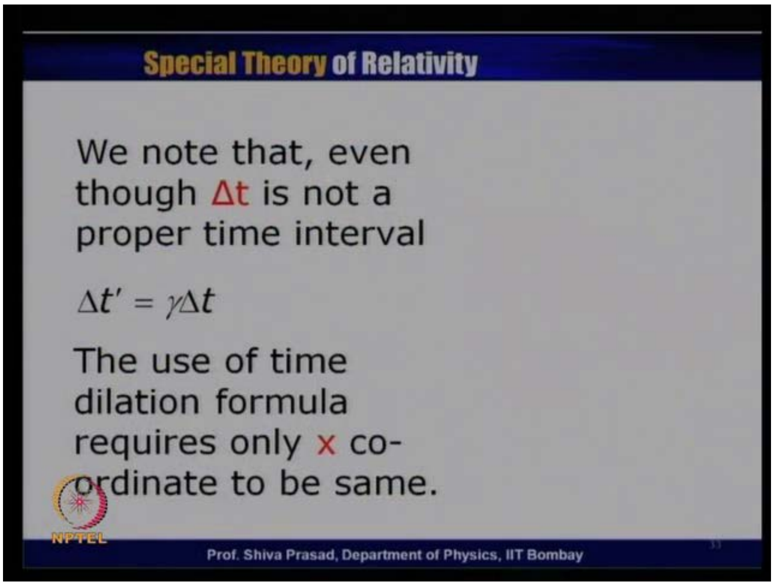


Even though strictly speaking delta t is not a proper time interval because these two events as for as S frame is concerned they occur at the same value of x, but they do not occur at the same value of y. So, two events are not occurring exactly at the same location. There is the difference in the y, but what is important for applying this particular expression of time dilation is that only the x co-ordinate need be same because remember for this particular thing if we have delta t prime.

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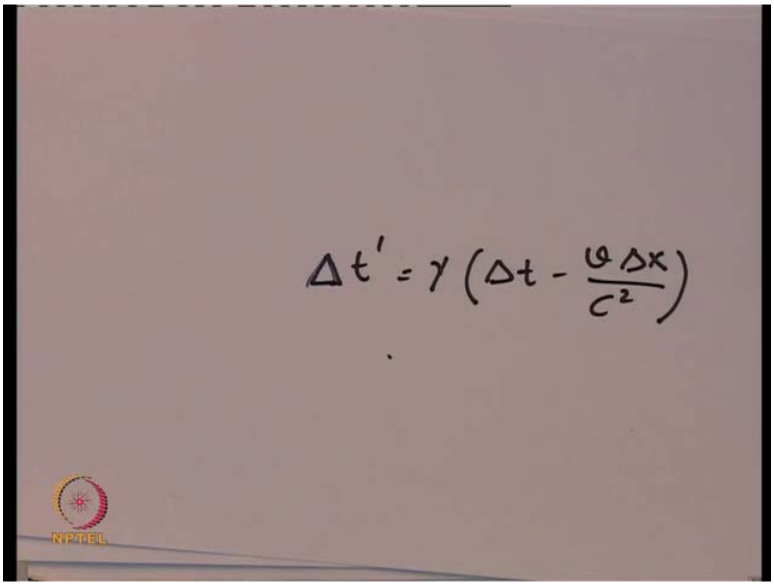

Is equal to gamma delta t minus v delta x divided by c square. So, if delta x is 0 delta t prime in term not be equal to delta t.

So, even though their y is may be different even though their z is may be different but, this apparent time dilation formula will work here because delta x for these two events turn out to be same. T here fore, strictly speaking in this case is strange that though these two events do not occur exactly at the same location in as still effectively I can use the time dilation formula to find out the time interval in S prime frame of reference just because the x coordinates of these two events are same. Now, is there a really a frame in which these two events occur exactly at the same position? You can very easily guess that this is the particle frame itself because if you are sitting in the particle then what you would notice that the top of the tower is coming towards you then the bottom is coming towards you while you are sitting exactly on the particle.

So as for as the particle frame is concerned these two events would really occur exactly at the same position. So, in particle frame of reference the time interval between these two events give you really be proper. So, if whatever we have said is consistent, if I calculate delta t in the particle frame of reference that must give me 0.72 micro second because it is in that frame that these two events occurs exactly at the same position. Therefore, delta x will be 0 delta y will be 0 delta z will be 0 and therefore, delta t prime delta t itself would be the proper time interval. Let us verify.

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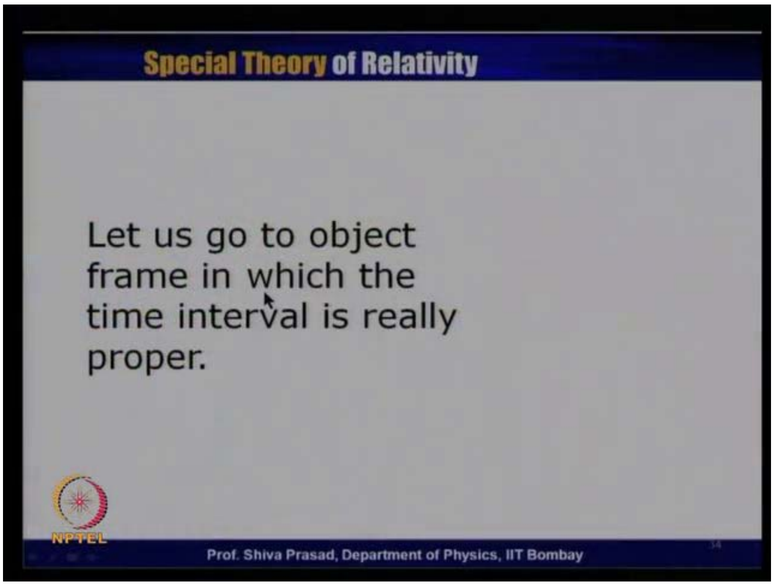

Let us go to the object frame in which the time interval is really proper.

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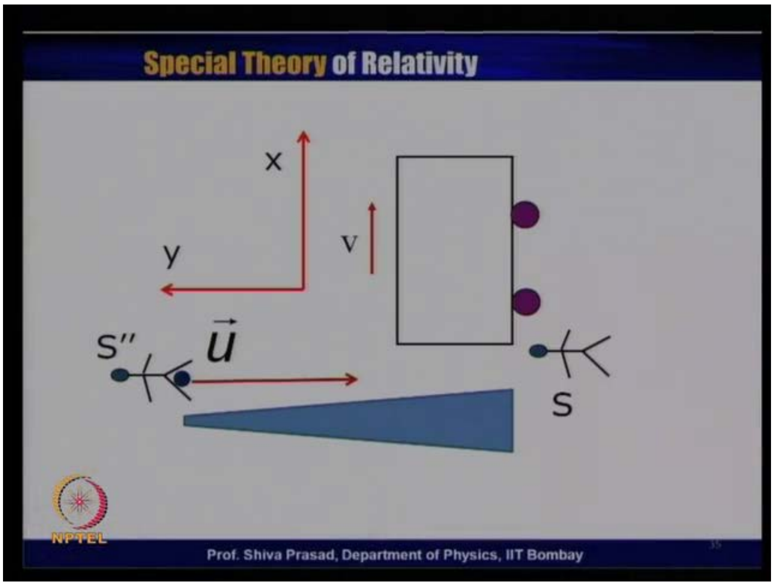


There is little bit problem here because once I go to the frame of reference of the particle particle moves actually along the y direction. So, what I have done I have tilted this particular figure by 90 degrees. I have just rotated just to make things comparison little easier. So I have put this as x axis and I have put this as y axis. Now, if I want to compare between S and S double prime. Let us call this particular frame of reference as S double prime. The particle frame of reference. This was my S prime frame of reference. This is my S double prime frame of reference.

So, now we has see that between S and S double prime, the relative velocity direction is along the y direction we be more correct it is along minus y direction because this is the y direction. Therefore, relative velocity is along minus y direction and remember in the Lorentz transformation when I am writing delta x prime. What is special about x? x is the direction of relative velocity. So, it is only the co-ordinate which is along the relative velocity direction that is what gets gamma delta things minus v delta t other two directions are same.

So, if I want to transform from S to S double prime frame of reference, it is the delta y co-ordinate which will get transformation delta x will be same as delta x prime, delta z is will be same as delta z prime because in that direction there is no relative motion.

Relative motion as for as S and S double prime is concerned is only along the y direction to be more precise along minus y direction.

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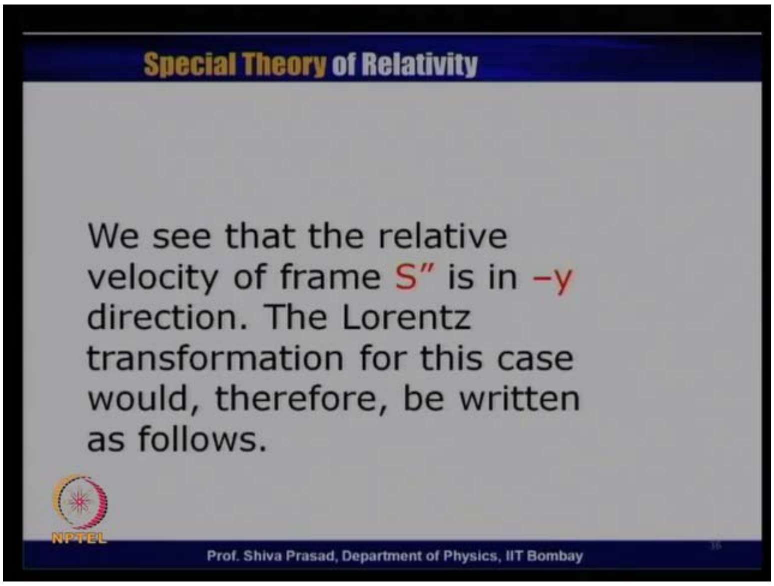


So, if I have to transform from S to S double prime prime frame of reference then in that particular case the transformation equation that I have to use to be some more different, which I will show just now. Let me just read this. We see that the relative velocity of the frame S double prime is in minus y direction. The Lorentz transformation for this case would therefore, will be written as this equation.

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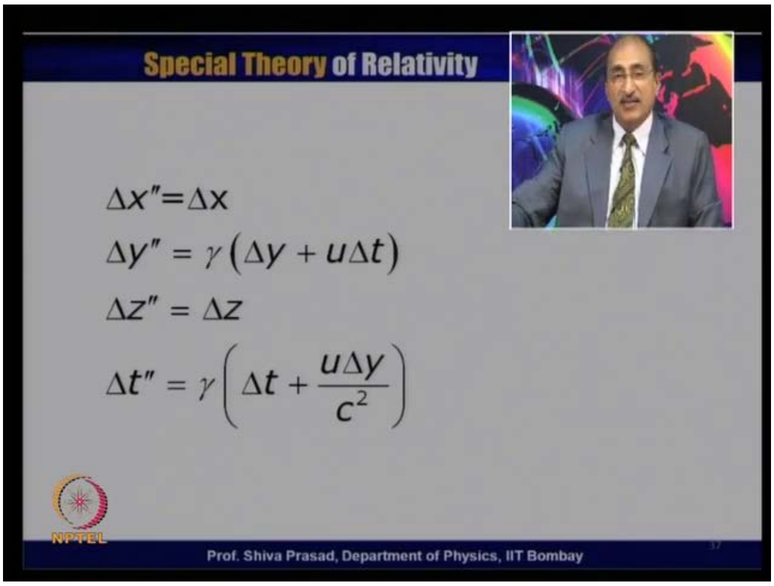

Now, delta x double prime will be same as delta x because in this direction there is no relative velocity. Delta z prime will same as delta z because in this direction there is no relative velocity. Delta y prime will be now equal to gamma of course, which will be determined because of the relative velocity between S and S double prime frame of the reference because it is in between these two frames that I am transforming delta y. I am using plus here because the relative velocity direction is in minus y direction. In the other case I have to get the other way. So, delta y and I have taken that particular velocity as u so it will be delta y plus u delta t.

Similarly, when I am writing delta t double prime there use to be delta x appearing here. But what was so special about x, for that was also depending upon the relative velocity direction. Here relative velocity direction is along the y direction. So, what will appear here is delta y and not delta x and of course, because u is along minus y direction. So, this equation will be given by gamma delta t plus u delta y divided by c square.

So, now let us substitute the value of delta x and delta t that we have calculated. Find out what is u and find out what will be the delta y double prime delta t double prime in S double prime prime frame of reference which happens to be the particle frame of reference, in which the two events occurs exactly at the same position.

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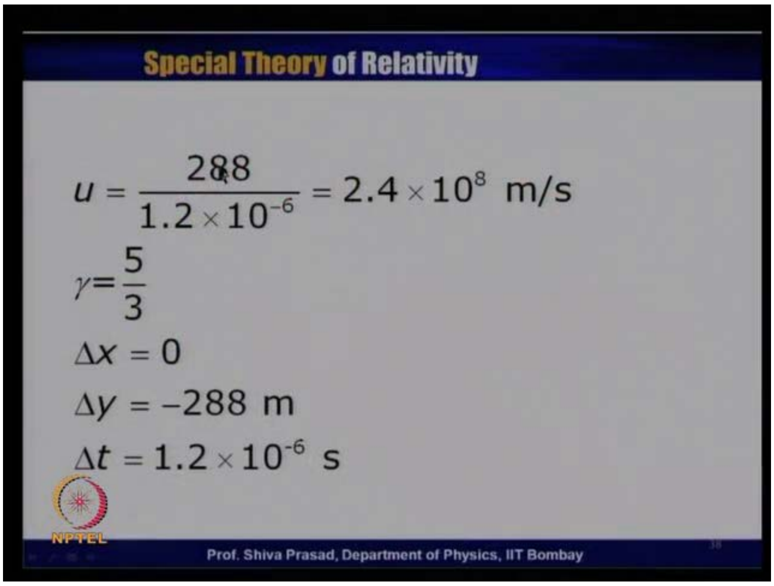

Let us first calculate u. We have already said that we are assuming that the particle is moving with constant velocity. So, we are always in the overall special relative velocity case. There is no x direction involve.

So, according to S observer this particle to 1.2 micro second to travel a distance of 288 meters which is the height of the tower. So, just divide the two you get, just take the ratio you get the speed of the particle as seen in S observer observer's frame which is 2.4 into 10 power 8 meter per second which is 0.8 c and for 0.8 c we know that gamma transfer to be equal to 5 by 3.

According to S observer we have already seen, the delta x is 0 delta y was equal to minus 288 meters delta t is equal to 1.2 into 10 power minus 6 second.

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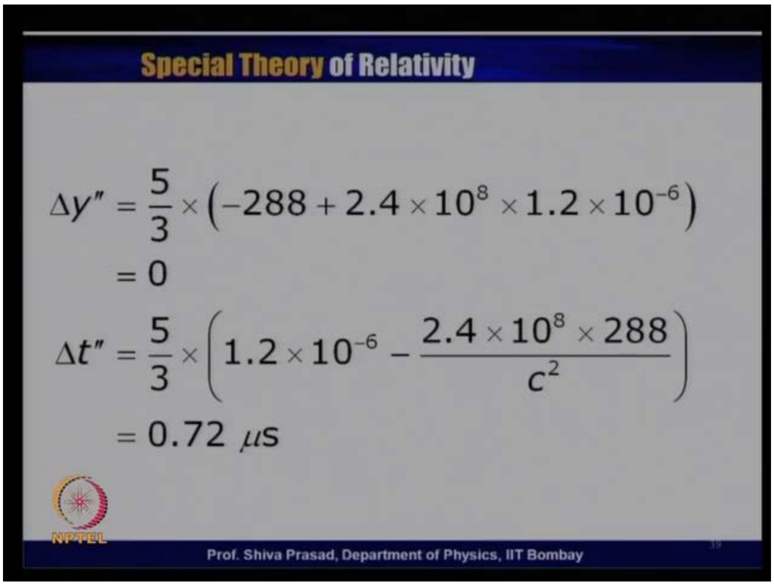


Everything in S frame transform to S double prime frame of reference. Delta y double prime is equal to 5 by 3 which is the value of gamma multiplied by delta y which is minus 288 plus relative velocity which is 2.8 into 10 power 8 meter per seconds multiplied by delta t which is 1.2 into 10 power minus 6 second. This gives me 0. I am not surprised because I know in this particular frame of reference these two particles indeed occur exactly at the same position. Therefore, delta x prime had to be 0 delta y prime had to be 0 delta z prime had to be 0 so, it is not surprising that we found out delta y double prime to be equal to 0.

Let us try to find out delta t double prime. If I want to find out delta t double prime again this is gamma which is 5 by 3 multiplied by what is that delta t. Which is 1.2 into 10 power minus 6 multiplied by the relative velocity between S and S double prime minus 2.5 into 10 power 8 and because delta y is minus 288 so, there is an minus sign here. 288 divided by c square you get delta t prime exactly equal to 0.72 micro second.

So, accord if an observer was sitting on the particle he would, he or she would really at find out that the time interval or the time taken for the rod or for the tower to cross this particular particle, that is what that particular person will observe. That time was 0.72 micro second and it is in that particular frame of reference that this particular time interval was indeed proper.

However, we did not we did not need this particular frame of reference to calculate the proper time interval between these two events. Even of might delta x is not 0 delta y is not equal to 0 delta z is not equal to 0 I can still find out delta t prime delta tau sorry delta tau to find the proper time interval between these two events.

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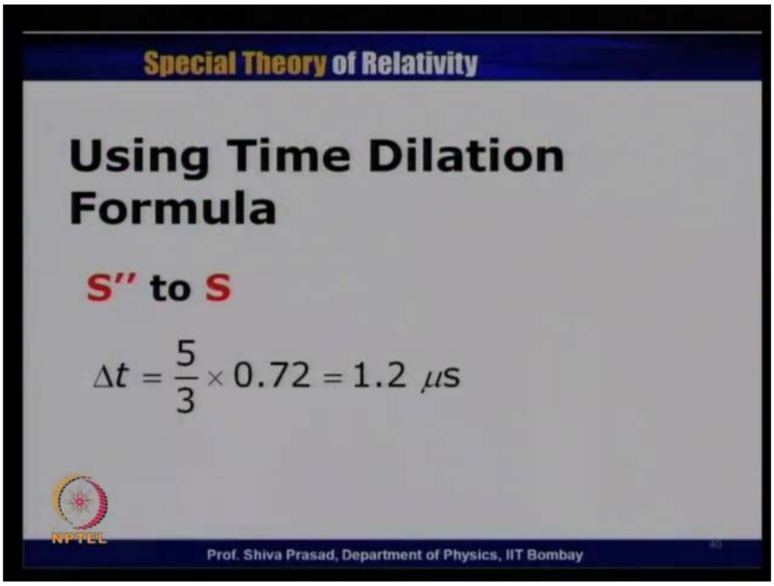


Now, let us check if I want to go from this particular frame of reference directly to S prime frame of reference. Okay. It means the frame of reference of the car. For particle frame of reference to car frame of reference. Remember if this is a proper time interval I can always apply a time dilation formula there also. So, let us not test whatever we are saying. Try to make a transition from S double prime frame of reference to S prime frame of reference not S frame of reference as we have just now seen and see whether I can apply the time dilation formula there to obtain the correct result.

So, now let us see what are all the cases in which I can apply in time dilation formula? I can definitely apply the time dilation formula for S double prime to S. As we have seen that in S double prime the time interval is proper and between S double prime and S the value of gamma is 5 by 3. So, I just applied here by delta t is equal to, this is the time interval between these two events. In S double prime frame of reference multiplied by gamma I get 1.2 micro second as I expected because it is S double prime frame of reference in which the time at time interval was proper. So, I am not talking of the time dilation formula language and if I find out if I can find out the frame of reference in which the time interval is proper then in additional frame of reference by multiplying by appropriate gamma I can find out the time interval. So, from S double prime I made a transition to S and for that I use this particular gamma which is 5 by 3.

So, delta t becomes equal to 5 by 3 multiplied by 0.72 which gives me exactly 1.2micro second which was the case which we have seen was the time interval observed in S frame of reference.

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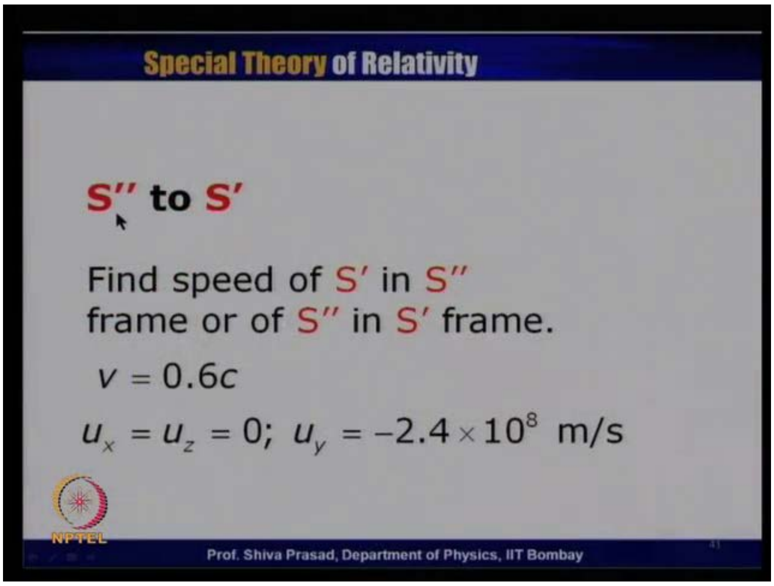


So, now we go from S double prime to S prime. Told. To do that I must find out a relative speed between S double prime and S prime frame of reference. So for that particular thing either I have to find out the speed of S prime in S double prime frame of reference or of S double prime in S prime frame of reference because initial problem was given more interrupts of S and S prime frame of reference so, let us choose these two frame of reference.

So, I know the particle speed which is the speed of S double prime in S frame of reference which is the ground frame of reference. Let me try to find out the speed of this particular particle in the train frame of reference which is S prime frame of reference. So, first I find the speed of the particle in S prime of reference that is what is the relative speed between S prime and S double prime. When I choose v, the v must be equal to 0.6 c which is the relative velocity between S and S prime because I am making a transformation from S prime S frame to S prime frame of reference.

In S frame of reference this particular particle which happens to be S double prime frame now, moves vertically downwards. So, it does not have x component of the velocity. It does not have z component of velocity. So, u x and u z must be equal to 0, but it has a y component of the velocity with we have as we have seen is minus 2.8 into 10 power 8 meters per second.

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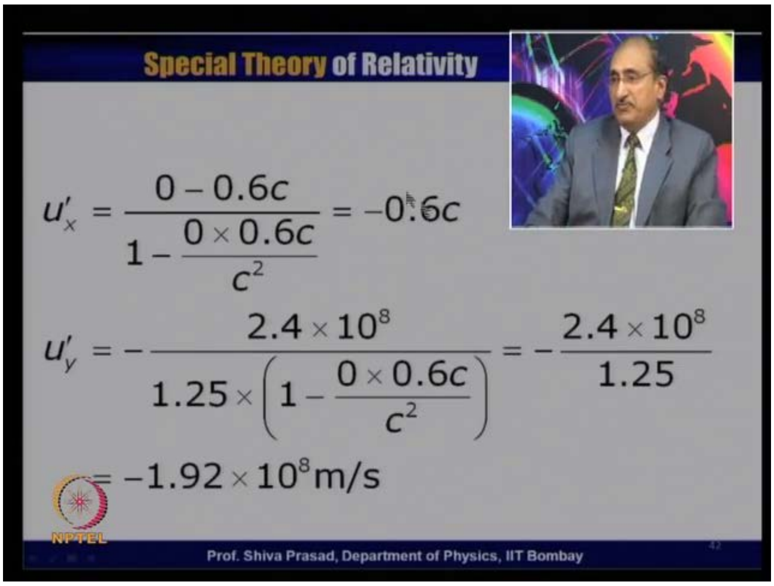


I make a velocity transformation using the standard expression of u x is equal to u x minus v divided by one minus u x v (( )) c square all those things. I get u x prime as minus 0.6 c. I use velocity transformation find out u y prime. I will get this as minus 2.4

into 10 power 8 divided by 1.25. I will get u y prime as minus 1.92 into 10 power 8 meters per second.

So, according to the observer in the train S prime observer the particle has not only x component not only y component, but it also has an x component which is even in a classical mechanics is obvious. So if I have to calculate now, the gamma value I have to use the complete work the components of use.

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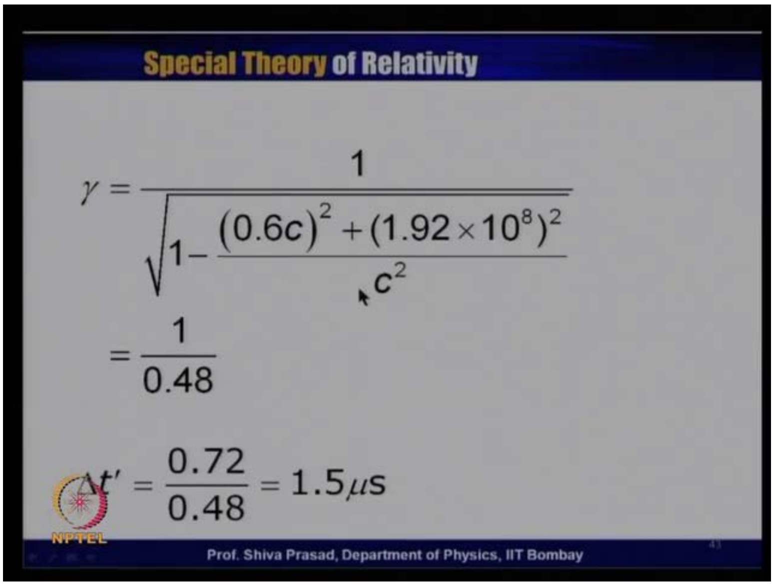


So, now let us calculate the value of gamma. This is the value of gamma that we have to use when we want to transform from S prime to S double prime frame of reference. So, for this value of gamma we have 1 divided by under root 1 minus the relative speed between S prime and S double prime. Which has two components with a bracket 0.6 c square plus 1.92 into 10 power 8 square divided by c square. If you calculate this number you get gamma is equal to 1 divided by 0.48. So, delta t prime which is the time interval between these two events as observed in S prime frame of reference, the train frame of reference will be the time interval as seen in S double prime frame of reference which happens to be a proper time interval multiplied by gamma which is 1.48. So, this particular time interval will turn out to be equal to 1.5micro second.

This is indeed the time interval that we had to calculated by making a transition directly from to S frame to S double frame. S frame to S prime frame of reference. So, let us summarize all these things in terms of a particular diagram because there so many frame there so many relative velocity there so many values of gammas one can tend tend to get confused. So, it is better to picturize the thing so, that will make it in our mind things clear.

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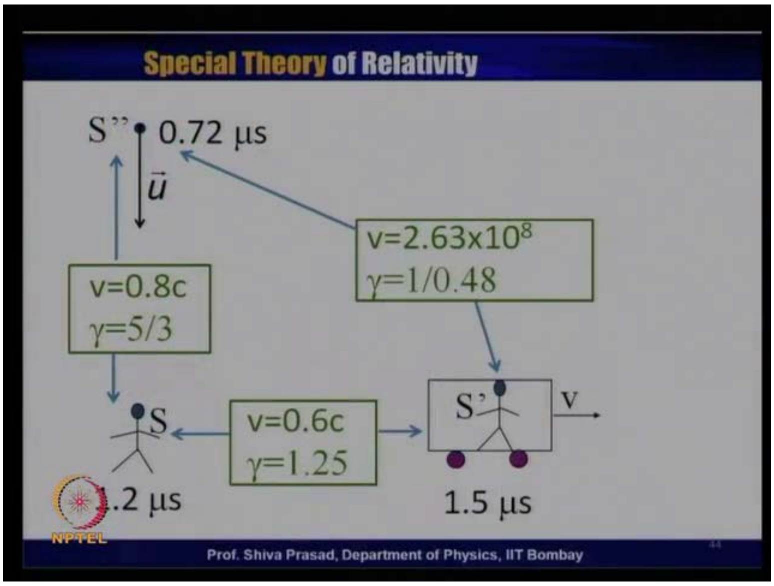


So, this is a picture which I am trying to show. We have three frames about which we have talked. One was this S frame which is the ground frame. There was another frame S prime frame of reference which is the frame which you are I am calling as a train frame. There was third frame S double prime frame of reference which is what I call as the particle frame of reference. Now, if at all we have to make a transformation from S frame to S double prime frame of reference, I would use this relative velocity v is equal to 0.8 c. In fact what I have written in my earlier transparency was u. This I have designed at as u and correspondingly the value of gamma that I will be using is 5 by 3. If I had to make a transition transformation from this frame to this frame, I would use v is equal to 0.6 c and correspondingly a value of gamma to be 1.25.

If I have to make a transformation from this frame to this frame then at v that I will be using 2.63 into 10 power 8. Though I have not calculated earlier but, it if you take those x component and y component and calculate the magnitude this is the magnitude that you will get and gamma will be equal to 1., 1 divided by 0.48.

Now, it is out of these three frames is only in S double prime frame of reference that the time interval between these two events was really proper. So, if I want to make a find out the time interval by using a time dilation formula then when I go from S double prime to S prime, S double prime to S frame of reference I have to use this gamma. So, this time interval 0.72 multiplied by 5 by 3 will give me 1.2.

If I want to go from this frame of reference to this frame of reference, I have to use this particular value multiplied by this gamma which is 1 divided by 0.48 this multiplied by 1 divided by 0.48 will give me 1.5 micro second. Normally, I should not have been able to wait a a use of time dilation formula from this frame to this frame because in none of these frames the time interval is proper, but in this case this is an example that we have given is a special example because it is so happens that a S frame of reference the two events occur exactly at the same value of gamma sorry same value of x. Therefore, it is sort of interesting that even in this particular case if I want to make a transition from S to S prime frame of reference, I can still use a time dilation formula by multiplying 1.2 micro second by a gamma of 1.25.

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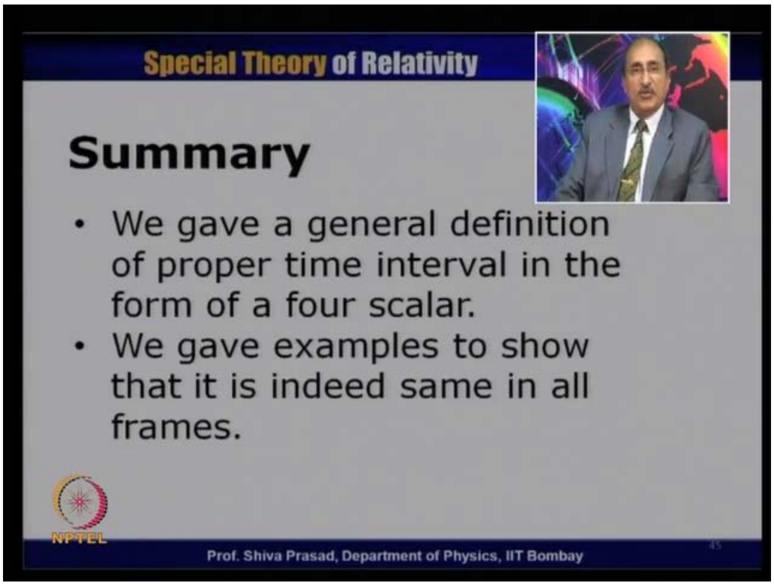


Now, in the end I was sort of summarize whatever we have discussed today. We have given a general definition of proper time interval in the form of a four scalar. Then we had given some examples and shown that it is indeed is a scalar. It means it does not change when I change my frame of reference.

Thank you.