Special Theory of Relativity
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Lecture - 12
Minkowski Space and Four Vectors

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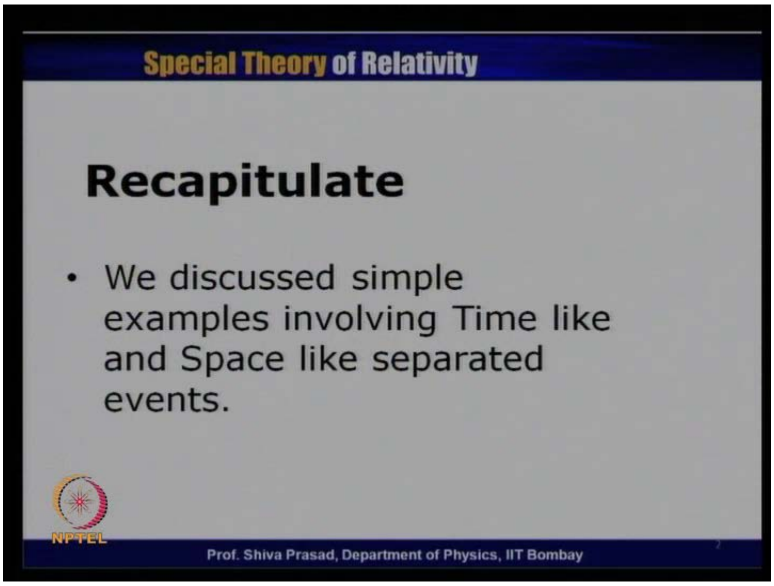

Let us start with the recapitulation of what we did in our last lecture. We discuss some simple examples involving time like, and space like separated events. And just discussed how in a particular frame condition, it is possible to find a frame. If it is a two events occurred the same time, but not at the same position or at the same position, but not at the same time.

After that we again took a simple example of a completely inelastic collision. This type of collisions are very well known in the classical mechanics, and showed that if a lot of transformation was the only requirement or only modification that we needed on the basis of the Einstein postulates. Then if momentum is conserved in a frame of reference, then it need not be conserved in another frame.

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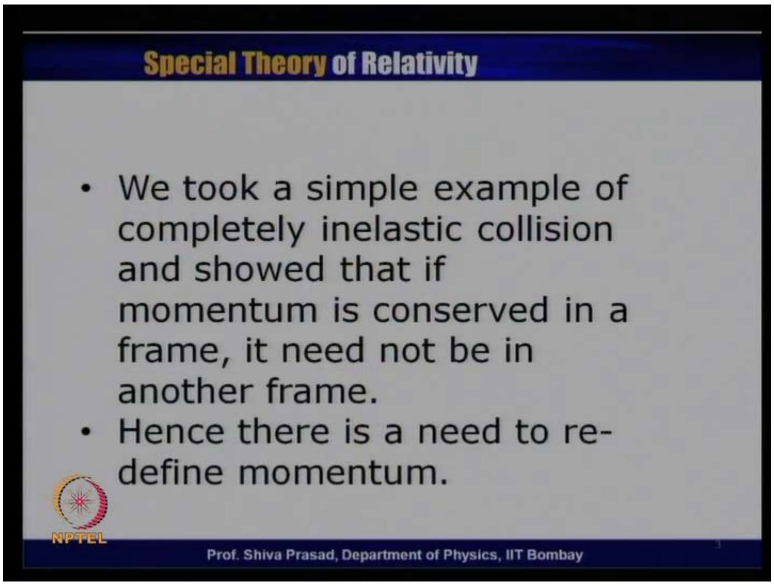

So, either we say that conservation of momentum is not a fundamental principle of physics, which I am sure many possible object or we need to redefine a momentum. So, now we will start looking for a particular way in which I can redefine momentum. There various ways in which one can get an idea of how to redefine the momentum. But what I will do? I will use the method of four vectors to evolve that particular concept. The idea of four vectors you know we introduce it, is that we will seen that the universality of the conservation of momentum would be assured. If we start following the way in which we work out the four vectors concept.

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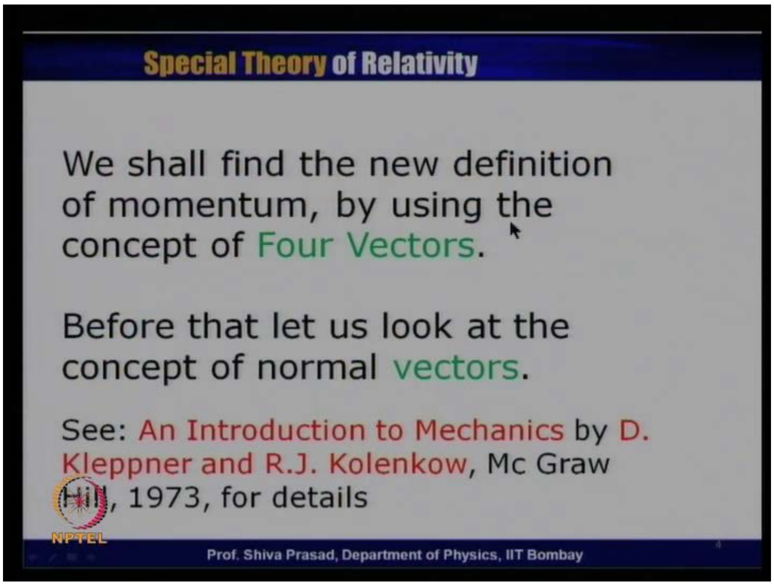

So, we shall as take a new definition by using the concept of four vectors. But before we do that, let us first took and discuss concept of normal traditional standard vectors which we have been learning right from our high school telling that the certain quantities which that are scalar the certain quantities which are vectors. So, let me first try to relook this particular aspect again and in a particular way. So, that I can lead this to the concept of four vectors, just to mention that the terminology etcetera, that I have followed for the particular concept of four vectors is what has been given in this particular book; the introduction of mechanics by Kleppner and Kolenkow.

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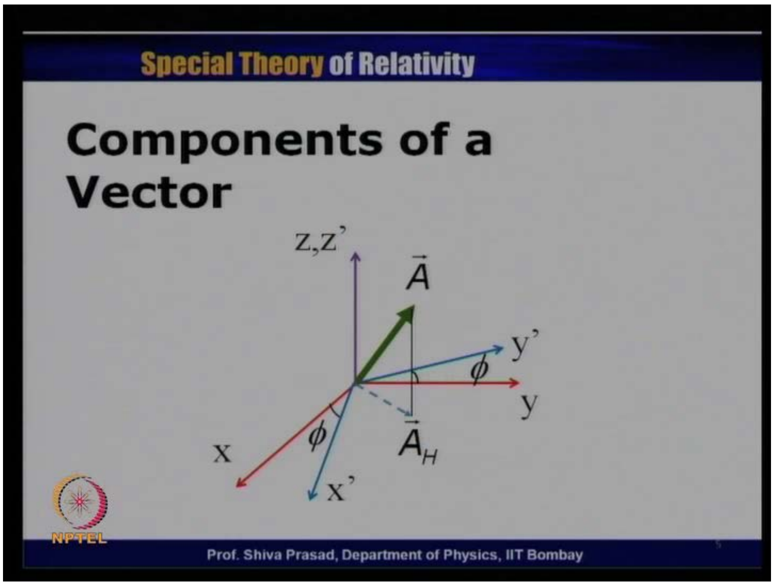

So, as I said, let us try to relook at the concept of traditional vectors. If you ask someone in high school what is a scalar and what is a vector? They would normally say that a vector has something which has both a magnitude and direction. So, it can be represented by something like an arrow was that many other things have to be associated with that. But there is the simple picture that every young student has in mind, while the certain quantity which do not have a direction associated with it.

For example, mass of a body, there is no direction associated with that upon there has a weight has something has direction, because weight is in terms of a force and that has a direction associated with this. So, normally we say that a vector can be represented by certain arrow which has the length of which represents the magnitude of that particular vector. And the way this particular arrow points that represents the direction of this particular vector.

So, what I have done in the particular picture I have drawn a vector which I am calling it as vector A. I have taken a particular set of axes which I am showing by this red part here. This is the x axis; this is the y axis and this is the z axis. Now I take another set of axes, let me remind you that I am not talking about the relativity. So, as of now so the concept of as soon as that we heard of relativity different that will bring in back later, but this let us take as totally independent example.

So, we have another set of vectors in which z axes same; this have x prime, y prime and z prime axes. While z prime and z coincides, the origin of the two sets also coincides. The only difference is that x prime is rotated with respect x axis by an angle of phi. Similarly y prime axes are rotated with respect to y by; obviously, the same angle phi. So, the only difference here is that this x prime and x they make an angle phi. Similarly, y and y prime make an angle phi as I said again z and z prime axis, axes are coinciding. Now, what I would like say, what I would like to represent by vectors, if I have these set of x y and z axes that I can always write my vector in the form of components.

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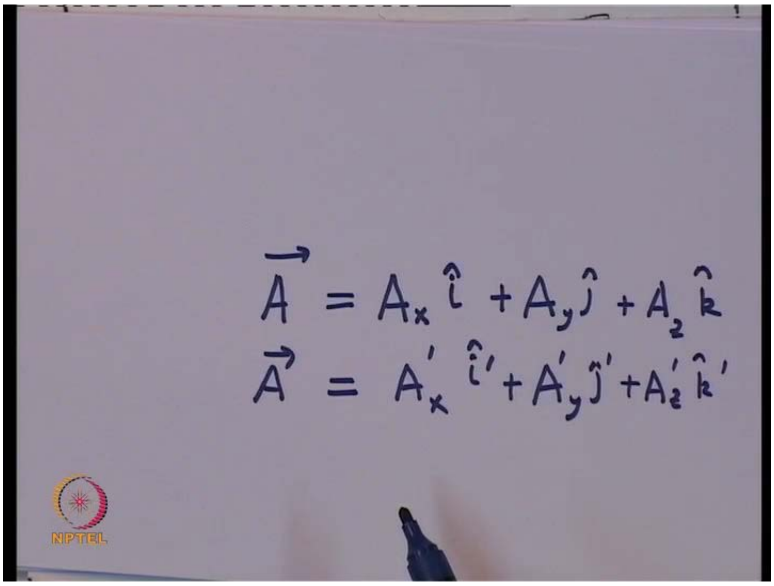

I can always write my vector A in terms of the components along with x y and z direction. I can write this as A x, and if I take a unit vector in the I direction. I can write this as A x i plus if A y is the component along the y direction. Then I will write this as

A j and if z is the component in the z direction. Then I can write this as A z k, where i j and k are unit vectors in the directions x y and z .This is the very standard way of writing a vector in terms of the components and the unit vector. Of course, in this case I have used the Cartesian coordinates system in which i j k are fixed direction along the x y and z axes.

Now, what is this particular figure represents that if phi does not use the set of x y and z axes. But decide to use x prime, y prime and z prime axes. Of course, as per the z component is concerned; this will remains same. But with respect to this x prime and y prime components, there these components would change, because now the direction of x prime is different from x. Similarly, the direction to the y prime is different from y therefore, this components which have earlier written would now change.

So, in principle the same vector to the vector is same which has the same magnitude, which points exactly the same direction, but depending upon which set of axes I decide to write the components, the components would change. So, same vector A which has written earlier could now be written as A x prime because this component has changed. Of course, this i would have also changed. So, I will write this is i prime, the y component has also changed I will write A y prime, j has also changed I will write j prime.

As for as z component is concerned it is not changed similarly, k is not changed for two little more general. Let me write this as still A z prime k prime where I will put a condition that A z is same as z prime and direction of k same as direction of k prime. So, same vector can be written in terms of two set A x Ay A z, and A x prime Ay prime and A z prime, which represents the components of this particular vector in the 3 directions that I have chosen to represent by vector.

So, I have chosen certain direction to represent my vector, and in those directions, if I take that component of these vector. Then I can write the vector in this particular way. Now, my question is that can I find, if I know A x Ay A z along with these set x y z. And if I know the angle phi, can I find out A x prime Ay prime A z prime. That is the components of the same vector along the x prime y prime and z prime direction. We can definite do that, it is not a very difficult problem, but anyway let us try to work it out.

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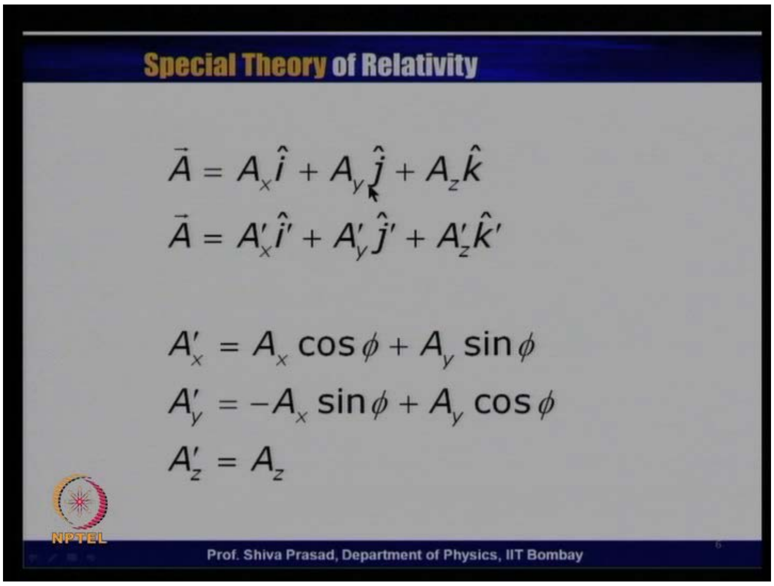

So, top thing I have written A is equal to A x i plus A y j plus A z k which have written also on the paper. Similarly, A is A x prime I prime plus Ay prime j prime plus A z prime k prime. Of course, we agree that A z is same as z prime and k is same as k prime. Now, what I insists that the relationship between A x prime, Ay prime, A z prime, and A x Ay A z is given by this particular question. This is not very difficult to you visualize. Let us look back into my whole figure, and try to see this is very very comparatively easy to derive this particular thing.

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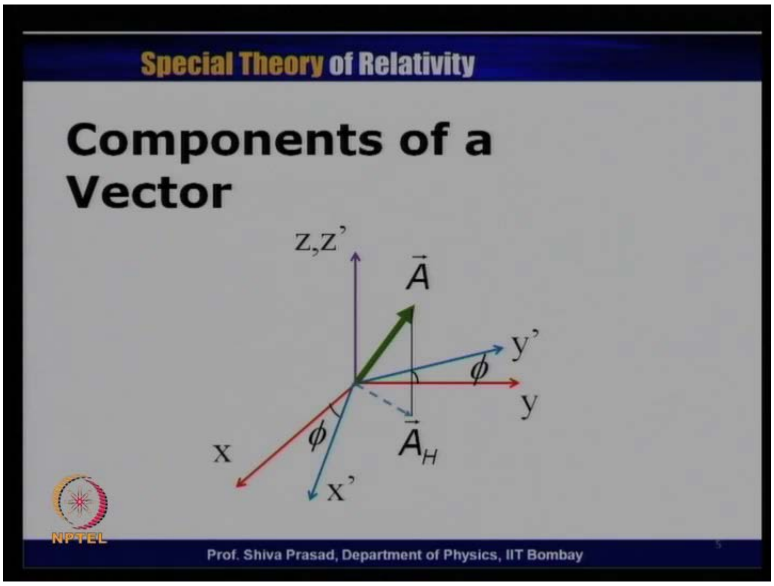

I know the component of that A along this particular direction which is the x direction. Say, as far as z is concern, it is the same solute not bother about it. Let us just look at this particular co dimensional picture.

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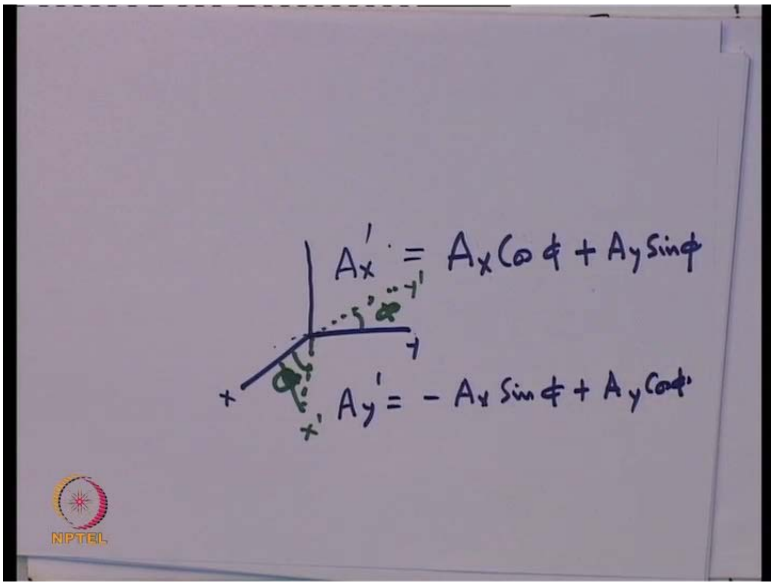

Let me write it here, this is one set of axes; this is another set of axes, which is rotated with respect to the first axes by an angle phi. This is also rotated with respect to this by the same angle phi. This is my x prime axes; this is my y prime axes, and this was my x axis, and this was by y axis. This factor the x component for somewhere here, have to find out my x prime component. See, if I take the projection of this particular component along the x prime direction I will get A x cos phi. Now, this is not the only component which gives the projection here. The y component will also give the projection here.

Now, if this is phi; this is 90 minus phi therefore, the projection of the y component on these x prime axes will be A y sin phi. So, this component will give the projection in this way. This particular component gives the projection this way. This component will give me A x cos phi. This component will give me A y sin phi. Of course, z axis is perpendicular to this particular plane or normal to this particular plane, it does not give any value component to this particular direction.

And any how we have discussed that as far as z component is concerned, this remains same. Whether I decide to take x y direction or whether I decide to take x prime, y prime direction. So, as we have seen that this A x cos phi plus Ay sin phi would be the actual component along the x prime direction. This I can write as A x prime. This is what I written here as A x prime is equal to A x cos phi plus A y sin phi. Now, let us look at the y component, if I take the y component which is y prime component rather. If I have to find out what is the component of this vector, this particular direction. I take the component along the x direction and the y direction.

Whatever is the component of this I have to take the projection on these y prime axes? So, whatever is the component which is A y, the projection of this will be A y cos phi. If I take x, the projection of that will be A x sin phi. But as we can say, that if I take the projection of this will actually project on the opposite side of it. So, the component of this particular part on this particular y prime direction would be in opposite direction of this particular component.

So, therefore, clearly A y prime would be given by minus A x sin phi plus A y cos phi. So, this is what I have written in this particular equation that A y prime is equal to minus A x sin phi plus A y cos phi. And of course, we had agreed that A z prime is equal to A z. So, what we have said that there is a vector depending on the set of axes that I have chosen to describe its components. The components may turn out to be different and their relationship would depend on the angle phi, and this is the relationship which I write here. This particular set of equations can be represented in terms of a matrix equation which I am presenting in my next transparency.

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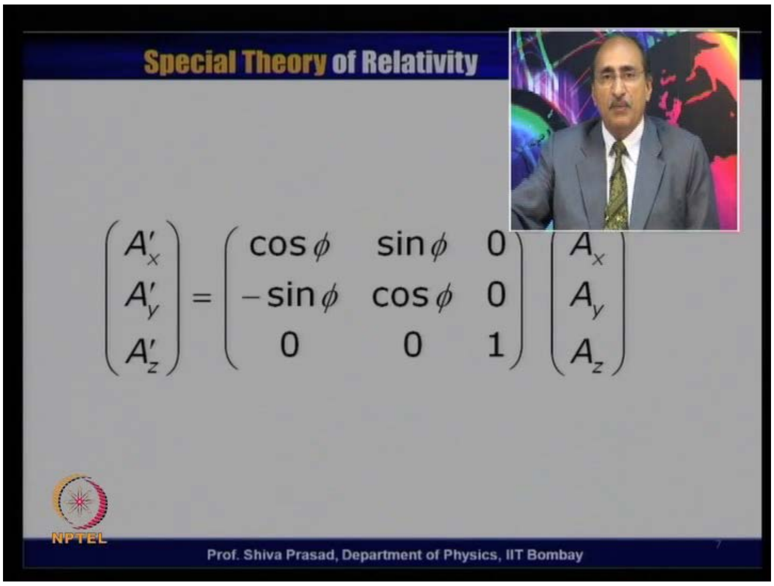

So, this is a matrix equation that I am writing, if I have open up the matrix I will all the 3 components, all the 3 equations which I have just now written in my previous transparency. This A x prime is equal to this multiplied by this; this multiplied by this plus this multiplied by this. So, this will give A x prime is equal to A x cos phi plus A y sin phi plus 0. This is the first equation. Second equation A y prime is equal to this multiplied by this plus this multiplied by this plus this multiplied by this.

So, this will become minus A x sin phi plus ay cos phi plus 0 which gives the second equation. Third equation; this multiplied by this plus this multiplied by this plus this multiplied by this, if I get A z prime is equal to A z. Therefore, this matrix equation gives me the set of all the 3 equations which I had it earlier. So, this single equation, single matrix equation represents how A x A y A z will transform once I rotate the axes by angle phi, keep inside axes same.

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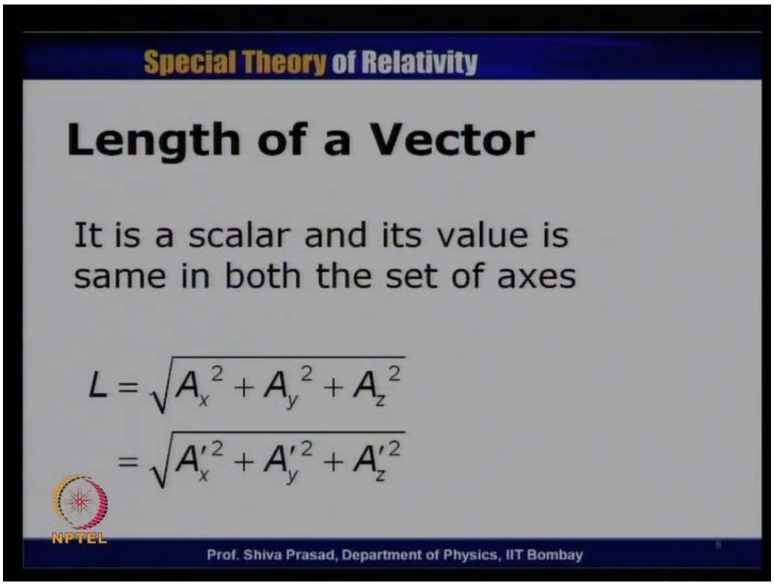

But one thing which I would like to mention that whatever we did, irrespective of why that have chosen my A x A y can become different they can be A x prime A y prime depending upon phi, their value this will be different. But all of you would agree that the length of vector will not change, their components may change irrespective depending up on the set of axes that I have decided to work with. But as per the length is concerned the length will remain same. Because length is a scalar, it will not depend on odds it has no components. So, irrespective of whatever set of axes it decide to describe my axes A x A y A z, but the length will remain same.

And, if you remember the length is defined as the magnitude or under root of A x square plus A y square plus A z square. So, this is what I call as a scalar that this length is a scalar. Because this will not change once I change my set of axes. So, this A x square under root A x square plus an A y square plus A z square must be same as the under the root of A x prime square plus A y prime square plus A z prime square. So, though the components change, but the length being a scalar does not change, once I change or else I rotate by set of axes. This is the key point which I would like to emphasize. This is probably very obvious, very simple, but never less as I said I want to put it in a way. So, that eventually we can generalize it into the form of the four vector. Let us take simple example very simple example as usually a high school example, but ideas will state my point.

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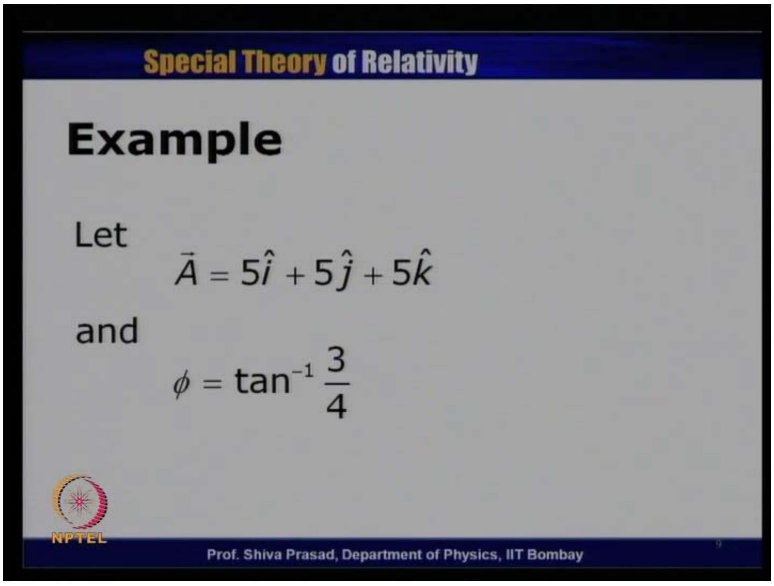

So, let us take one specific vector A as 5 i plus 5 j plus 5 k, some arbitrary vector which have chosen nothing very special about this numbers and; obviously, 5 5 5 are the components of this particular vector along x direction y direction and z direction. And I have taken just to make number simple, the angle phi as tan inverse 3 by 4. Now, let us assume that this set of axes has been rotated about z axes, the way I have described earlier by an angle phi which is given by this particular value which is tan inverse 3 by 4.

So; obviously, the same vector a would now be represented by different component different set of components, and that set of components I can find out by the transformation equation that I had just now written.

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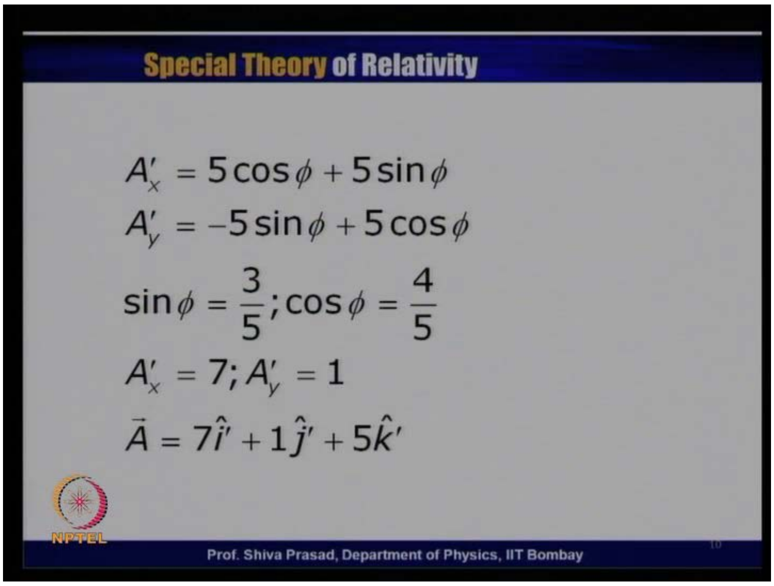

So, now, let us try to find out A x prime A y prime A z prime, using the transformation equation that we have just now described, just now written. So, my A x prime was equal to A x cos phi plus Ay sin phi. A x Ay both happen to be 5 in the example which I have given. So, this becomes A x prime is equal 5 cos phi plus 5 sin phi. Similarly A y prime will be given by minus A x sin phi plus A y cos phi. Again A x and A y being same as 5. So, A y prime becomes minus 5 sin phi plus 5 cos 5. we earlier written that time phi is equal to 3 by 4.

So, I can very easily find out what is sin phi and cos phi? And this sin phi will turn out to be 3 by 5 and cos phi will turn out to be 4 by 5. As you can see sin square phi plus cos square phi would equal to 1, 3 square plus 4 square equal to 5 square. And sin phi divided by cos phi is 3 by 4. So, these are the values of sin and cosine of phi I substitute these values in this particular equation. If I look at this top equation for cos phi I substitute 4 by 5. So, here, cos phi gets replaced by 4 by 5, 5 cancel. So, what remains here, in this particular factor is just 4. Here you have 5 sin phi, sin phi is 3 by 5, again this 5 cancels out. Here you left with 3.

So, this was 4 here plus 3 here. So, A y prime becomes equal to 7 whatever units we have described, let us not bother about the units. Now, A y prime is equal to minus 5 sin phi, sin phi was 3 by 5; this 5 cancels out. So, this becomes minus 3 plus 5 cos 5, cos 5. I substitute 4 by 5, 5 cancels out. So, what remain here is 4. So, this is minus 3 plus 4, what is left here is plus 1. So, A y prime is equal to plus 1. So, this is what I have written A x prime is equal to 7 A y prime equal to 1. And of course, A z prime is equal to A z which I have not specifically written.

So, it means A z prime will be equal to 5. So, the same vector now with respect to the rotated set of axes can be represented as 7 i prime plus one j prime plus 5 k primes. So, as we have seen that these set of numbers have changed. But as I have assisted that though these 3 sets of numbers have changed, their length would not change. And if you know the length is given by under root A x square plus A y square plus A z square.

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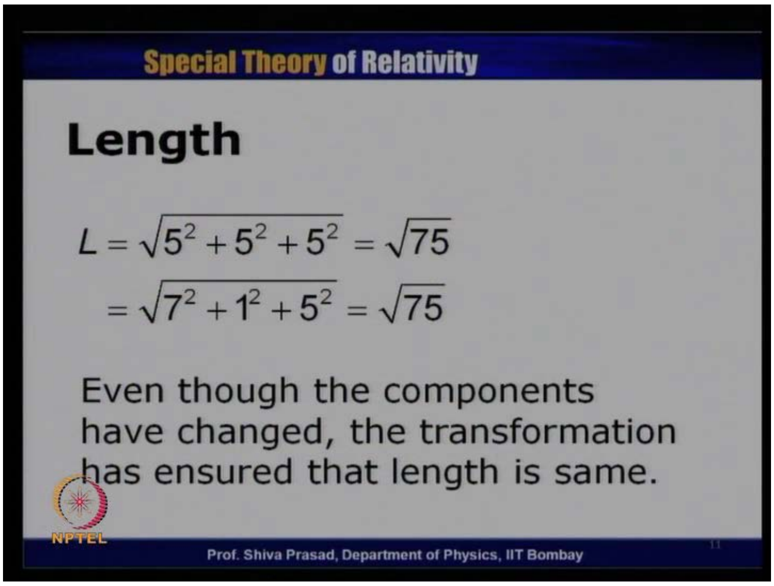

So, in the first case, when I had chosen x y and z set of axes? A x was 5, Ay was 5, A z was 5. So, the length will be 5 square plus 5 square plus 5 square; this becomes to 25 plus 25 plus 25 becomes equal to under root 75. In the second case, when I was presenting the components in the set of x prime, y prime, z prime. So, A x prime was 7, A y prime was 1, A z prime was 5. So, the length will be given by 7 square plus 1 square plus 5 square, under root 7 square is 49, 1 square is just1. So 49 plus 150 plus 25 under root 75.

So, as you have seen that the components have changed, but in a way as so as to maintain the length same, in fact my transformation as I showed that the length of the vector is not changed. We should not change because this is the vector, and its length is a scalar which cannot change, which set of axes you have chosen to describe your vectors components now, let us generalized this particular idea little more. Let us have 2 vectors instead of 1 vector. Let us suppose we have vector A and vector B, and I choose to describe these vectors A and B, interrupt of components of a set of axes i j k, just like before x y z which are unit vectors i j k.

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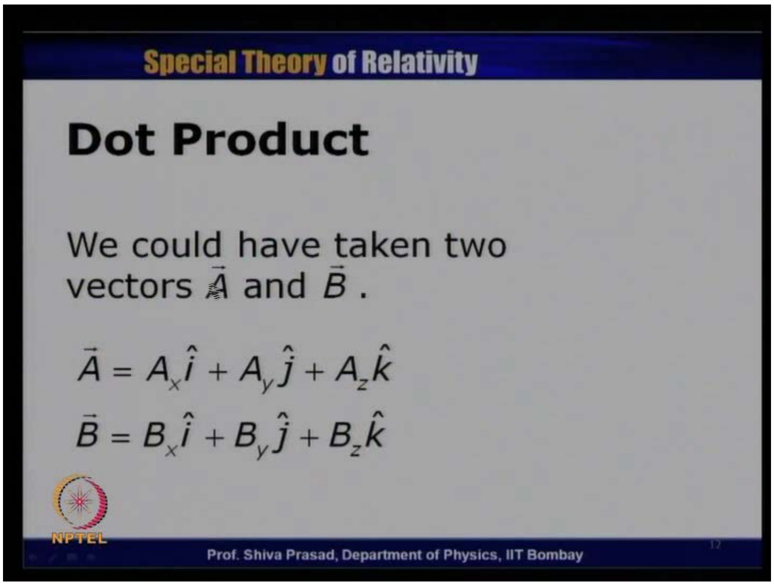

So, the 2 vectors A and B, exactly the way I have described can now be written as A x is equal to A x i plus A y j plus A z k. Now choosing the same set of axes the vector B can be written as B x i plus B y j plus B z k, very, very standard way of writing the vectors. Choose the set of axes, take the components, and write in terms of unit vectors along those particular directions x. We have been doing all those things essentially from my school.

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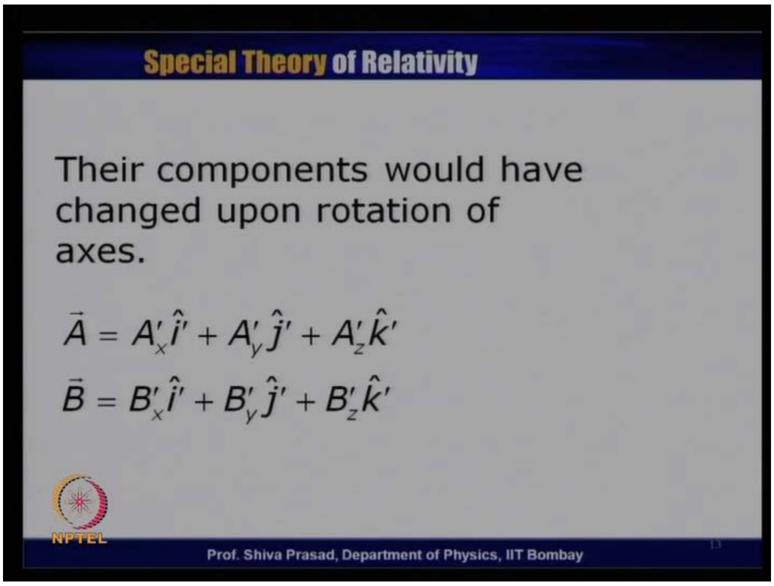

Now, instead of these axes, if I would have chosen to describe another set of axes which are x prime, y prime, z prime with unit vectors i prime, j prime, k prime, where this x prime, y prime, z prime axes are related to this x y and z axis. Exactly in the same way as I have described earlier, it means rotated with respective to z axes, then the components will change. And again I write the same vector a as A x prime i prime plus A y prime j prime plus A z prime k prime. Similarly B, B x prime i prime B y prime j prime B z prime k prime. Now, if I take a dot product of these 2 vectors I know the dot product is also the scalar. So, the components might have changed, once I have decided to represent the same vector with respect to the components of different set of axes. But their length would not change I mean the dot product would not change; the dot product would not change, because that is the scalar.

So, the transformation would ensure that the dot product will not change. Hence, we must have A x B x plus A y B y plus A z B z must turn out to be equal to A x prime B x prime plus A y prime B y prime plus A z prime B z prime. The transformation would ensure this problem can be worked out easily am sure that really happens.

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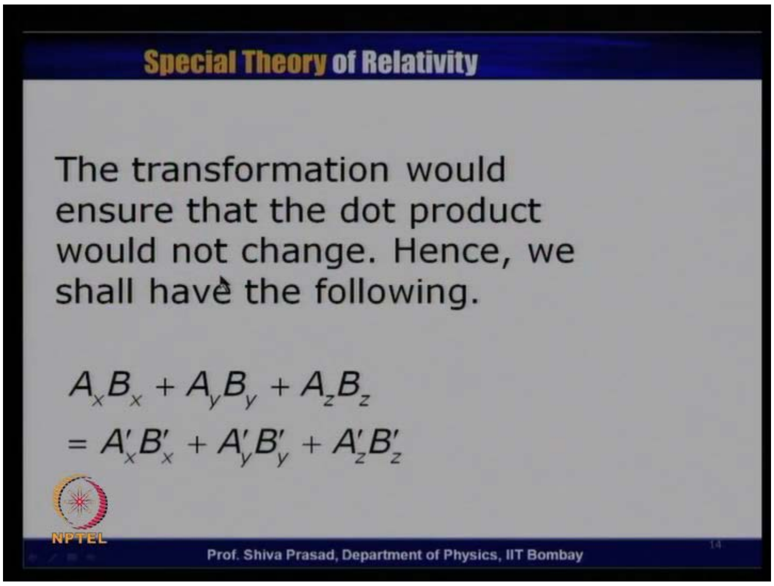

This has to be something is extremely; obvious, that they the dot product to the same vector can be represented as a set of components in a different fashion by if I choose different set of axes. But the length remains to be scalar which would not change. Similarly, the dot product of the 2 vectors could not change, because that happens to be a scalar quantity. Now, let us go little further talk about cross product. We know that the product of 2 vectors is defined in 2 different fashions. The dot product is a scalar, while the cross product is a vector.

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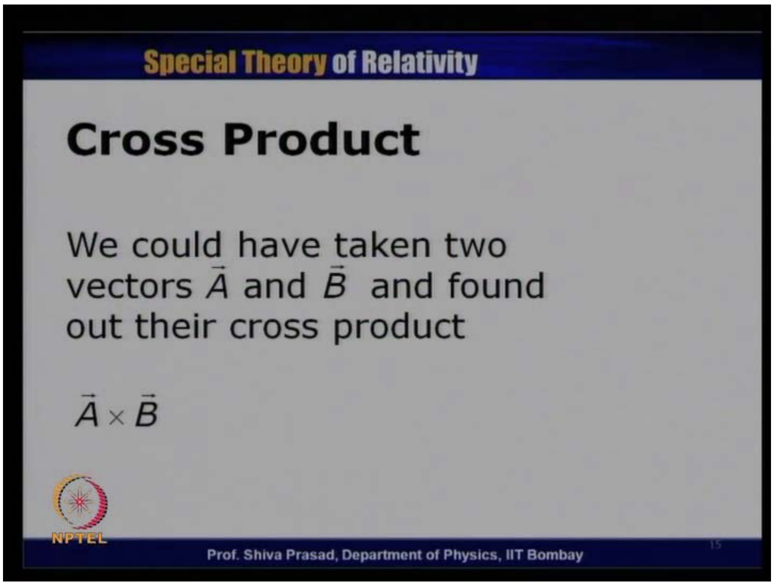

So, if I choose the same two vectors A and B, and take a cross product which is A cross B, this will turn out to be a vector quantity. Now I can again choose to describe A and B in terms of either x y z or x prime y prime z prime correspondingly I can write A cross B. But because A cross B happens to be vectors to be a vector. So, if I take the x y z components of A cross B. And take x prime y prime z prime components of A cross B. They will also follow the same relationship same transformation equation which I have described earlier.

In a nutshell what I am trying to say that if I have a set of these number. If they represent a vector once I change my x y z axes to x prime y prime z prime axes that we have described. Then they always form then they always follow the same transformation equation. On the other hand, if the quantity happens to be a scalar quantity, then this scalar quantity will always remains same irrespective of what set of axes I have chosen to describe my original vectors with.

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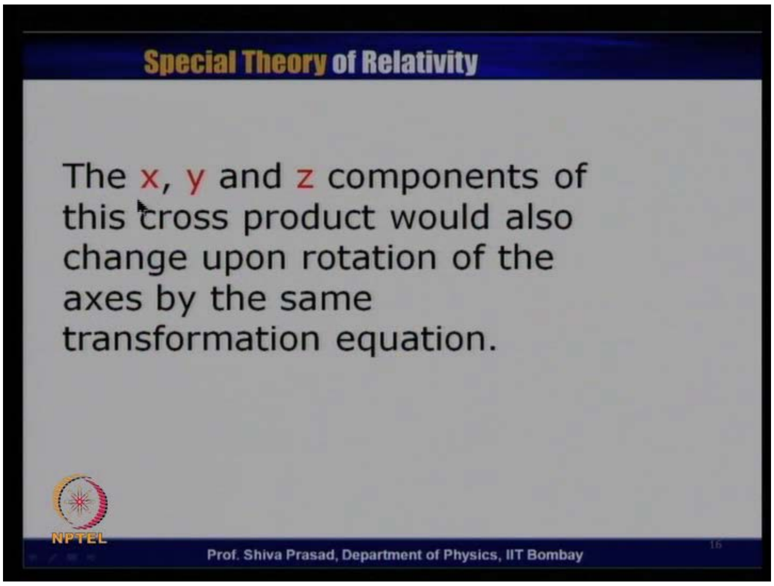

So, that is what I written the x y and z components of this cross product would also change upon rotation of the axes by the same transformation equation, which were used to change the components of vector A and vector B? So, rather holding my ear like that I will hold it like that, and define or describe vector in a different fashion.

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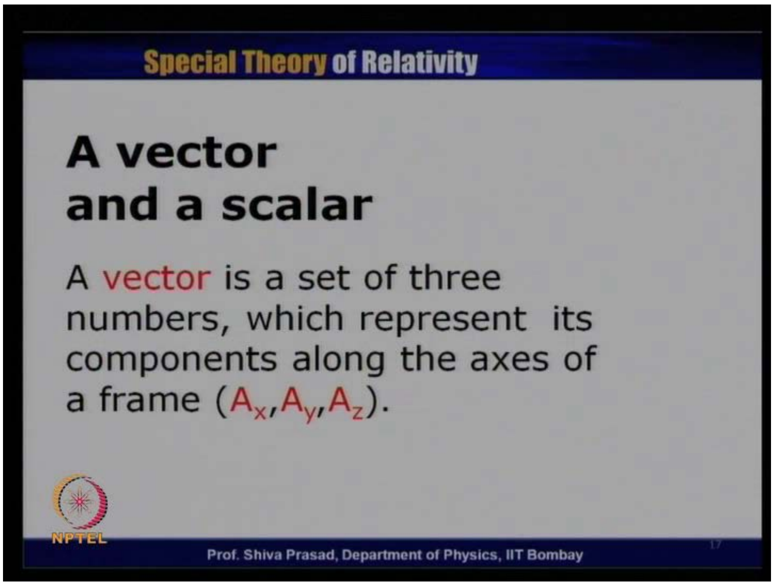

I will say a vector is a set of 3 numbers which represents its components along the axes of a given frame or axes of a different a particular set. And I will say that a vector is a set of 3 numbers A x Ay A z which I keep in my mind are the components of this particular vector along a given set of axes.

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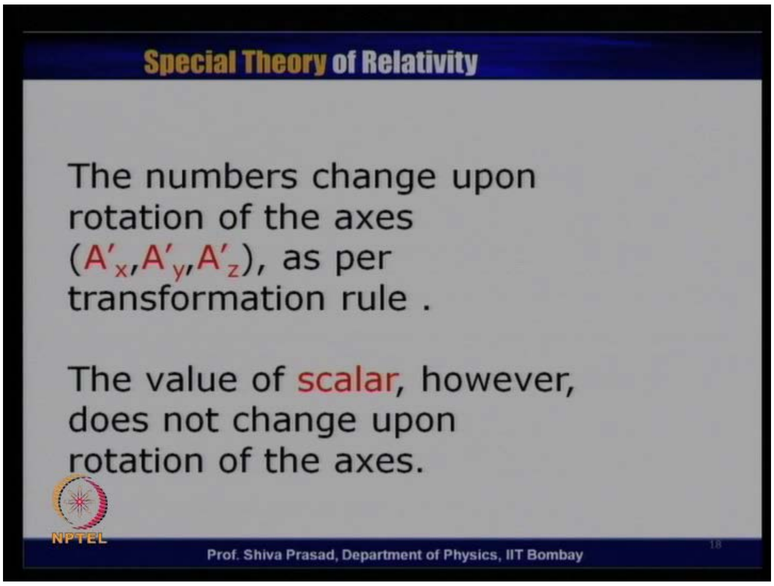

These numbers if I choose in different rotated set of axes would change to A x prime A y prime and A z prime. But will always obey the transformation rule which I have described earlier. However, if we have a scalar; this is scalar would not change upon the rotation of axes, this will remain same. I think we had enough introduction of the traditional vectors.

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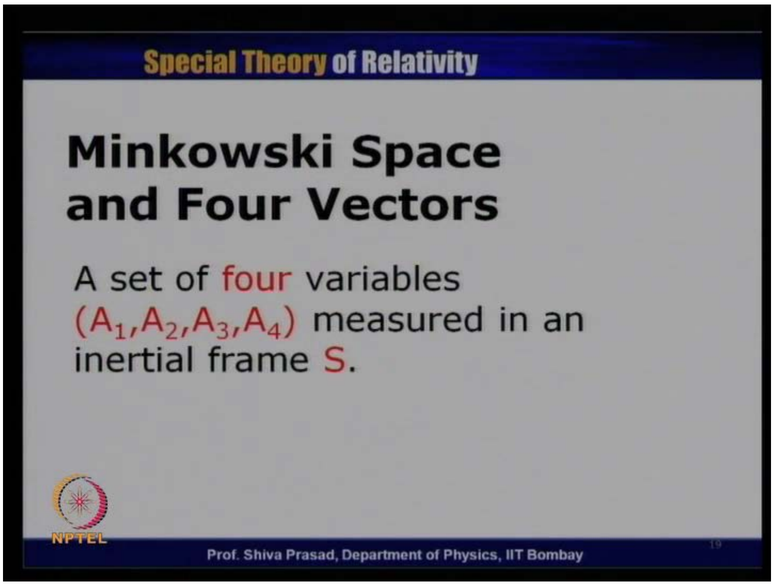

Now, let us try to extend this idea to A 4 dimensional space which is generally called Minkowski space. And in this particular space a vector is described by not 3 components, but 4 components. That is why it is called A 4 vector. So, as we have said a traditional vector can be described in terms of 3 components. A Minkowski four vector is described in terms of 4 components. So, let us assume that we have one particular four vectors and its components are 4 which are A 1, A 2, A 3, A 4.

Like, when we say vector is a set of 3 numbers A x Ay A z. A 4 vector has its a set of 4 numbers A 1, A 2, A 3, A 4. We are talking something which is little more abstract, and these components are measured in a given frame of reference as I come back to my relativity, and as I as prime are described exactly the same way we have described earlier in Lorentz transformation. So, we still have the same S and S prime as we have described now in Lorentz transformation I come back to that is equation. So, A 1, A 2, A 3, A 4 as 4 components or something are certain 4 variables which have been measured in S.

Then a set of these numbers, 4 numbers would we called A 4 vector provided they satisfy certain transformation equation. Just take the parallel with what we have done in the case of vectors. I described in a vector 3 numbers which obey a certain transformation equation when I rotate the axes. Now I say A 4 vector is a set of 4 numbers which transform, when I go for my frame S to S prime as described in Lorentz transformation. It means there is a relative velocity between S and S prime and all those things. Then of course, this number change, but they will change in a way which is described by a particular transformation matrix. So, we have a set of 4 numbers, 4 variables A 1, A 2, A 3, A 4 which are measured in an inertial frame S.

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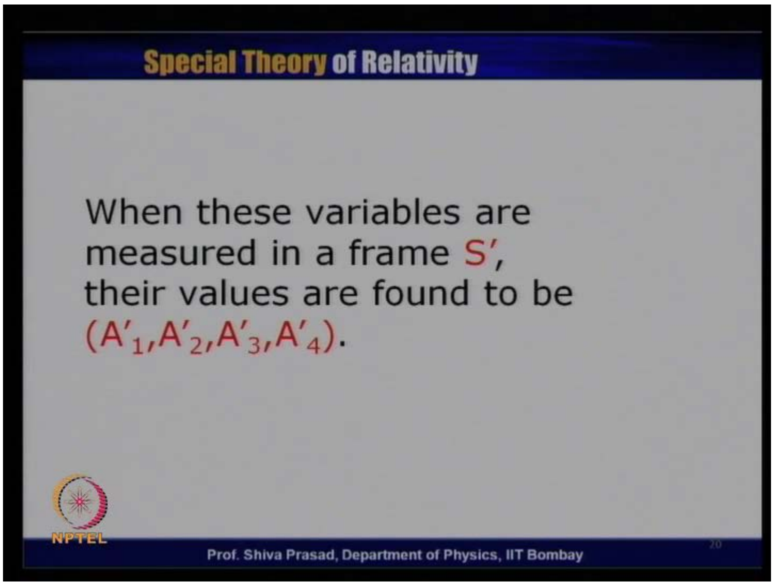

When these variables are measured in a different frame S prime, their values would in general change, like in the case of traditional vectors when I rotated these set of axes the component change. Similarly, if I choose to describe my vector in a different frame as prime frame, these quantities A 1 prime A 2 prime A 3 prime A 4 prime would change. So, same vector is now represented by different numbers A 1 prime A 2 prime A 3 prime A 4 prime which is now measured in S prime frame earlier they were be measured as S frame.

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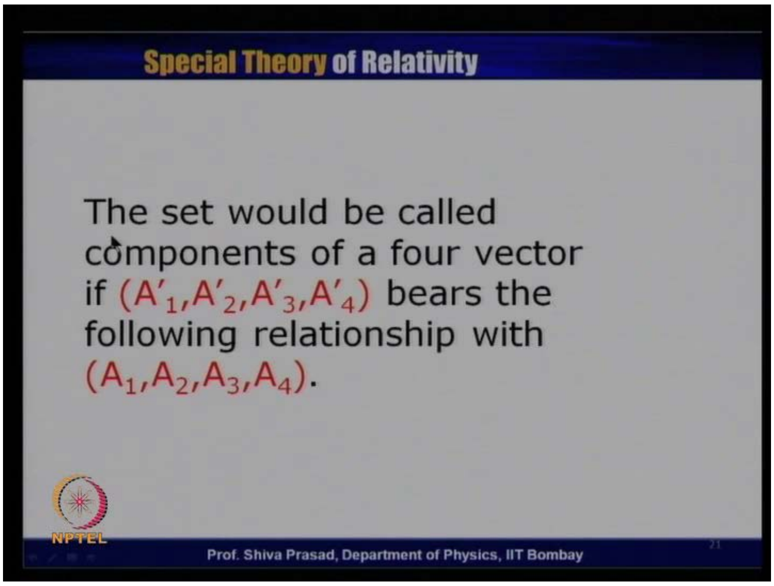

This set of numbers either you want to call A 1, A 2, A 3, A4 or A 1 prime A 2 prime A 3 prime A 4 prime would be called components of A 4 vector. If A 1 prime A 2 prime A 3 prime A 4 prime bears the following relationship with A 1, A 2, A 3, A 4. And this relationship like I have described in case of traditional vectors by A 3 by 3 matrix. Now I describe this in terms of by 4 by 4 matrixes which is given in the next transparency.

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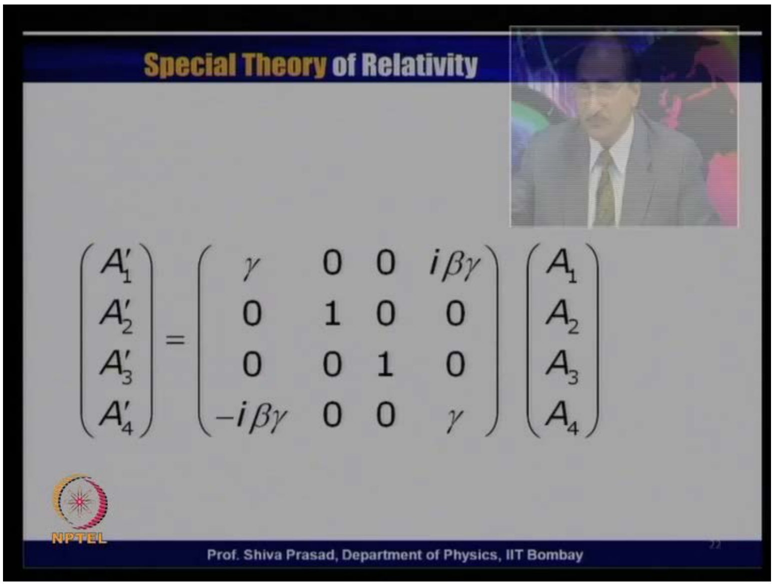

So, this is my transformation equation I have set of 4 variables A 1 A 2 A 3 A 4 which are measured in frame S. I measure the values of the same quantities in a different S prime I get the value A 1 prime A 2 prime A 3 prime A 4 prime, if these 4 numbers are related to these 4 numbers or other variables given by this particular matrix 4 by 4 matrix. Then I will call these 4 set as forming A 4 vector or a Minkowski vector. Of course, I can open it; will open it just now to when I go it ahead. But this is I want to assert is a definition of A 4 vector we just is a set of the 4 numbers like a traditional vector can be described as a set of 3 numbers. Here we are talking of the set of 4 numbers.

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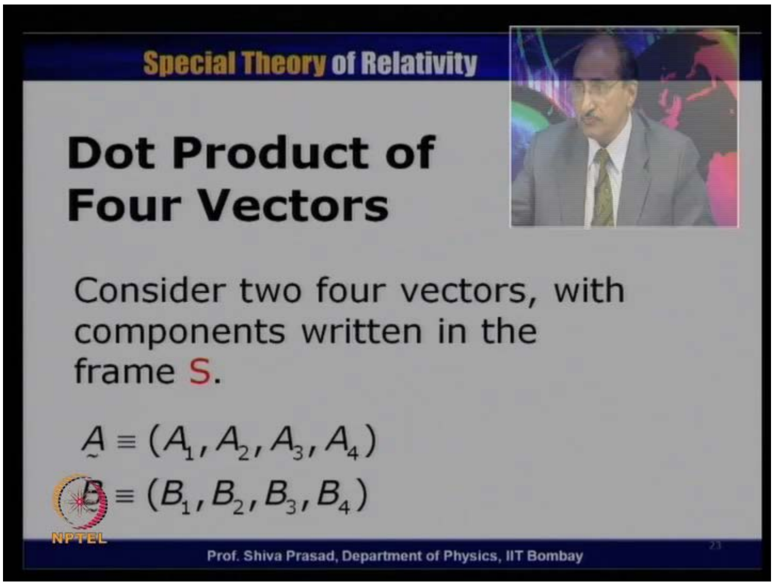

Now, like I have described the dot product of 3 vector I can also define a dot product of 4 vectors. So, now consider two four vectors, this is not a very standard way of writing it, But we have just written that below A there is a curve sort of thing, sort of v v thing, just to describe this is A 4 vector. So, this 4 vector A has 4 components A 1, A 2, A 3, A 4. Take another 4 vector which has also component B 1, B 2, B 3, B 4. Both are 4 vectors it means both obey exactly the transformation equation, if I choose to describe these components in a different frame S prime. Now I define the dot product of these two vectors exactly in the same way as we have describing the tradition, just extending to the fourth dimension.

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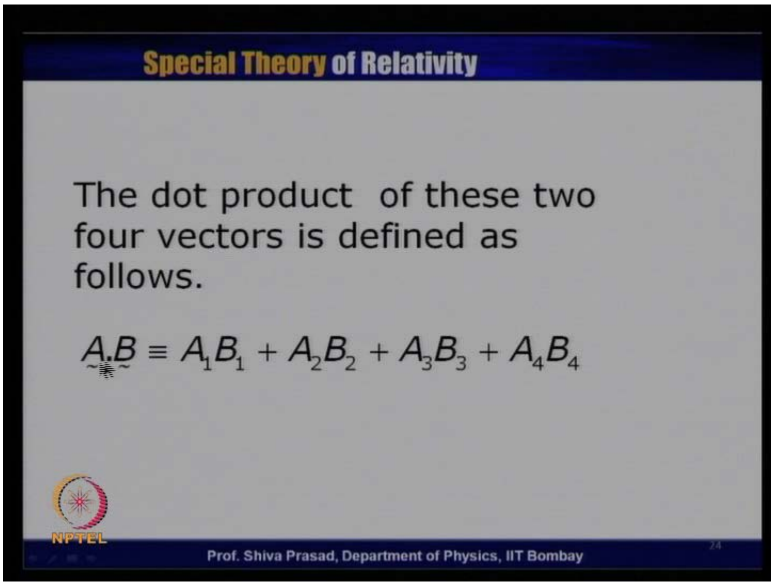

So, dot product I will define as A dot B as A 1 B 1 plus A 2 B 2 plus A 3 B 3 plus A 4 B 4. Remember in case of traditional vectors, create just 3 component and for A x B x plus A y B y plus A z B z. Here we have 4 components, we write this as A 1 B 1 plus A 2 B 2 plus A 3 B 3 plus A 4 B 4. This is the way I define the dot product of two four vectors.

And what I want to assert is that the transformation equation that I have described earlier for the 4 vector would ensure that this dot product would not change if I change from S to S prime frame. Like the dot product of two traditional vectors did not change upon the rotation of the axes, that number turn out to be same I would now wish to show that this dot product remains same. Even if I change my frame of reference from S to S prime provided these vectors transform the way I have written my equation.

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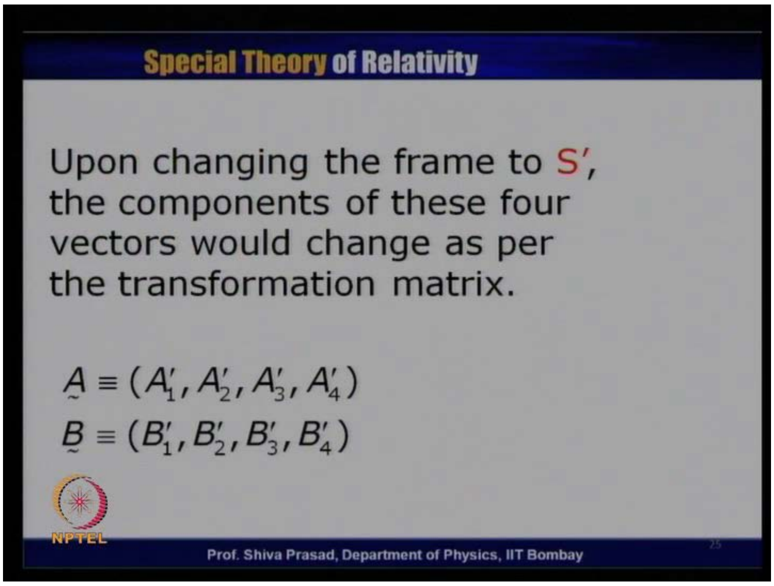

So, I have written that upon changing the frame to S prime. The components of these 4 vectors would change as per the transformation matrix. And now, you have a different set of 4 members. So, same 4 vector A would not be described as A 1 prime A 2 prime A 3 prime A 4 prime. Exactly the way when I rotated the axes the component of the vectors becomes different. Similarly, 4 vector B could not be described in terms of 4 different numbers B 1 prime B 2 prime B 3 prime B 4 prime.

But now, we shall show that the dot product of these 4 vector would not change it means I would definitely get A 1 B 1 plus A 2 B 2 plus A 3 B 3 plus A 4 B 4 equal to A 1 prime B 1 prime plus A 2 prime B 2 prime plus A 3 prime B 3 prime plus A 4 prime B 4 prime. Because this happens to be what we call as A 4 scalar, and a scalar quantity even though components may change. But that particular scalar quantity their magnitude would not change, that is what the crux of the idea? See after all this looks little more bit more abstract.

But let us realize know, why we at all evolve the concepts of vectors to make our life simple. So, when you introduce a concept of vector dot product, cross product, initially they all looks surprising. But then we realize how much physics had become simple. See for example, if I have to define a force in Lorentz force I just write v cross once I know want is my cross product I know magnitude direction everything I do not have to describe anything more. So, it is much easier to describe my physics with respect to standard cross product. Similarly, it becomes much simpler to describe the relativity, if in terms of we write things in terms of four vectors. That is the reason we have introduce this particular concepts of 4 vectors.

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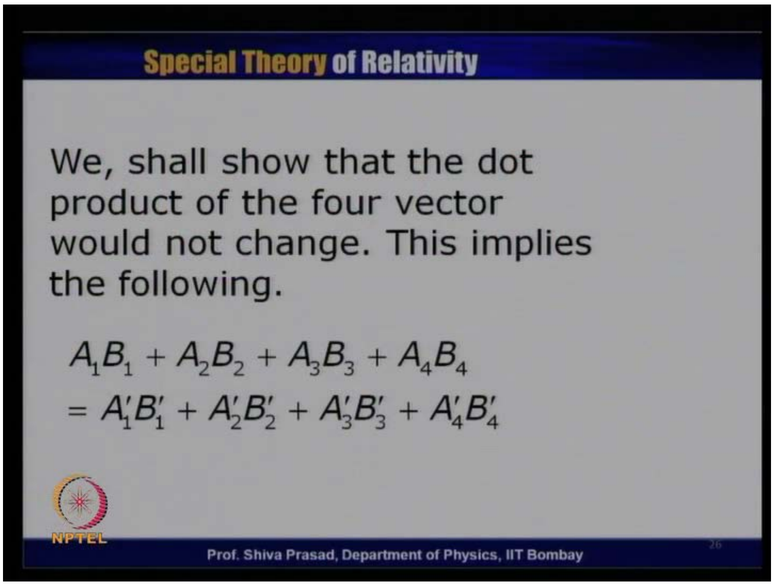

Now, let me try to show that this particular identity is maintain that is A 1 B 1 plus A 2 B 2 plus A 3 B 3 plus A 4 B 4 is actually equal to A 1 prime B 1 prime plus A 2 prime B 2 prime plus A 3 prime B 3 prime plus A 4 prime B 4 prime is a small amount of mathematics.

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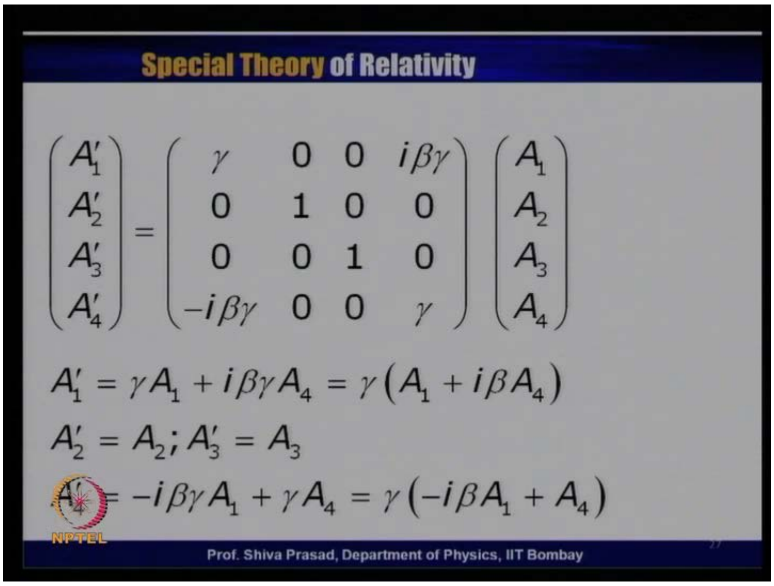

Let us just bear. This was my original transformation matrix. Let me open it up I will give 4 equations out of it, because this is 4 by 4 matrix. First equation is A 1 prime is equal to gamma times A 1 plus 0 times A 2 plus 0 times A 3 plus i beta gamma times A 4. Of course, I want must mention the beta and gamma are same, which we have describe in terms of relativity. That is beta is equal to v by c, and gamma is equal to 1 upon root 1 minus v square by c square. We know about, we have used this beta and gamma in number of times. A 2 prime is equal to 0 times A 1 plus one time A 2 plus 0 times A 3 plus 0 times A 4. Similarly, A 4 prime is equal to minus i beta gamma times A 1 plus 0 times A 2 plus 0 times A 3 plus gamma times A 4 is a standard way of opening a matrix of multiplying a matrix.

So, I have to multiple this matrix, then I equate each component. So, my A 1 prime will become gamma A1 plus i beta times gamma A 4. A 2 prime will become equal to A 2, A 3 prime will become equal to A 3, A 4 prime must be equal to minus i beta gamma A 1 plus gamma times A 4. All I have done is taken out of this. So, this gamma can be written as A 1 plus i beta A 4, just taken this gamma out of it. Similarly I take this gamma out. So, this will become gamma in bracket minus i beta A 1 plus A 4. So, this is the way A 1 prime would be obtained; this is the way A 4 prime would be obtained, if I know A 1 and A 4. Of course, A 2 and A 3 are simple, because they just happen to equal to A 2 prime and A 3 prime. Now, let us these values in the dot product that I have defined earlier.

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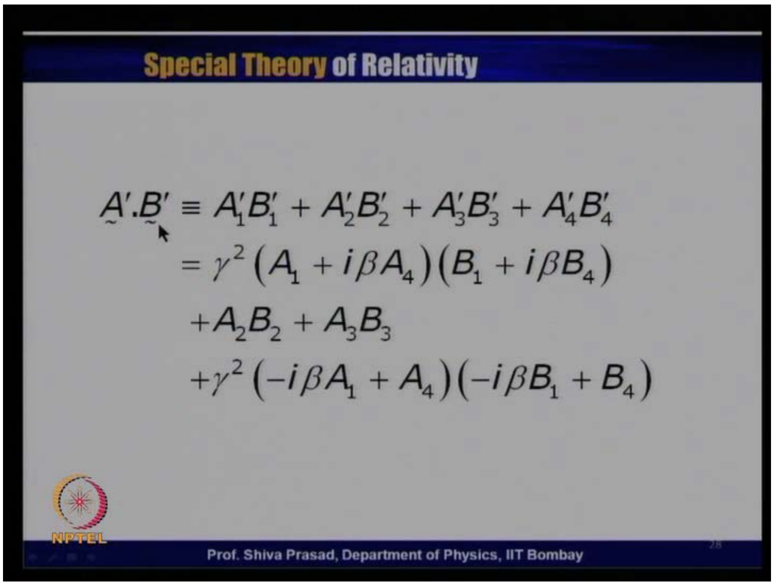

So, I start with A dot A prime dot B prime, and this by the definition is written by this quantity. So, what I do I substitute for A 1 prime and B 1 prime? Write this in terms of A 1 and B 1. We just now seen that A 1 prime is gamma times A 1 plus i beta A 4. Similarly, B 1 prime will be gamma times B 1 plus i beta times B 4. The two gamma gets multiplied to get gamma square, what is remaining here is A 1 plus i beta A 4 bracket multiplied by B 1 plus i beta times B 4.

About A 2 prime same as A 2, B 2 prime same as B 2. So, this remains just A 2 B 2. Similarly, A 3 prime is just A 3, B 3 prime is just B 3. So, this remains A 3 B 3. Just look at A 4 prime. A 4 prime, earlier we have written as gamma times minus i beta A 1 plus A 4. Similarly B 4 prime is written as minus i beta times B 1 plus B 4, could the 2 gammas multiplied I get gamma square. So, this is the way I have written after that these steps are simple. I just multiplied this two I just multiple this two, and show that this will turn out to be equal to A dot B.

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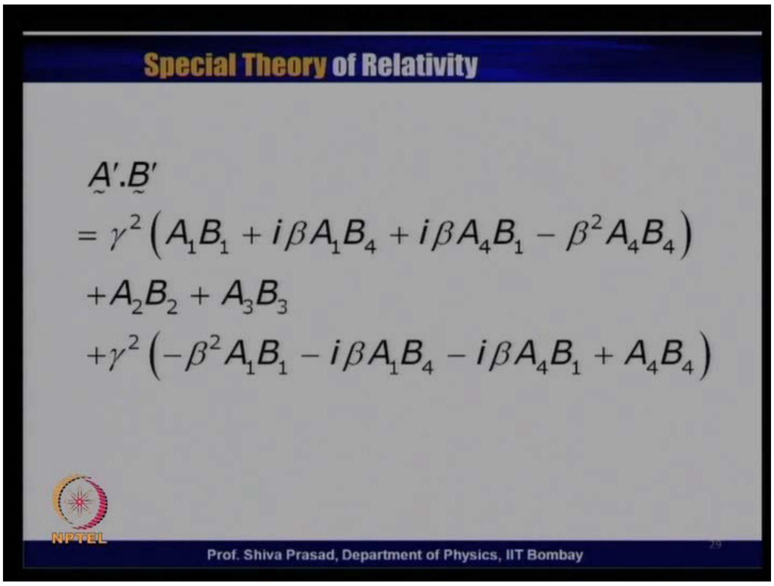

If I just open it here, this is what I will get. Let me try to write it here. We had gamma square A 1 plus i beta A 4. This was multiplied by B 1 plus I beta times B 4. We have gamma square, if I multiply this; I get A 1 B 1, multiply this I get i beta A 1 B 4, multiply this by this I get i beta A 4 B 1. Multiply these two i square becomes minus 1 minus beta square A 4 B 4. So, this is what I have written here A 1 B 1 plus i beta A 1 B 4 plus i beta times A 4 B 1 minus beta square A 4 B 4. This quantity is exactly identical, similarly I can open the other two other bracket also which is again very, very simple, exactly the same way. And I can write it in this particular fashion or in starting collecting term.

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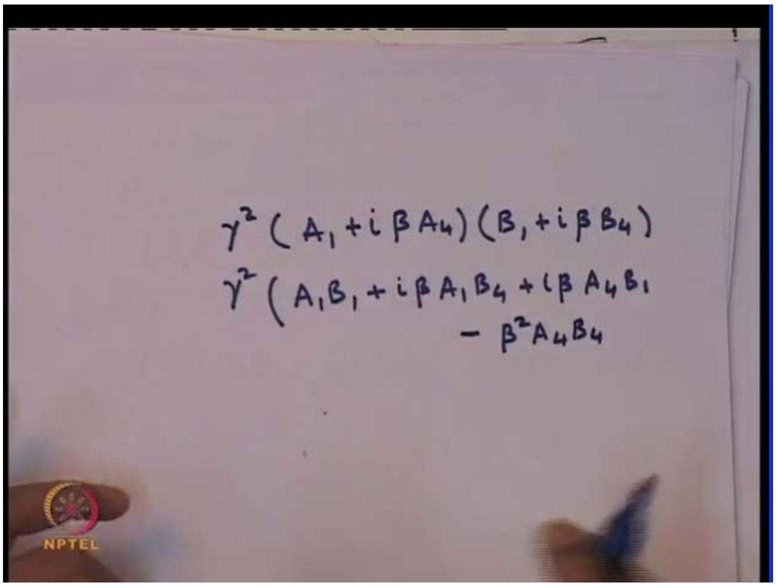

I will take A 1 B 1 there is an A 1 B 1 here. See, if I take A 1 B 1 out, this will have 1 minus beta square. I realize that these quantities i beta A 1 B 4 there is a minus i beta A 1 B 4; this will cancel with this. There is i beta A 4 B 1 and there is minus i beta A 4 B 1 this will also cancel with this. Then you are left with A 4 B 4 and there is minus beta square A 4 B 4. So, if I take A 4 B 4 common again I will get in bracket 1 minus beta square. This is what I have written here, same A dot A prime dot B prime will be gamma square. This gamma square anywhere out here.

So, this is gamma square A 1 B 1 multiplied by 1 minus beta square plus A 4 B 4 multiplied by 1 minus beta square, all other terms cancelled out. And of course, plus you have A 2 B 2 plus A 3 B 3, all other terms involving A 1 and B 4 and all those things they have all cancelled out.

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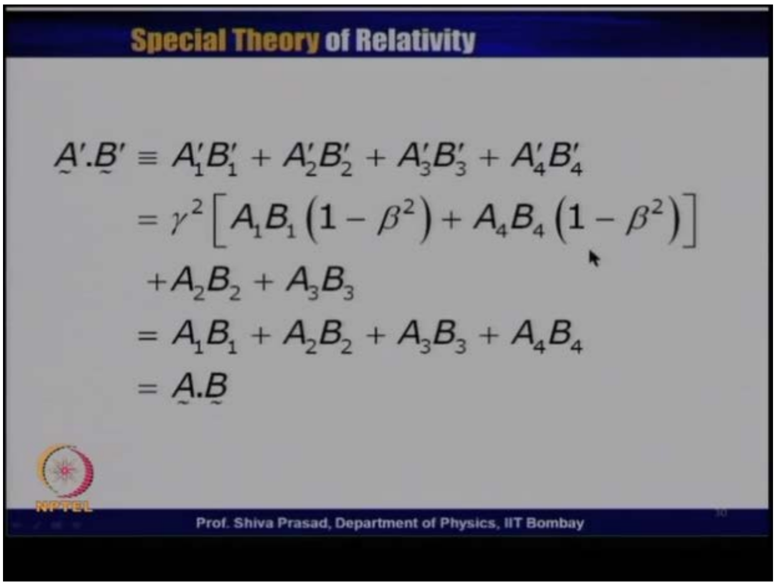

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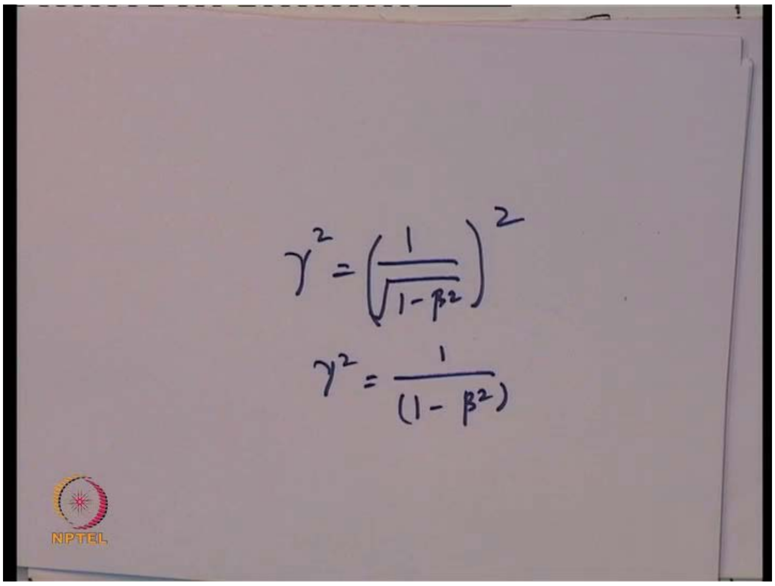
e 

Now, you realize that gamma square is equal to 1 minus 1 minus beta square whole square. So, gamma square is equal to 1 upon 1 minus beta square standard beta was equal to v by c as we know. So, here we have gamma square for which I can write as 1 upon 1 minus beta square. So, if I come back to this particular transparency. If I write 1 minus 1 divided by 1 minus beta square, 1 minus beta square will cancel here; 1 minus beta square will cancel here.

And this equation simply becomes A 1 B 1 plus A 2 B 2 plus A 3 B 3 plus A 4 B 4 which is nothing but the dot product of A dot B. So, I have shown that using this transformation equation A 1 prime dot B 1 prime; A prime dot B prime is equal to A dot B. So, though the components have change, but they were always changes in such a fashion. So, as to make the dot product same the transformation equation assures this particular thing that the dot product is a scalar is a four scalar. So, it does not change its value.

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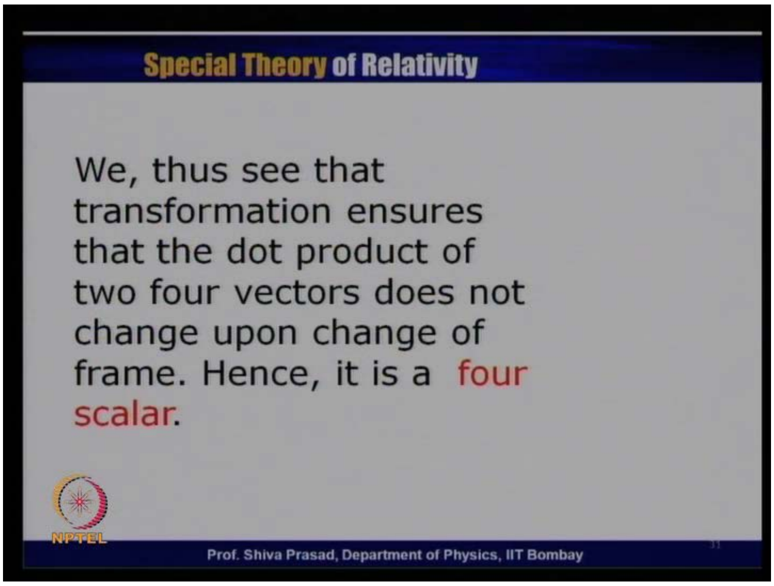

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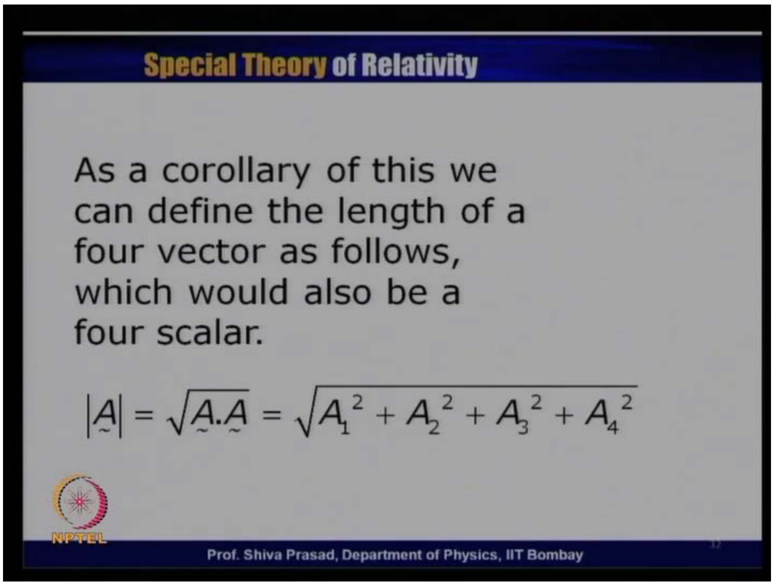

So, we, thus c that the transformation ensures showed that the dot product of two four vectors does not change upon the change of the frame. Hence, it is what we call a four scalar. As the corollary of this I need not take a dot product of A with B. I can take the dot product of A with own self I can write A dot A. And can define a length of the four vector like we describe the length of the traditional vector. So, if I take the length of a traditional four vector or the length of the four vector. This I can define as under root of A dot a it means dot product of this vector with its own self. And as we have seen from the definition I can write this as under root of A 1 square plus A 2 square plus A 3 square plus A 4 square.

And of course, this ensures that the length of the four vector would also not change once I change my frame of reference. So, if I go from S to S prime, and if A 1 A 2 A 3 A 4 happen to be to the component of the four vector. Then A 1 square plus A 2 square plus A 3 square plus A 4 square under root will not change, if I change my frame of reference. So, this becomes sort of the universal quantity that by changing the frame of reference, this will not change. We have talked so much of in abstractness. Let me come closer to something which we are familiar with it is the standard Lorentz transformation.

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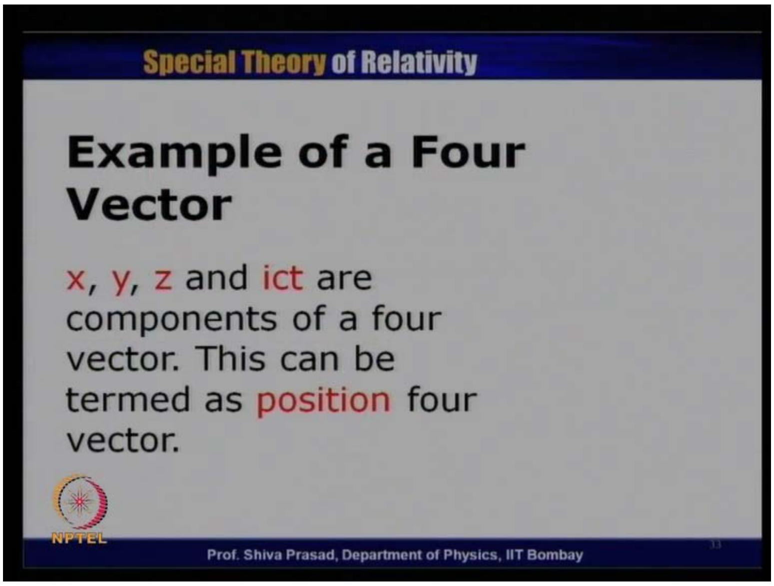

Now, I have given example of a four vector. I call first I show that x, y, z and ict, where I is imaginary number under root minus 1. They will form the component of a four vector. This is what I call it as position of a four vector x y z ict.

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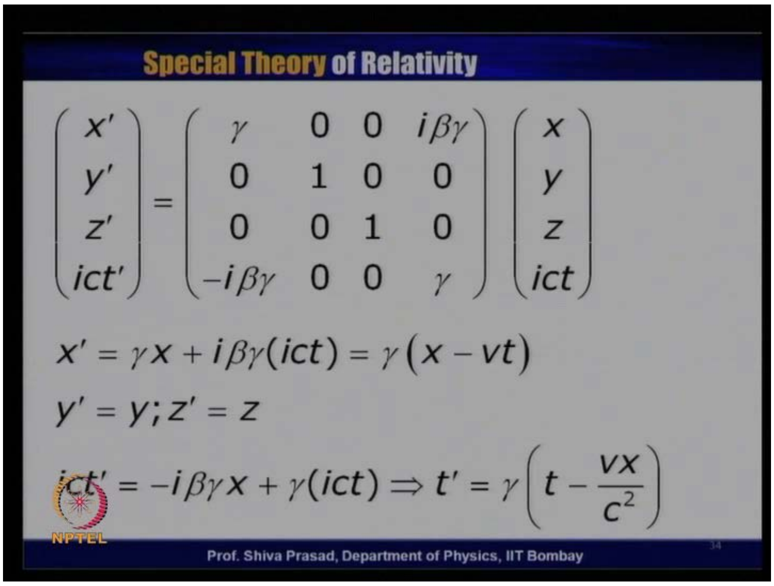

If they have to far a four vector, then if I change my frame of reference from S to S prime. The way I have described in the Lorentz transformation x will change to x prime, y will change to y prime, z will change to z prime, t may change to t prime, And the new 4 components, new components of the 4 vector will be given my x prime y prime z prime ict prime. And they must obey this transformation equation, if they really are components of four vector. Let just try and test. So, x prime must be equal to gamma times x, these are zeros plus i beta gamma times ict. This is why I have written gamma x plus i beta gamma times ict i square becomes minus 1, beta is v by c gamma I can take out; this v upon c, c would cancel it.

This you will just get it as gamma x minus v t which is the first equation of the Lorentz transformation. I know I really know that knowing x and t, this is the way x prime going to transform when I change my frame form S to S prime. Of course, y prime these are all zeros, y prime is equal to y, z prime is equal to z which are also parts of the transformation, Lorentz transformation equation. Let us look at the fourth equation, ict prime should be equal to minus i beta gamma times x plus these are zeros gamma times ict.

So, I have written gamma time’s ict, gamma I take common I take it out. This is minus i; this is will cancel with this i. There is c here; this c also I would like to cancel here. I canceled out here; this c if I cancel; this c will go away. Then this beta was equal to v by

c. There is no c here in the numerator; there is no c in the numerator here. So, if I have take c out and cancel it. I have to divided it by c this beta was equal to v by c.

So, this will become v by c square, with this negative sign this v remaining in the numerator. This will become minus v x by c square, ic I have already cancelled, gamma is taken out. So, this becomes t. So, this equation will lead to be t prime is equal to gamma t minus v x by c square, which is the fourth equation corresponding to the transformation of time in Lorentz transformation. So, I know that these equations are correct, because these equations have been described by Lorentz transformation. Hence I know that x y z and ict have to transform when I change my frame of reference by this transformation equation. Therefore, x y z ict are the components of four vector.

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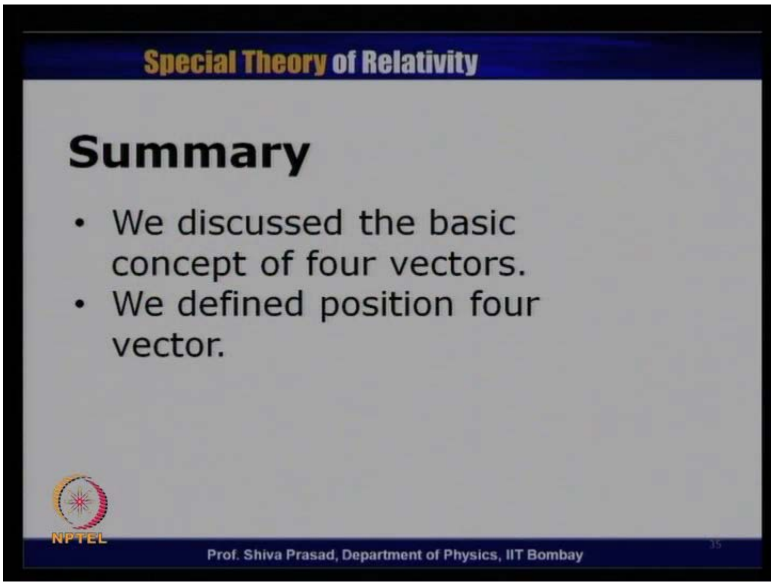

So, then I will summarize whatever I have discussed. We have discussed the basic concept of four vector, as I generalized from the definition of the standard traditional 3 vectors. Then finally, I described the position four vector. We will go ahead later evolving the concept of four vectors to other quantities.

Thank you.