

Electromagnetic Theory
Prof. D. K. Ghosh
Department of Physics
Indian Institute of Technology, Bombay

Module - 2
Electrostatics
Lecture - 8
Electric Field and Potential

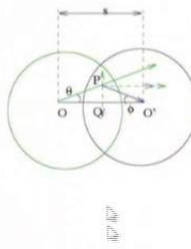
In the last lecture we had talked about gauss's law of electrostatics basically what we said is the amount of flux that is that comes out of any surface a close surface is equal to essentially the amount of charge that is contained within that surface and divided by epsilon 0 this surface which we take need not be a physical surface, but one could think of it as an artificial boundary which a geometrical surface that we have in mind and the gauss's law is true irrespective of whether the surface is real or is an imaginary surface as we said that the physical content of gauss's law is identical to that of coulomb's law; however, its lot more difficult to work directly with coulomb's law where as in certain situations particularly those were there are enough symmetry of the problem one finds application or using gauss's law is lot more convenient.

Today we will continue our application of gauss's law to couple of problems and after that we will be discussing the concept of potential.

(Refer Slide Time: 01:46)

ELECTROMAGNETIC THEORY

Field in region of intersection of two oppositely charged spheres


$$\text{Field at P} = \frac{Q}{4\pi\epsilon_0 R^3} (\overline{OP} - \overline{O'P})$$
$$E_y = 0$$
$$E_x = \frac{Q}{4\pi\epsilon_0 R^3} \overline{OO'}$$

Prof. D K Ghosh, Department of Physics, IIT Bombay

So, now, let us look at this interesting problem of two charged spheres which are oppositely charged. Charge density is uniform in each one of them and we are looking at the field in the region of intersection of the two spheres. Now we had already seen that when we look at the electric field within a material within a surface inside a surface, Gauss's law that we apply that says we have to consider the amount of charge that is contained therein.

So, we had also seen that if I have a sphere of radius R containing a charge Q then if the charge density is uniform that is if the charge is uniformly distributed over the surface, the electric field at a point R which is less than the radius R of the sphere is actually linear where as linear with the distance from the system where as if the distance at which we are trying to get an expression for the electric field is outside the sphere then of course, like Coulomb's law it sort of goes as $1/R^2$.

So, let us look at this problem in which I have two spheres whose centers are at O and O' and I am looking at the electric field at the point P here. So, what I do is this that I know according to superposition principle the electric fields due to distribution of charges different charge distributions simply add up and the point P is inside the sphere one as well as it is inside this sphere two.

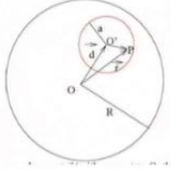
So, therefore, according to what we have learnt from application of Gauss's law the field at the point P due to this sphere is given by $\frac{Q}{4\pi\epsilon_0 R^3}$ the capital R is the radius times \vec{OP} . I told you this is linear and the other one is $\vec{O'P}$ now notice that the center of the second one is at O' and the minus sign is because the charge density is opposite now. So, therefore, what happens is the following that we if you look at how much is $\vec{OP} - \vec{O'P}$ you notice this is just a by vector addition and it turns out that this is along the vector $\vec{OO'}$. So, in other words the y component or the perpendicular component of the electric field will become 0 and the x component is $\frac{Q}{4\pi\epsilon_0 d^3} \vec{OO'}$.

So, now notice that this only depends on the distance between the two centers in other words the field in the intersecting region is uniform it does not matter where inside the intersection you want to evaluate the field. So, this is a method of producing uniform field.

(Refer Slide Time: 05:19)

ELECTROMAGNETIC THEORY

A uniformly charged sphere with a cavity : Use of superposition



Field due to Filled sphere = $\vec{E}_1 = \frac{Q}{4\pi\epsilon_0 R^3} \vec{r} = \frac{\rho}{3\epsilon_0} \vec{r}$

Field due to smaller sphere = $\vec{E}_2 = -\frac{\rho}{3\epsilon_0} (\vec{r} - \vec{d})$

Net field = $\frac{\rho}{3\epsilon_0} \vec{d} = \text{constant}$

Prof. D K Ghosh, Department of Physics, IIT Bombay

Let us take a slightly different example which is an application of the principle of superposition this is a sphere of radius r and inside that there is a sphere of smaller radius which I have called it as a , but this sphere is not concentric. So, there is a cavity there is a cavity of radius a at located at the point O prime and we are interested in finding out how much is the field inside the cavity.

Now, look at this problem. So, what we do to begin with is this supposing the charge density of the bigger sphere is ρ and I have a cavity now I can make that sphere the bigger sphere into a uniform sphere without cavity by filling up the cavity with a charge density ρ , but; however, remember that cavity does not have any charge. So, what I do is mentally I assume that the cavity is equal to a charge density ρ plus ρ and a charge density minus ρ 1 over the other.

So, that the charge density is 0 now. So, therefore, my original problem with the cavity is now split into two different charge distribution one charge distribution which is this sphere of radius capital R with center at o containing a uniform charge distribution ρ and we had seen that in such a case the field at an arbitrary point r will be given by Q by $4\pi\epsilon_0 R^3$ and you recall the density of charge ρ is the total charge Q divided by 4π by 3 capital R cube if you put that in the electric field due to a uniformly distributed charge density ρ at the position vector small r is given by ρ by $3\epsilon_0$ vector r .

so that is due to the bigger sphere not having a cavity, but containing a charge density ρ now I need to superpose this with the field due to a sphere of radius a , but having a charge density $-\rho$. So, notice this that since I am interested in finding the electric field at P which is at a vector position vector \vec{r} with respect to O then what I am interested is in the vector $\vec{O'P}$ now this $\vec{O'P}$ is nothing, but vector \vec{r} minus vector \vec{d} where vector \vec{d} is the displacement vector from the center of the bigger sphere to the centre of the smaller sphere and. So, therefore, the field due to the smaller sphere E_2 is $-\rho$ which is the charge density there by $\frac{1}{3\epsilon_0} \nabla \cdot \vec{r}$ using the superposition principle the field at the point p is simply the sum of these two and you notice that vector the term proportional to the vector are cancels out and I am left with ρ by $3\epsilon_0$ vector \vec{d} which is which only again depends on the distance between the point O and the point O' the center of the cavity and the field at any point within the cavity then is constant.

(Refer Slide Time: 09:25)

ELECTROMAGNETIC THEORY

Coulomb Force is conservative

$$\vec{F} = \frac{\vec{r}}{r^3}$$

$$\vec{\nabla} \times \vec{F} = \nabla \left(\frac{1}{r^3} \right) \times \vec{r} + \frac{1}{r^3} \vec{\nabla} \times \vec{r}$$

$$= -3 \frac{\vec{r}}{r^5} \times \vec{r} + 0$$

$$= 0$$

Prof. D K Ghosh, Department of Physics, IIT Bombay

So, these are two applications of the gauss's law that we find will be of interest let me now come back come to a slightly different issue that is the nature of the electric field we had seen that the electric field which is a inverse square law, but a central force is essentially conservative force if you recall when we discuss vector calculus we had said that the conservative force is characterized by the curl of the vector field becoming 0 now one can easily calculate what is the curl of this vector field.

So, del cross F now this is the ratio of a vector or multiplication of a vector with a scalar. So, del cross of this quantity is first term is gradient of one over r cube cross r vector plus one over r cube comes out and del cross r now you can easily calculate. Firstly, the gradient of one over r cube is along the vector r. So, therefore, r cross r will give me 0 and a direct calculation will tell you that del cross of the position vector r is also equal to 0. So, del cross of F is equal to 0 which means that the force is conservative.

(Refer Slide Time: 11:01)

ELECTROMAGNETIC THEORY

Electrostatic Potential

$$\nabla \left(\frac{1}{|\vec{r} - \vec{r}'|} \right) = - \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|^3} dV' = - \frac{1}{4\pi\epsilon_0} \nabla \int_V \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} dV'$$

$$\phi(r) = - \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} dV'$$

Prof. D K Ghosh, Department of Physics, IIT Bombay

Now if you recall that if the force is conservative that is if the curl of a vector field is 0 we have also said that it means that the field can be expressed as a gradient of a scalar function and the conventionally one defines the electrostatic field F instead of just the gradient minus the gradient of a scalar field which is the potential field that is called a potential field first we will be talking about what is potential at great length in this lecture, but let us look at first how does one express it for the case of coulomb's field. So, basically what I need is remember that in the coulomb field force I have vector r by r cube or if I am looking at a field at the point r prime due to a charge placed at the point r I need vector r minus r prime by r minus r prime cube.

Now, you notice that this quantity is nothing, but the gradient of one over the distance between the charge and the position r prime where I am interested in calculating the electric field there is minus sign comes in because it is gradient of 1 over r minus r prime now if you look at the expression for the electric field for a continuous charge

distribution we had seen that this is given by $\frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|^3} dV'$ this is straight forward extension from the coulomb's law and we have just now seen that this quantity incidentally this should have had a $\vec{r} - \vec{r}'$ vector here. So, this quantity $\vec{r} - \vec{r}'$ vector by $|\vec{r} - \vec{r}'|^3$ is nothing, but negative gradient of this quantity.

So, therefore, I can write this as minus the gradient of this and the gradient is taken with respect to the first variable \vec{r} . So, therefore, it comes out and. So, therefore, I define potential ϕ which is by this expression here that my potential ϕ is given by $\frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} dV'$. So, that is that is what the expression for the potential electrostatic potential for a continuous charge distribution is fair enough.

(Refer Slide Time: 13:59)

ELECTROMAGNETIC THEORY

Electrostatic Potential

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|^3} dV' = -\frac{1}{4\pi\epsilon_0} \nabla \int \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} dV'$$

$$\nabla \cdot \vec{E} = -\frac{1}{4\pi\epsilon_0} \nabla^2 \int \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} dV'$$

$$= -\frac{1}{4\pi\epsilon_0} \int \rho(\vec{r}')(-4\pi)\delta(\vec{r} - \vec{r}') d^3r'$$

$$= \frac{\rho(\vec{r})}{\epsilon_0}$$

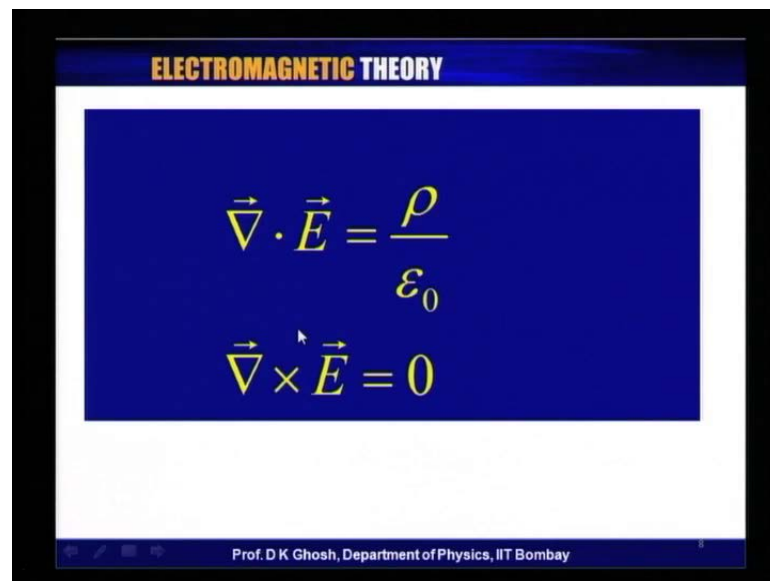
Prof. D K Ghosh, Department of Physics, IIT Bombay

So, we have now let us look at some consequences there of. Firstly, if I calculate the divergence of the electric field the divergence of the electric field is given by del dot of this quantity here which is del square of this del dot del is del square now what I do is this that the what I require now is to take this del square because this is a derivative with respect to the first variable \vec{r} inside. So, because the integral is with respect to dV' prime. So, I get del square of $\frac{1}{|\vec{r} - \vec{r}'|}$ and if you recall we had during our module on calculus we had shown that del square of $\frac{1}{r}$ is minus four pi times delta of r . So, therefore, del square of $\frac{1}{|\vec{r} - \vec{r}'|}$ is minus for pi times delta of $|\vec{r} - \vec{r}'|$

prime where this delta actually it is a delta cube because it is a three dimensional delta function is the Dirac delta function.

Now, once I have a derived delta function inside an integral the integral is easy to do the integral then is simply the value. So, the 4π 4π will cancel out cancel out and I will have 1 over ϵ_0 and this only point contributes namely r prime equal to r . So, I get ρ r by ϵ_0 .

(Refer Slide Time: 15:44)



So, del dot of E is rho by epsilon 0 and del cross of E is equal to 0 are the two major equations of electrostatics that we have learnt. So, far let us return to the fact that the electric field can be written as a negative gradient of a potential function.

(Refer Slide Time: 16:11)

ELECTROMAGNETIC THEORY

Potential

$$\int_{\text{Ref}}^r \vec{E}(\vec{r}') \cdot d\vec{r}' = - \int_{\text{Ref}}^r \vec{\nabla} \phi \cdot d\vec{r}'$$
$$= - \int_{\text{Ref}}^r d\phi = -\phi(\vec{r})$$
$$\phi(\vec{r}) = \frac{q}{4\pi\epsilon_0 r} \quad (\text{for a point charge at origin})$$

Prof. D K Ghosh, Department of Physics, IIT Bombay

So, now, let me start with $\vec{E} \cdot d\vec{r}$ now what is $\vec{E} \cdot d\vec{r}$ now suppose you are bringing in a unit charge now you remember that by definition of electric field if I have a charge q the force that is exerted on the charge is q times the electric field strength at a point. So, therefore, what I do is this that first purely mathematically. So, suppose I integrate it formally from some reference point to the point r $\int_{\text{Ref}}^r \vec{E} \cdot d\vec{r}$ \vec{E} is written as minus gradient of a scalar function ϕ $\vec{E} = -\vec{\nabla}\phi$ now we had seen that this quantity this quantity is nothing, but $d\phi$ this is by definition of the gradient.

So, therefore, this is minus the integral from the reference point to r of $d\phi$ which is minus ϕ at r plus ϕ at the reference point, but at the reference point let me define the value of the potential ϕ to be equal to 0. So, that it simply becomes minus ϕ of r now supposing you looked at electric field due to a point charge at the origin the I know what is \vec{E} of vector \vec{r}' which is simply q by $4\pi\epsilon_0 r'^2$ and in that case this integral can be vary trivially done and by comparing the result of this integral with the potential I find that ϕ of r for a point charge at the origin is given by q by $4\pi\epsilon_0 r$.

(Refer Slide Time: 18:15)

ELECTROMAGNETIC THEORY

Potential- Relation to Potential Energy
Bring a test charge q from the point A (reference point) to the point P . As the electric field exerts a force on q , work has to be done

$$W = - \int_{\vec{r}_A}^{\vec{r}} \vec{F} \cdot d\vec{l} = -q \int_{\vec{r}_A}^{\vec{r}} \vec{E} \cdot d\vec{l}$$
$$= q \int_{\vec{r}_A}^{\vec{r}} \vec{\nabla} \phi \cdot d\vec{l} = q(\phi(P) - \phi(\text{Ref}))$$
$$= q\phi(P)$$

Prof. D K Ghosh, Department of Physics, IIT Bombay

Now, So, the question is this there are two issues to be discussed as you realize the name has a lot of similarity with another term with which are familiar namely potential energy now it is somewhat unfortunate that this quantity which is whose negative gradient is the electric field is called a potential because though there are there is some relationship between the potential and the potential energy these are actually two different things and, but; however, this name has stuck and. So, we need to also we will be talking about what is this relationship and we need to be clear in our mind the difference between the term potential and the potential energy.

So, first thing is that suppose I want to bring in a test charge q and let us say my reference point is some point a and I want to bring it to some point P whose position with respect to some arbitrarily chosen origin is vector r now if the electric field is E this test charge q at any point r experiences a forces which is q times E the electric field at the point now if the electric field is the electric field exerts a force.

Now if I want to bring the charge from a point let us say r_a which is the position vector of the reference point a to the point r I have to do work against this force and hence the work done by external agency which is me is minus the work done by the charge. So, which is w that is the work done by the external agency is minus $F \cdot dl$ from r_a which is the reference point to the point r .

So, which is same as minus $q \int_{r_a}^r \mathbf{E} \cdot d\mathbf{l}$, but we had just now seen that \mathbf{E} is negative gradient of the potential. So, it becomes $q \int_{r_a}^r \text{grad } \phi \cdot d\mathbf{l}$ which is integral simply of $d\phi$. So, what you get is that it is q times the potential at the point P minus the potential at the reference point which we will take it to be 0. So, it becomes q times the potential at the point P .

So, let us look at what are the consequence there off. So, what if you look at this expression you find that the work done in bringing a charge from a reference point where we define the potential to be 0 is q times the potential at the point to which it is being brought in. So, this amount of work that we have done the external agency has done; obviously, becomes the potential energy of this system or we can say it is the potential energy associated with the charge q now remember the potential energy of is of the system.

So, what it means is there is a source charge it could be a distribution of electric charges which is producing that electric field and this term this work that has been done is the contribution of the to the potential energy of this source charges and this charge. So, in another words this is the potential energy of the system which can be associated with the charge that is being brought in now if I take the charge to be a unit charge the test charge to be a unit charge then you notice that the work done is simply the potential at the point p in other words what it means is that potential can be interpreted as the potential energy associated with a unit charge at a point. So, that is the relationship between the potential and the potential energy.

(Refer Slide Time: 23:17)

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$
$$\vec{\nabla} \cdot (-\vec{\nabla} \phi) = \frac{\rho}{\epsilon_0}$$
$$\boxed{\nabla^2 \phi = -\frac{\rho}{\epsilon_0}} \quad \text{Poisson's Eqn.}$$
$$\nabla^2 \phi = 0 \quad \rightarrow \quad \text{Laplace's Eqn.}$$

So, let us look at what we the first equation that we wrote down was divergence of the electric field there are two equations which we talked about one was curl of the electric field was 0 and divergence of the electric field is rho over epsilon 0 now notice we have said the electric field can be written as negative gradient of the potential. So, this is del dot minus grad phi is rho by epsilon 0 which means del square phi. So, remember the del square is the Laplacian operator is minus of the charge density divided by epsilon 0.

This equation is known as the Poisson's equation we will have occasion to discuss this Poisson's equation in detail now supposing I am looking at this solutions of the Poisson equation that is I am trying to evaluate the potential at some point and supposing in that region there are no charges that if there are no charges then this equation becomes del square phi equal to 0 this is in the source free region source free region and this equation which we will be discussing in detail later is known as the Laplace's equation. So, Poisson's equation is to determine the potential where there are sources available and Laplace's equation is when the source it is in a source free region.

(Refer Slide Time: 25:22)

ELECTROMAGNETIC THEORY

Potential of a line charge

$$\int_S \vec{E} \cdot d\vec{S} = |E| 2\pi r l = \frac{\lambda l}{\epsilon_0}$$
$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0} \frac{1}{r}$$
$$\varphi = -\frac{\lambda}{2\pi\epsilon_0} \ln r + \text{constant}$$

Prof. D K Ghosh, Department of Physics, IIT Bombay

Let me look at a few examples of calculation of the potential I take a simplest possible example namely that of a line charge an infinite line charge. So, what I do is this that I have a line charge I which I have taken as the positive charge and its infinite now two I know how to evaluate the field and this is simply done we have done it earlier by explicit calculation from the coulomb's law, but what we do is we apply gauss's law here. So, let me take a cylinder of radius small r and length l and this is the Gaussian surface and I enclose the charge within it.

So, notice since the Gaussian gauss's law says that the flux integral $E \cdot dS$ is equal to the amount of charge that is enclosed by the surface this is an imaginary surface in this case the amount of charge that is enclosed; obviously, is if λ is the charge density is λ times the length l of this cylinder which is symmetrically placed with respect to this long wire line charge.

So, the flux e now notice the flux is due to electric field on the surface the by symmetry it is very clear that the electric fields must be radially outward from the line charge. So, therefore, the contribution to the flux from the top and the bottom surfaces will be 0 because the normal directions are perpendicular to the direction of the electric field. So, the only thing that we have to consider will be the charges will be the contribution to the flux from the curved surface of the cylinder and the magnitude of the electric field; obviously, is the same everywhere.

So, therefore, it is magnitude λ times the curved surface area which $2\pi r$ times the length of this cylinder. So, that tells me the magnitude of the electric field is λ by $2\epsilon_0 r$ and of course, in the radial direction.

Now, this is if you want to write it as negative gradient of the potential now electric field is simply a function of r . So, therefore, gradient boils down to essentially a derivative with respect to r . So, as a result the potential ϕ is given by $-\lambda$ by $2\pi\epsilon_0$ logarithm of r and of course, a constant of integration now notice unlike that of a point charge where we could take the reference point at infinity that is the point at infinity would be taken as the point where the potential will become 0 because potential expression was 1 over r I cannot do that for a line charge because the logarithm r equal to 0 the logarithm is not defined. So, one has to take this reference point in case of a line charge little more carefully and if you want this constant to vanish then you can sort of artificially take some unit r is equal to 1 and say that the 0 of the potential is at r is equal to 1 what is your r equal to 1 you have to sort of course, decide.

(Refer Slide Time: 29:22)

ELECTROMAGNETIC THEORY

Screened Coulomb Potential

$$\phi = \frac{q}{4\pi\epsilon_0} \frac{e^{-r/\lambda}}{r}$$

$$\vec{E}(\vec{r}) = -\vec{\nabla} \phi(\vec{r}) = -\frac{q}{4\pi\epsilon_0} \hat{r} \frac{\partial}{\partial r} \left(\frac{e^{-r/\lambda}}{r} \right)$$

$$= \frac{q}{4\pi\epsilon_0} \hat{r} \frac{1}{r^2} \left(1 + \frac{r}{\lambda} \right) e^{-r/\lambda}$$

Prof. D K Ghosh, Department of Physics, IIT Bombay

13

So, this is the potential due to a line charge as a second example I would consider a rather well known case this arises this is called screened coulomb potential this arises in many cases for example, when you have a charge put in inside a semiconductor medium then what happens is because of the fact that the medium itself is a dielectric we will be discussing this in detail the interaction between charges in the medium is not bare

coulomb, but the charges are screened charges are screened by an exponential factor and the potential instead of being just 1 over r it is given by q by 4 pi epsilon 0 e to the power minus r by lambda lambda is some quantity some length parameter at which the strength of the potential reduces by a factor of 1 over e. So, instead of just 1 over r potential I have a e to the power minus r by lambda by r this also is sometimes known as Yukawa potential because of its importance in some other problems of nuclear physics. So, let us look at how does one calculate how does one calculate the electrical field due to such a screened coulomb potential. So, let me work it out.

(Refer Slide Time: 31:12)

$$\begin{aligned}\phi &= \frac{q}{4\pi\epsilon_0} \frac{e^{-r/\lambda}}{r} \\ \vec{E} &= -\nabla\phi = -\frac{q}{4\pi\epsilon_0} \hat{r} \frac{\partial}{\partial r} \left(\frac{e^{-r/\lambda}}{r} \right) \\ &= \frac{q}{4\pi\epsilon_0} \hat{r} \frac{1}{r^2} \left(1 + \frac{r}{\lambda} \right) e^{-r/\lambda} \\ \vec{\nabla} \cdot \vec{E} &= \frac{q}{4\pi\epsilon_0} \vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2} \left(1 + \frac{r}{\lambda} \right) e^{-r/\lambda} \right) \\ &= \frac{q}{4\pi\epsilon_0} \left[\vec{\nabla} \cdot \left(\frac{\hat{r}}{r^2} \right) \left(1 + \frac{r}{\lambda} \right) e^{-r/\lambda} \right. \\ &\quad \left. + \frac{\hat{r}}{r^2} \nabla \cdot \left[\left(1 + \frac{r}{\lambda} \right) e^{-r/\lambda} \right] \right]\end{aligned}$$

So, electric field now first thing to I have notice is the potential phi is q by 4 pi epsilon 0 e to the power minus r by lambda divided by r now electric field is negative gradient of that. So, minus grad phi, but remember grad is because this function depends only on r. So, grad is nothing, but d by d r and the direction is along the unit vector r. So, this is minus q by 4 pi epsilon 0 the direction r and d by d r of this quantity here namely e to the power minus r by lambda divided by r it is a rather trivial differentiation to be done. So, I take d by d r. So, 1 over r gives me minus 1 over r square and exponential differentiated chain rule differentiation. So, straight forward it gives me q by 4 pi epsilon 0 unit vector r 1 over r square I take out this common and I will get as 1 plus r by lambda into e to the power minus r by lambda. So, this my electric field.

(Refer Slide Time: 32:46)

ELECTROMAGNETIC THEORY

Screened Coulomb Potential

$$\begin{aligned} \nabla \cdot \vec{E} &= \frac{q}{4\pi\epsilon_0} \nabla \cdot \left(\frac{\hat{r}}{r^2} \left(1 + \frac{r}{\lambda}\right) e^{-r/\lambda} \right) = \\ &= \frac{q}{4\pi\epsilon_0} \left(\nabla \cdot \left(\frac{\hat{r}}{r^2} \right) \left[\left(1 + \frac{r}{\lambda}\right) e^{-r/\lambda} \right] + \frac{\hat{r}}{r^2} \nabla \cdot \left[\left(1 + \frac{r}{\lambda}\right) e^{-r/\lambda} \right] \right) \\ &= \frac{q}{4\pi\epsilon_0} \left(-\nabla^2 \cdot \left(\frac{1}{r} \right) \left[\left(1 + \frac{r}{\lambda}\right) e^{-r/\lambda} \right] + \frac{1}{r^2} \frac{\partial}{\partial r} \left[\left(1 + \frac{r}{\lambda}\right) e^{-r/\lambda} \right] \right) \end{aligned}$$

Prof. D K Ghosh, Department of Physics, IIT Bombay

Now, let us look at the what is the charge distribution what is the we are interested in finding out what is the charge distribution that gives rise to this eclectic field and for that we need to calculate the divergence of this del dot of e and equate it with rho by epsilon 0. So, let us look at this a little detailed calculation, but fairly straight forward once again electric fled only has radial component. So, calculating divergence would be easy. So, let us look at first this is q by 4 pi epsilon 0. So, I have got del dot of this quantity in the here which is r by r square there are three terms there one plus r by lambda into e to the power minus r by lambda.

So, we need to do this differentiation remember del dot of a vector times a scalar is scalar multiplied by del dot of that vector plus gradient of the scalar dotted with the vector itself. So, therefore, what I get is the following the this is equal to q by 4 pi epsilon 0. So, let me take this as let me take this as my one term. So, which is first term is del dot of r by r square which I could write as vector r by r cube into 1 plus r by lambda e to the power minus r by lambda plus the vector r by r square times gradient of this quantity namely 1 plus r by lambda times e to the power minus r by lambda.

So, you note this thing coming back to this here this term here this is what I have written down now vector r by r square is nothing, but the minus the gradient of 1 over r. So, therefore, this term can be written as minus del square of 1 over r there is no dot there into this term plus whatever we have written down here namely r by r square and. So,

notice that this is r by r square dot gradient of this and since the gradient is also along the unit vector r . So, I get a one here 1 by r square d by $d r$ of this thing and being a straight forward differentiation you can do.

(Refer Slide Time: 36:04)

ELECTROMAGNETIC THEORY

Screened Coulomb Potential

$$\begin{aligned}\nabla \cdot \vec{E} &= \frac{q}{4\pi\epsilon_0} \left(-\nabla^2 \cdot \left(\frac{1}{r} \right) \left[\left(1 + \frac{r}{\lambda} \right) e^{-r/\lambda} \right] + \frac{1}{r^2} \frac{\partial}{\partial r} \left[\left(1 + \frac{r}{\lambda} \right) e^{-r/\lambda} \right] \right) \\ &= \frac{q}{4\pi\epsilon_0} \left(4\pi\delta^3(\vec{r}) \left(1 + \frac{r}{\lambda} \right) e^{-r/\lambda} - \frac{1}{r\lambda^2} e^{-r/\lambda} \right) \\ &= \frac{q}{4\pi\epsilon_0} \left(4\pi\delta^3(\vec{r}) e^{-r/\lambda} - \frac{1}{r\lambda^2} e^{-r/\lambda} \right) \\ &= \frac{\rho}{\epsilon_0} \\ \rho &= q\delta^3(\vec{r}) - \frac{q}{4\pi\lambda^2 r} e^{-r/\lambda}\end{aligned}$$

Prof. D K Ghosh, Department of Physics, IIT Bombay

So, what do you get out of it the once again you recall that del square of 1 over r del square of 1 over r is minus four pi times a del of function. So, if you do this calculation this straight forward differentiation first term which is del square of 1 over r gives you four pi delta cube r into one plus r by λ e to the power minus r by λ and this term simply gives you minus 1 over r λ square e to the power minus r by λ , but remember delta function only contributes at r is equal to 0 so, but r times delta cube of r is 0 .

So, this term actually does not exist. So, this is what I am left with this is what I am left with that there is a delta function at the origin multiplied by this quantity and times this thing. So, you notice this and of course, I put r is equal to 0 here also this will also become a one and this has to be equated to ρ by ϵ_0 . So, the charge density ρ is given by q times delta cube r that is in delta function at the origin minus q by $4\pi\lambda^2 r$ e to the power minus r by λ the reason for this long exercise is to tell you that one can from the knowledge of electric field get the charge distribution and vice versa the electric field can be obtained from a knowledge of for example, potential. So, why talk about a potential notice that if potential is a scalar quantity.

So, therefore, if I am looking at calculating electric field due to a charge distribution if I am directly dealing with vectors I have to do vector superposition vector additions that is of all the sources at a whatever is the electric field they give at a point I have to make vector addition and that is a little more difficult problem on the other hand if I deal with potential these are scalars. So, there can be just add it up nicely and having added them up I can take its negative gradient and that will be then that will then give the electric field at that point. So, dealing with scalars is one of the greater advantages of introducing the concept of a potential. So, let me let me illustrate now by calculating potential for a couple of cases.

(Refer Slide Time: 39:19)

ELECTROMAGNETIC THEORY

Potential due to a dipole

$+q$ $-q$

d

$\vec{p} = qd \hat{p}$

$R_{\pm} = r \pm \frac{d}{2} \cos \theta$

$\frac{1}{R_{\pm}} \approx \frac{1}{r} \left(1 \mp \frac{d}{2r} \cos \theta \right)$

P

\vec{R}_{+}

\vec{r}

\vec{R}_{-}

θ

Prof. D K Ghosh, Department of Physics, IIT Bombay

So, one very important problem which will be dealing with lot more detail later is that of a dipole a dipole is basically two charges q and $-q$ they are separated by a distance d which for an ideal dipole is infinitesimally small. So, by convention the direction of the dipole dipole is a vector it is from the negative charge to the positive charge this has to be sort of understood that dipole moment P is the magnitude of the charge q times the distance between them these are two equal magnitude charges one positive one negative.

So, q times d along this vector from minus charge to the plus charge which is what I called as a unit vector \hat{p} now let us calculate we are interested in calculating the field and the potential due to such a dipole. So, I will calculate it by the potential method that we talk about. So, let us talk about a point p which let me orient this dipole along the z

direction and the point p is at a distance r from the origin making an angle theta with the z axis.

So, therefore, this point p is at a distance R plus from the positive charge and R minus from the negative charge now you can just drop perpendiculars from this charges to this O P and. So, R plus is actually less than r the. So, R plus is this distance at given a positive because it is a positive charge I have written plus the way I have given it this is not really greater it is less than R plus is less than the vector r and R minus is greater than the distance O P.

So, as a result R plus minus R plus is R minus d by 2 cos theta and R plus is this R minus is this distance which is R plus d by 2 cos theta now what I need is 1 over r pulse minus. So, I take the divide 1 by this and. So, therefore, what do I get.

(Refer Slide Time: 41:49)

ELECTROMAGNETIC THEORY

Potential due to a dipole

$$\begin{aligned} \phi(r) &= \frac{1}{4\pi\epsilon_0} \left(\frac{q}{R_+} - \frac{q}{R_-} \right) \\ &= \frac{q}{4\pi\epsilon_0 r} \left(1 - \frac{d}{2r} \cos \theta - 1 + \frac{d}{2r} \cos \theta \right) \\ &= \frac{qd \cos \theta}{4\pi\epsilon_0 r^2} = \frac{p \cos \theta}{4\pi\epsilon_0 r^2} = \frac{\vec{p} \cdot \hat{r}}{4\pi\epsilon_0 r^2} \end{aligned}$$

Prof. D K Ghosh, Department of Physics, IIT Bombay

So, what do I get is phi of r is 1 over 4 pi epsilon 0 q by R minus because it is a opposite charge by R minus. So, as a result I can put this approximation there one and one will cancel out and I will be left with I will be left with q d cos theta q d cos theta divided by 4 pi epsilon 0 r square there is a r there and there is a r there. So, this is equal to p cos theta by 4 pi epsilon 0 r square because p is nothing, but q times d, but recall that the angle between p and vector p and r is theta. So, as a result p cos theta is vector p dot r by 4 pi epsilon 0 r square.

(Refer Slide Time: 42:57)

$$\begin{aligned}\phi(\vec{r}) &= \frac{\vec{p} \cdot \hat{r}}{4\pi\epsilon_0 r^2} \\ \vec{E}(\vec{r}) &= -\nabla\phi(\vec{r}) \\ &= -\frac{p}{4\pi\epsilon_0} \nabla\left(\frac{\cos\theta}{r^2}\right) \\ &= -\frac{p}{4\pi\epsilon_0} \left[-\hat{r} \frac{2}{r^3} \cos\theta + \hat{\theta} \frac{(-1)}{r^2} \sin\theta \right] \\ &= \frac{1}{4\pi\epsilon_0} \left[\frac{\hat{r}}{r^3} \frac{2\vec{p} \cdot \hat{r}}{r^2} + \frac{p}{r^3} \hat{\theta} \sin\theta \right] \\ &= \frac{1}{4\pi\epsilon_0} \frac{3(\vec{p} \cdot \hat{r})\hat{r} - \vec{p}}{r^3}\end{aligned}$$

So, we have obtained an expression for the potential phi due to a dipole at a point r and that is given by vector p dotted with r by 4 pi epsilon 0 r square now I can calculate the electric field at the point r because this is nothing, but negative gradient of the potential now notice the gradient because the potential depends now on not only r, but it also depends upon the angle theta there is no phi dependence. So, as a result I need to know what does the gradient operator look like in spherical polar coordinates.

(Refer Slide Time: 43:53)

ELECTROMAGNETIC THEORY

Field of a dipole
The electric field of the dipole can be calculated from potential.
Gradient in spherical-polar :

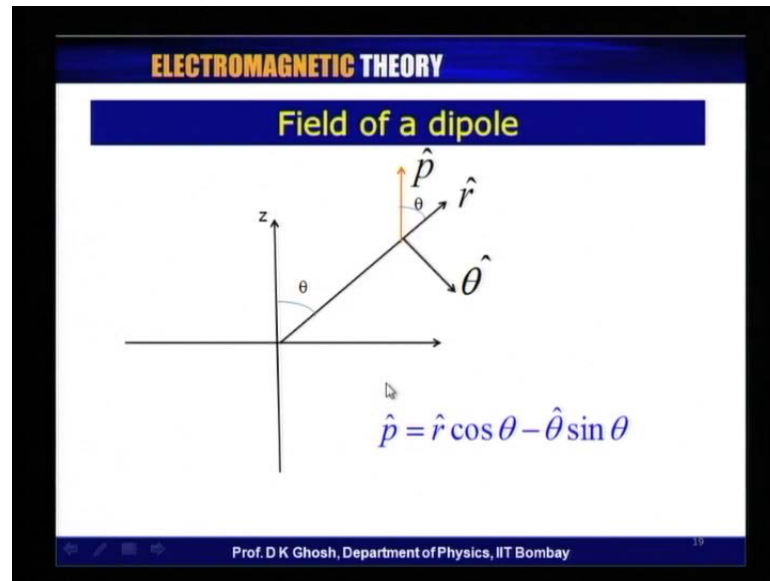
$$\vec{\nabla} = \hat{r} \frac{\partial}{\partial r} + \hat{\theta} \frac{1}{r} \frac{\partial}{\partial \theta} + \hat{\phi} \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi}$$

$$\vec{E} = -\frac{p}{4\pi\epsilon_0} \nabla \frac{\cos \theta}{r^2}$$

Prof. D K Ghosh, Department of Physics, IIT Bombay

But. So, this is the expression for the gradient in the spherical polar coordinate r d by $d r$ plus unit vector θ 1 over r d by $d \theta$ this term is of not much interest to us here because the potential expression does not have any ϕ dependence, but this is the term which would be important if there is an azimuthal dependence. So, this is 0 . So, I am interested in calculating this and this is $\cos \theta$.

(Refer Slide Time: 44:25)



So, this is very fairly straight forward to calculate now notice this the. So, the this quantity that we are interested let us rewrite it is minus p by $4 \pi \epsilon_0$ gradient of $\cos \theta$ divided by r square which is equal to minus $4 p$ by $4 \pi \epsilon_0$ we had seen that the gradient is vector r unit vector r times d by $d r$ the partial with respect to r of 1 over r square. So, 1 over r square gives me minus 2 by r cube times $\cos \theta$ then I have a θ 1 over r and now d by $d \theta$, but d by $d \theta$ of $\cos \theta$ is simply $-\sin \theta$. So, I got 1 over r square which is already there times $\sin \theta$, but with the minus sign because the cosine being. So, let me put this minus here.

Now, look at this gives you the geometry this gives you the geometry I have oriented my dipole along the z axis this is the radial direction. So, this is the direction of p and you recall that the tangential unit vector θ is always defined in the direction of increasing angle. So, as a result since this is the θ angle θ . So, the direction of increasing θ is this which means if I want if I have a vector p the unit vector p I can resolve it along the radial direction along the radial direction and the tangential direction by saying

vector \hat{p} is vector unit vector $\hat{r} \cos \theta$ minus unit vector $\hat{\theta} \sin \theta$ this is an expression which will require.

(Refer Slide Time: 46:44)

ELECTROMAGNETIC THEORY

Field of a dipole

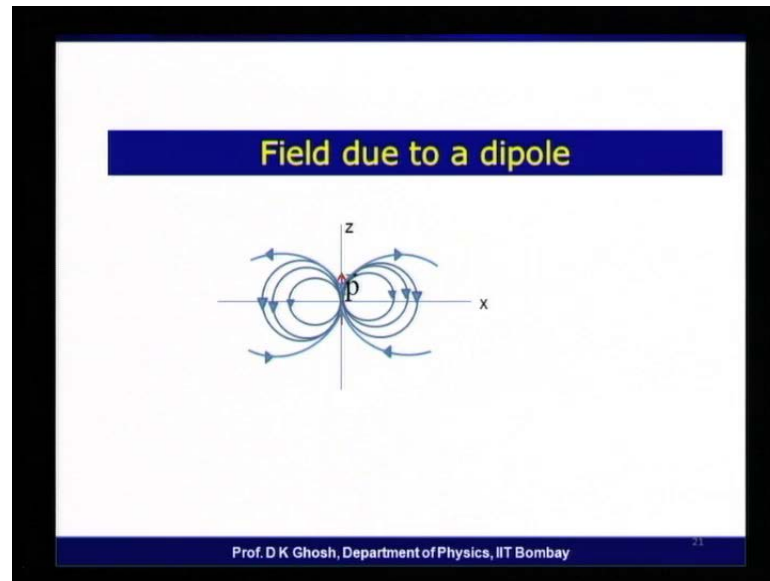
$$\begin{aligned} \vec{E} &= -\frac{p}{4\pi\epsilon_0} \left[\hat{r} \frac{\partial}{\partial r} \frac{\cos \theta}{r^2} + \hat{\theta} \frac{1}{r^3} \frac{\partial \cos \theta}{\partial \theta} \right] \\ &= \frac{p}{4\pi\epsilon_0} \left[\hat{r} \frac{2 \cos \theta}{r^3} + \hat{\theta} \frac{1}{r^3} \sin \theta \right] \\ &= \frac{1}{4\pi\epsilon_0} \left(\hat{r} \frac{2 \vec{p} \cdot \hat{r}}{r^3} + \frac{1}{r^3} \hat{\theta} \sin \theta \right) \\ &= \frac{1}{4\pi\epsilon_0} \frac{3(\vec{p} \cdot \hat{r})\hat{r} - \vec{p}}{r^3} \end{aligned}$$

Prof. D K Ghosh, Department of Physics, IIT Bombay

So, let us look at what we have got. So, notice that there are the minus signs gone away. So, I am left with p by $4\pi\epsilon_0$ unit vector \hat{r} by r^3 there is now notice this what is $\cos \theta$ $\cos \theta$ is $\vec{p} \cdot \hat{r}$. So, let me write this two times unit vector \vec{p} dotted with vector \hat{r} or I will let me take this away. So, vector \vec{p} dotted with the unit vector \hat{r} divided by r^3 plus plus because this minus has been already absorbed there 1 over r^3 1 over r^3 the \vec{p} that as to come inside and I have got $\theta \sin \theta$ fair enough.

So, this quantity is equal to 1 over $4\pi\epsilon_0$. So, let us look at what do I get out of this remember we said that the unit vector \hat{p} is $\hat{r} \cos \theta$ minus $\hat{\theta} \sin \theta$. So, what I do to the second term to this term here is write my $\theta \sin \theta$ as unit vector \hat{p} or rather vector $\hat{r} \cos \theta$ minus unit vector \hat{p} and if you just add them up properly what do you get is three times vector \vec{p} dot unit vector \hat{r} along the vector \hat{r} minus the vector \vec{p} divided by r^3 this is this is an expression which is independent of your coordinate system because it is all written in terms of the vector is the rather important expression for the field of a dipole if I now try to plot what is the electric field I could take this expression and plot the electric field like we have been doing for any vector field.

(Refer Slide Time: 49:29)



This is the way it looks like it should not come as a surprise to you because the dipole is basically one positive charge and one negative charge. So, the field lines must lead the positive charge and enter the negative charge. So, this the way the electric field due to a dipole looks like.

So, let us quickly summarize as to what we have done today we have introduced the concept of a potential we have realized that electric force the coulomb force being a conservative field of force the curl of the electric field is 0 and hence the field itself can be written as a gradient of a quantity which we call as the potential in definition we take a negative gradient, but that is convention now potential is related to potential energy, but is not the potential energy potential is measured in joules per coulomb which is called a volt.

So, essentially potential at a point is the amount of potential energy associated with a charge test charge unit test charge at that point potential is something like pressure when a liquid is flowing. So, for example, if you have a pipe through which one end of the pipe has a higher pressure than the other end the liquid will flow from the high pressure region to the low pressure region the potential is a very similar concept if a positive charge is at a higher potential than a negative charge, it will have a tendency to go into region of lower potential. Potential is a scalar quantity superposition principle can be

applied much easier and it is easy to deal with because once I have added up the scalar potentials I can then take the gradient and calculate the electric field.