Special Theory of Relativity
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Lecture - 10

A Problem Involving Light and Concept of Causality

In our last lecture we had worked out a long example.

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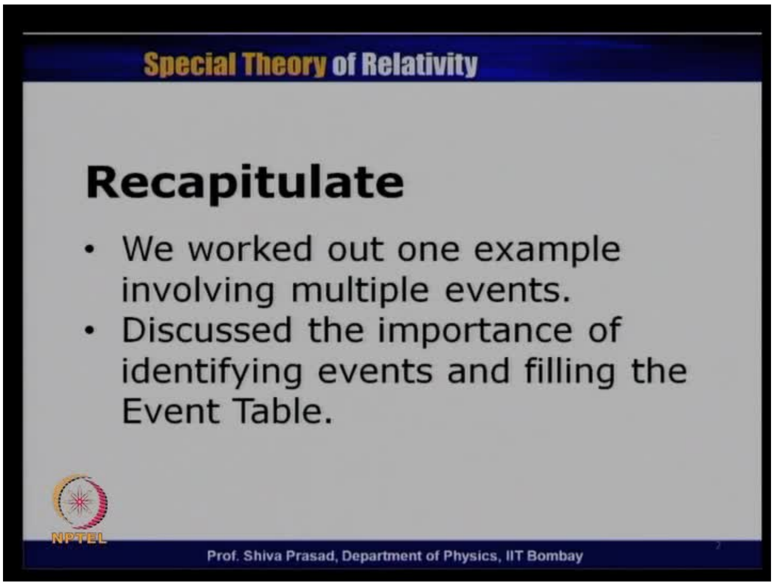

The idea of working out that example, was just to give you some feeling of how multiple events problems have to be solved. And we had discussed specifically, the importance of identifying the events and filling the event table. So, in the most of the relativity problems, involving kinematics it is important to realize what are the events and then write the co ordinates of these events including time; time of course, will be treating as co ordinates as been telling.

And then fill the event table of course, event table can be filled with whatever information is been given, in a frame. If the information is not available in that frame but, in a different frame that information has to be trans translated back to the given frame, by using an appropriate task formation. So, that is what is the basic trick of working out the problems of relativity, look events, fill the event tables find out the information available in the frame. Whatever are not available in the frame but, available in different frame bring them back to your frame by using an appropriate task formation, that is the way to eventually work out these problems.

Now, in today lecture we will starts with giving one more example, involving the speed of light before we going little bit ahead in the course. So, example is comparatively simple, we have been working out these type examples, when there is a train in which particular observer sitting and that particular observer emits light. But, in all earlier problems, the direction of the first speed of the light or direction of the velocity of the light was always along the motion of the train or opposite to the motion of the train which we have called is x axis.

So, c was always pointed out was always pointing out either in pulse x direction or in minus x direction. Now, we will work out the different examples, in which the speed of li in which the light will be pointing out in the y direction that is the direction which is perpendicular to x direction and towards the top. So, this is what is the example, which I am just describing.

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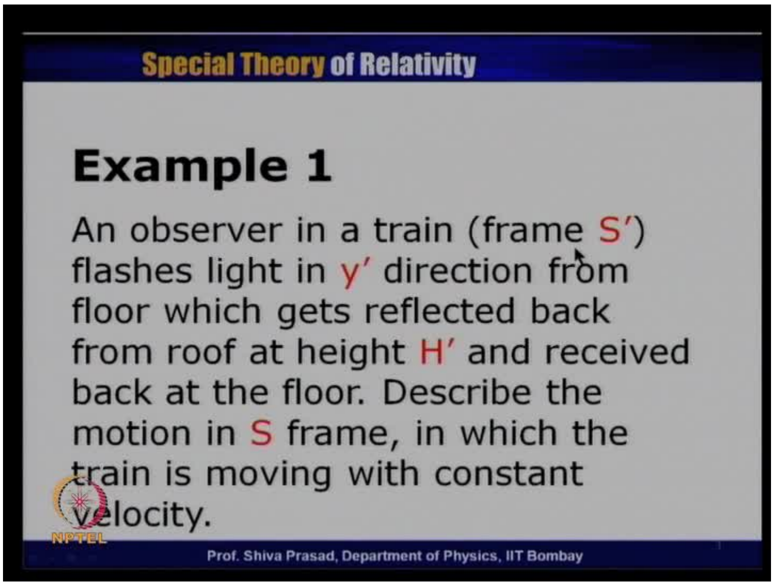

An observer in a train which we are calling frame as S prime, flashes the light in Y prime direction, from the floor which gets reflected back from the roof at height of H prime. So, that roof at the top which at the height of H prime, I will show just a figure just now, and is received back the light is reflected back from the top and received back at the floor of the compartment, of the train.

Now, what we have to do is to describe the motion in S frame which I am calling as an ground frame or the frame, in which the particular train is moving. And view of course, the train is moving with the constant velocity because, we are always in the realm special theory of relativity. So, in our case, the velocity is always assumed to be constant, so this the problem which really essential problem but, a way of explaining various concepts ones more, with a slightly different type of example.

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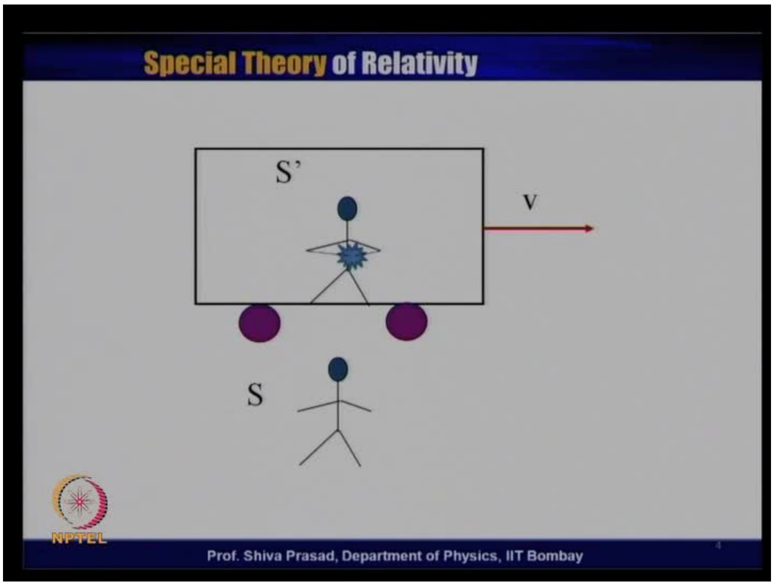

So, this is the figure which I have drawn here of course, there is one particular person with a light source and then, let us assume that the light source is right here, at the floor of the compartment, this is the train compartment, which is moving to the right with the velocity V as seen by an observer S. And as you known that this direction we always call as x direction, let us assume the y direction is the long the top. So, this particular light is been flashed from the floor, the light goes up is S seen by the observer, vertically up and then comes back.

This is a situation, which is S observed by observer S prime in this particular train. Now, question is that, there is an observer setting on the ground, in this ground frame which I am calling as S frame this train is moving with the velocity V. How, this particular person would perceive this particular lights motion that is what we. In fact, we worked out many type similar types of problem classical mechanics earlier, the professor who ever familiar with classical mechanism was be familiar that, a particular person sitting in the train the ball vertically, upwards and the ball comes back and reflected back.

And same of the events or same experiment is been seen or visualized by the observer sitting on the ground, how the perception will come different. Now, everything assumption ally similar except that the instead of ball, we have light and because, we are talking of light. So, we are talking special theory of relativity and use the relativistic task formations, otherwise the problem is somewhat similar. So, let us define our events we have emphases quite a bit on the importance of identifying the events.

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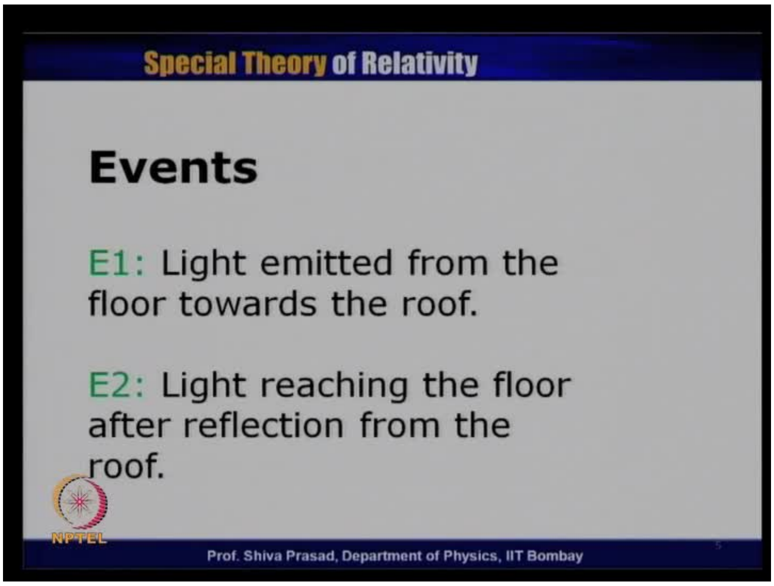

So, here event 1, event 1 is light emitted from the floor of the compartment of the train towards roof. So, light is thrown upwards, so this is event number 1, the light been emitted from the roof from the floor of the compartment and train towards the roof. Then the light goes up comes back and receive back at the ground, so when it is received back we called that as event 2. So, my event 2 which have written here, E 2 is light reaching the floor after reflection from the roof.

So, light been thrown, even light been thrown event number 1, light goes up comes back received at the ground that is event number 2. Now, let us first look how an observer S prime could observer because, the experiment has described with respect the to an observer S prime, which is sitting in the train. Observer in S prime, will always feel the light is always going vertically upwards.

So, initially when it goes upwards then the direction of the velocity is along the pulse y direction. When it first goes up and it gets reflected back, the light starts moving downwards and the velocity is now, downwards is minus y direction that that is what will be the perception of the observer sitting in the train.

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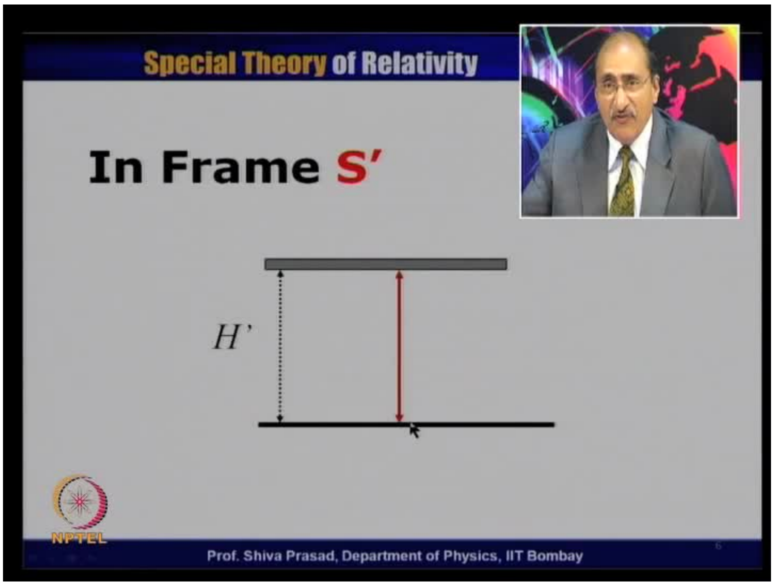

So, this is what I have shown in this particular figure, this is with the reference to the observer in the train, which I am calling as S prime. Has been given in the example that this particular height is H prime, this arrow shows the direction of the velocity initially, the velocity of light initially which goes upwards, this arrow shows when veloc when the light is reflected back and tries to come back towards the floor.

So, initially the light moves upwards which is the plus y direction plus y prime direction to be more precise. And then it comes back to the floor, which we call in minus y prime direction.

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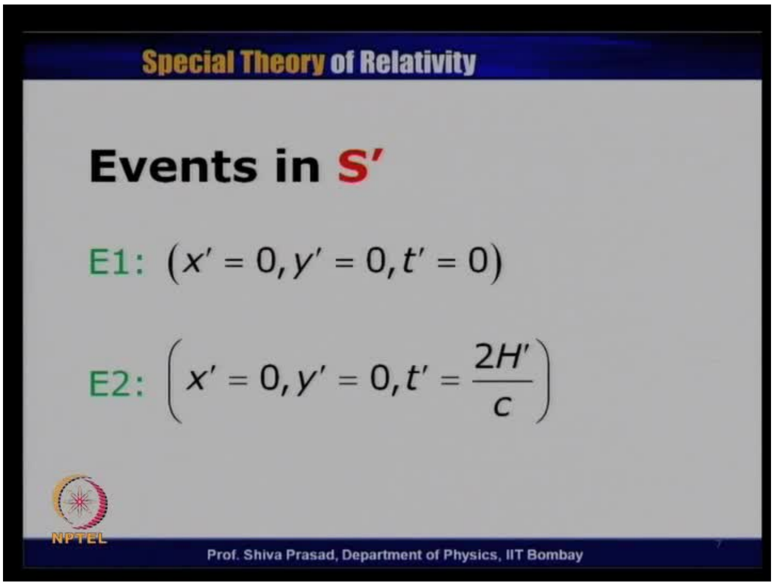

So, now, let us tries to write the events in this particular for these two events in S prime frame of reference. So, let us assume that light is emitted from the origin, so at the point when the light was emitted that point was the origin of S prime, in this particular case z axis is not involved because, we are looking only at the x direction and the y direction. So, I am not writing the z co ordinate but, we got the origin, so I can write that x prime equal to 0 and y prime equal to 0.

Let us, also assume the time according to the S prime observer was also 0 when light was emitted. So, it is easy to write the co ordinates of event one, which is x prime is equal to 0 y prime is equal to 0 and t prime is equal to 0. So, the light was emitted from the origin of S prime and when light was omitted emitted at that time the work of the observer S prime was showing a time equal to 0. Now, event 2 was discussed is that light goes upwards and comes back.

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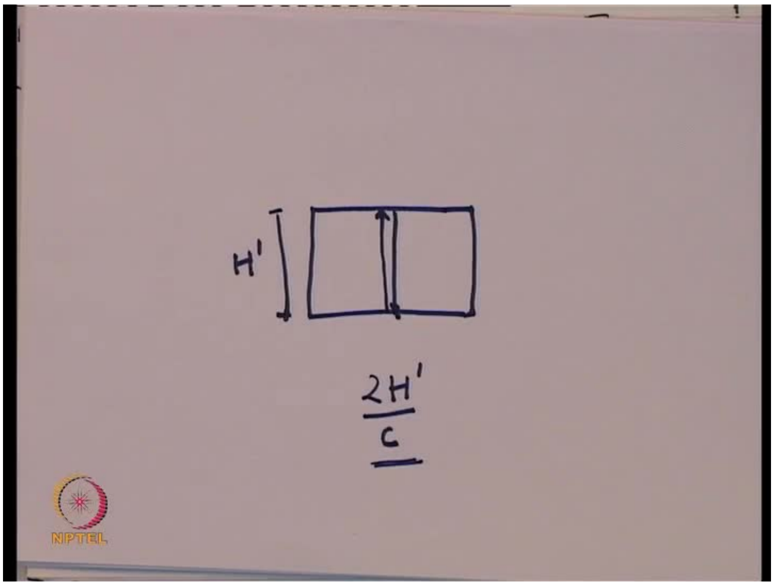

So, light goes upwards this distance is H prime then light comes backwards, again it travels at the distance H prime, remember the every information has been given in S prime frame of different. So, the total distance that light would travel, will be H prime going up and H prime while coming down. So, total distance is 2 H prime and we know that the light travels with the speed of light therefore, if I have to find out the time taken by the light to go up and come back, will be the total distance 2 H prime that the light has traveled divided by the c the speed of light.

So, it means the second event must have occurred at a time, which is 2 H prime divided by c. When the light is received back at this particular point remember that point is again at the origin because, if light goes vertically upwards and comes down it is received back exactly at the same point. Therefore, x co ordinates and y co ordinates of the second event has not changed because, both these events occurred exactly at the same point.

The light is emitted from this point, goes up goes up and comes back in the particular point, both these events light been emitted and light been received, occur at the same point which is the origin of S prime frame of reference. But, the second event occurred later then the first event because, light to be finite among time to go up and come down. So, the second event must ever occurred at a time 2 H prime divided by c.

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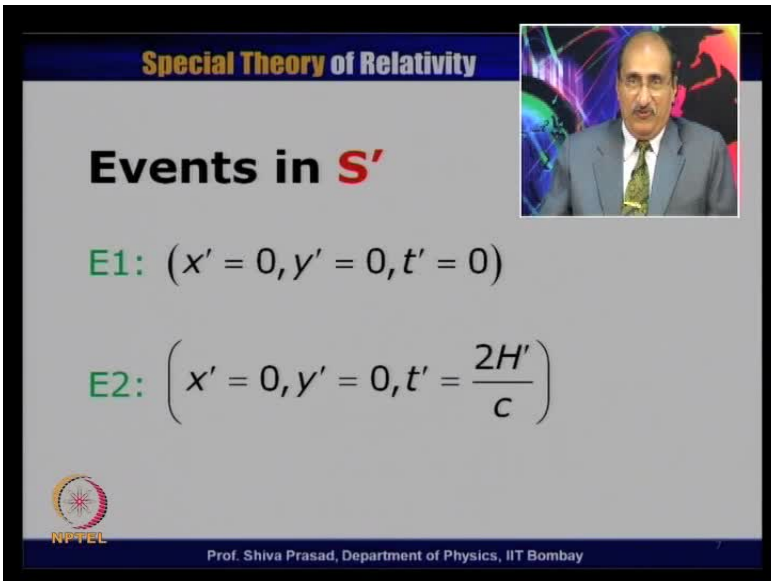

So, this is what I had written here this particular transparency. That for the event number 2 x prime is equal to 0, y prime is equal to 0 because, the event does occurred at the origin and it occurs at time t prime is equal to 2 H prime divided by c. So, this is these are the co ordinates of the event as seen in S prime frame of distance.

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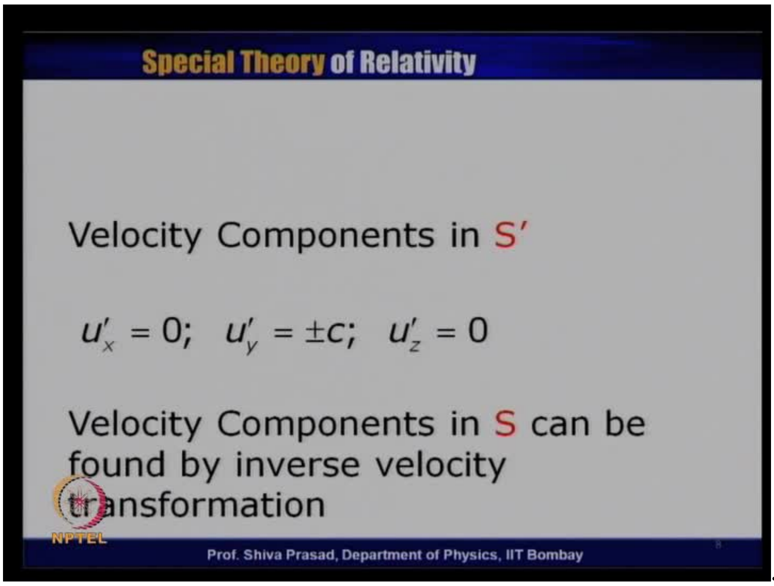

Now, let us look at the velocity components. Again with reference to an observer in S prime frame of reference, when the light is moving upwards we had all the direction pulse y direction. So, the light had only a y component, upwards when the light was returning down again it has only the y component because, light has as seen in S prime frame of reference, is only moving either in pulse y direction or in minus y direction, it has no component of velocity in x direction.

That is what an observer sitting on the train, sees that light goes vertically upwards, vertically downwards. So, light never moves in horizontal direction or any other direction, so; obviously, it means that u x prime must be equal to 0 because, it has no component, no x component of velocity and it has no z component of velocity. So, u z prime is also be equal to 0, it is only u y prime which is not equal to 0 and this is pulse c while going upward and it is minus c while going downwards, while coming downwards.

So, I have written u y prime is equal to pulse minus c where, the signs have to be interpreted appropriately, by choosing pulse sign wherever I am referring to the motion of light going upwards and I have to choose minus sign, when the light is coming downwards. So, these are the velocity components of the light has seen in S prime frame of difference. Now, my next question is that, what will the velocity component as seen by an observer on the ground which I am calling is S frame.

Remember, it is a disk ground frame the disk train is moving with the velocity v. Now, remember all the information has been given in the S prime frame of reference and if I have to find out the information S frame, we have to use inverse transformation. So, what I do, I would use an inverse velocity transformation to find out what will be the velocity components of the light has seen is S prime frame and I am sorry seen in S frame, which the ground frame of difference.

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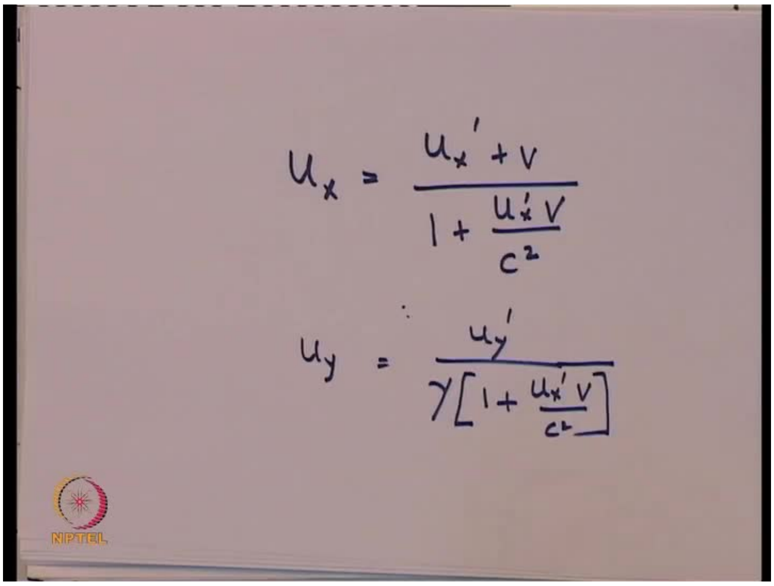

So, let us write the velocity transformation equations, velocity transformation equations are u x prime. In fact, I will write u x is equal to u x prime pulse v 1 pulse u x prime v by c square, this is as for as the transformation of x component is concerned. Remember, we have used plus sign here and I had used plus sign here because, this is an inverse transformation.

Similarly, I had u y is equal to u y prime divided by gamma 1 pulse u x prime v divided by c square I am not writing for z component because, as for the z component is concern this is sort of irrelevant as for this particular example is concern. So, let us use this particular example, this particular equation transformation equation and try to find out u x and u y and to see it what are those values, as it will seen in S frame of reference.

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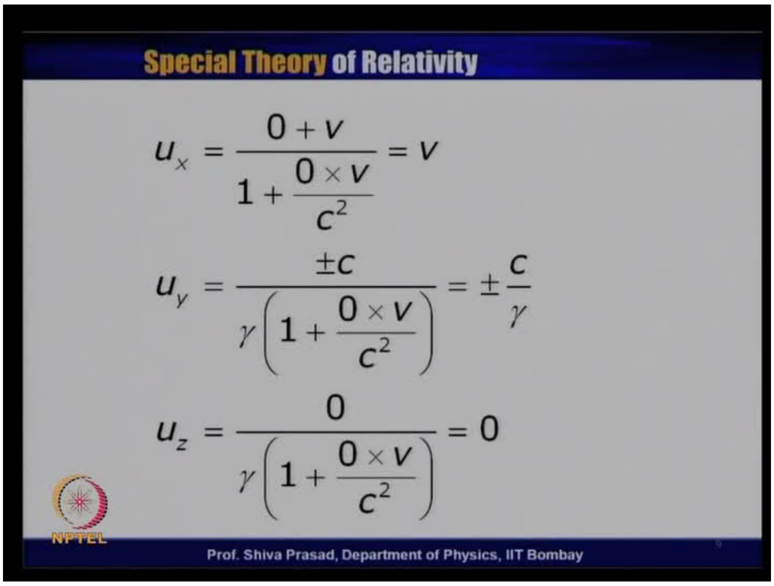

So, let us look at the equation, we have u x we had u x prime which was equal to 0 remember, in S prime of different there is no x component of the velocity. So, this is 0 v is the relativity velocity between the frame, so it is anyway present here, divide by 1 pulse u x which is 0 multiplied by v divided by c square. So, this particular thing gives you 0, so in denominator you are left with 1 and this becomes 0 pulse v divided by 1 which is just equal to v.

So, u x turns out to be equal to v which is the relative velocity between the frame. Let us look at u y this pulse minus c again, we said that the sign have to appropriate interpreted pulse will be when it going upward, minus when it is going downwards. So, I just putting one equation pulse minus c divided by gamma multiplied by the same factor here, which here any we discussed one. So, what you get is pulse minus c divided by gamma.

So, the y component of the velocity of the light as seen in S frame of reference is pulse minus c by gamma of course, u z is anyway 0 because, the u z prime becomes 0. Remember, the transformation equation of y and z component look appear very, very similar. So, what we have done is no found out the velocity component, as seen by an observer in S frame.

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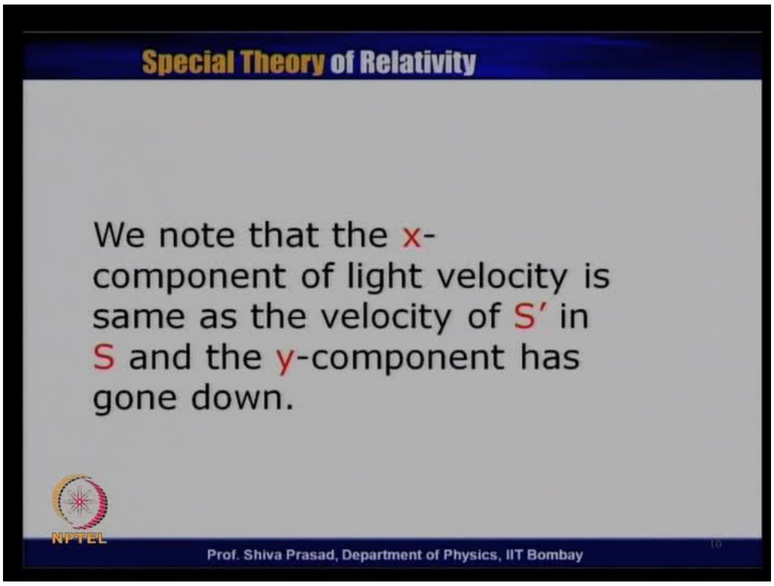

We just note that according to the observer in S frame, the light is also travelling in x direction it also a component in x direction.

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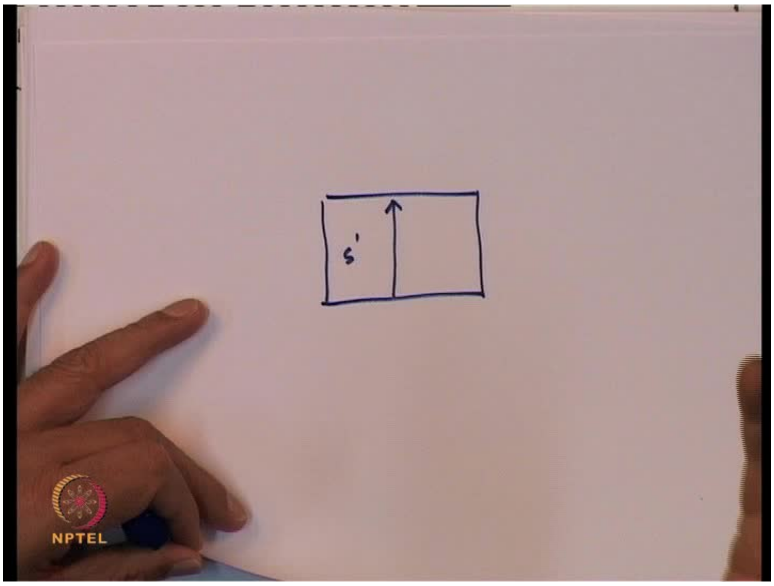

So, give similar to what we had observed classically that if a ball is thrown vertically upward as seen by S prime this ball does not really move upward as seen by the observer here. But, also develop x component of the velocity, which is same as the velocity of the train that is what we seen in classical mechanics, also that is what we are seeing here.

Also that this particular light now, has developed the x component and this x component is same as the velocity of S prime as seen in S which is the same velocity of the train.

But, now in contradiction to the classical mechanics we have found even the y component has changed. And the y component is now, pulse minus c by gamma remember, this has to be done because, as seen in S frame also the speed of the light has to be same which is c. So, if it has picked up the velocity component in the x direction, in the y component the velocity component must come down in order to maintain the speed of light to be see, also in S frame of reference that is, what this transformations have to do because, that is the first postulate of that is the postulate of special theory of relativity.

Now, let me just first check whether I am really getting the velocity of like to be c as seen in the S frame of reference we let us first to a quick exercise because, that was I expect y component has come down, x component has picked up from 0 it gone to v, y component has come down but, I expect that still the speed light must remain c in S frame of reference let us verify it.

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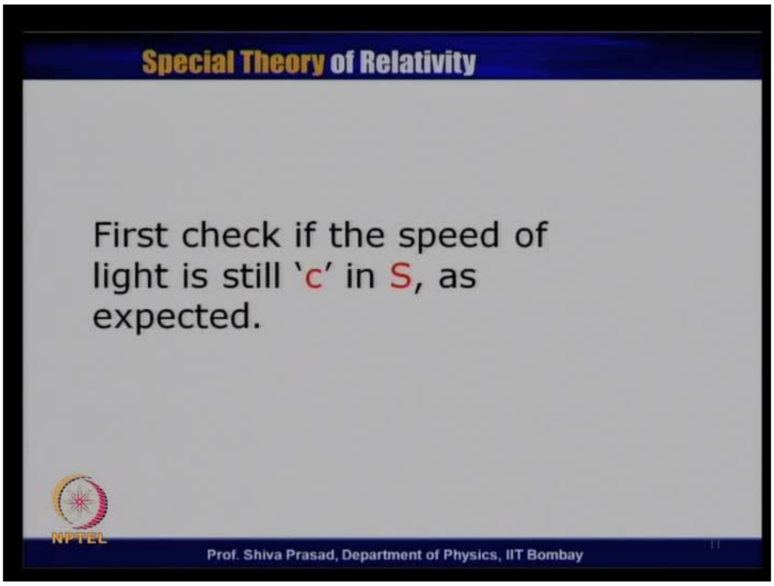

So, as I have said first check, if the speed of light is still c as seen in S as expected as we expected as per the postulate of special theory of relativity.

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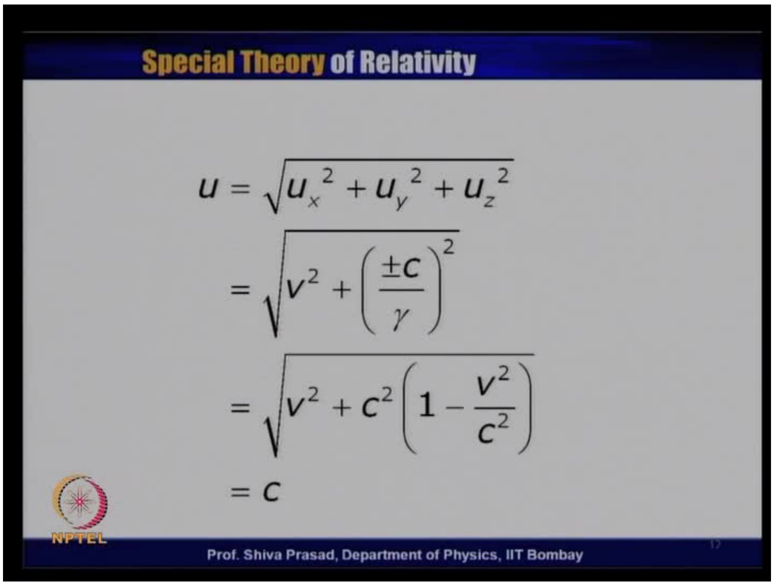

So, when I calculate the magnitude of the speed of the light, I will use this particular standard formula u is equal to under root of u x square pulse u y square pulse u z square u z is anyway 0. So, let us note particular thing, we have seen that u x is just equal to v relative velocity of with frame for u x square I put v square, for u y square we just now, seen that u y was pulse minus c by gamma. There these signs have to be appropriately interpreted but, ones I square it whether it pulse sign or minus sign both will make it pulse. So, this particular equation would just reduce to v square pulse c square by gamma square. Now, what I do is just substitute the value of gamma square.

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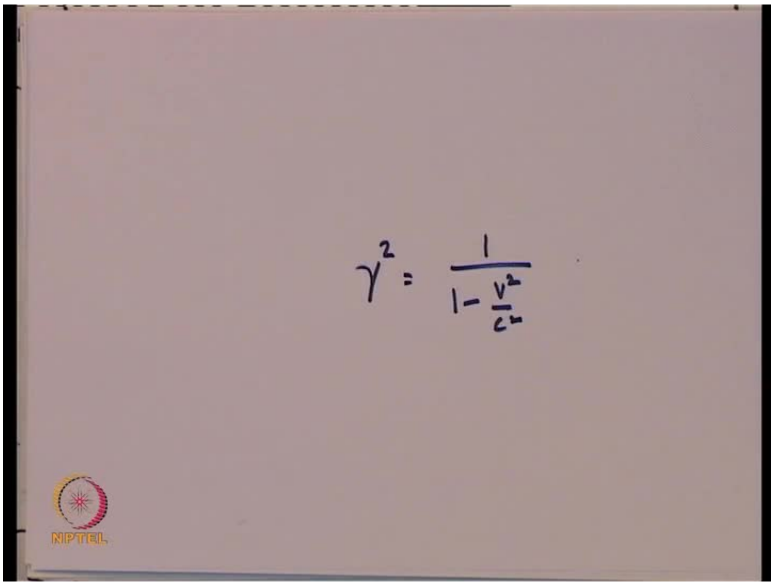

As you remember, gamma square will be equal to 1 upon 1 minus v square by c square because, this gamma square in denominator. So, this 1 minus v square by c square will come into the numerator that is what I written in this particular transparency.

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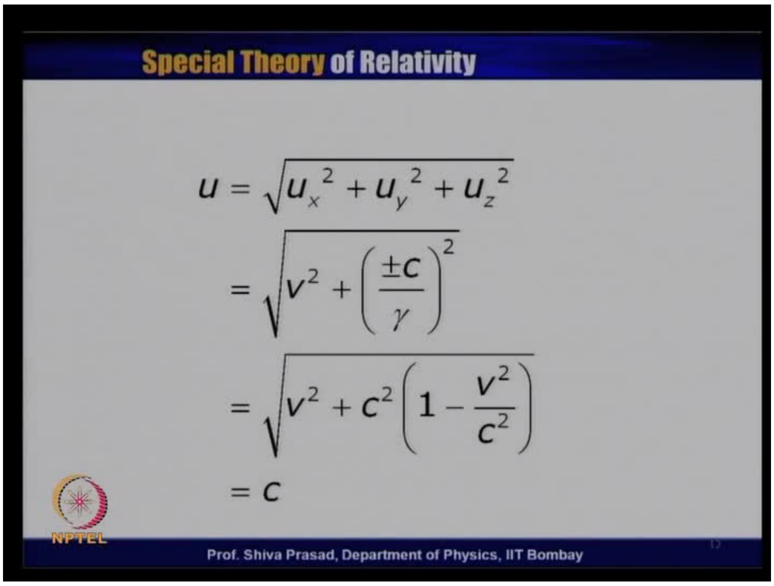

This is v square pulse c square by gamma square and this particular gamma square has been written as the c square is still here and this gamma square has written in written as 1 minus v square by c square. If you expand here, you will get v square pulse c square and this c square gets multiplied to this particular factor, this become minus v square. So, you will have v square minus v square that will cancel giving you just C Square and u becomes equal to c.

In fact, becomes pulse or minus c and depending upon the way, you look out and I looking only the magnitude therefore, I am putting only the pulse sign. So, what I have shown, is that the speed of light in the turns out to c as seen in observer s as expected from the second postulate from the postulate of special theory of relativity.

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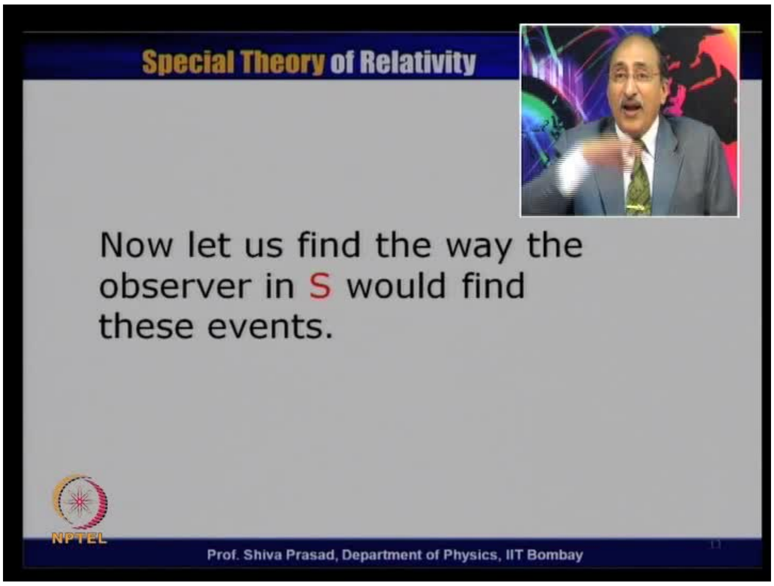

So, this is what I written now, let us find now let us find the way the observer in S would find these events. Now, let look and how the observer S would find these events now, according to the observer S the light has picked up x component of velocity, it means the light is not going vertically upwards but, it is going at an angle. And the angle will depend on the value of gamma of course, because, u y depends on the value of gamma. So, if the observer in the S frame wants to look up the motion of the light.

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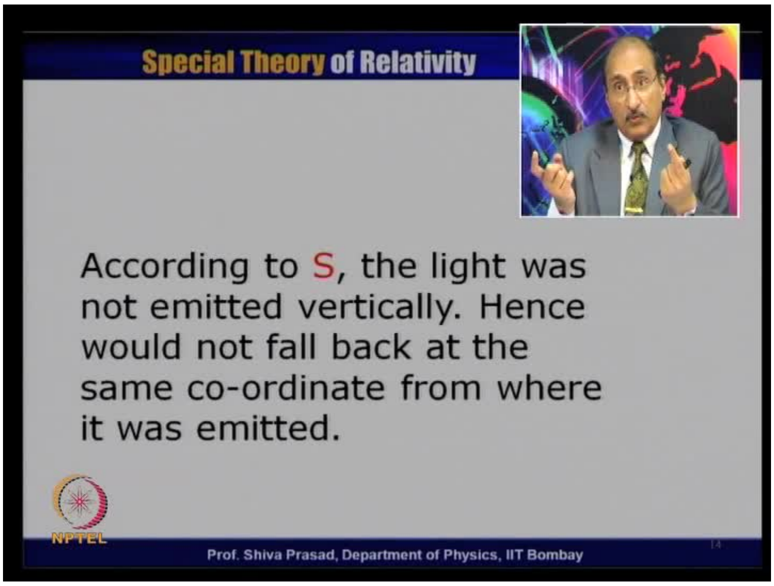

This is what the he would probably visualize, he would see that the light has an x component and also has a y component and is not emitted vertically. What I have written according to S the light was not emitted vertically. Hence would not fall back at the same co ordinate where it was emitted because, the light does not go upward goes like this, then it falls back it have to fall in different point in x is according to him the x co ordinate must have changed for the second event.

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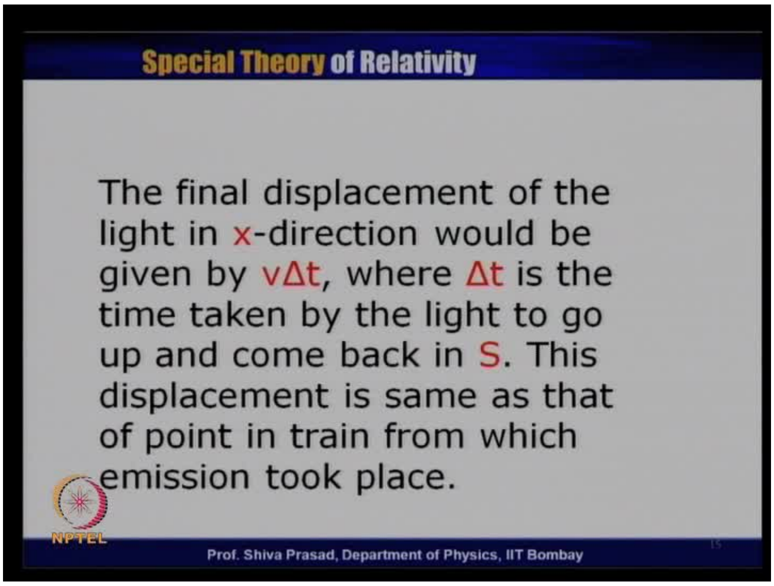

So, the interpretation of this particular observer in S would be that the second event has occurred not at the same value of x but, has shifted and shifted by how much, it depends on how much time light took to go up and come back, as seen by observer in S frame. Then whatever was the x component velocity, in that frame multiplied by that particular time, let me just explain the point little more.

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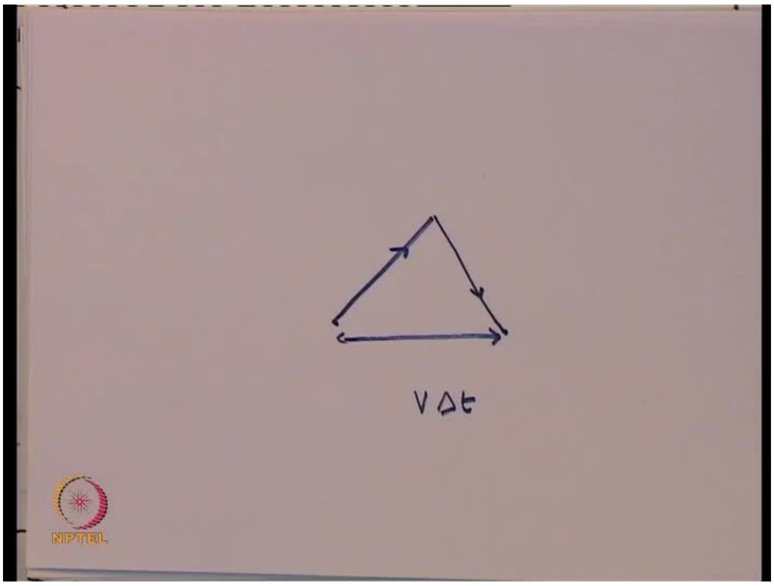

So, according to the S observer the light is moving at an angle like this, it comes back and does not hit at the same point, same co ordinate as it has started. So, a light will go like this and come back like here, if I to find out what is the displacement, this displacement would depend on the x component of the velocity light which is v. And the time taken by the light go up here and come back here, as seen by the S observer if the time interval is delta t then this distance must be equal to v delta t. Because, v is the x component of the velocity and delta t is actual time taken for the light to go up and come down.

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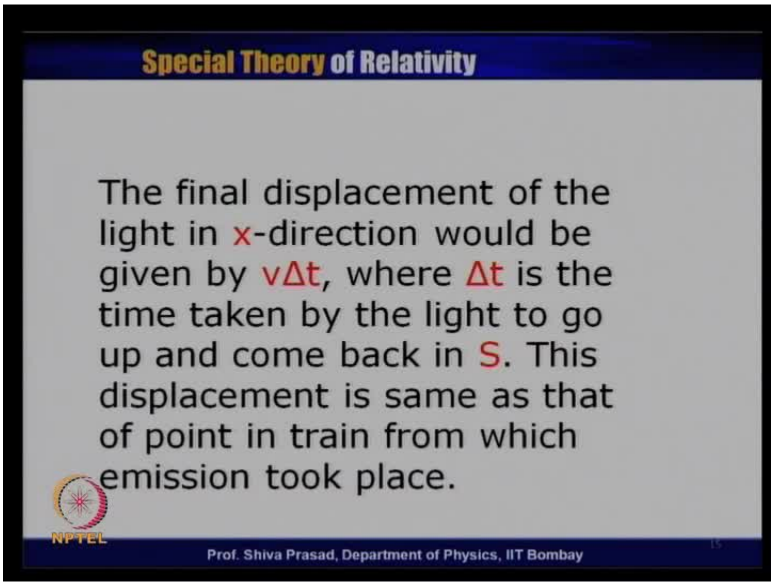

So, it is not written the final displacement of the light in x direction, would be given by v delta t, where delta t is the time taken by the light to go back in s frame. And this displacement happens to be same as the displacement of that particular point of the train.

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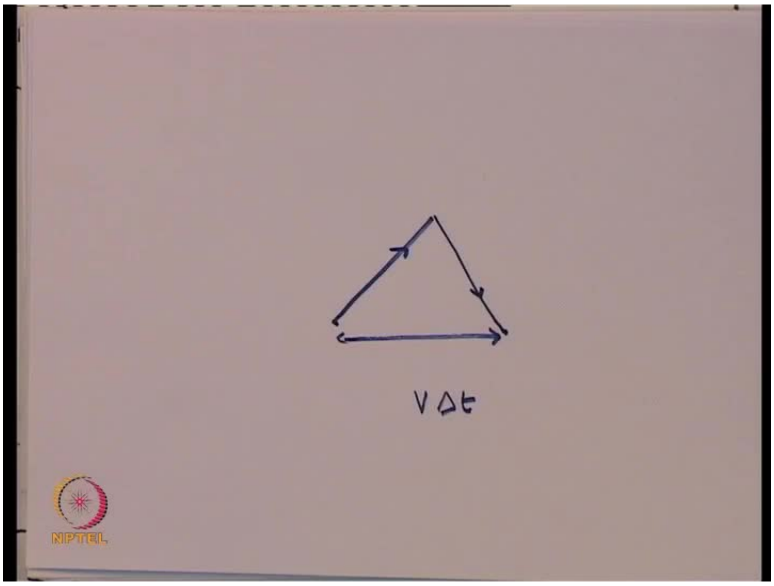

Because, the noise observer S at this point, this particular point also is moving with the same speed, which is relative speed of the train. So, this point also moves exactly by the same distance v delta t and the second event occurs also at the same shifted by the exactly the same x co ordinate. So, according to the observer S prime this point also moves by the distance v delta t and the shift in the x co ordinate of the two event is also equal is to v delta t, that is was the observer in s frame would see. So, I have put this particular thing in the picture.

Remember, this height H is going to be same in the two frames as in S frame because, as we discussed in one of the earlier example, there is no change in the y component. The length contraction occurs on the x component, as per the y component is concerned because, y prime is always equal to y is not a transformation. So, as for as going up and going down concern the heights are seen to be same. So, though I am writing H to be to make it the size to be seen by an observer in S frame.

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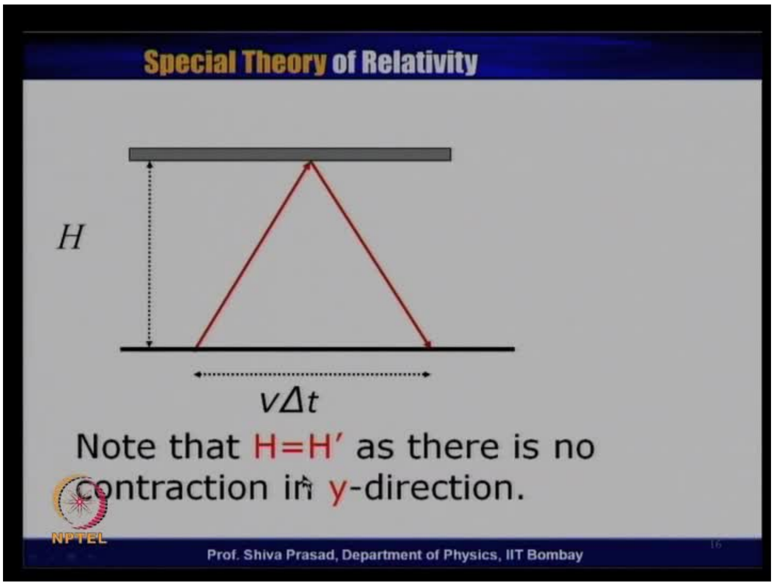

But, we know that H has to same value as H prime because, this distance is along the y direction. So, an observer in an S frame sees, that light goes this way comes back like this and the x difference in the x co ordinate of the two events, will be v delta t it does not occur at the same point or same co ordinate from which it was turn. The second event occurs at the different co ordinate, this is what will be the perception of observer in S frame.

Of course, this particular point from which the light was thrown that also moves by the second one same amount. So, as far as the observer S is concerned he will also feel that, the same point from the same point on the train from which the light was emitted, it is received back also by the same, at the same point, but this point as well as the light both are moved along the x direction. So, that was I said move the H equal to H prime there is no contraction in the y direction. Let us calculate the delta t to find how much have been the displacement again I can use a inverse transformation.

Delta t is equal to gamma delta t prime, this is pulse sign because, we talking of inverse transformation v delta x prime divided by c square. This is the inverse velocity transformation delta x prime was 0 because, as for as the x prime is concern the co ordinate of the both the events are same therefore, x prime was 0. So, this will becomes 0 and delta t prime was equal to 2 H prime by c and H prime equal to H. So, I have written as 2 H by c this becomes gamma 2 H by c.

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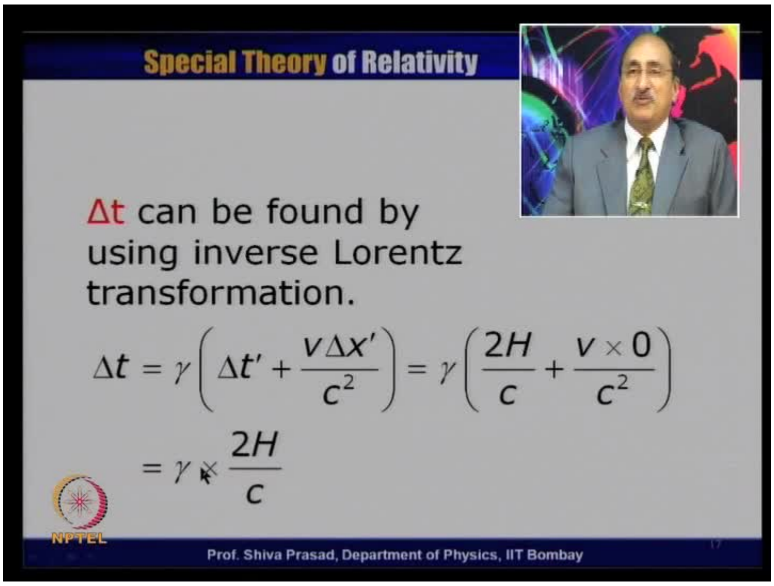

So, according to observer in S frame, the time interval between these two events is not 2 H by c but, gamma times 2 H by c you will realize that this is the same as time dilation formula, which has to happen because, S prime frame of reference both events occur at the same point. So, the time interval in S prime frame of reference was a proper time interval between these two events therefore, in S frame this time interval must turn out to be dilated and that is what we saying, that this time interval turns out to gamma multiplied by 2 H by c.

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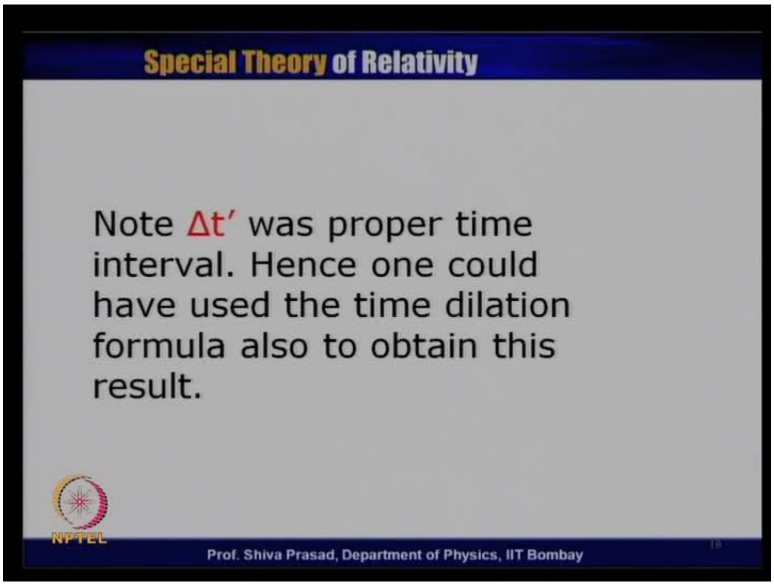

So, this is was I written, note the delta t prime was proper time interval. Hence, one could have used time dilation formula directly for obtaining this result but, we need not really written, the lots of transformation if your fast, we can immediately realize that this is the proper time interval as in S prime frame of reference. So, in S frame you just multiply by gamma I will get back the time interval and delta t of course, it can also be observed in different fashion, we can also find out delta t prime slightly different fashion. If you look from the perception of the observer in S frame of reference.

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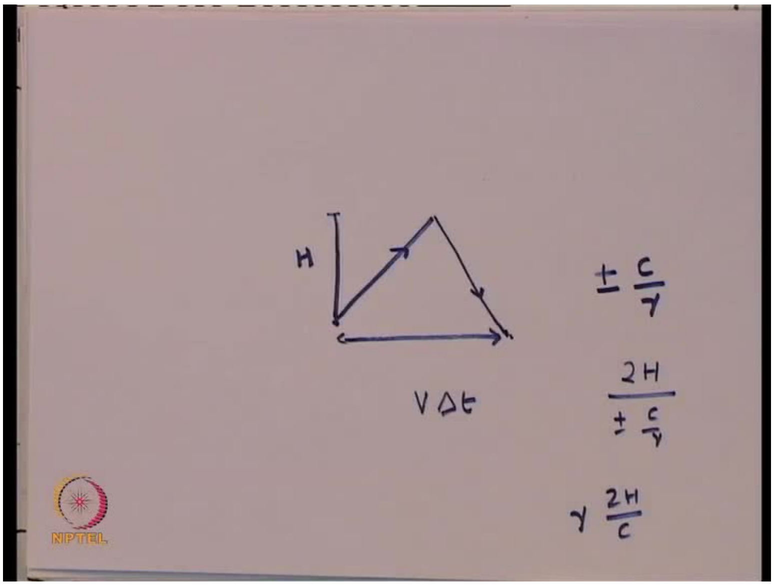

This particular light traveled in vertical distance of H. So, vertical component of the velocity ver vertical distance travel by the light was H and the vertical component velocity was pulse minus c by gamma. So, find out the distance if I know the speed, the vertical component of speed I can find out how much time will be taken. So, the total distance traveled by the light total vertical distance travel by the light, as seen in the frame of reference was H going up, H coming down and this is achieved by a velocity y component of the velocity, which pulse minus c by gamma.

Therefore, I can always find out that total time will be 2 H divided by the y component of the velocity which is pulse minus c by gamma. So, this gamma goes upwards and this becomes 2 H by c to gamma because, I am only looking only at the (( )) time interval pulse minus does not make any difference. So, I can also find out this particular time interval delta t by using this particular way of doing it, I could have also done a direct inverse transformation on delta x and obtained, exactly the same result which I have shown in this particular transparency.

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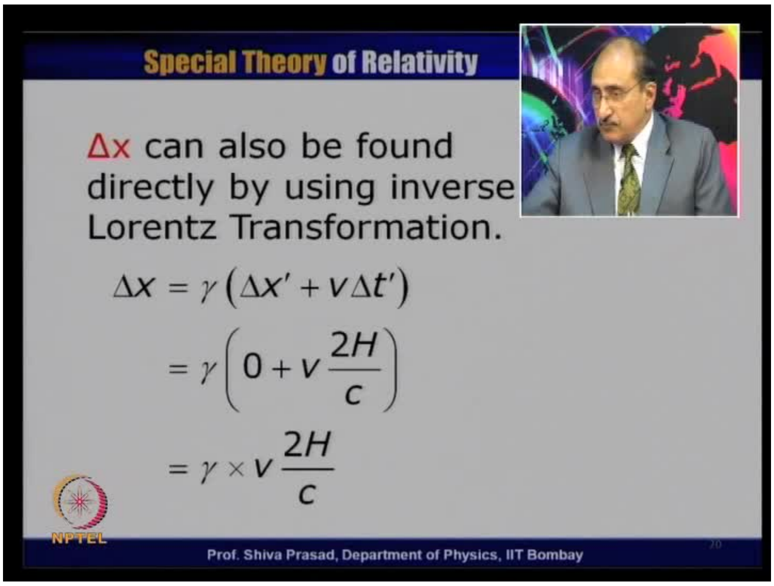

Delta x can also be found directly by using inverse Lorentz transformation, delta x is turned out equal to be gamma delta x prime pulse v delta t prime delta x prime equal to be 0 delta t prime equal to 2 H by c. Again I remain that is x prime is equal to H therefore, this is equal to gamma times v multiplied by 2 H by c we get the same result. So, if you know the delta x value, we can always find out the v delta t is the same result.

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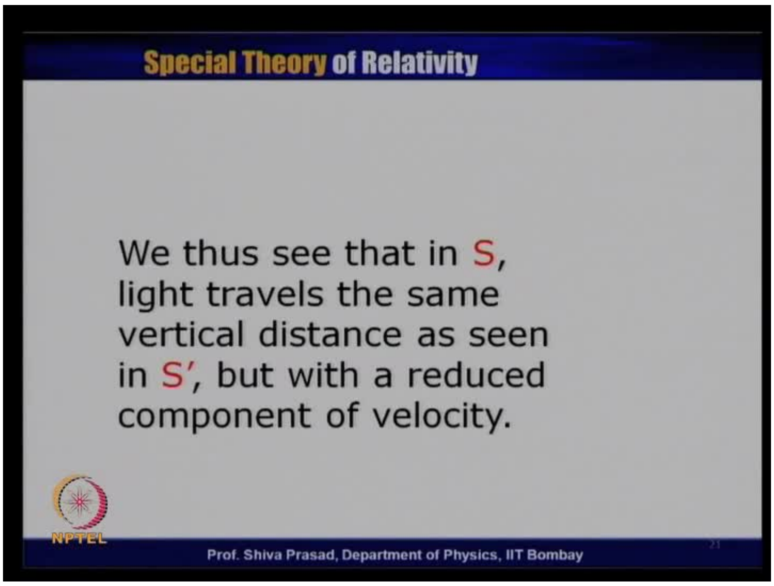

So, we thus see that in S light travels with the same vertical distance as seen in S prime but, with a reduced component of velocity.

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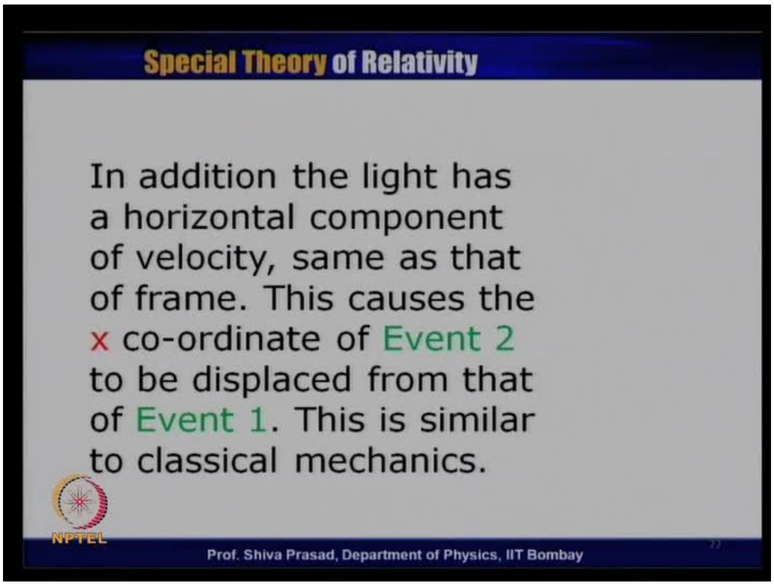

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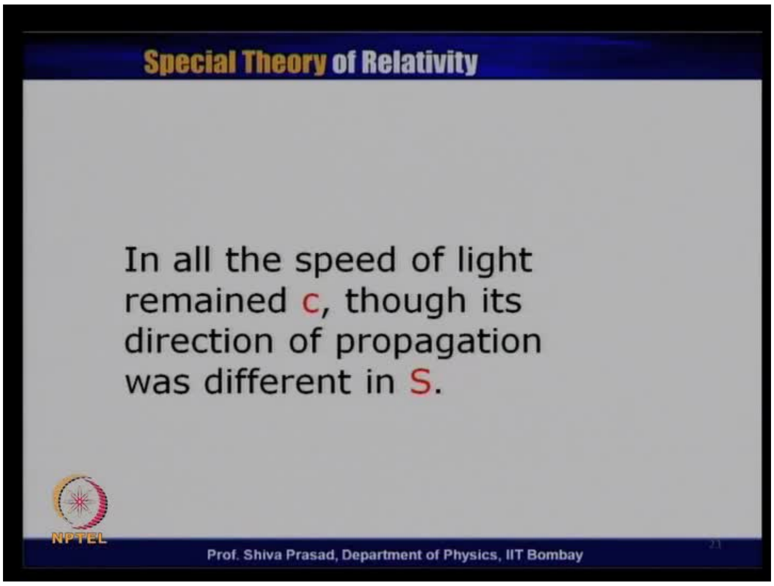

In addition the light has horizontal component of velocity same as that of frame. This causes the x co ordinate of the event 2 to be displaced from that of event 1, from that event 1 as in as frame. This is similar to classical mechanics because, the classical mechanics also this event will appear to be a displaced value of x, the difference here of course, is that in relativity they component of the velocity has changed as seen in S frame of reference to make the speed of light same.

In all the speed of light remained c though it is direction of propagation was different in S. Now, let us look into the slightly different issue, generally it is effective almost everyone sort of knows that one of the outcome are special theory of relativity, was that speed is greater than the speed of light were prohibited. We always said that information does not or should not travel or could not travel, with the speed greater than speed of the light this was consider as the one of the outcome the special theory of relativity. We are not discussed this point, so far. So, this one particular point which was mathematical which has we see it very easily that almost in every expression that we are using transformation the gamma appears which is equal to 1 upon the root 1 minus v square by c square.

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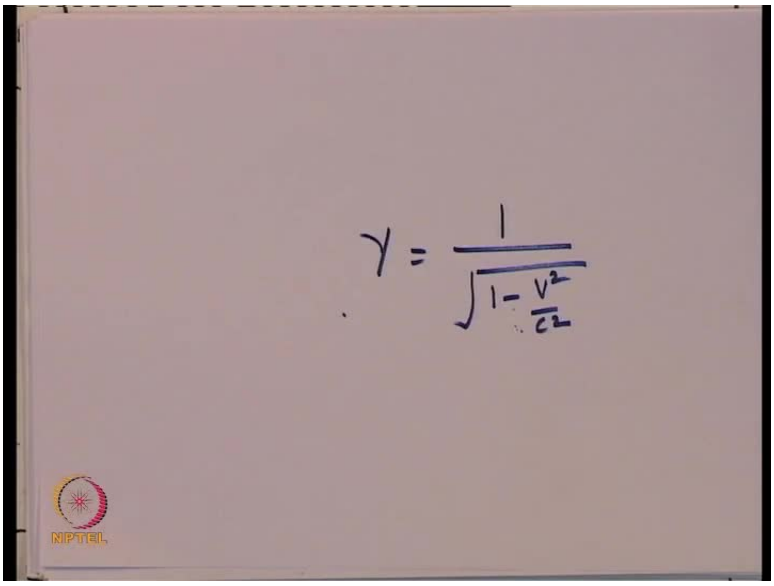

If v really happens to be larger than c then this particular factor, would turned out to be larger than 1 therefore, this when I take under root the gamma would turn to be imaginary. When gamma turn to be imaginary almost all the transformation that I am talking about x transformation, t transformation everything there may be an imaginary number and do not really know, how to handle all these imaginary number or how to physically interpret. So, there is an mathematical reason that if we had really v is less than c, probably will comfortable because, we would not bother about any thing one of the simplest way of looking into this particular thing.

But, there is an another important physical reason of why we think that is speed is greater than speed of light should not be possible. And let me some spend time now, to explain this particular aspect and this I will explain by using what I call as the order of events. So, let me first explain by basic things before I come to this particular concept of speed greater than or speed being greater than or speed smaller than speed of light.

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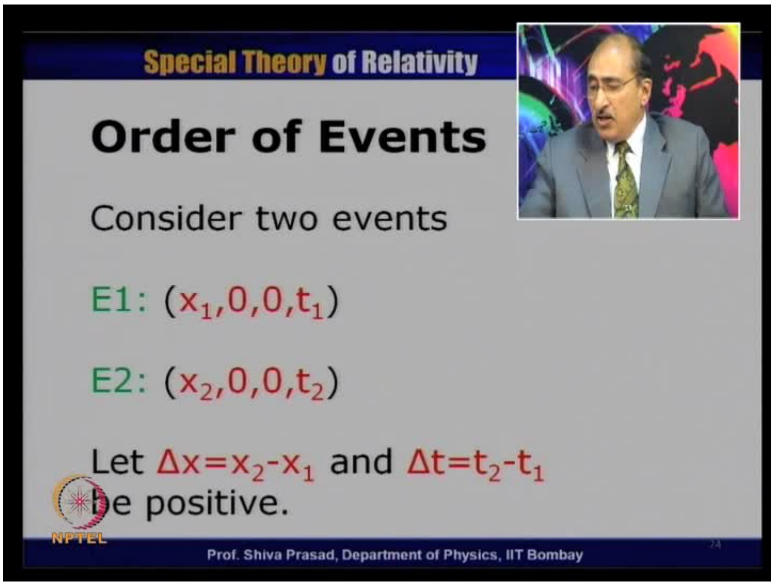

So, let us consider a two events, any two events and these two event let us suppose now I am writing the full expression let us call event number 1, E 1 which appears at a co ordinates x given by the x 1, y co ordinate is 0, z co ordinate is 0 and it occurs at a time t 1 any arbitrary event. Let us look at the second event, second event occurred at co ordinate x 2 y and z I am taking 0 to make things simple and occurs at a time t 2.

Now, for simplicity let us assume that x 2 minus x 1 is positive which I am calling as delta x. So, delta x I am defining x 2 minus x 1 similarly, let me assume that t 2 minus t 1 is positive t 2 minus t 1 I am deciding, defining as delta t. So, I am assuming that the delta t and delta x both are positive remember, both the events are seen by the particular observer in a frame S and two events have occurred such a fashion that event delta x is positive, delta t is positive, delta y delta z is equal to 0 that I never bother.

Because, y co-ordinates and z co ordinates are assumed to be 0 or to be same or to presides I have written to 0, so x takes 0. Now, my question is that if delta x is positive and delta t is positive, is there a frame in which delta x can be communicative if of course, if I go to different frame call it as delta x prime. Delta x prime has been communicative and delta t prime has been communicative one or both of them communicate, is it possible.

So, let us discuss this particular aspect little bit more in detail, that if delta x is positive and delta t is positive is it possible to have a frame, in which delta x can change sign delta x prime, becomes negative or delta t can change the sign becomes t prime becomes negative or both can be communicative, is it possible to have any such frame. So, this the question that I am trying to understand and trying to concentrate on this particular thing.

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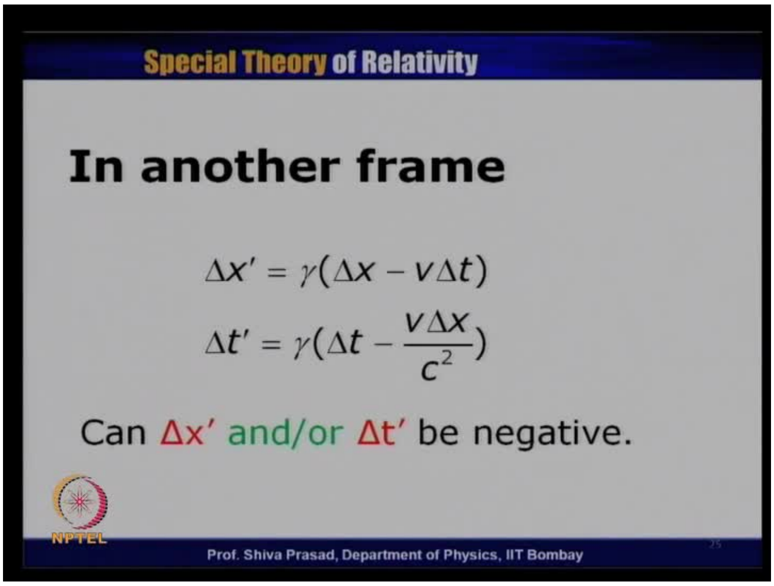

So, let us look at the Lorenz transformation. The Lorenz transformation have applied in deferential form, this we have discussed even earlier at rather I writing x I can use x 2 prime, x 1 prime taking the difference but, I have taking and writing in a differential form. So, I am writing delta x prime, delta t prime directly I am transforming and this is given by Lorenz transformation, which is delta x prime is equal to gamma delta x minus v delta t.

Similarly, delta t prime is equal to gamma times delta t minus v delta x divided by c square, exactly the same as the Lorentz transformation I repeat the question, can delta x prime and or delta t prime be negative. Now, let me divide this particular problem into two different cases, one I will call case 1, another I will call case 2 and I separately look into the case 1 and case 2 and trying to answer this question is that, if delta x to be positive delta t is positive can delta x prime delta x prime be negative.

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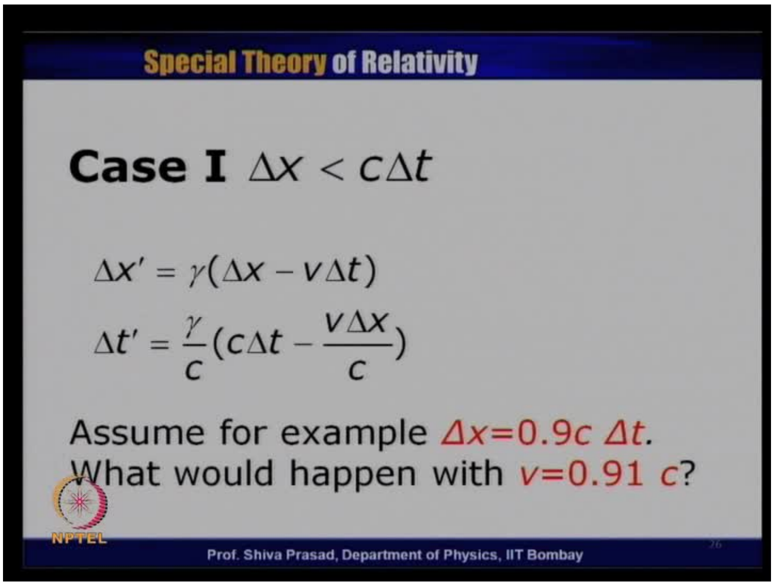

So, let us look at the case 1, which is delta x is less than c delta t. So, we specifically at looking at one particular case where delta x is less than c delta t that is my case 1. I have rewritten this equation delta t prime equation have slightly rewritten, I have taken c out of that particular bracket. So, this becomes gamma bar c because, I have taken c out of the bracket, so there is a c appears here. So, this becomes c delta t remember if I expand then it becomes c delta t and there was c square here, I have taken one c out here this becomes v delta x divided by c.

So, same equation I have written just to make things little simple interprets I have taken the c out of the this particular bracket. So, this is the same equation which I have written slightly different fashion, let us take specific example let us assume that delta x is equal to 0.9 times c delta t. So, the delta x is less than the c delta t because, delta x is only 0.9 times delta t.

Now, let us assume that the v is equal to 0.91 c which is slightly larger than this particular factor of 0.9. So, I have chosen them for particular value of v which is 0.91 c slightly larger than this factor 0.9.

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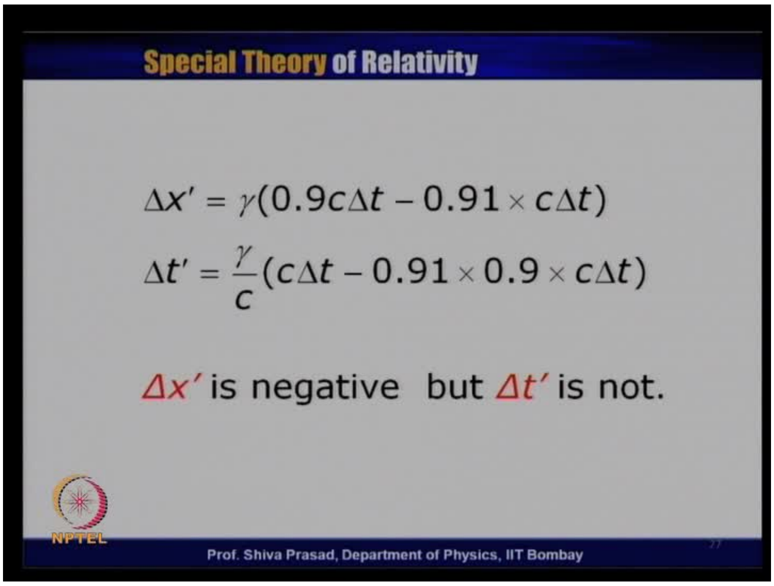

Let us, look and substitute at this particular equation. Let me write this particular equation again.

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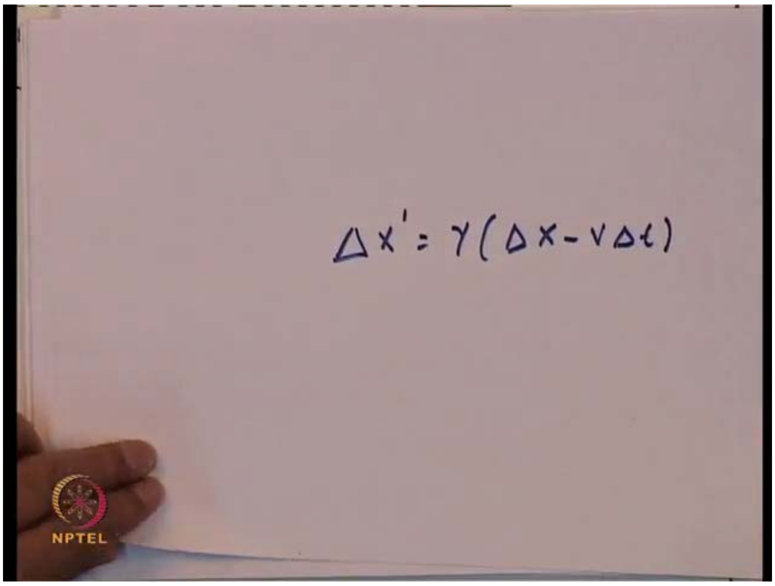

Delta x prime was is equal to gamma delta x minus v delta t.

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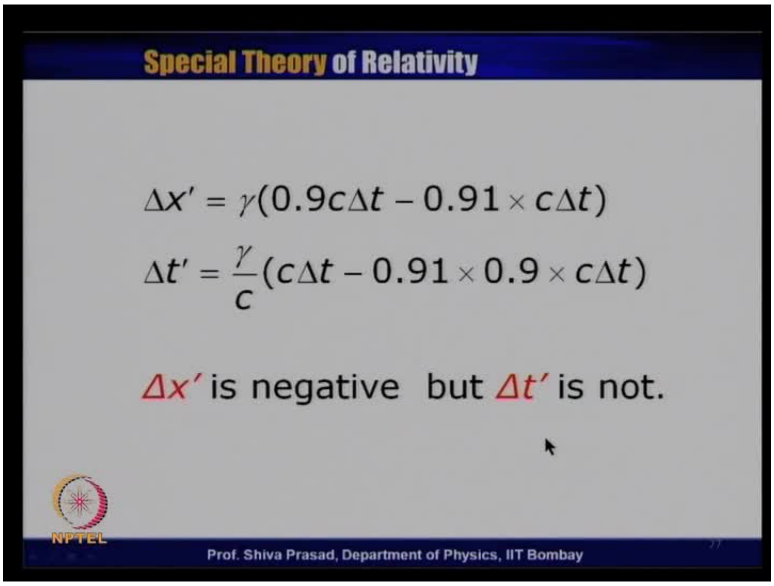

Now, delta x have taken as 0.9 c times delta t, we have taken as 0.91 times c, this c delta t.

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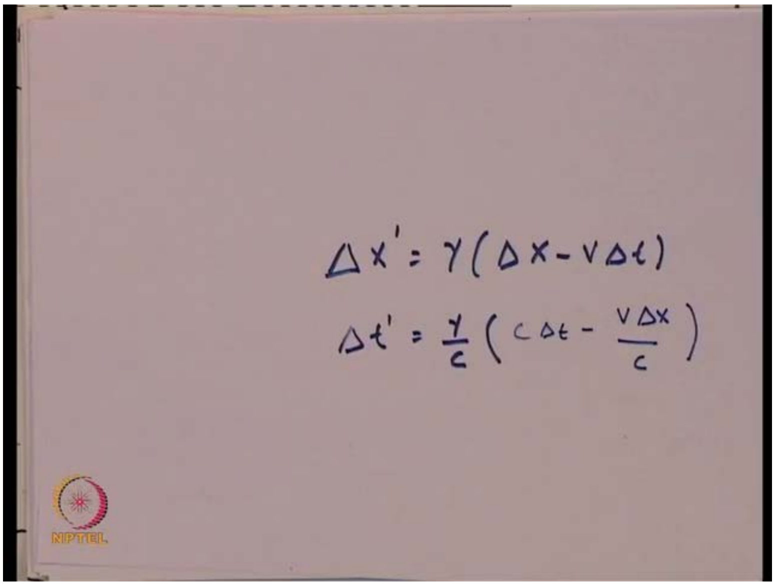

Let me substitute for delta t prime gamma divided by c into c delta t minus v delta x by c.

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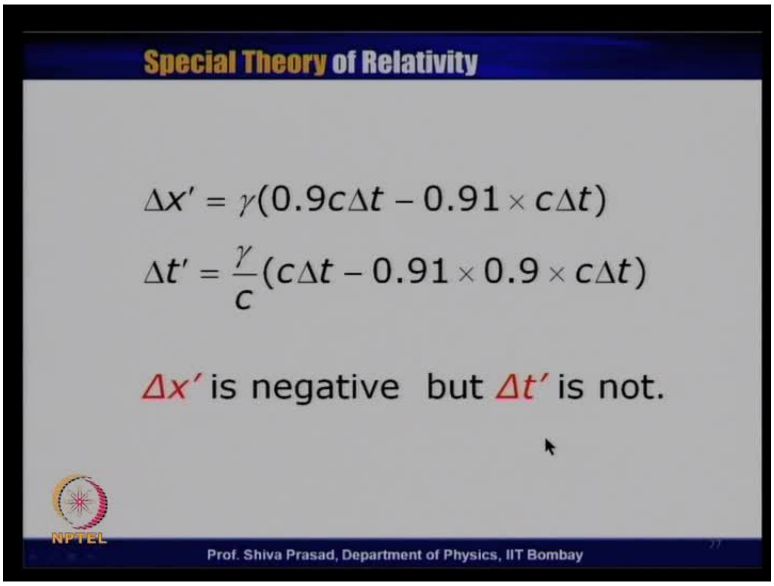

I put it here in the transparency gamma by c, c delta t minus v which we have taken as 0.91 c, that c cancels the down c. And this particular delta x was 0.9 times c delta t are you can very easily see that is if I take c delta t out here, out of the bracket and this becomes 0.9 minus 0.91. So, delta x prime has became negative, look at delta t prime if I take c delta t common here, this becomes 1 minus 0.91 multiplied by 0.9 when I multiply 0.91 by 0.9 I will get the factor which is less than 1.

Therefore, this delta t prime is still not negative is positive. Because, here you have 1 minus something which is less than 1, instead of 0.91 I could have used even a larger value but, still I can always find one particular factor here, which is slightly larger than this to make delta x prime negative. But, because, of this factor which I have used here, is less than 1 whatever value of v I used here, whatever this particular factor I use here I will always get this product to be less than 1, until this factor what I am using here, the ratio between v and c exceeds 1.

So, remember here we have something which has less than 1, in order that if I want to make this more than 1 this factor should definitely exceeding one. Otherwise if I am multiply this by factor still less than 1 this factor will always be turning out to be less than 1. And remember, delta t prime can became negative, only if this factor becomes greater than 1 and all to that this factor becomes greater than 1 and this factor here, is less than 1, this factor here must be greater than 1. So, let us try to consolidate I have said, delta x prime is negative but, delta t prime not.

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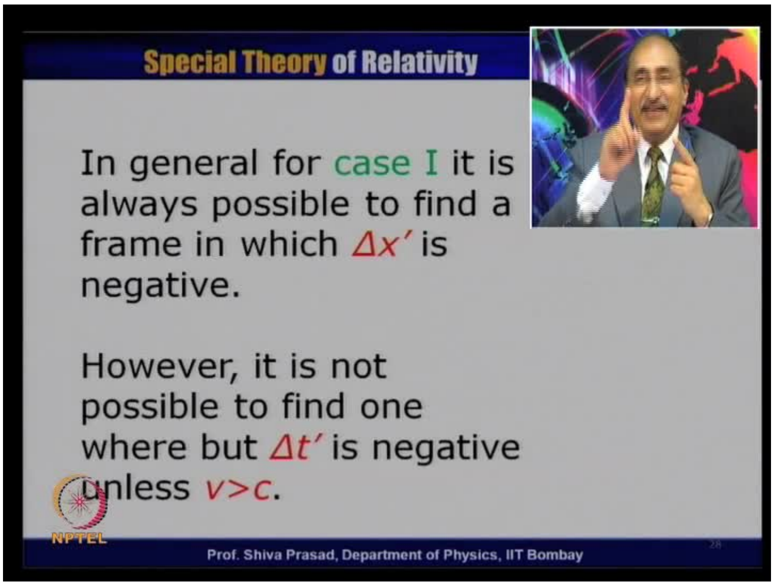

In general for case 1, it is always possible to find a frame in which delta x prime is negative, even it was 0.9 or it was 0.92 then I would have used v is equal to 0.93. But, I will never able to make delta t prime negative unless v becomes larger than c because, only that particular case the ratio between v and c will exceed one. And then is multiplied by a factor which is less than 1, can yield a value which is greater than 1, two factors both are which are less than 1, when they are multiplied they will never give a value greater than 1.

Therefore, in this case delta t prime will never be about to be negative, irrespective of what we use, what we you use unless v is larger than speed of light. But, as far as delta x prime is concerned you could always used in appropriate value v which is less than c and can it still make delta x prime negative.

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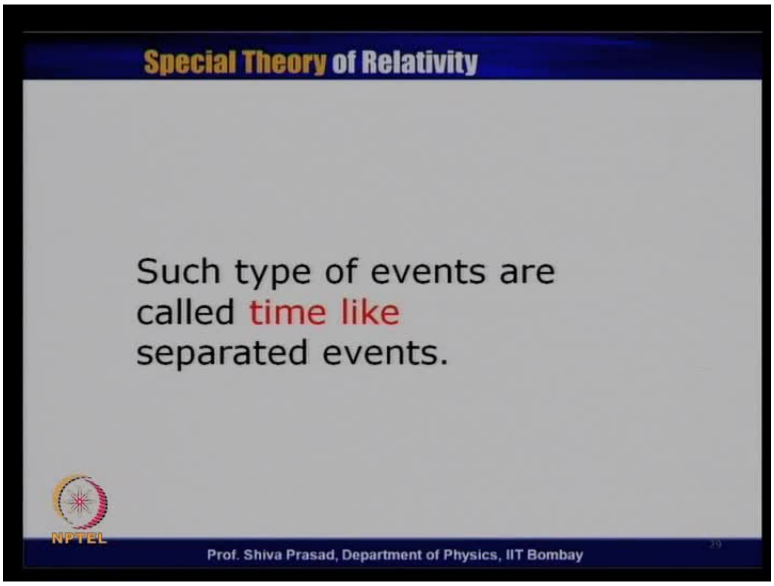

Such type of events are called time like separated events these events are separated as time like time is, so strong is a pure time interval cannot be reversed, the position can be reversed between the two two events I can always find a frame of reference. In which the position of the event can be re re reversed delta x prime can become negative but, I never able to find out a frame in the delta t prime changes the sign unless of course, the velocity of frame difference exceeds speed of light.

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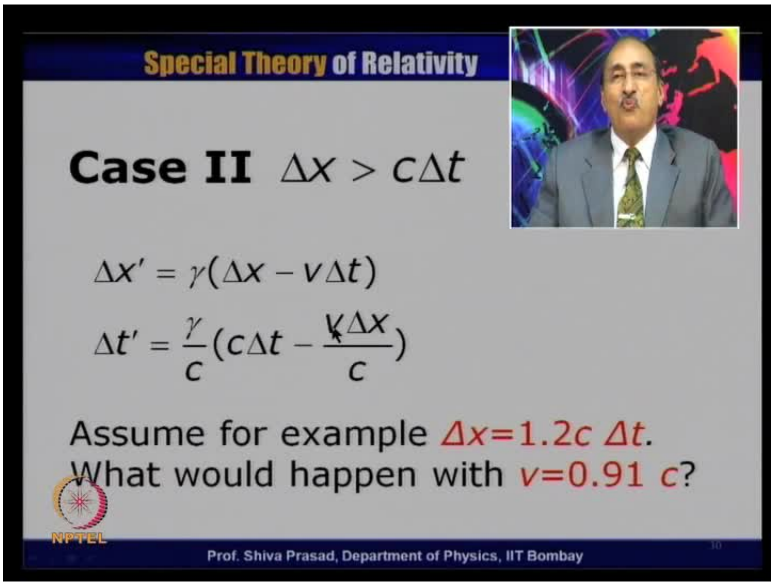

Now, let us look out the case 2. Case 2 is a case when suppose delta x is greater than c delta t, I write exactly the same equation delta x prime is equal to gamma times delta x minus v delta t. I write delta t prime is equal to gamma by c, c delta t minus v delta x by c, exactly the same equations which I have used for case one. But, now because, delta x have assume to be larger than c delta t. So, let us take one example, delta x assume to be 1.2 times c delta t.

So, delta x has becomes larger than c delta t, we still take v is equal to 0.91 c and let us see what happens. We substitute the value delta x is equal to 1.2 c delta t here and for v I substitute 0.91 c. Similarly, here for delta x I choose 1.2 c delta t and for v I choose 0.91 c

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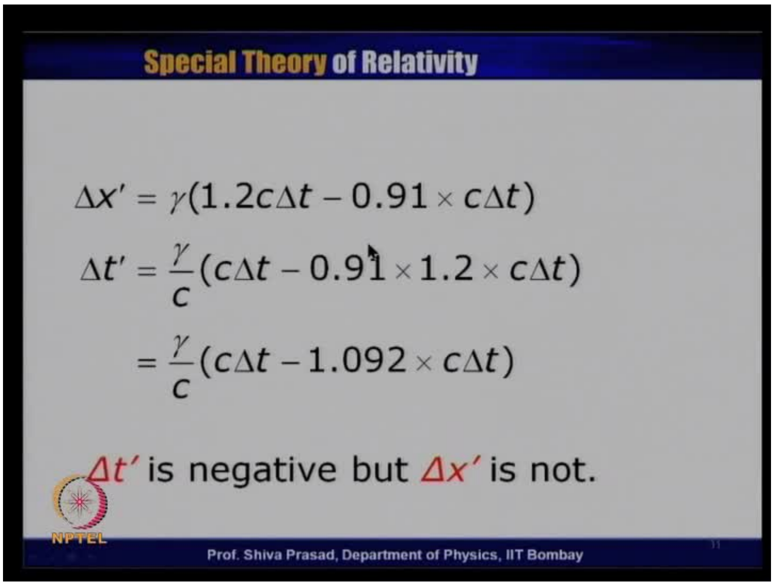

I put it in this equation. Here, if we take c delta t common out of the bracket you will find that here 1.2 and here we have 0.91 this of course, is positive if you look at the delta t prime here c delta t here 0.91 multiplied by 1.2 multiplied by c delta t. If just multiplied by 1.2 by 1.91, you get 1.092 this factor is larger than one if I take c delta t common outside the bracket, you will get 1 minus 1.092 which makes it negative. So, delta t prime will turn out to be negative.

So, my conclusion is that at in this particular case delta t prime is negative but, delta x prime is not. Now, when I say delta x greater than delta t could be I could have be anything but, remember because, this factor is larger than one, unless we itself becomes larger than c this will always remain positive. Therefore, delta x prime will always be positive here, instead of 1.2 if this particular factor was not 1.2 but, what something less I could have still chosen some particular value because, some particular value still greater than 1. I could have chosen appropriated value v, to make this particular factor greater than 1.

So, even if it not 1.2 but, 1.1 I make this factor slightly larger. So, I can still find one particular frame, some particular frame of reference with a very large value of v in which delta t prime will not be negative. So, my conclusion is that, in such type of cases you will never be find a frame of reference, in which delta x prime turns out to be negative, unless that frame moves with the speed greater than speed of light. But, delta t prime you can always find out some frame of reference which still moves with speed less than speed of light but, for which delta t prime will turn out to be negative.

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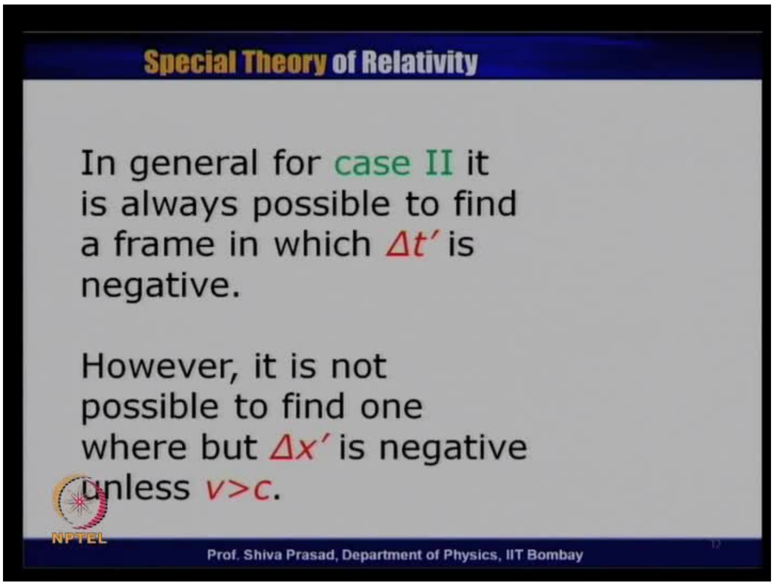

So, this is what I have written here. In general for case 2 it is always possible to find a frame in which delta t prime is negative; however, it is not possible to find one, when delta x prime is negative unless v is greater than c.

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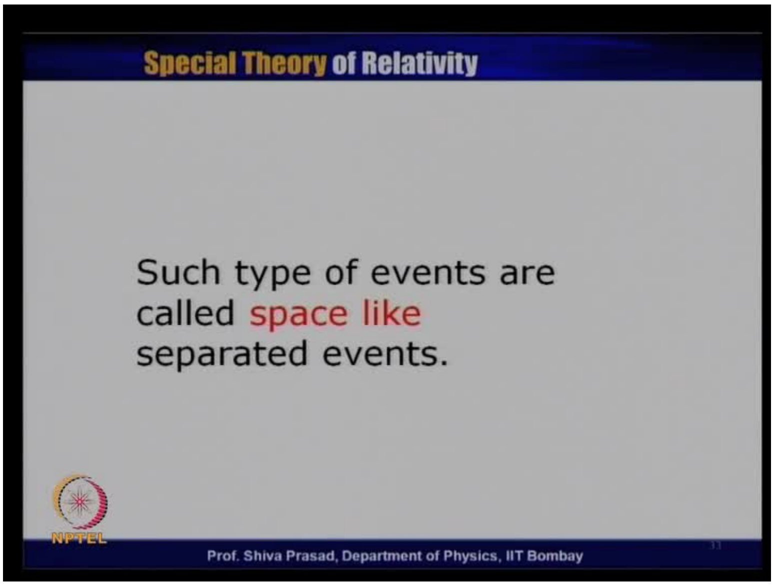

Such type of events are called space like separated events because, this interval is pure space the, space cannot be reversed, the space is strong this cannot be reversed, time interval can be reversed, their sign can be reversed but, the space sign cannot be reversed. So, these are called space like separated events. Now, we have talked about sign changing this and that you know what is that mean, you know what is the physical significance of that. So, let us come back to the particular significance of what mean by delta x prime changing the sign, delta t prime changing the sign become negative.

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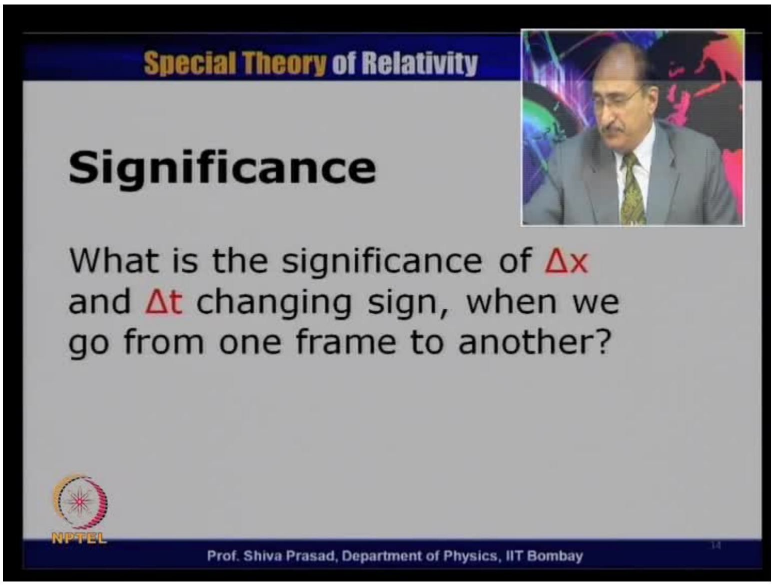

So, what is the significance of delta x and delta t changing the sign, when we go to from one frame to another frame.

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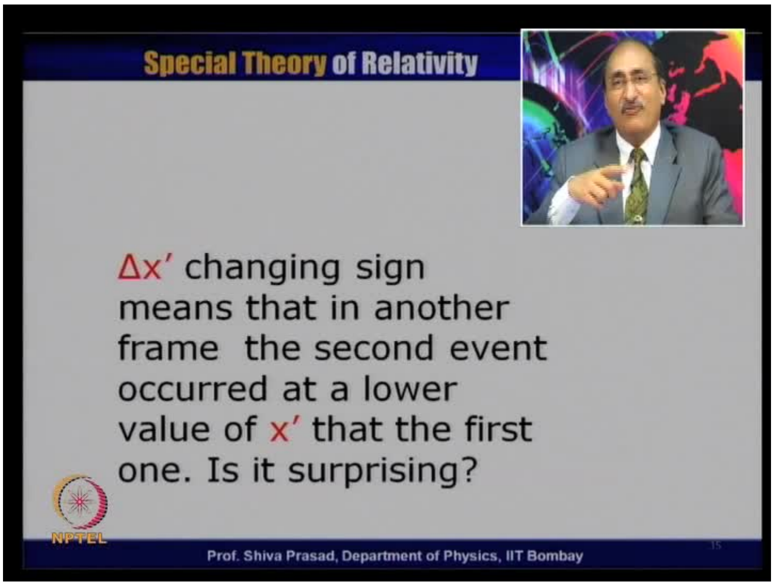

Let us, first look at delta x, delta x prime be becoming negative in another frame. Essentially means that the second event occurred at a value of x which is smaller than the first one.

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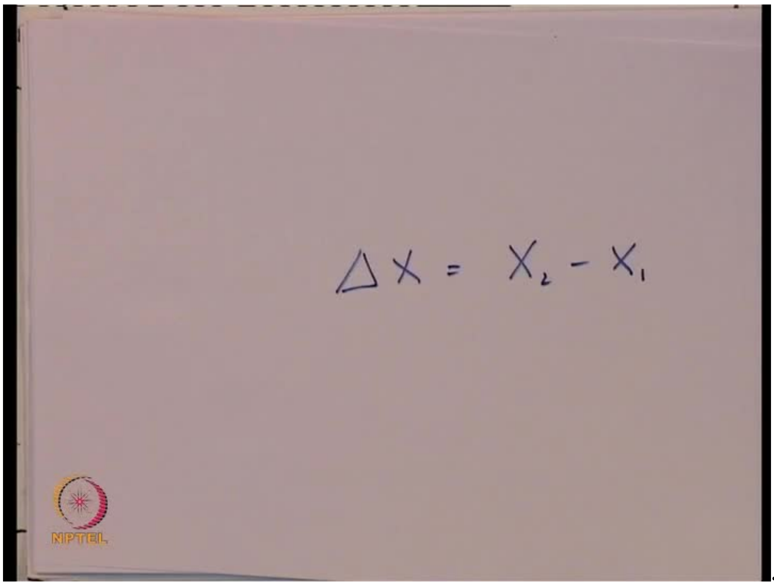

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See, remember delta x was equal to x 2 minus x 1. So, if x 2 becomes smaller than x 1 then delta x turned out to be negative, so all that it means, that another frame of reference x 2 turns out to be smaller than x 1 that is not very surprising. Let us, just look out an example, which is simple classical example let us forget relativity let us look classically, even look classically even classically this is possible to see that delta x could change sign.

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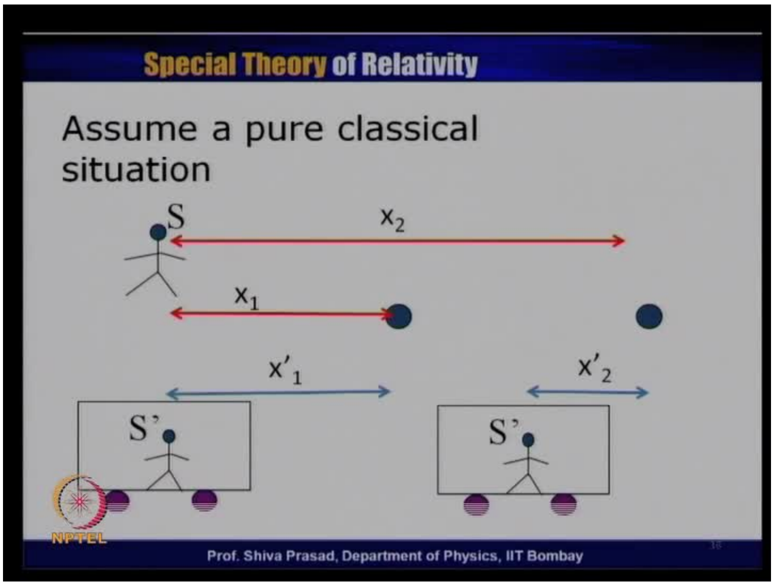

This figure looks some what complicated but, let us try to understand it, let us suppose this is an event which occur here. This is the frame S in which is sees that events occurred at the distance of x 1 from him or her and let us assuming that an lighting striking here, at a given time. And it occurs at the distance of x 1 format it is a second lighting striking and further away from him, at a time later than this.

So, x 2 is larger than x 1, so delta x was positive because, this event occurred later than this event. So, t 2 was larger than t 1 therefore, delta t is positive, so as for as S is concerned delta x is also positive, delta t is also positive, Remember I am only talking classical thing, nothing of relativity of here, there is an another person train which is moving with respect to S.

When this is moving then event number 1 occurred this particular person was close to this observer S therefore, he finds that this event occurs at x 1 prime. Of course, if there in classical mechanics if there exact a line then x 1 should be equal to x 1 prime is let us forget about that particular issue, we see that this will occur at x 1 prime. Now, second event occurs little later, during this particular time this compartment was moving towards the right hand side.

Should depending upon the delta t x is possible that this event let us occur, let us 10 minutes afterwards and during 10 minutes this particular train has moves from here to here, will has from move here to here. The second event occur only at this one distance because, he is also moves respect to us. So, I call it as S prime observer, the coordinate of the second event is x 2 prime, while the coordinate of the first prime was x 1 prime.

So; obviously, x 2 prime smaller than x 1 prime, could depends on what is the speed and what is the time difference between this two events, which always possible. That he finds that this particular event occurs, closer to him then when the first event occurred, so according to the observer S prime delta x has been assigned. So, even classical mechanics possible delta x was positive in S frame delta x prime will be communicative in S prime frame of reference. So, there is nothing surprising in delta x change in sign as I change my frame of reference.

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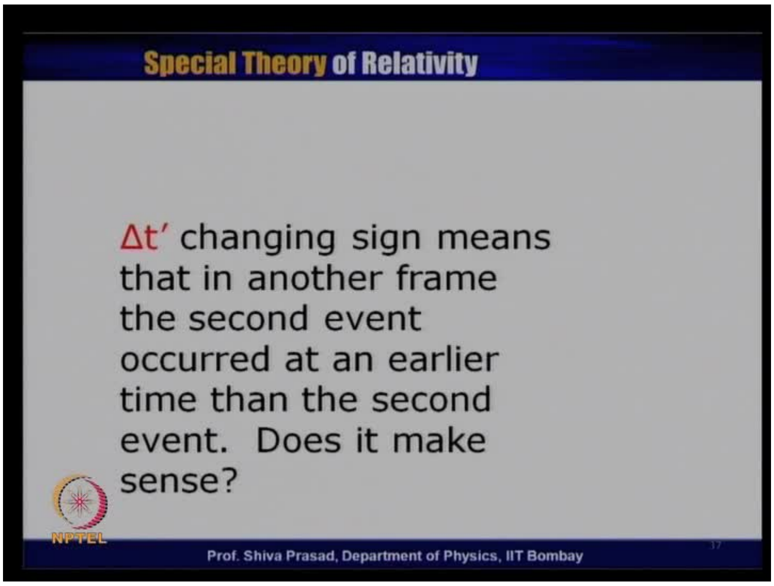

But, what does changing sign of time interval means, it mean the second event occurred earlier in a different frame of reference then the first event. So, as far as two lighting comes, if the second observer observes that the second lighting of the occurred first and the second lighting occurred second, that is we mean delta t changing sign, does it make sense. Now, if we are talking of two events, which are totally uncorrelated, which has no bearing with each other.

Then if it changes I do not bother for example, I say event number 1, in particular train departing from New Delhi railway station. And event number 2 a plane flying off from New York airport after some moment of time, this event 1 and event 2 are, so totally uncorrelated. That particular plane and this particular train departing has no relation with respect to each other, the departure of that particular plane does not depend on the departure of the train.

But, let us consider some correlated events, in which there is a cause and effect event number 1, train is starting from New Delhi railway station, event number 2 train reaching at Mumbai railway station this event number 2. Now, unless the train would have started from New Delhi railway station, it would have never reached the Mumbai station. So, these are related events, occur as the second event depends on event number 1, take another example.

I shoot somebody, with the gun and that person dies. Now, unless I would have shot that person would have not died. So, his death which I call as event number 2 and if I call my shooting the gun as event number 1, event number 2 has to occur because, of event number 1 had occurred, if event number 1 had not occurred then event number 2 will not occur. Now, if delta t prime changes sign, it means these orders can also change sign.

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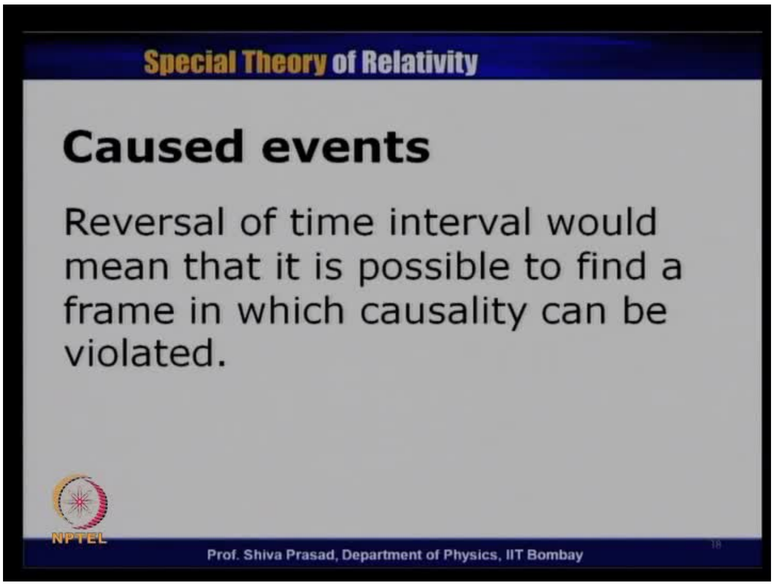

Let us just see I am calling this event as caused events. Reversal of time interval would mean that it is possible to find a frame in which causality can be violated, let me explain this particular point little bit.

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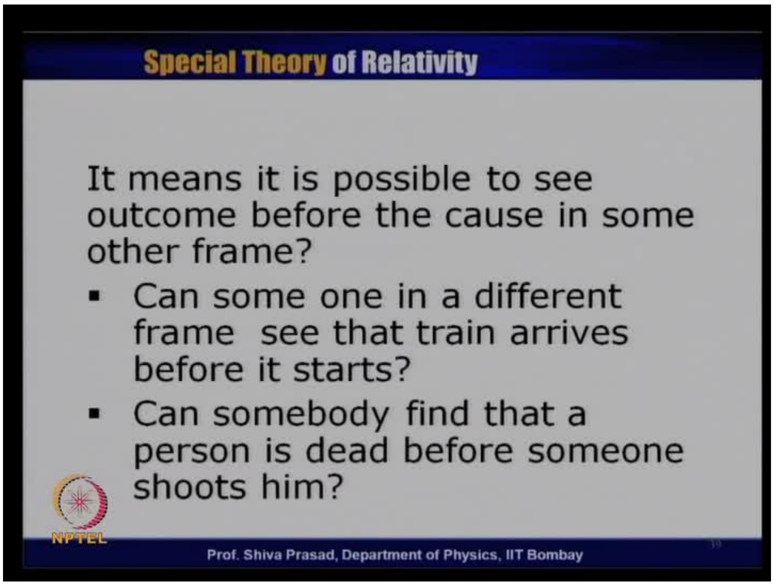

If delta t prime changes sign it means, it may be possible that is some other frame of reference, one could see a train arriving before it starts or in some other frame one could see that a person dies before somebody has shooting. That is what changing the sign delta t prime, this what call as causality because, event number 2 is the effect the cause is the event number 1 because, of the event number 2 occurred.

On normally we never see, in our daily life that causality is violated I should not be able to physics physical idea says that, should not be possible for me to find out the frame of reference. In which these two events which are one which the cause of another, in which the time interval could change because, would mean that in some other frame of reference a person found to be dies before, he was shot which we will not be accept.

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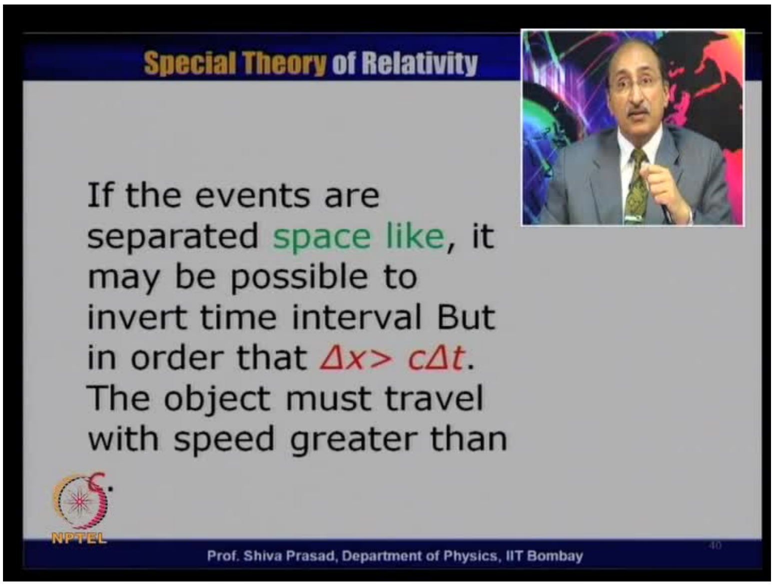

Now, if the events, the two events I am talking related events as space like events only that particular case is possible to involve the time, delta t prime make delta t prime to be negative. But, order the events are space like delta x must be greater than c delta t, it means the distance between the two these events must be larger than the c delta t. It means, if I am shooting the gun here and the person is dying at the little distance let us a 2 kilometer away then this distance should be larger than the c delta t. It means the bullet must have travelled with speed greater than c delta t, then only it is possible that the events the correlate events become space like.

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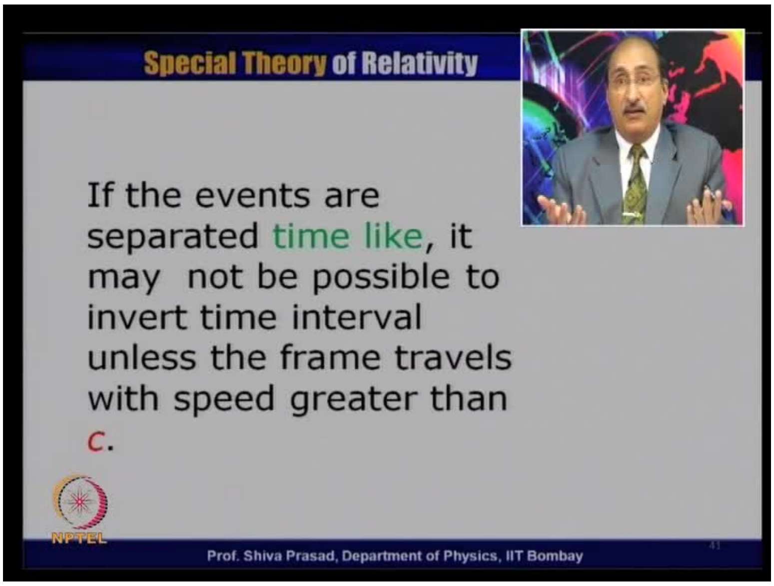

And if the events are time like where delta x is less than c delta t any way it is possible to invert time interval unless than c delta t anyway it is not possible to involve the time interval unless v is larger than c. So, if we say that if v larger than c are not correlated, it will sustain, it will make causality in that it will not possible for us to see that the cause comes later and effect comes earlier in any other frame of reference.

Therefore, if I say that if I restrict my speed to speed less than speed of light, that the most equal to the speed of light I maintain the causality. That is what ley let to the conclusion that speed greater than speed of light should not be acceptable in special theory of relativity.

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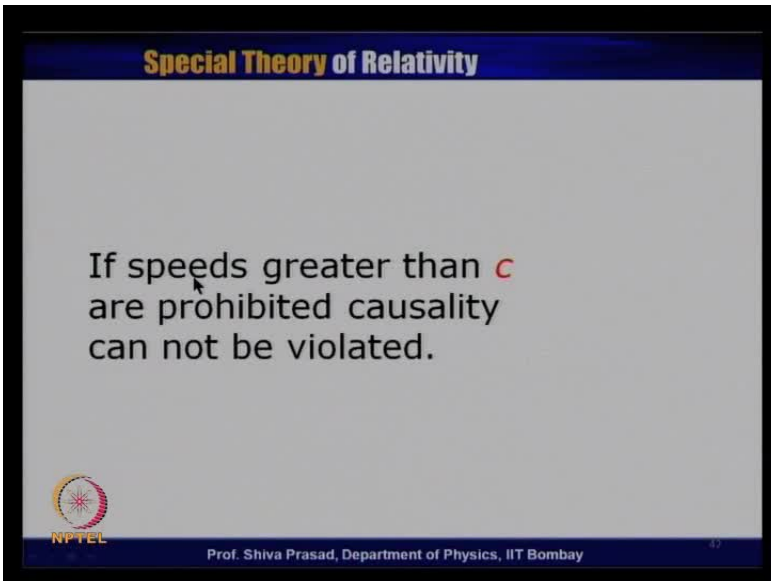

So, this what I had say as conclusion, if speed greater than c are prohibited causality cannot be violated.

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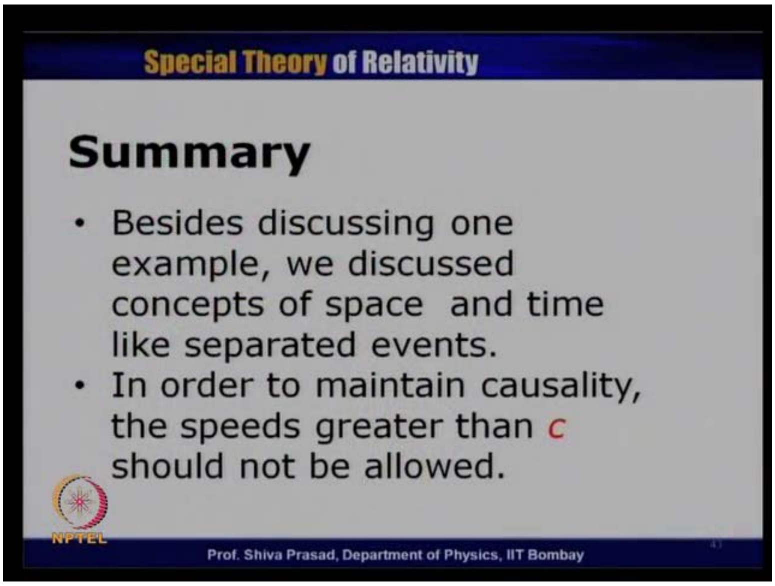

And then I will give the summary of whatever we discussed today, we of course, discuses one example. In which we had included the speed of light and taken the speed of light and in two different frame of reference. And then we have said in order to maintain causality, the speed is greater than c should not be allowed.

Thank you.