Special Theory of Relativity
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Lecture - 11
Problem involving Causality and Need to Redefine Momentum

Hello, in our last lecture, we had described time like and space like events, we had discussed that, what are the basis on which it was postulated that space greater than speed of light, would not be allowed in special theory of relativity. So, let us quickly recapitulate, we had given an example of pool events and we had said that if delta x is the coordinate difference between these two events, and delta t is the time difference between these two events.

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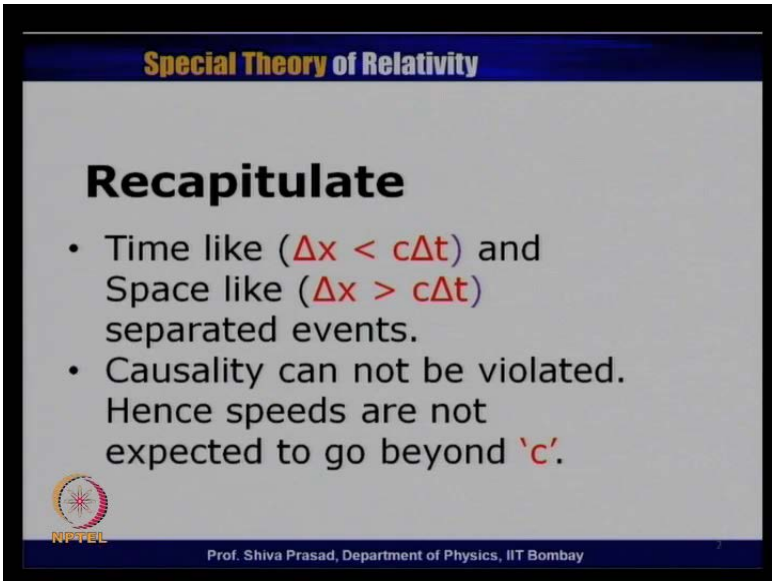

Assuming delta x and delta t to be positive then if delta x turns out to be smaller than c delta t, we call these events as time like separated events. Just to remember, when we say, time like, it is time factor delta t, which is large. Similarly, if delta x turns out to be larger than c delta t then we call these events as space like separated events, when we say space like, delta x is suppose to be larger, just to remember.

And we had mentioned that, in case we want the causality not to be violated it means, if in a frame of reference two events occur, the second event is an out come of the first

event. First event is the cause of the second event, then you would like that in any other frame of reference, we should always have the same order of the events. We cannot see in any other frame that whatever has whatever is the result or whatever is the out come has come first and the reason for that is coming later.

They have given an example that suppose, I shoot somebody then in my frame of reference, shooting of the gun, if it is event number 1 and that person being hit by the bullet is event number 2. Obviously, event number 2 has to occur after event number 1 then we should not be able to find out a frame of reference, in which it so happens that, that persons observes that, the person is hit by the bullet first and the bullet is shot later.

So, the same order of events must be maintained, if first event is the cause of the second event so this is what, we called causality. So, we do not expect from the physics point of view that, causality should be violated and that is possible only, if we do not allow space greater than speed of light, so that is what, is the physics reason for this particular conclusion.

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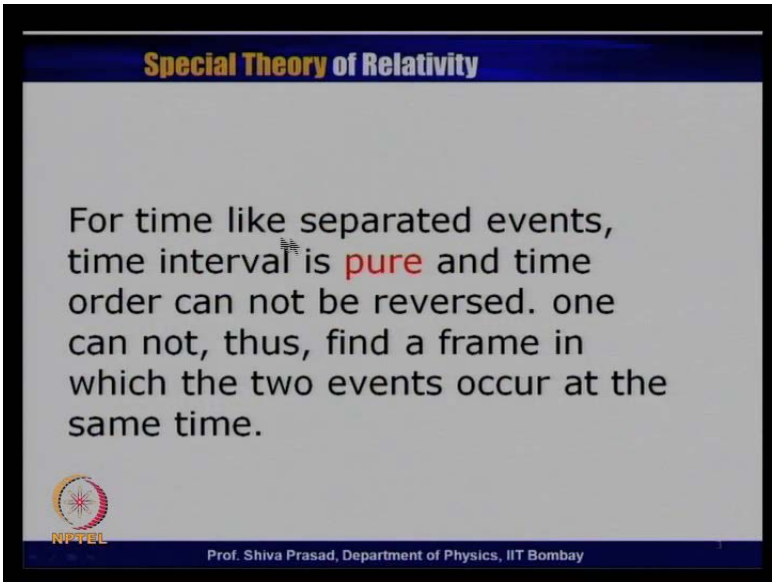

Just to mention again, I am writing here, for time like separated events, time interval is pure and time order cannot be reversed of course, if it means that, time order cannot be reversed it means that, we cannot find out a frame of reference, in which delta t 0. It means, we cannot find a frame of reference, in which the two events occur at the same time.

Similarly, for space like separated events, space interval is pure and position order cannot be reversed by again the same logic, if it has to be reversed it means that, we cannot also not find out the frame of reference, in which these events occur at the same place. So, that is what, I have written, for space like separated events, space interval is pure and position order cannot be reversed, one cannot thus find a frame, in which the two events occur at the same place.

So, this is what, we had discussed essentially in our last lecture, the most general behavior of these space like and time like, they will be discussing little later. So, let us postpone this particular discussion, until we evolve certain other concepts, at the moment what I would like to give, is a few examples of space like and time like events and the cause and outcome related events, just to make our ideas clear. So, let us go to one or two very, very simple examples.

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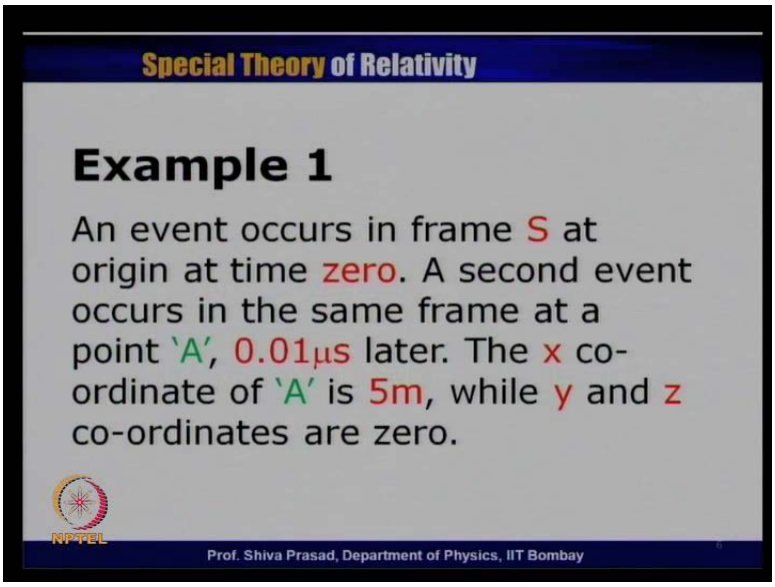

Let me take the example number 1, let us assume that an event occurs in a frame of reference S, at origin at time zero and event it could be for example, lightning, striking or anything, whatever we can imagine of. So, one event occurs in a frame S at origin, origin means x is equal to 0, y is equal to 0, z is equal to 0 and I also assume that, this occurred at time t is equal to 0.

Again the same frame S, a second event is formed to be observed, which is at a point A and the time difference, it means it occurs at 0.01 micro second later, micro is 10 to

power minus 6 second. So, it is 1 into 10 to power minus 8 seconds later, another event is found to be observed at point A, not at the origin and the x coordinate of A has been given as 5 meters, while y and z coordinates are same.

So, we have two events, one occurring at x is equal to 0, another occurring at x is equal to 5, the first event occurring at time t is equal to 0 and the second event occurring at time t is equal to 0.01 micro second.

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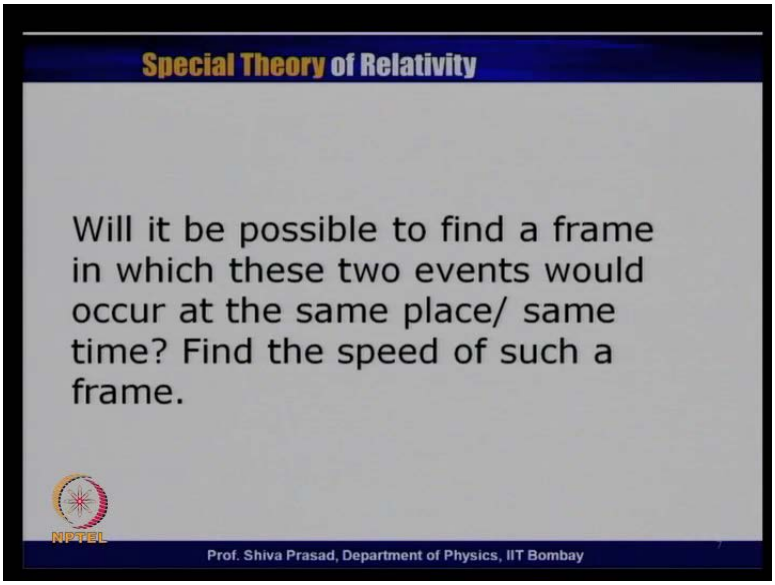

The question is, will it be possible to find a frame, in which these two events would occur at the same place slash same time, find the speed of such a frames. So, first we have to resolve, whether it should be possible to find a frame of reference, in which these two events occur at the same place or at same time or whatever it is. And once, we find out then we have to find out the speed of that particular frame of reference, in which these events either occur at the same place or at same time, depending upon the situation.

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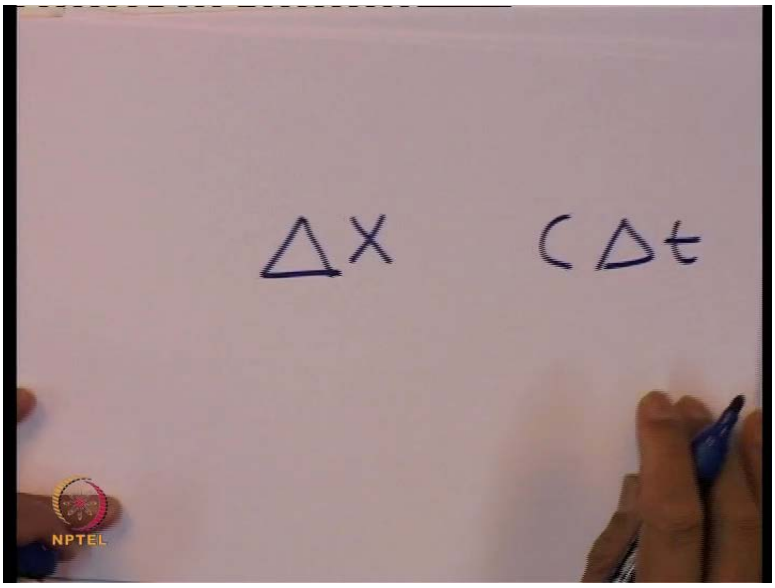

The problem is essentially simple, all we have to do is, to take delta x and compare it with c delta t. If delta x happens to be larger than c delta t, then we know that, it is a space like event, if it happens to be smaller than c delta t then we know that, it is a time like separated events. Then accordingly, we can conclude, whether it is possible to find a frame of reference, in which these two events occur at the same position or at same time so we have given everything so let us look at this particular thing.

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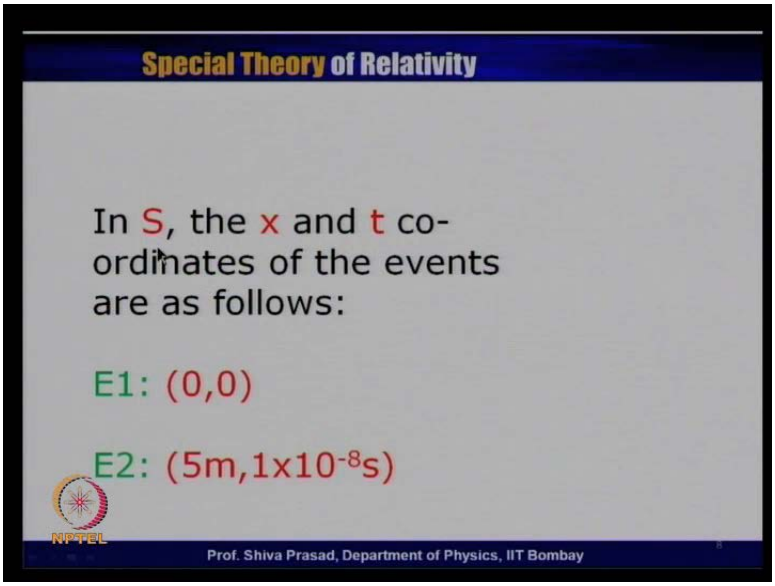

In S, the x and t co-ordinates of the events, I have just listed, I have removed y and z reference of y and z coordinates because that is not really material as far as this particular problem is concerned. Because, in both the events, y and z's are 0 so with a event number 1, which occurs at x is equal to 0 and t is equal to 0, S has been given in the question where, event number 2, which occurs at point A, which is at a distance of 5 meters along the x direction from the origin.

So, x co-ordinate is 5 meters and as we have mentioned that, it occurs at a time 0.01 micro second it means, the time was 1 into 10 to power minus 8 second. So, we have written by event table, my event number 1 is occurring at 0 0 and event number 2 is occurring at 5 meters and 1 into 10 to power minus 8 second.

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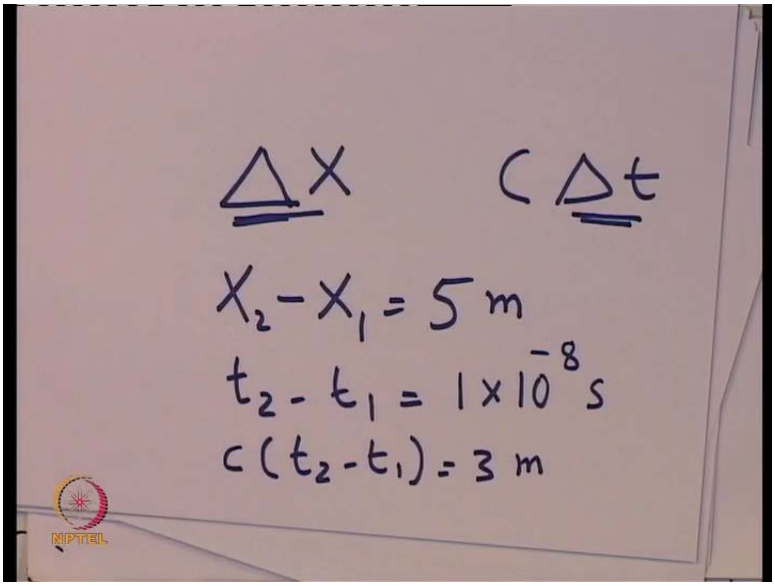

Now, all I have to do is, to find out this delta x then we have to find out this delta t then multiply by c and compare the two, that is all I have to do. Anyway, it is very easy to see that, for the first event x was equal to 0, for the second event x was 5 meters so; obviously, delta x is 5 meters, x 2 minus x 1 is 5 meters. Similarly, t 2 minus t 1, t 1 occurred at time t is equal to 0 I mean, event 1 occurred at time t is equal to 0, event number 2 occurred at time 0.01 micro second so t 2 minus t 1 there will be 1 into 10 to power minus 8 second.

If I am multiply this by c, to compare with this and if I take c is equal to 3 into 10 to power 8, will be 3, 10 to power 8 will cancel out 10 power minus 8 so this will become 3

meters. So, as I see very clearly that, delta x is 5 meters while c delta t is 3 meters that is what, I have written in this particular transparency. Delta x is equal to 5 meters, delta t is equal to 1 into 10 to power minus 8 second, c delta t is therefore 3 meters therefore, delta x is greater than c delta t it means, these events are separated space like.

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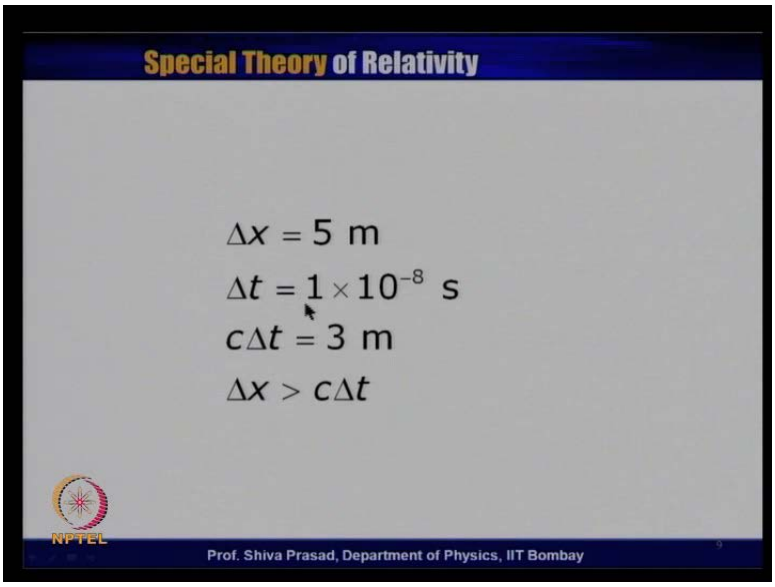

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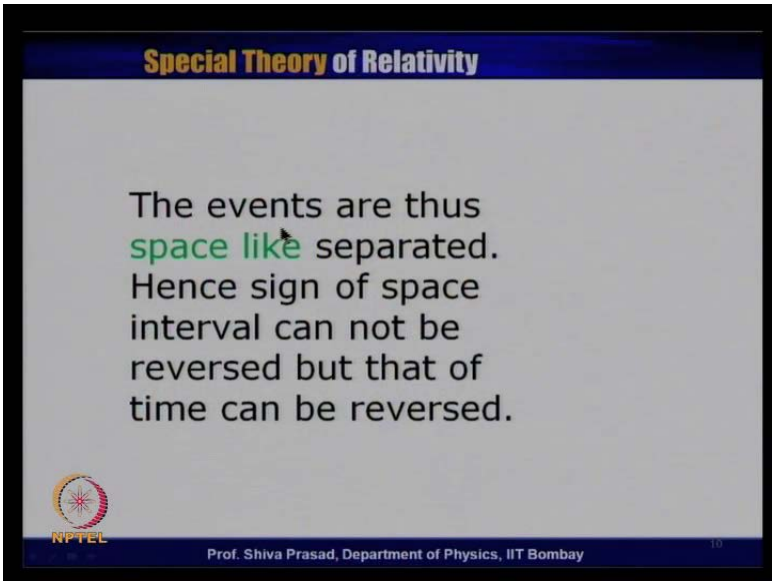

Because, delta x is turning out to be larger than c delta t and if the events are separated space like, it is not possible reverse the position order of the events it means, it is not possible to find a frame of reference, in which these two events will occur at the same

position. Though it is possible to find out the frame of reference, in which these two events will occur at the same time that is, you see and verify whatever we are telling. This what I have mentioned just now, the events are space like separated hence, sign of space interval cannot be reversed but that of time can reversed. How do we find this particular thing, just use Lorentz transformation.

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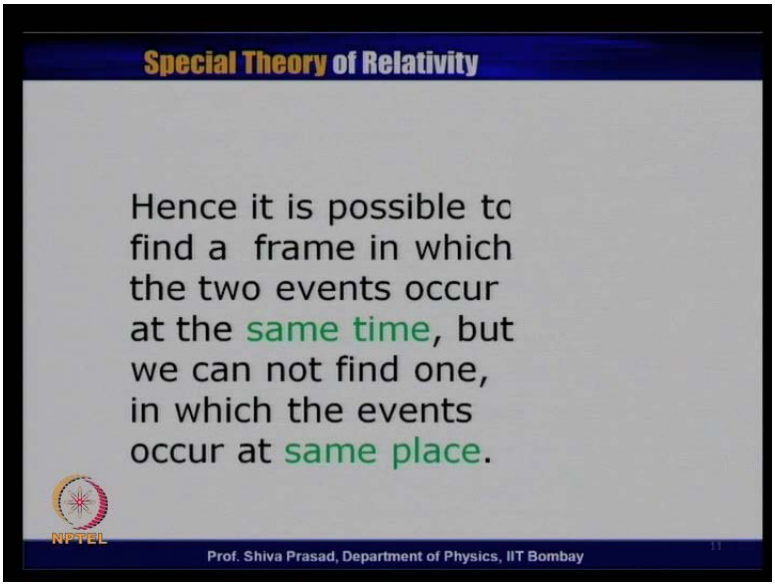

This what I have mention again hence, it is possible to find a frame, in which the two events occur at the same time but we cannot find one, in which the events occur at the same place.

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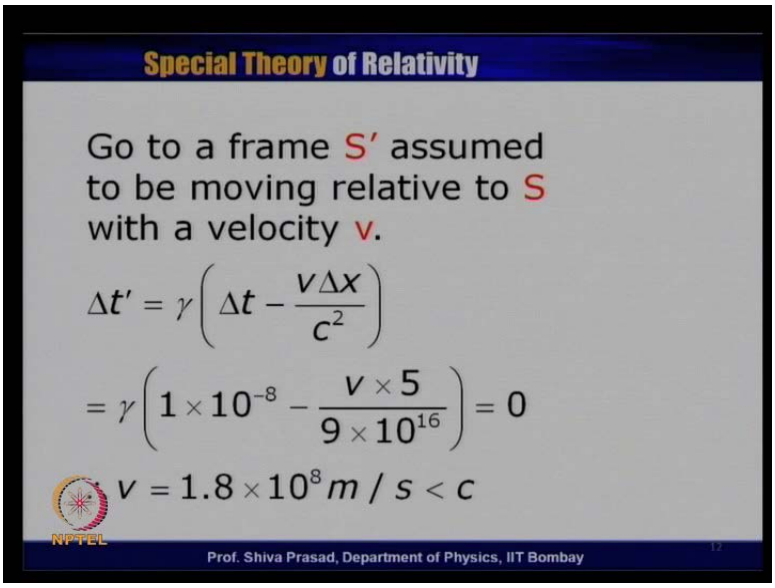

Now, we go to Lorentz transformation assume that, there is a frame reference, frame of reference S prime, which is moving a relative to S with a velocity v and I know that, it is possible to have delta t prime is equal to 0. All I have to is, to find the speed of that particular frame of reference, for which delta t prime will turn out to be 0.

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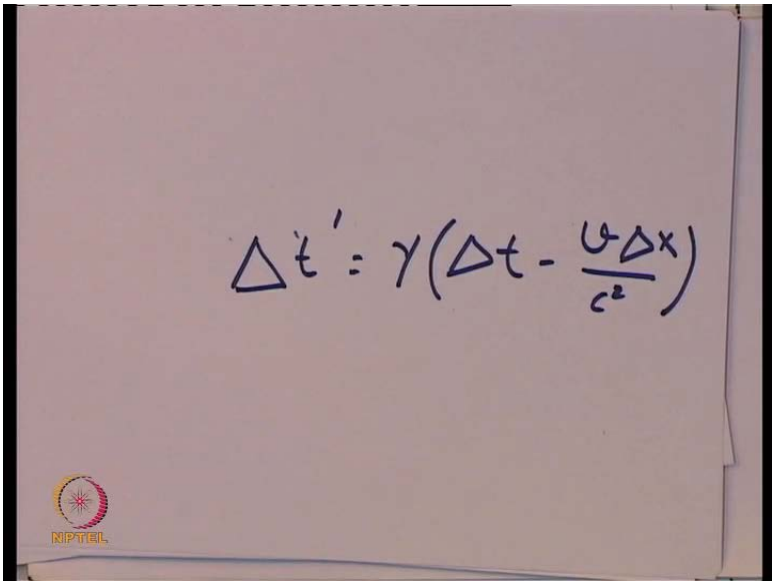

So, I write delta t prime is equal to gamma delta t minus v delta x by c square, this is standard Lorentz transformation. Now, what I have to do, I know my delta t, I know my delta x, how to find out a v, for which delta t prime is equal to 0. I substitute delta t prime

is equal to 0, substitute therefore, delta t 1 into 10 to power minus 8 second, I substitute for delta x 5 meters, solve for v, this is what, I have done in this particular transparency.

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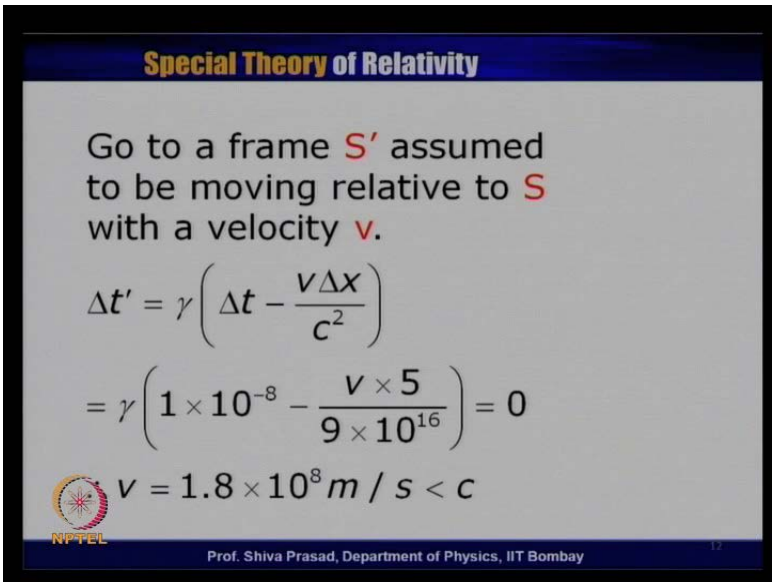

I have written delta t prime is equal to gamma delta t minus v delta x by c square, for delta t I have substituted 1 into 10 to power minus 8 second, for delta x I have substituted 5, for c square I have substituted 9 into 10 to power 16 assuming of course, that c is equal to 3 into 10 to power 8 meters per second. Just solve this equation, we get v is equal to 1.8 into 10 to power 8 meters per second obviously, this is speed, this is smaller than speed of light.

It means, it is physically possible to find out the frame of reference, in which these two events would occur at the same time it means, the time difference between these two events will turn out to be 0 in this frame of reference. And that frame of reference would be moving relative to S, with a speed of 1.8 into 10 to power 8 meters per second. Obviously, it is clearer that, if v would have been larger than this particular value, if you look at this particular equation, if v would have been larger than this particular value, delta t prime would turn out to be negative.

And time order of the events would get reversed, which is possible in this particular phase because the events are space like separated and not time like separated. Now, some of you can ask question, why cannot I do the same exercise and try to see, whether I can

really make delta x prime equal to 0 and if whatever we have said is correct, I should be able to get a speed greater than speed of light.

Then only, it will be possible for me to make delta x prime is equal to 0. So, let us just look into that to satisfy ourselves that, whatever we are saying is correct.

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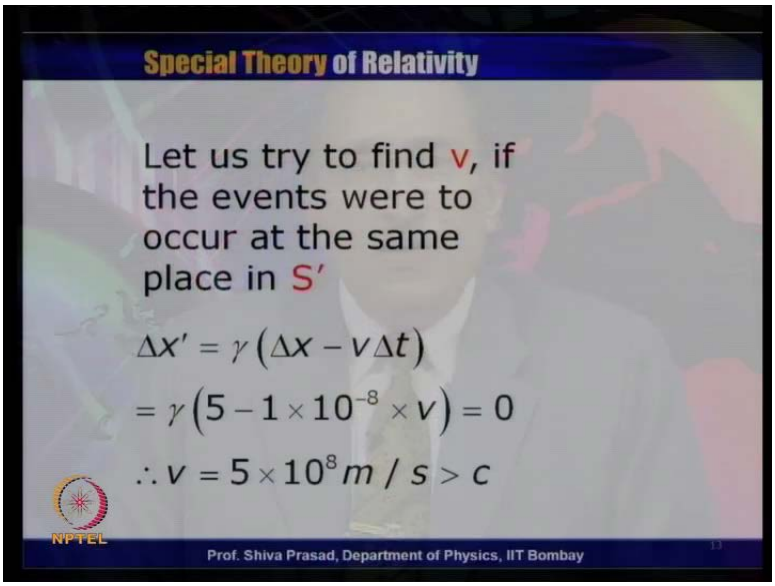

So, I do exactly the same thing instead of, delta t prime making equal to 0, I try to make delta x prime equal to 0. So, let us try to find v, a speed of frame of reference, in which I assume if at all possible that, these two events occur at the same position. So, now, instead of writing delta, the transformation Lorentz transformation corresponding to time, I will write a Lorentz transformation corresponding to space.

So, that is what it gives here, delta x prime is equal to gamma delta x minus v delta t, which is the Lorentz transformation corresponding to the x coordinate. I substitute my numbers, delta x y is equal to 5 meters, I substituted here 5 meters, delta t was equal to 1 into 10 to power minus 8 second, I substituted the value of 1 into 10 to power minus 8 second here multiplied by v.

I solve for this particular thing, we can see that gamma will turn out to be is equal to 0, this one when divided by 5 make will will make this 5 and this 10 to power minus 8 then it goes to numerator, it will become 10 to power 8.

And v will turn out to be 5 into 10 to power 8 meters per second, which is obviously greater than speed of light. So, it means, if at all it was possible for us to have a frame of reference with v greater than 5 c, which in this particular case is 5 into 10 to power 8 meters per second then it was possible for us to find these two events at the same position.

But, as we have already seen that, on the ground of physics and also some extent on the mathematical ground, we have rejected speeds greater than speed of light. Therefore, it I would generally conclude that, it is not possible to find out a frame of reference, in which these two events occur at the same position.

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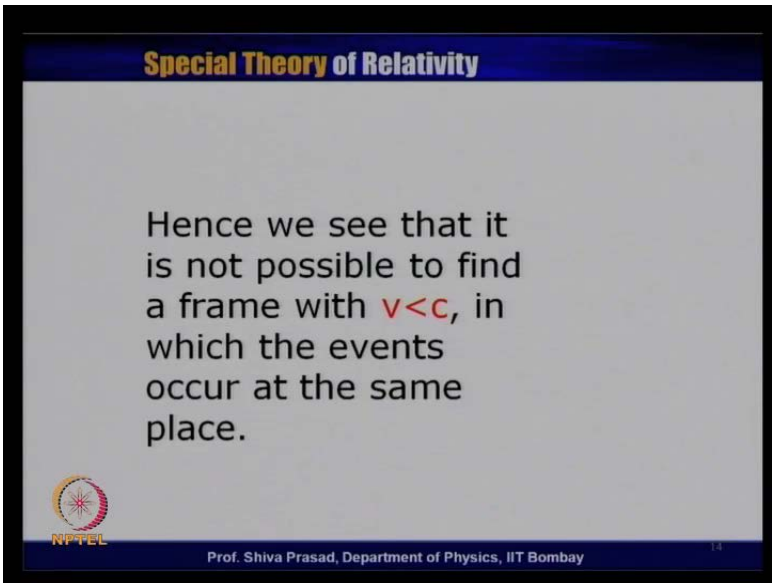

This what, I have written here hence, we see that it is not possible to find a frame with v less than c, in which the events occur at the same place. Now, we have just now said that, it is possible in the particular case to revert the time order, to reverse the time order, it is possible to find out a frame of reference with v less than c, in which the time order delta t turns out to be negative.

It means, events number 2 occurs before event number 1, we have said also that, this is not possible if these two events are cause and outcome related. It means event number 1 and event number 2 have to be totally independent events, event number 2 cannot be a cause of event number 1, if this particular condition has to happen, let us just look and try to clarify our ideas little further.

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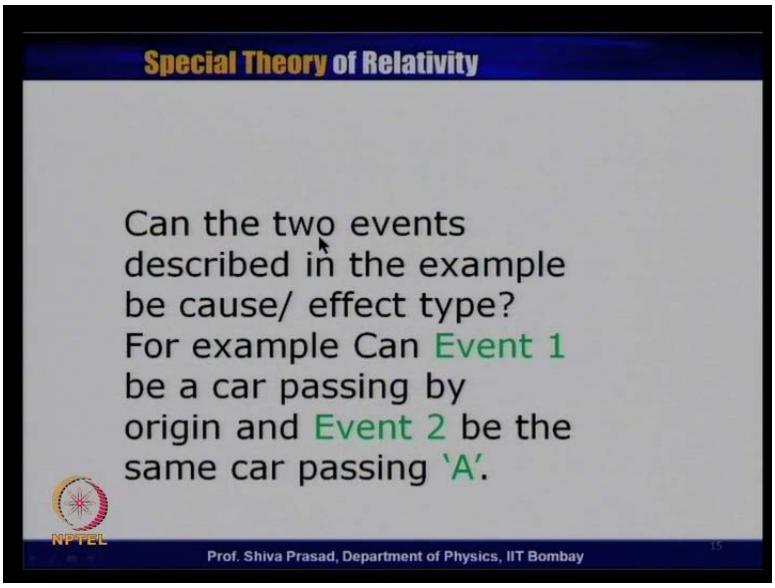

That is what, I have written here, can the two events described in the event be cause and effect time for example, let us imagine a particular case.

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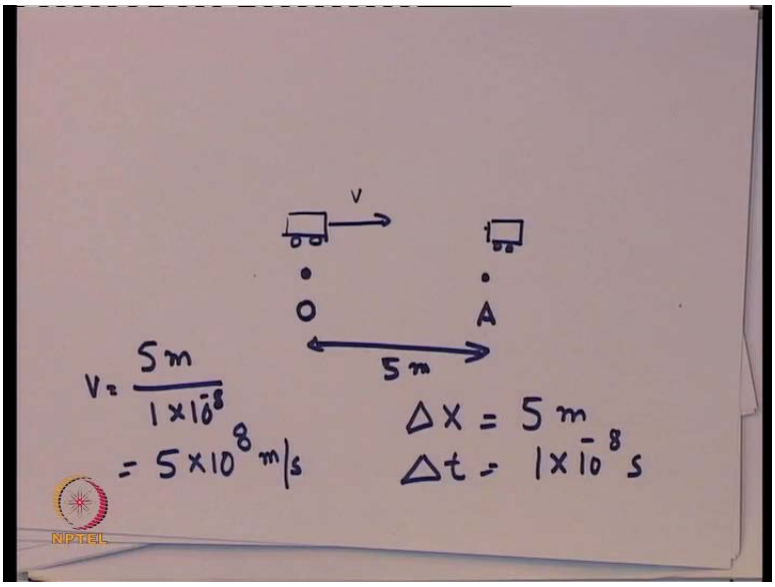

That, this was my origin O and this was my point A, and this distance is 5 meters, everything being described in S frame. Now, let us assume that, event number 1 was one car, which is moving to the right was been found at origin, we have solved many such problems. So, let us assume that, events number 1 was that, this particular car, was found

at x is equal to 0 and this car is moving with a constant velocity v along the plus x direction.

And let us assume that, event number 2 is that this same car being found here at x, is equal to A. Now; obviously, these two events are cause and outcome related, if the car was not starting from this particular point, car would have never this particular point A. Therefore, for event number 2 to occur, event number 1 was essential, the car must have passed through this particular origin and must have come to point A.

So, these two events are related, event number 1 is sort of a cause for event number 2 because car must pass through the origin then only it has to reach at A. Now, if this happens, can these two events lead to situation, which is given in this particular problem, involving delta x is equal to 5 meters and delta t involved in equal to 1 into 10 to power minus 8 second.

Is it possible that, these two events are separated of course, delta x is given as 5 meters and the time interval between these two events was 1 into 10 to power minus 8 second. That is what, I have written here in this transparency, can the two events described in the example be cause effect time. For example, can the event 1 be a car passing by origin and event 2 be the same car passing at A.

If this happens it means, and if the time separation between these two events is 1 into 10 to power minus 8 second it means, the car must have taken 1 into 10 to power minus 8 second in of course, S frame, we are talking of only one particular frame, which is S frame. So, it means, the car must have taken 1 into 10 to power minus 8 second to reach from O to A and the distance between O and A is 5 meters.

Therefore, the speed of the car must be 5 meters divided by the time, which is 1 into 10 to power minus 8 second, which will give you 5 into 10 to power 8 meter per second, which is obviously, larger than speed of light. Therefore, if these two events for example, were related like this then this car must travel with speed greater than speed of light and if this is not happening then it is not possible in a situation like this, for this particular situation to arise where, delta x turns out to be larger than c delta t.

And remember, only in a situation where, delta x is larger than c delta t, it is possible to revert time interval and because these two situation cannot lead to this particular delta x

be greater than c delta t. Therefore, we always say that, time order cannot be reversed as far as, these two events are concerned.

Because, these two events relating to the motion of the car cannot lead to this situation and only if they though have let to situation of delta x being greater than c delta t, it would have been possible for me to find a frame of reference, in which the car would have reached A first before starting from O. Hence, you can see that, by limiting the speed to a value less than or equal to c, we have avoided a situation, in which causality would be violated.

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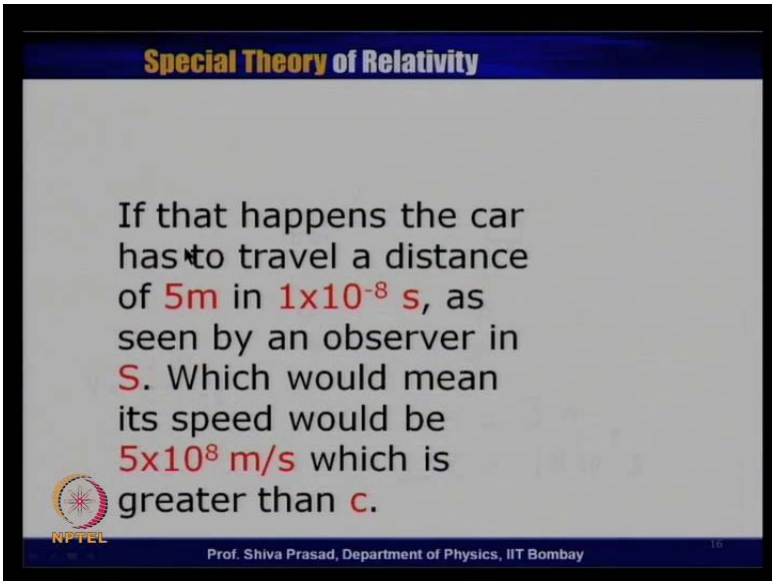

Same thing I have written here, in this particular transparency, if that happens, the car has to travel a distance of 5 meter in in 1 into 10 to power minus 8 second, as seen by an observer in S, which would mean it is speed would be 5 into 10 to power 8 meters per second, which is greater than c.

Now, let us take a situation, make it little more realistic, let us assume that car is not really traveling with speed greater than speed of light. So, I change my, my delta x is fix as 5 meters, I change my delta t, let me still call my event number 1 as the car passing through O and event number 2 as car passing through point A. But, I assign a speed to the car, which is less than speed of light, this what I have written here.

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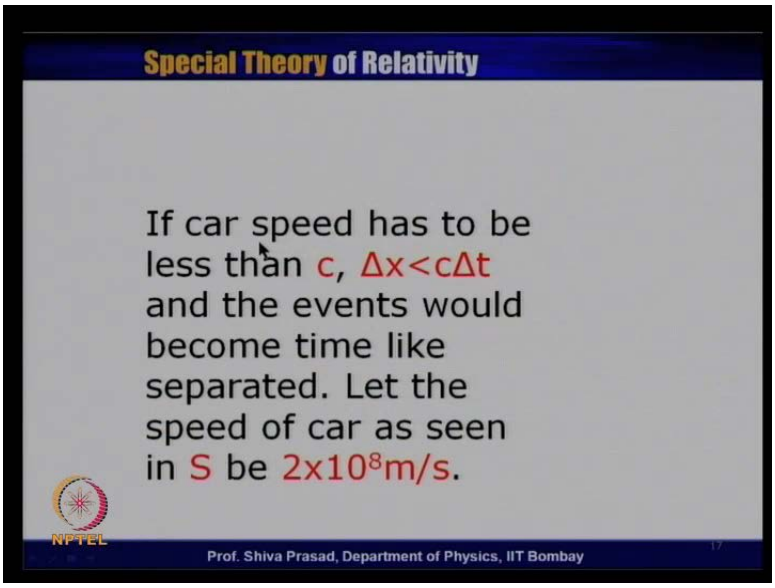

If car has to have speed less than c then delta x has to be less than c delta t and the events would become time like separated. If the events are becoming time like separated, as we have discussed, it is not possible to revert the time order in any other frame of reference. And therefore, it is not possible to find the violation of causality it means, we will not be able to find a frame, in which the car reaches A first before staring from O.

So, let us this, let us assume that the speed of car, as seen in A I am sorry as seen in S is 2 into 10 to power 8 meter per second, this is some value, which I have taken some arbitrary value, which is of course, less than speed of light. Of course, delta x is still 5 meters and if the car's speed happens to be 2 meters per 2 into 10 to power 8 meters per second. I can find out, how much time this particular car would take, to reach point A which is of course, given by the distance, which is 5 meters divided by the speed, which is 2 into 10 to power 8 meters per second, as I have described earlier. So, this time delta t would be given by this, which I can simplify 5 divided 2 is 2.5 into 10 power minus 8 second. So, in this case, we will find that, delta t turns out to be equal to 2.5 into 10 to power minus 8 second now, let us compare delta x with c delta t.

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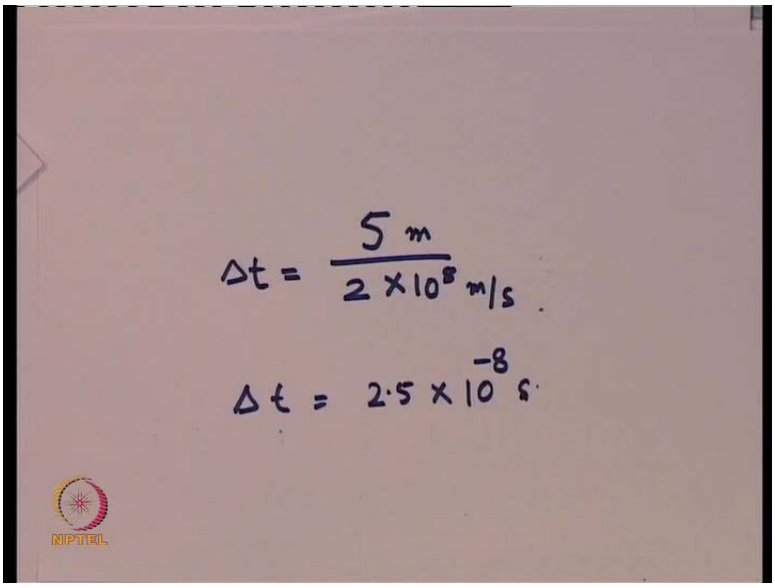

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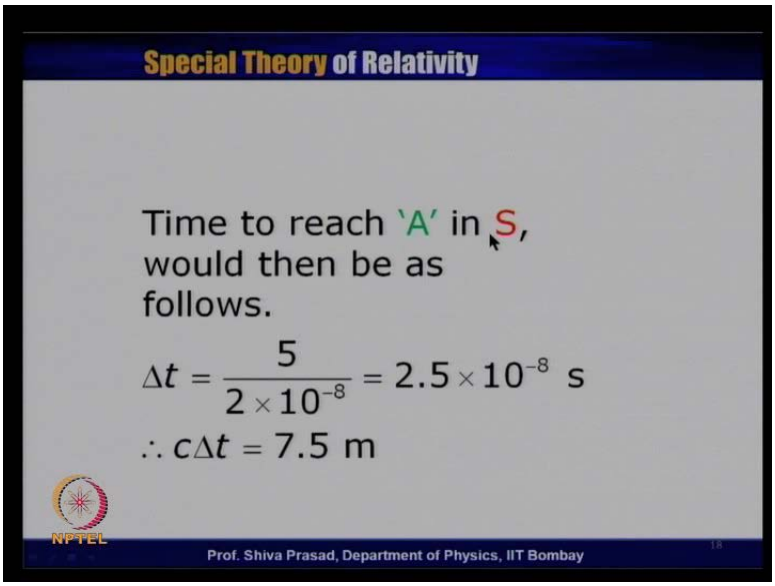

That is what, I have written here, time to reach A in S would then be given by 5 into 2 into 5 divided by 2 into 10 to power minus 8, which is 2.5 into 10 to power minus 8 second. I have to just multiply it by c, to find out what is my c delta t, if I take c is equal to 3 into 10 to power 8 meters per second, this 10 to power 8 and (Refer Slide Time: 25:11) this 10 to power minus 8, their product will lead v 1.

So, 2.5 multiplied by 3, which gives me 7.5 meters obviously, c delta t is larger than delta x and therefore, these are time like separated events. Now, it is possible to find out

a frame of reference, in which these two events occur at the same position but now I will not be able to find a frame of reference, in which these two events occur at the same time, let us just see.

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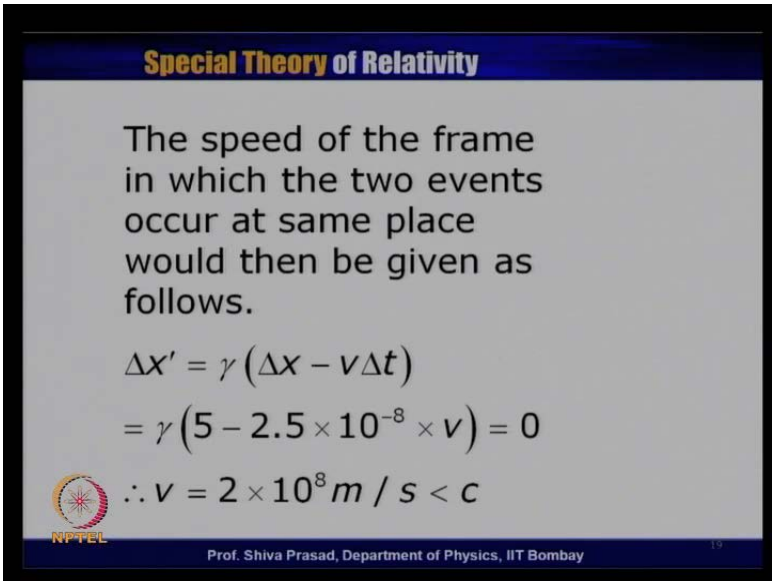

The speed of the frame, in which the two events occur at the same place now, I do exactly the same thing but slightly different numbers. Delta x still 5 meters but delta t has now, become 2.5 into 10 to power minus 8 second, I use exactly the same expression delta x prime is equal to gamma delta x minus v delta t. Delta x being 5 meters, delta t being 2.5 into 10 to power minus 8 second multiplied by v, I can solve for v, this gamma would of course, cancel because there is a 0 here.

This 5 divided by 2.5 will give me 2, which is 2 into 10 to power 8 meters per second, which is of course, smaller than c, this result was also obvious, this turns out to be same as the speed of the car, which we know. Because in the speed of the car, when if a person sitting on in the car will observe both the events to be occurring at the same position, we have noted many such problems earlier.

That, if a person is sitting in the car, he will find that O is approaching him and the point A is approaching him and as far as, his concern, he or she's concern, both the events are occurring at the same position. Therefore, delta x is 0, which I am able to find out of course, in this particular case, with a realistic speed, which is less than c, which is equal to 2 into 10 to power 8 meters per second.

But now, if I want to make delta t prime equal to 0, I will not be able to do it, unless I allow speed is greater than speed of light, let us try and see.

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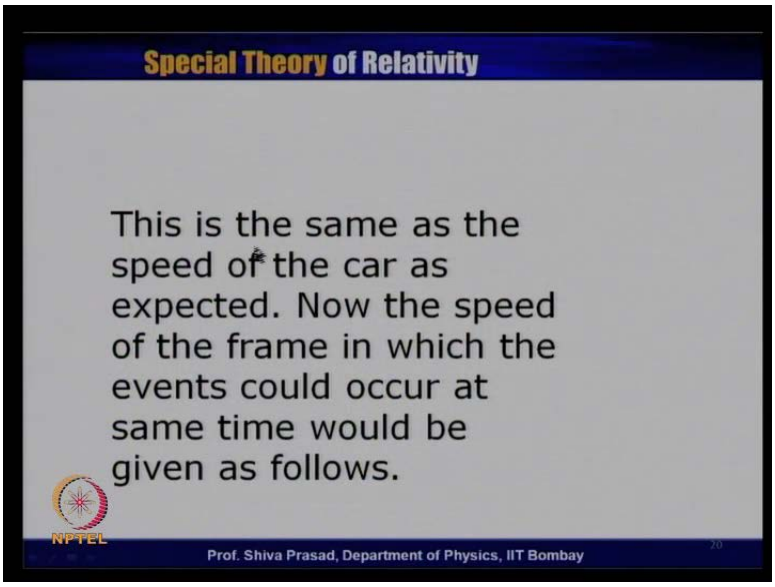

Of course, just the same narration, which I have said earlier, this is the same as the speed of the car, as expected. Now, the speed of the frame, in which the events could occur at the same time would be given as follows.

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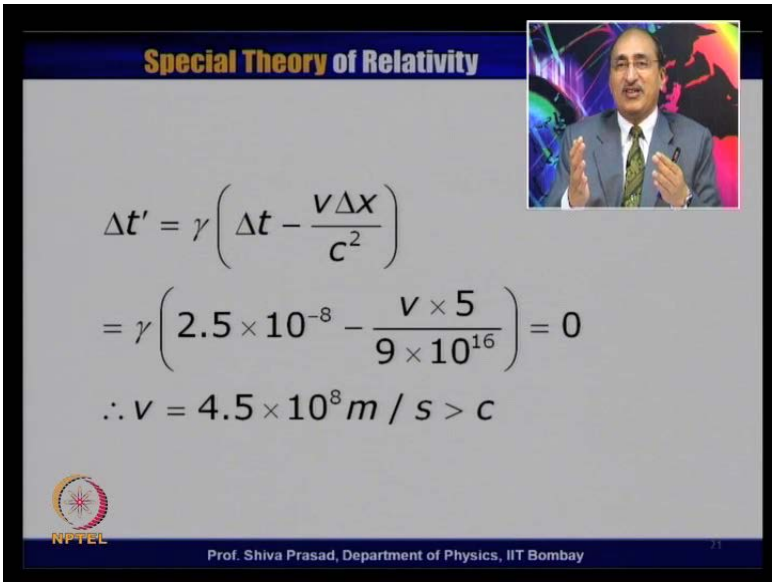

Which is given by, delta t prime is equal to gamma delta t minus v delta x by c square, I substitute the new numbers. For delta t, I have 2.5 into 10 to power minus 8 seconds,

delta x is same 5 meters, I workout this particular equation, I will get 4.5 into 10 to power 8 meters per second, which is obviously larger than speed of light.

So, only if they would having the frame travelling with this much speed then only, it would have been possible for us to find out that, the two events occur at the same time. Because of physical reasons, I have avoided, I have said, the speeds like this are not allowed therefore, I would conclude that, it is not possible to find a frame of reference, in which the two events occur at the same time.

And of course, it would mean that, it will not be possible to find a frame of reference, in which the time orders are reverse. Therefore, causality is is excepted, it is not rejected, causality has to be has to maintain, has to be maintained and we have to maintain this causality, as we have seen that, we have to restrict speeds to speeds less than equal to speed of light.

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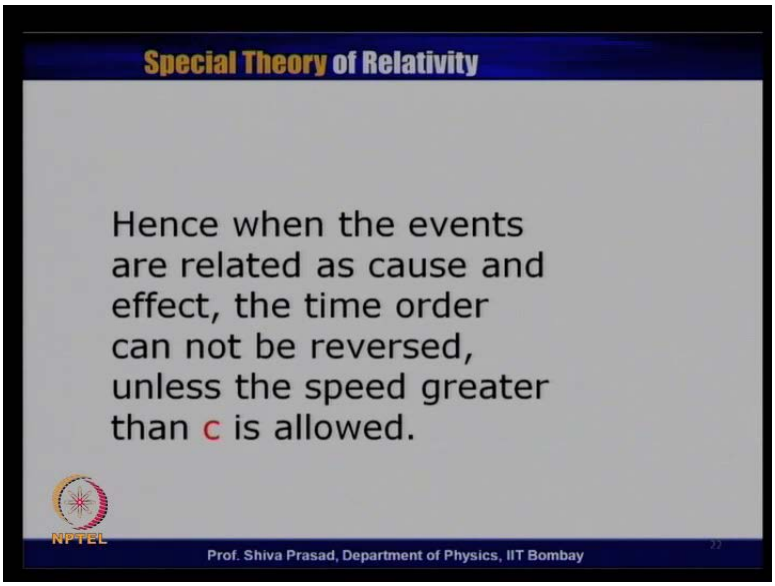

What I have written hence, when the events are related as cause and effect, the time order cannot be reversed, unless the speed greater than c is allowed. Instead of this particular train, you could have taken any other example and still you could have seen that, the causality will not be violated. And in the case, they has their their events, which are related as cause and effect then they will necessarily turn out to be time like events and in that case, time order cannot be reversed.

Let us take one more example, this is still a simpler example and example like this, we have been discussing earlier.

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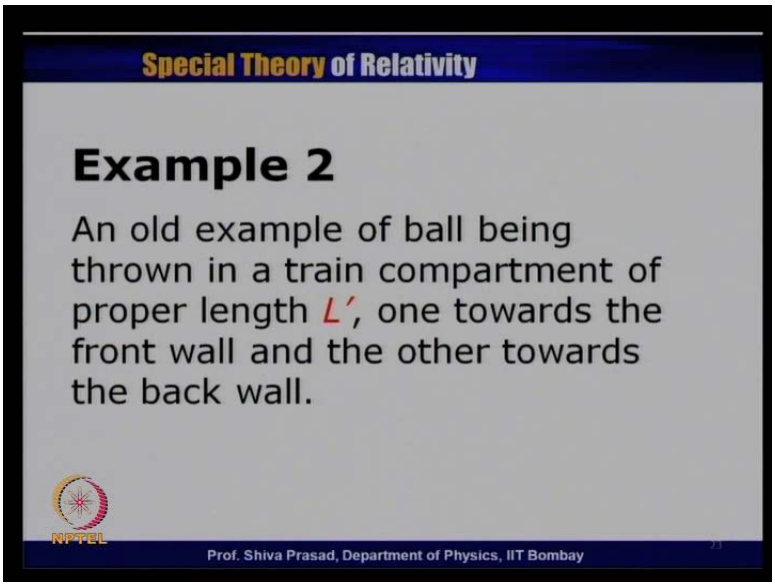

There is an old example of a ball being thrown in a train compartment of proper length L plane it means, there is a particular compartment, which is moving, it is a relative to earth assuming, earth to be a inertial frame of reference. And the length, as measured by an observer in the compartment is L prime because that is a proper length because the compartment is at rest and in it is own frame. One is ball is thrown towards the front wall and another is thrown towards the back wall.

This is what, I have shown in the picture, similar picture we have drawn earlier that, this is a person, which throws a ball to the first, towards the front of a wall, the front being described like this. If this is the direction of the speed then this wall is the front wall, this wall is the back wall so person throws simultaneously one ball in this side, another ball on this particular side. And my event number 1 and event number 2 are corresponding to the events of the ball being received at the two walls.

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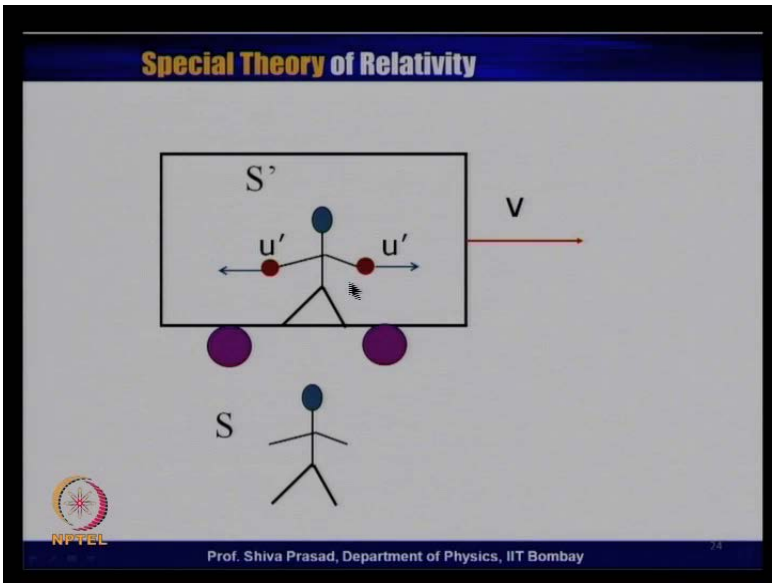

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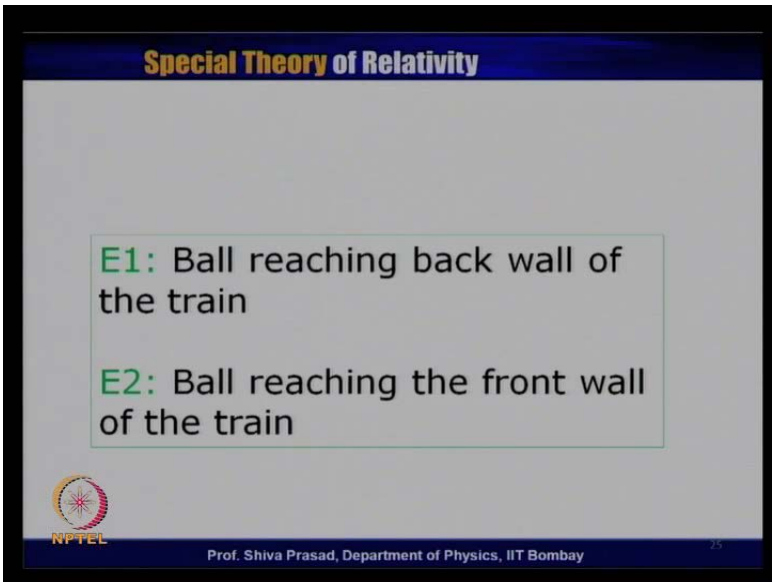

So, this is what, I have described here, event 1, ball reaching the back wall of the train, event number 2, ball reaching the front wall of the train. Now, if we remember this type of problem, we have done with ball then we have done with speed of light, light being flashed at the two ends, we have done many times. Because, the ball is being thrown from the center of the compartment therefore, each ball has to travel a distance, a horizontal distance along the x direction, as L prime by 2.

And both of them travel with the same speed, as seen in the S prime observer, as seen in the S prime frame of reference. Therefore, both the walls, both the balls would reach the wall together it means, delta t between these two events will be equal to 0. As per the first event is concerned, I assume that the center of the compartment is at the origin then the first event occurs at a distance of minus L by 2, second event occurs at a distance of plus L by 2.

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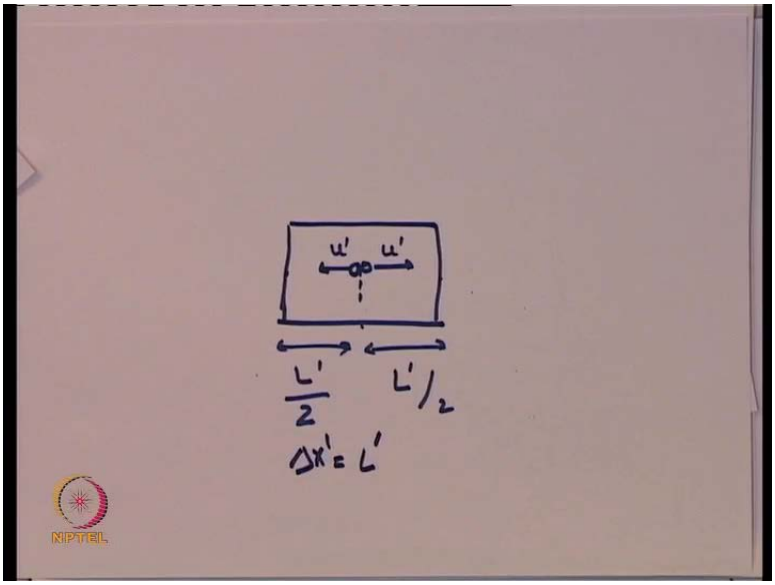

Let me draw my compartment here, this is the center point, a ball is thrown through the right and a ball is thrown through the left, this distance is L prime by 2, this distance is L prime by 2. Therefore, if I calculate this speed as I said u prime, this speed I have said as u prime, if I have to calculate the time that it takes then all I have to do should divide this L prime by 2, by this speed and because this distance is same as this distance, this speed is same as this speed.

So, the time when this ball hits here and the time when this ball hits here, these two times must be same. So, if this I call as t 1, if this, I call it, this time I call as t 2, t 2 minus t 1 must be equal to 0. But, this event as far the first event is concerned, it occurs before the origin, at a distance of minus L prime by 2 and at a distance L prime by 2 but it is in the negative sign. This events occur in the plus x value at a distance of L prime by 2 so if I take this as the difference, the difference will be turning out to be L prime so therefore, delta x prime will be equal to L prime.

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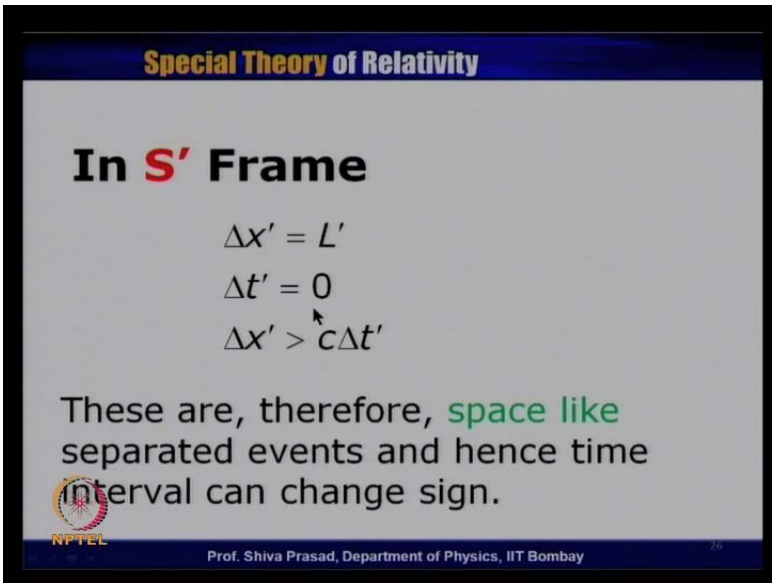

So, this is what, I have written here, the delta x prime is equal to L prime and delta t prime is equal to 0 of course, delta t prime is obviously equal to 0. Therefore, delta x prime is greater than c delta t prime because L prime is positive and non zero. Therefore, because delta x prime is greater than c delta t prime, these events are space like separated events and therefore, it possible that, the time order gets reversed as far as, these events are concerned but it is not possible that, the position order gets reversed.

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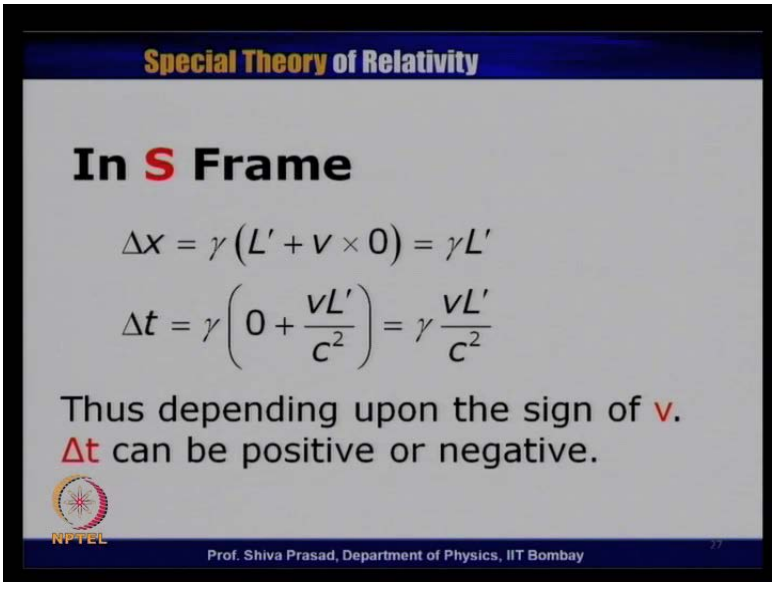

Let us assume as frame, which is the gone frame an observer is observing while sitting in the ground, this particular train. I applied the Lorentz transformation of course, I have to apply inverse transformation here because my S prime frame of reference is moving with speed v. If I say, delta x equal to gamma delta x plus v delta t, plus sign we got this inverse transformation, delta x was L prime, delta t prime was equal to 0 therefore, delta x will turn out to be equal to gamma L prime.

I would not discuss this particular issue because we have discussed this particular issue earlier, as we have said that, it is not possible to find, when this particular x, delta x is 0. But, I have escaped, this case I have taken a specific example shown so if I take delta t, delta t is equal to gamma delta t prime plus v delta x prime divided by c square, delta x prime is L prime, this is 0.

So, delta t turns out to be equal to gamma v L prime by c square so we can see that, the time difference S, in a S frame of reference will turn out to be v gamma v L prime divided by c square. Of course, depending upon v, v happens to be positive, this delta t will turn out to be positive, if v happens to be negative because I can always assume that, this particular train was moving other way that, this delta t prime will turn delta t will turn not to be negative.

Therefore, depending upon the sign of v, delta t can be positive or negative, as we have seen this particular example. Now, I would, all I would like to emphasis that, these two events, although they appear to be thrown by the same person, from the center of the box and same time, these are not cause and effect related events. Because, the motion of the first wall is independent of the motion of the second wall, it is not because that the ball one must thrown, that ball two has to be thrown.

It just matter that, we decided that person sitting there, decided to throw the two balls at the same time otherwise, he could have thrown one ball earlier than the second ball. In no way, the motion of throwing of the first ball is related to the throwing of the second ball A for, as far as, these two events are concerned, they are not cause and effect related. Though it appeared as with the same man is throwing the two balls, may be one is outcome of the second, of the cause of the second event, this is not correct because these two big balls being thrown are sort of independent each other.

Hence, it is possible to reverse the time order but I could have gone into this slightly different situation. When I could have made these two events later, let us see, let me first read, what ever I have said.

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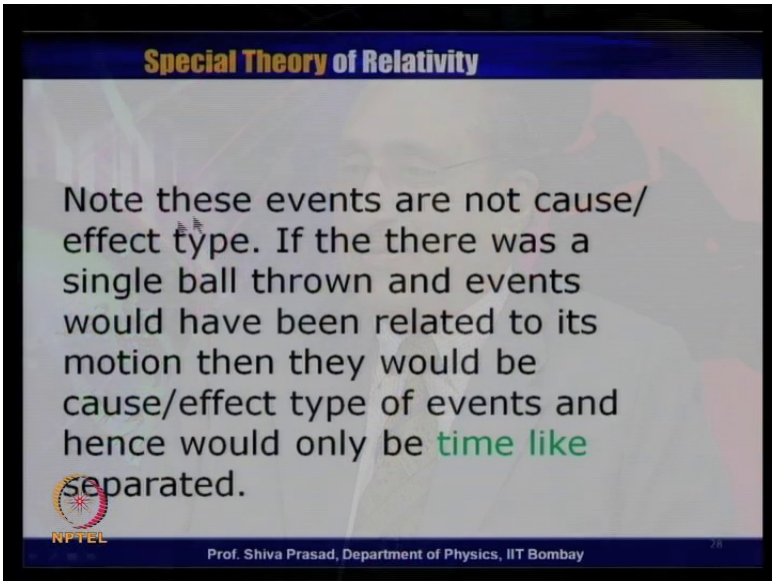

Note, these events are not cause and effect type, if there was single ball thrown and the events would have been related to the motion of that single ball, then they would have been cause and effect related and then only they will be time like separated.

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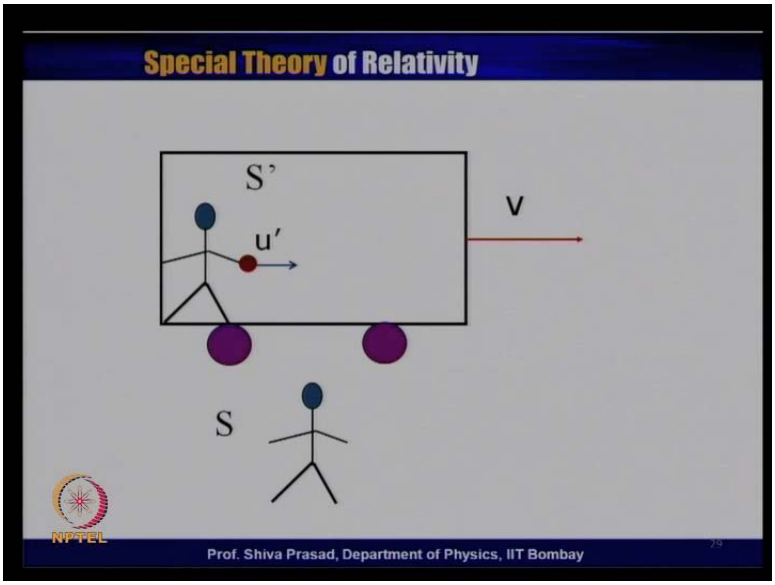

Let take an example here, suppose, this person would have been sitting at the end of the compartment and was throwing a ball from this particular end towards the front and event number 1 would have been throwing the ball and event number 2 would have been receive, ball being received at the front end. Then these two events are cause and effect related because unless this ball would have been thrown, the ball would not have reached here and in this case, we will find that the events are time like that is what, I wanted to emphasis.

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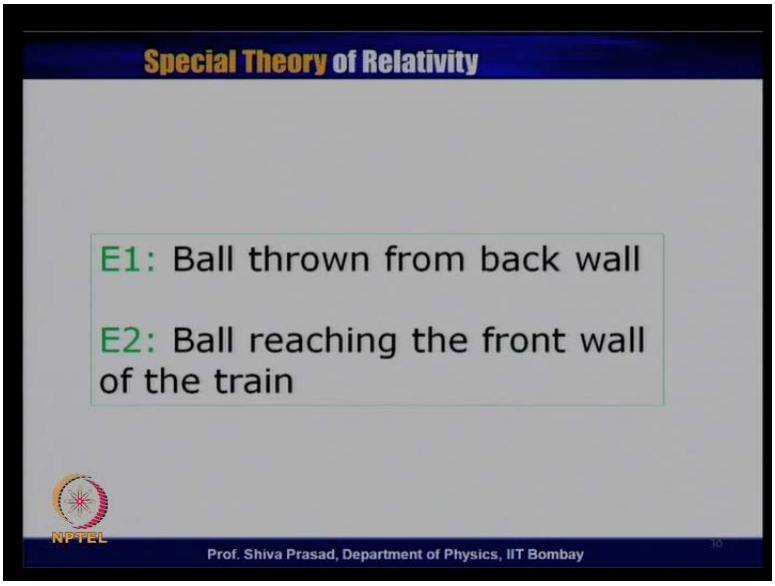

So, let look at this situation and assumes that, event number 1 is ball thrown from the back wall and event number 2 is ball reaching the front wall of the train. I can find out exactly the same way, what are delta x and delta t corresponding to these two events of course, delta x prime is L prime. Because the first event occurred at the back of the compartment.

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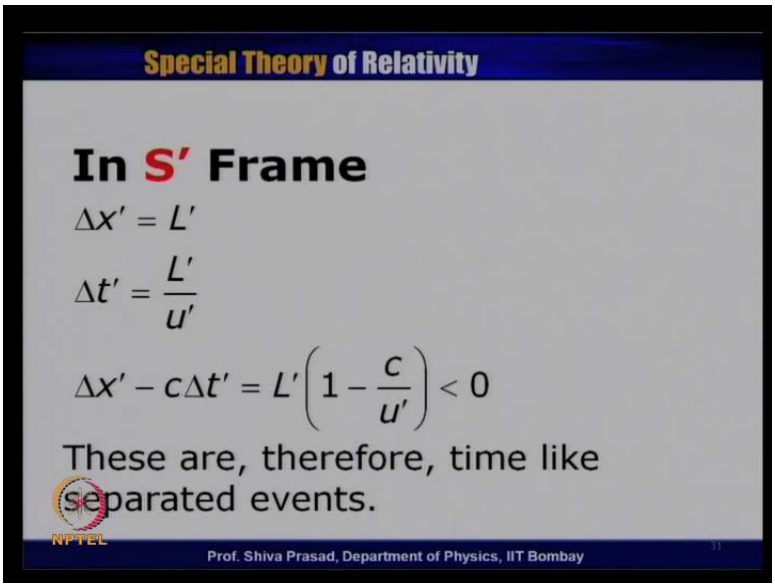

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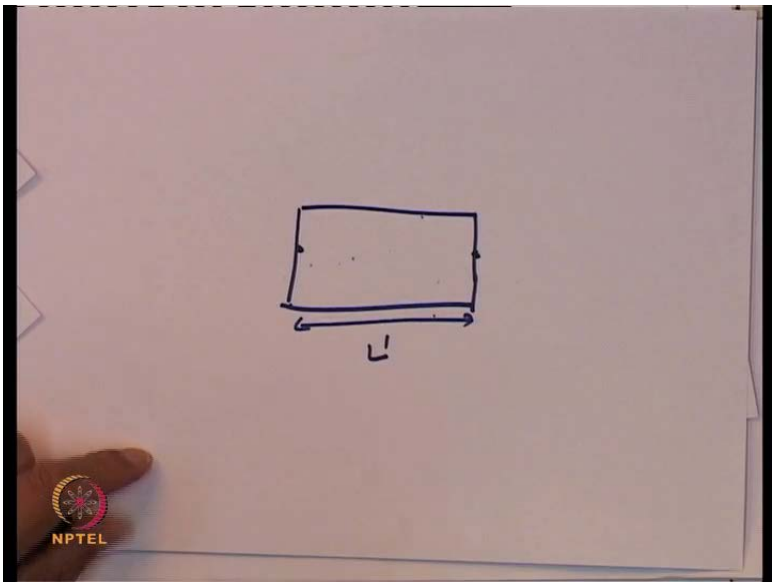

The ball was being thrown here, so first event occurred here let assume that my origins now here. So, the first event occurred at this particular point, second event occurred at this particular point, these two are separated by distance of L prime. Therefore, delta x, S seen in observer S prime, the compartment frame is obviously L prime so delta x prime has not change, which is L prime.

But, but the ball takes a finite amount of time to go from this particular end to this particular ends now, delta t prime is not equal to 0, delta t prime is equal to the, will be

equal the time taken by the ball to travel a distance from here to here in the x direction and that distance given by L prime divided by this speed.

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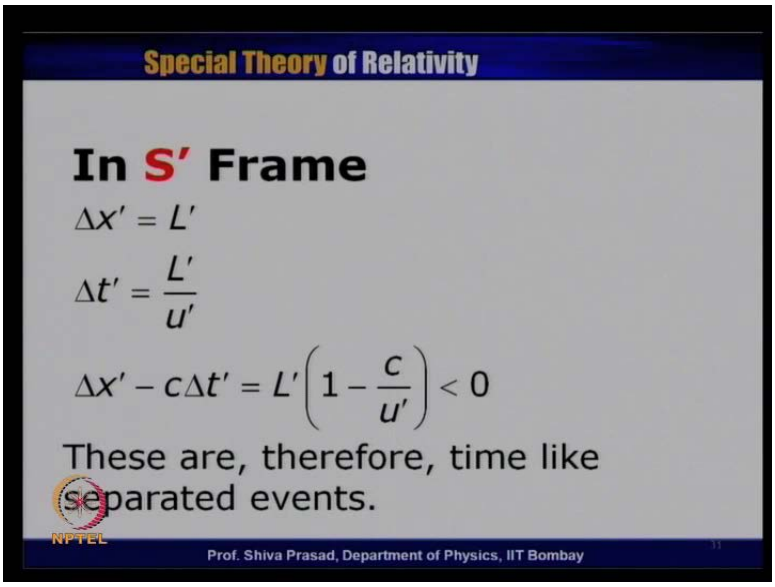

This is what, I have written here in this particular transparency that, delta x prime though is equal to L prime but delta t prime is now L prime divided by u prime because this is the time that, this ball will take to go from the back end to the front end, as seen in observer, as prime. And if I calculate delta x prime minus c delta prime, I have to multiply this by c and if I multiply by this c and take L prime common so I can write this, I take this L prime out.

So, in the bracket, I will left with this L prime has to be taken out so this will evolve here, L prime has to been taken out so it will be 1 divided by u prime, I have multiplied by c so to be c divided by u prime. And because c is expected to be larger than u prime therefore, this quantity is negative therefore, c delta t prime is larger than delta x prime. And these two events become time like separated events then it is not possible to find a frame of reference, in which the time order of this two events will gets reversed.

Now, let us try to calculate delta t, just to see that, this cannot become negative so let us, let me comeback to the frame S, apply inverse Lorentz transformation.

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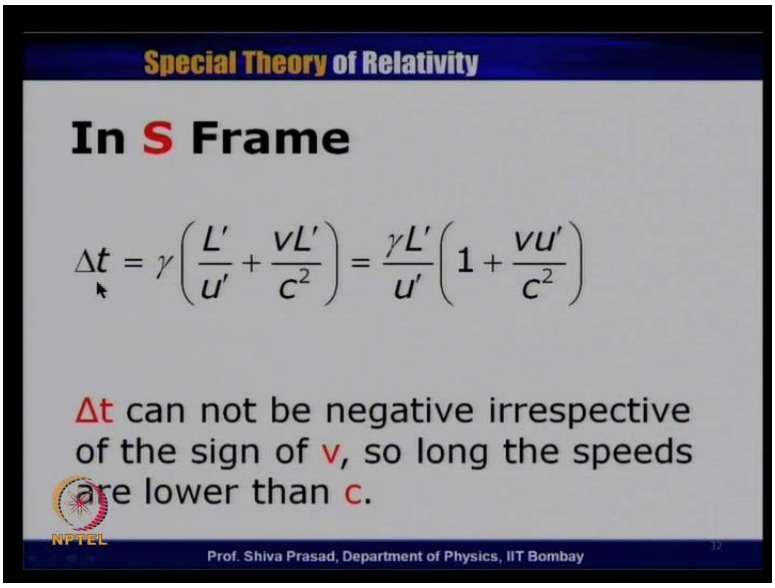

I write delta t is equal to gamma delta t prime, which is L prime divided by u prime, as we just now, discussed plus because this was an inverse transformation then v delta x is any way here, delta x was L prime divided by c square. So, this should this is what, will be the value of delta t, as seen in S frame of reference, what I am doing, gamma is any way out of this particular bracket. I take L prime also, out of this particular bracket and I also force u prime outside this particular bracket.

I take L prime divided by u prime, common from this particular bracket, if I take here then this factor will become 1, if I am taking L prime anyway, L prime here but there is no u prime here so I have to multiply by u prime. So, this factor becomes equal to gamma L prime divided by u prime multiplied by 1 plus v u prime by c square. As you can see that, this delta t cannot be negative of course, it cannot be negative, if v is positive but even if v is negative, this particular factor because v and u prime, both are going to be smaller than c individually.

Therefore, this factor cannot exceed c square therefore, this factor has to be always less than 1 and even if it is of negative sign, the whole quantity delta t will always lead me in positive value. Hence, what I have written, delta t cannot be negative, irrespective of the sign of v, so long this speeds are lower than c and therefore, it is not possible to find a frame, in which the time order get reversed.

This is what was sort of discussion of giving some example about time like and space like events. Now, let us go little ahead, in Lorentz transformation, enough ,as it given me all the information about relativity or do I require something more. What I would like to mention here is that, this was only the first step, just doing Lorentz transformation is not enough in order that, the postulate special theory relativity that, all the laws of physics remains same in all the frames.

It is necessary to do something more and what I would like to say is that, atleast I would have to redefine my momentum. So, now, I will give you a very simple example, it is a classical mechanics, this particular example solved in a very, very simple fashion, which we call as, completely inelastic collision. In a completely inelastic collision in a classical mechanics, we always say that, the momentum has to be conserved but the mechanical energy is not conserved.

Actually, if you take the total energy that, should be conserved there also but it is converted into some sort of energy, which is not really mechanical. And traditionally, in the classical mechanics, whenever we are solving collision problems, we apply conservation momentum of course. And conservation of energy, which is happens to be the conservation of mechanical energy, only those cases where, the collisions are elastic.

It is possible to have non elastic collisions or the example, which I have gave you as completely inelastic collisions, in which the two bonds come together and gets stuck to each other, in which we do not conserve the mechanical energy, we conserve only the momentum. Now, we take this particular example of completely inelastic collision, as given in a particular frame and of course, in this particular frame, we will say that, momentum is conserved.

Now, what I will like to show that, if I change my frame of reference with different frame of difference then in that particular frame of reference, momentum need not be conserved, if we just take only the Lorentz transformation into consideration. Therefore, something else is need to be done, to make conservation of momentum, which we believe to be one of the fundamental principles of physics to be valid in all initial frames. So, I take one specific example, the example is essentially very simple but even in this particular example, we will show that, this violets the universality of conservation of momentum

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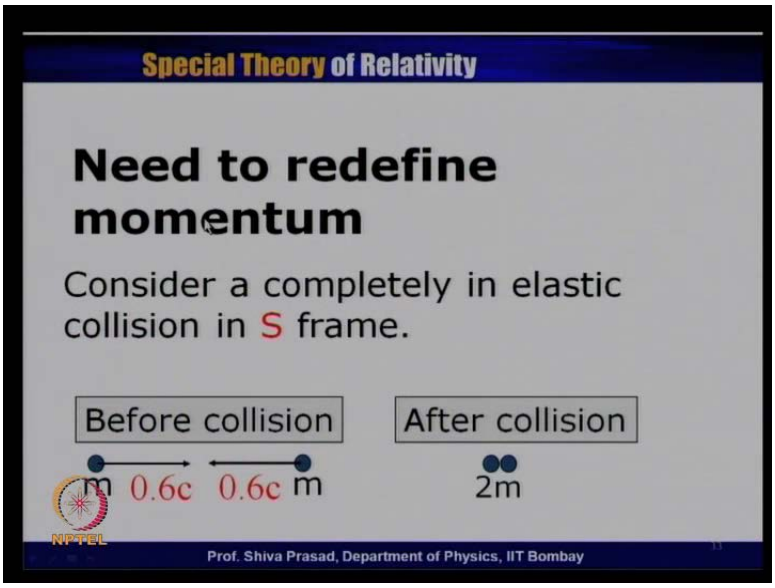

So, this what the example I have, title I have to need to redefined in momentum, consider a completely inelastic collision in S frame. So, let us assume that, there is a frame S in with one particular particle of mass m, is found to trouble to the right with a speed of 0.6 c, in the same frame another mass travels to the left with the same speed, which is 0.6 c. Now, the observer S finds that, these two masses collide and then get stuck to each other so what we call, completely inelastic collision.

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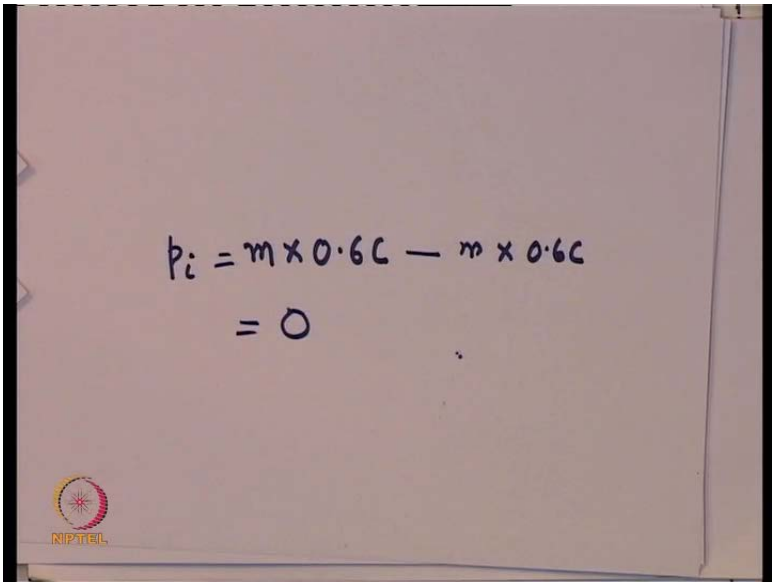

So, this was a situation before collusion where, one mass was travel to the right with the speed of 0.6 c, another mass exactly identical, was travel into the left with speed 0.6 c. They have a head on collision and then two masses get stuck together so what is remaining is, just a single mass of total mass 2 m. The two masses stuck together and this particular mass comes to rest, and as you can very easily see that, this will conserve momentum.

Because, if I take the momentum of the first particle, that will be given by m into 0.6 c, if I take the momentum of the second particle, the magnitude of that will also be m into 0.6 c. But because this speeds are in different direction, opposite direction therefore, the momentum direction, momentum being that will be opposite so this will be the net momentum because this magnitude is same as this magnitude so this should be equal to 0.

So, this will be what, I called as a initial momentum it means, according to conservation momentum, the final momentum must also be equal to 0 because mass is 1 0 obviously. Therefore, it means, the mass must come to rest, because then only, it is possible for momentum to be conserved or to become 0. So, this speed has to become 0. And that is what, I am saying that, after the collision, the two masses are come together and got stuck, and have come to rest.

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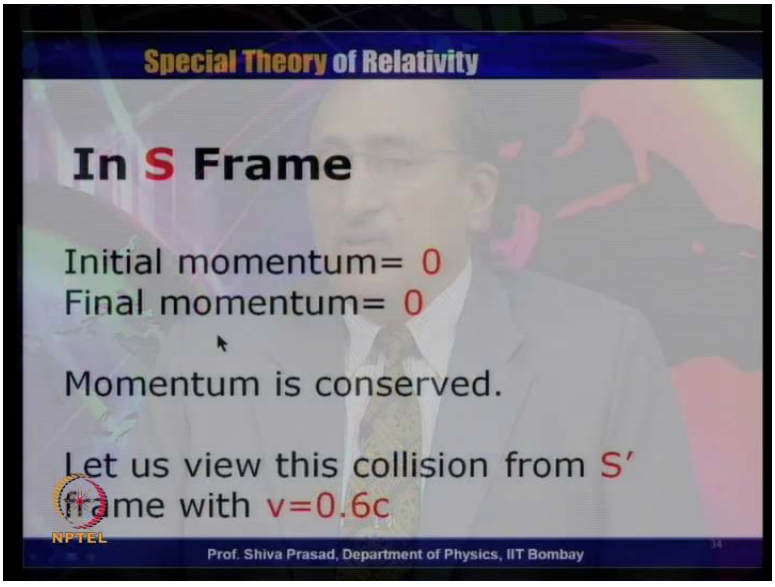

Therefore, the final momentum is also 0, so we have a situation where, initial momentum is also equal to 0, final momentum is also equal to 0 therefore, momentum is conserved. So, as you have seen just now that, initial momentum is 0, final momentum also 0 therefore, in S frame, the momentum is conserved. Now, let us try to look this particular collision, from one particular frame on specific frame, S frame, which happens to be moving with the same speed S, the initial speed of the first particle, which is 0.6 c. And we show that, in this particular frame, this conservation is valid, even if you can show violation in one particular frame of reference, the law gets valid.

So, just one example contrary is enough to say, that this particular law is valid so let us go to S prime frame of reference, in which the relative speed v is 0.6 c.

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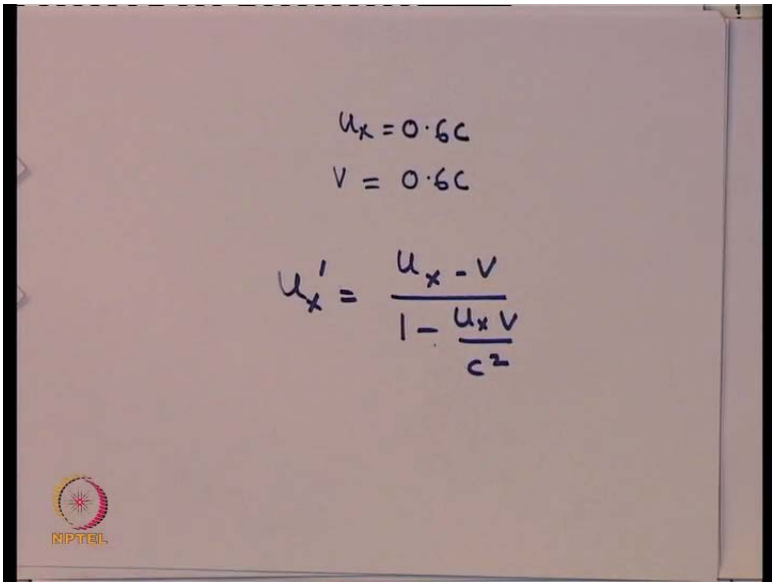

As we have seen, this tender formula for velocity transformation is u x minus v divided by 1 minus u x v by c square. In this particular case, the relative speed within the frames has been chosen as 0.6 c, if we applied this particular thing to the first particle, that first particle has a u x, which is equal to same as 0.6 c. I substitute these two quantity in this particular formula, to find out this speed of this particular particle in S prime frame of reference, which is what I have done here.

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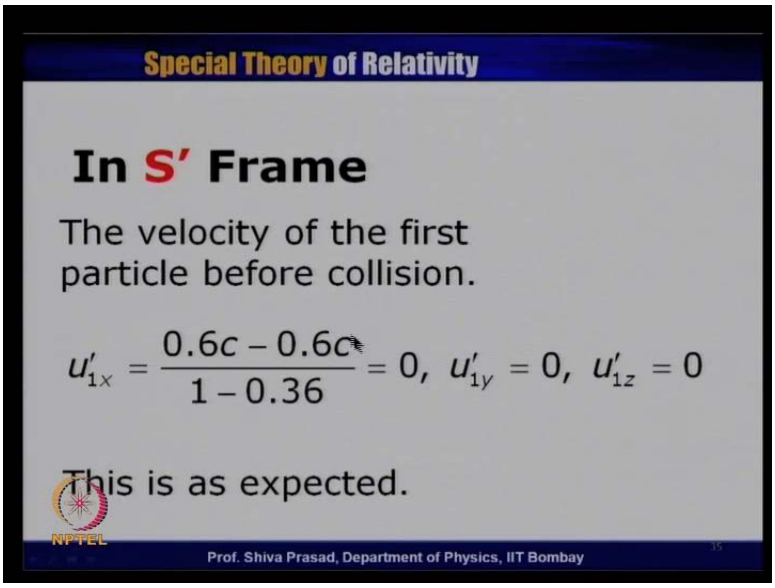

So, u x was equal to 0.6 c, v was equal to 0.6 c divide by 1 minus u x into v divided by c square, c square gets cancel, this becomes 1.36, what as you can see, numerator we have 0.6 c minus 0.6 c, this gives me 0. So, irrespective of the denominator of course, denominator is not equal to 0, I will get for the first particle this S component of this speed is equal to 0, which is expected.

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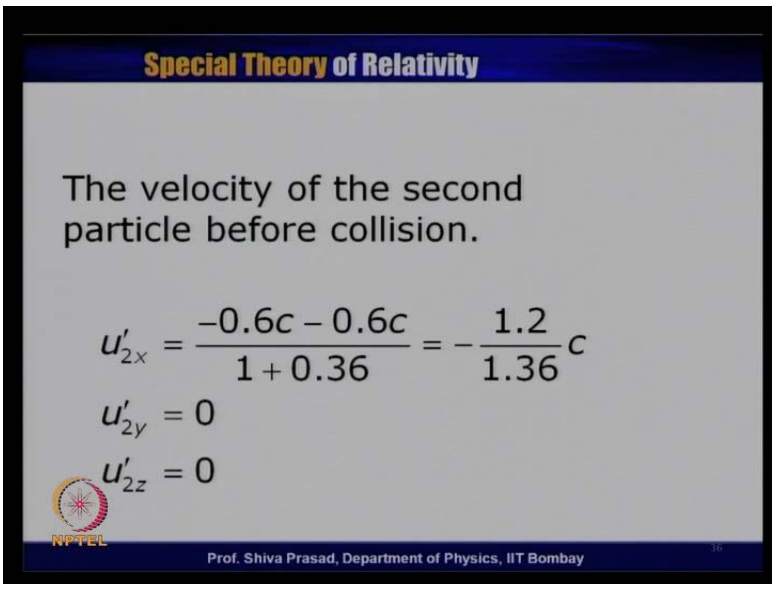

Because, if a frame of reference is moving along with the particle, with moving with the same speed of the particle that particular in that particular frame of reference, the particle

would appear to be at rest. It would not be appear to be moving therefore, I am getting this speed to be 0, which is obvious.

Now, let us calculate the velocity of the second particle before the collision. This particular particle was having the same speed but in a different direction so my u x was equal to minus 0.6 c. So, therefore, I write here as minus 0.6 c, v was any where 0.6 c and because this is negative so this particular sign becomes plus. So, I get 1 plus 0.36, which when I calculate is turning out to be minus 1.2 divided by 1.36 c.

So, observer S prime would notice that, this speed of the second particle of course, is in minus x direction and is equal to 1.2 divided by 1.36 c. Now, let us try to calculate this speed of the two particle, which of course, now turn into a single particle, after the collision. This particular particle was at rest in S frame of reference so let us try to calculate this speed, here the speed in S frame was 0, here v is 0.6 c.

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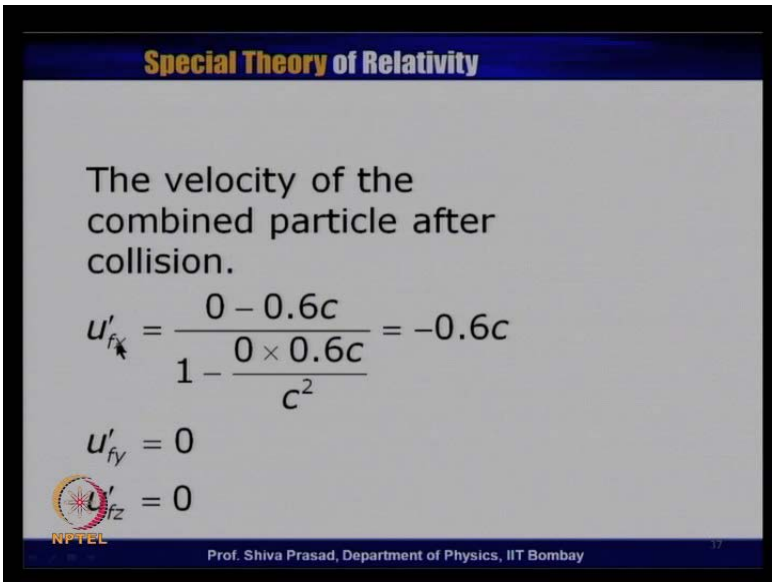

Therefore, which I am saying is the final speed, u f x prime is equal to 0 minus 0.6 c, which is v divided by 1 minus u, which is equal to 0.6 c divided by c square. This quantity becomes 0 because of, 0 here, denominator is 1 and you get minus 0.6 c therefore, the speed of the two particles, two combine particle after the collision, as seen in S prime frame of reference will be turning out to be equal to be minus 0.6 c.

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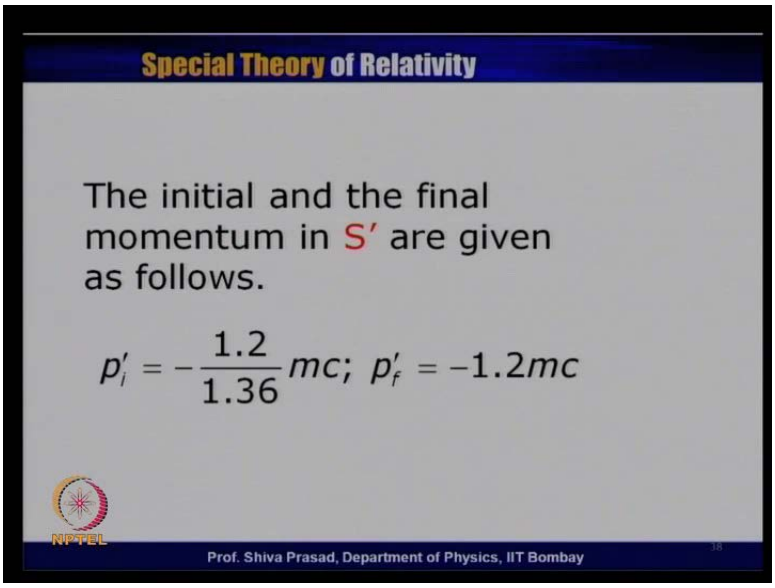

Now, let us try to calculate the momentum of the particle, the initial and the final momentum, the first particle in S prime frame of reference was having a 0 velocity, so obviously, it has a 0 momentum. The second particle was having a speed of minus 1.2 divided by 1.36 c, this multiplied by the mass, the momentum of the second particle, which happens to be the total initial momentum as observed in S prime frame of reference.

This initial momentum as observed in S prime frame of reference, which will be equal to minus 1.2 divided by 1.36 m c. Now, final momentum, there is only one particular particle, which has a mass 2 m and that particular particle, as we seen in the previous transparency, was moving with the speed of minus 0.6 c in a S prime frame of a reference.

So, this multiplied by the mass of the particle which is of 2 m , the final momentum is turn out to be equal to minus 1.2 m c, as we can clearly see that, this is not same as this. According to S frame observer, the momentum of the particle has not conserved so according to an observer S frame of reference, this collision really let to conservation of momentum.

But, if everything what ever I am saying is true then according to an observer S prime frame of reference, the momentum was not conserved in this particular process. Hence, either we say that, conservation of momentum is not a fundamental principle of physics,

which I am not sure, anyone of us will agree, which means that, we must change we must do something else so that, the momentum is also conserved in S prime frame of reference.

Therefore, we need to redefine our momentum so in the event, we will summarize, what we have discussed today. We discussed two examples related to space and time like separated event and specifically, we saw the cause and outcome related events. Then finally, we discussed one simple example of completely inelastic collision and we said that, we must redefined our momentum.

Thank you.