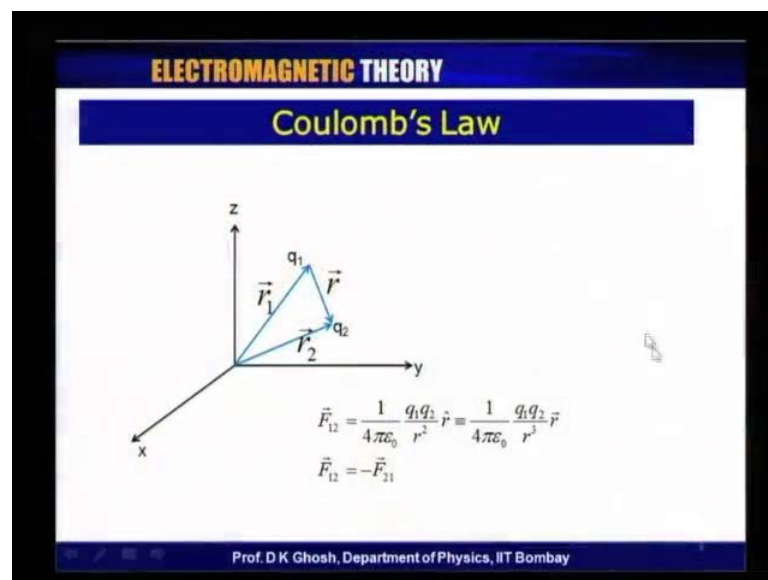


Electromagnetic Theory
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Module - 2
Electrostatics
Lecture - 6
Electric Field Potential

In the last module consisting of about five lectures, we have laid the foundation or mathematical foundation for getting into the electromagnetic theory. We will now begin with a discussion of electrostatics. As the name suggests that this is phenomena associated with charges which are not in motion. So, the we will be talking about electric field and consequently or a potential which arises from such a field. We begin with the basic principal of electrostatics, which is a charge q it attracts or repels other charges depending upon the sign of the charge, and the force between charges is given by what is known as Coulomb's law.

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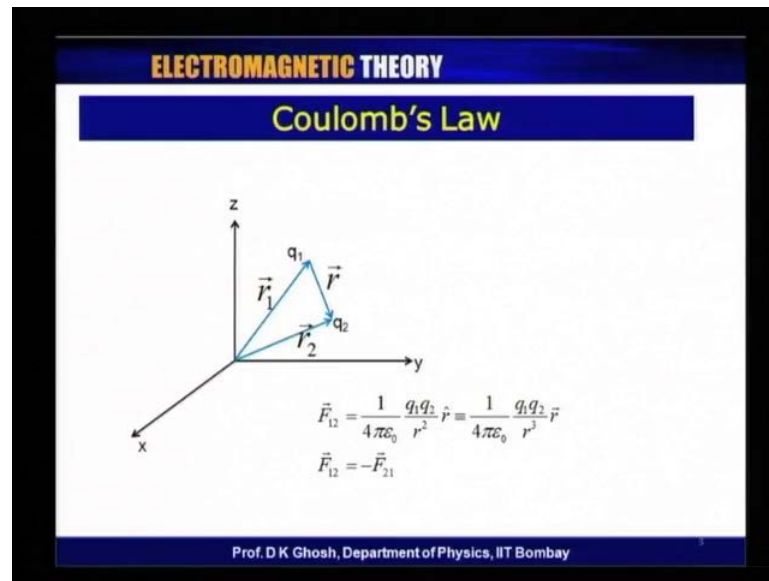


So, if you look at this diagram you will realize that there is a charge q_1 I have kept at the position r_1 and there is a charge q_2 at the position r_2 . The origin is absolutely arbitrary, because the form of the law does not depend upon choice of origin. The force between q_1 and q_2 is proportional to the product of the charges q_1 into q_2 and is inversely proportional to the square of the distance namely r between them. The direction the

direction of the force is along the line joining the two charges. It is and there is of course, a multiplicative constant which is in SI units, it is usually written as 1 over $4\pi\epsilon_0$ where ϵ_0 is known as the permittivity of the free space.

I am assuming that these charges are interacting in vacuum that is with no medium. There being there, what happens when there is a medium in which these charges are located is something which we will be talking about much later in these lectures.

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Now, if F_{12} is the force on the charge 2 due to the charge 1 then by Newton's third law, the force 1 due to charge 2 is just the negative of that. That is the famous action reaction principle, and we have F_{12} is equal to minus F_{21} .

(Refer Slide Time: 03:05)

ELECTROMAGNETIC THEORY

1. Inverse square force
2. Like charges repel, unlike attract
3. Long range force
4. A central force (magnitude depends only on distance and direction along the line joining the charges.
5. Permittivity of free space ϵ_0 :

$$\frac{1}{4\pi\epsilon_0} = 8.9874 \times 10^9 \approx 9 \times 10^9 \text{ N-m}^2 / \text{C}^2$$
$$\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 / \text{N-m}^2$$

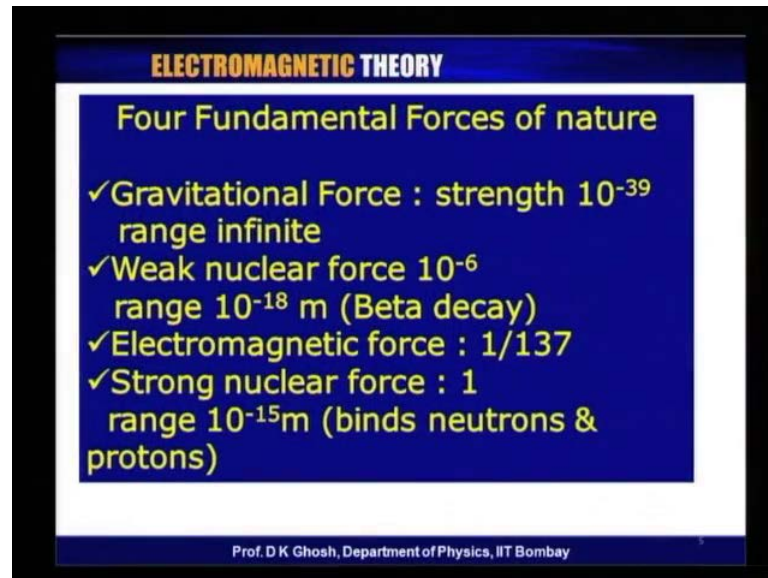
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So, let us summarize what are the properties of the Coulomb's law force. Number one to notice that, the force is inversely proportional to distance and it is, well second thing is that if you look at the form then it is clear that the force is repulsive if it is between like charges and is attractive. If the charges happened to be dissimilar that is, one is positive the other one is negative. The another characteristic of this charge is, it is a 1 over r square force, which is essentially a long range force. Long range forces are those whose range is essentially infinite. That is the force really never becomes 0 except in at infinite distance.

The other point to notice, that the force is central force. The characteristic of the central forces that it is a force whose magnitude depends only on the distance between the two charges and the second thing is its direction its direction is along the line joining the two charges. Along the line joining the two charges can imply that it is attractive or repulsive depending upon the mutual signs of these two charges. The constant or proportionality which in standard international or S I units is written as 1 over 4 pi epsilon 0. The 1 over 4 pi epsilon 0 to a great deal of precision is 8.9874 into 10 to 9, but for all practical purpose we can take it as 9 into 10 to 9 Newton meter square per Coulomb square. And the epsilon 0, itself is the 8.854 into 10 to minus 12 Coulomb's square by Newton meter square.

Now, before we begin the process of electrostatics, let us look at this force little more in detail. It turns out that in the nature, we have basically four types of forces. They are they go by the name four fundamental forces of nature.

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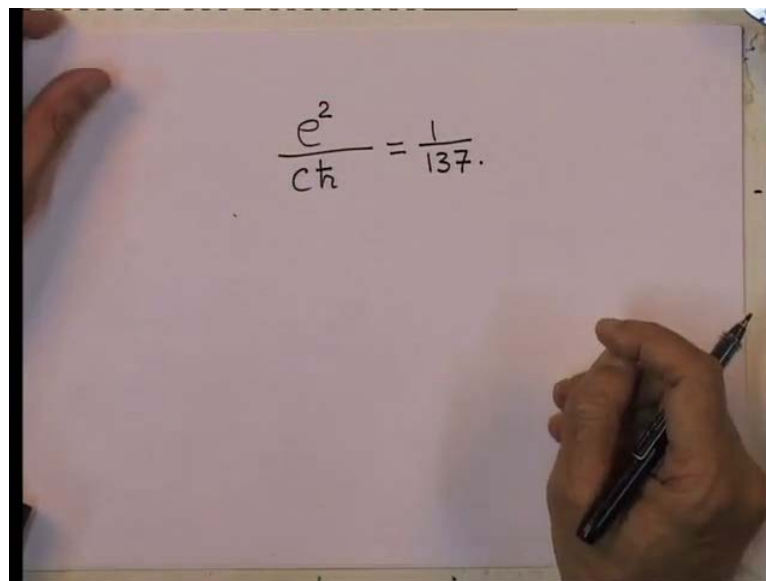
The weakest of the force is what keeps our solar system, the planets, all the planets, the sun everything at their place and it is because of the gravitational force this is an attractive force like electromagnetic force. However, it is of infinite range. It is a long range force. However its strength is very small. Now, in a relatively speaking the, there is a force which is known as the strong force. I will come to a discussion of that a little while later. But assuming that strong force is has a unit 1 then the gravitational force has a magnitude which is 10 to the power minus 39 times that of the force which binds the nuclear the nuclear on together inside a nucleolus. So, it is an extremely weak force the next weaker force again in this course will have not much to do with it, is what is known as a weak nuclear force.

This is a force which is responsible for beta decay and it has a range which is fairly small 10 to the power minus 18 meters and it is called weak nuclear force and its strength if gravitational force has strength of 10 to minus 39. The weak nuclear force has a strength of 10 to minus 6. Therefore, the stronger than gravitational force, but you know it is still a weak force. The third force is the one with which we are involved in this course which

is the electromagnetic force of course, in this part this module will be talking mostly about electrostatics. What is electromagnetic? About it we will come back much later.

So, electric magnetic force has a relative strength of 1 over 137 1 over 137 is a number which is the magnitude of the fine structure constant e^2 over $c h$ cross h cross being the plank's constant. So, the electromagnetic force has a strength relative strength of e^2 over $C h$ cross. C is the speed of light h cross is a plank's constant and this number to a great deal of accuracy is one over one thirty seven e is the electronic charge.

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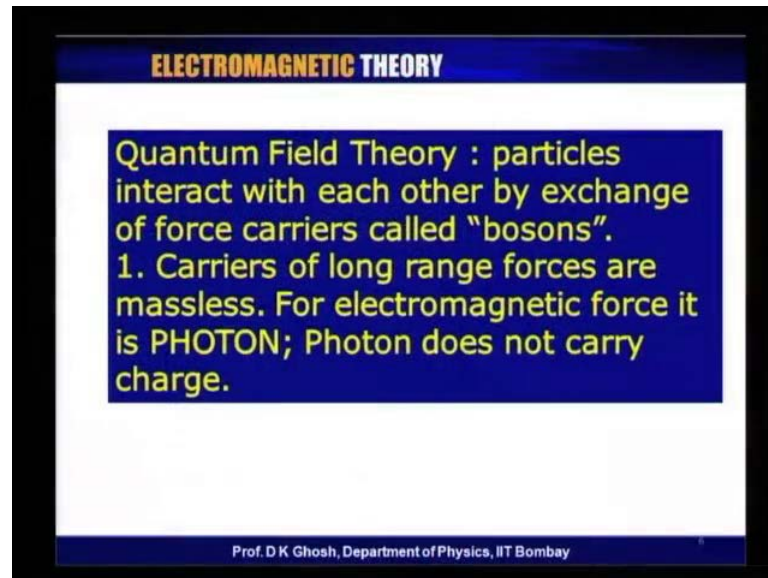

$$\frac{e^2}{c h} = \frac{1}{137}.$$

So, it is something like 10 to minus 2 of the order of 10 to minus 2, but the strongest of them all is what is known as the strong nuclear force. You all know that inside a nucleolus there are protons and neutrons. Neutrons are neutral objects where as protons are positively charged objects. Now, in a nucleolus these are bound together. In spite of the fact that the protons all have similar charges and the electrostatic or electromagnetic force between them is repulsive and neutron has no charge at all. So, and the gravitational force is the only other force that could come in for neutral objects and it is extremely weak force, but nevertheless the nucleolus is bound together.

Now, nucleolus is bound together by a very special force and its name is strong force - strong nuclear force. It exist between neutrons protons, protons protons, and neutron neutron. It is an extremely short range force. Its range is of the order of 10 to the power minus 15 meters., which is also called a fermi, which is also called a fermi and that is

typical nuclear dimension. Now, this force is I am taking its strength as equal to 1. So, compare to this force strong nuclear force, the electromagnetic force is about 1 by 100 I mean 200 of magnitude smaller.

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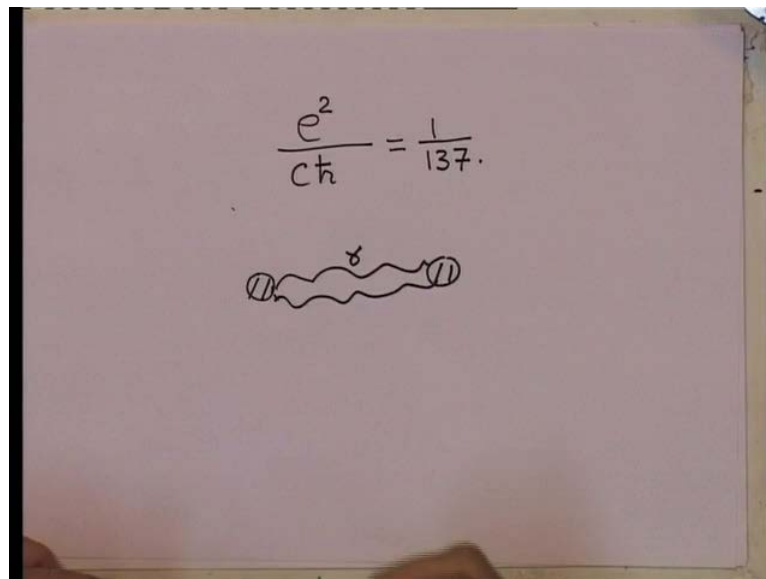
Now second thing is that there is a problem that we come across which I will be briefly touching upon little later and it appears that if you go from the classical theory to the quantum theory which is known as the quantum of field theory knowledge of it is not important for our purpose. But it turns out that the force between two objects any two objects is mediated by a third party, which is normally known as the carrier that is the crude, explanation of that is, that supposing you are looking at the interaction between let us say two charged bodies, then the picture is this two charged bodies are continuously exchanging particles and there by remaining in touch with each other and the fact that the force is propagated from or force exist between two objects is because of the fact that they continuously exchange particles and this particles are known as Bosons.

They incidentally, Boson's the name is after the famous Indian scientist Satyendra Nath Bose and so this Bosons are exchanged. Now, for reasons that I cannot go into in this course, that if a force if the force between two objects is of finite range. Now, we already talked about strong force and the weak nuclear force. If it is a finite range then the exchanged particles have masses. That is their their particles which have mass. On other hand if the force is of long range namely of infinite range, as it is for the gravitational

force and the electromagnetic force. Then the particles which they exchange happen to be mass less.

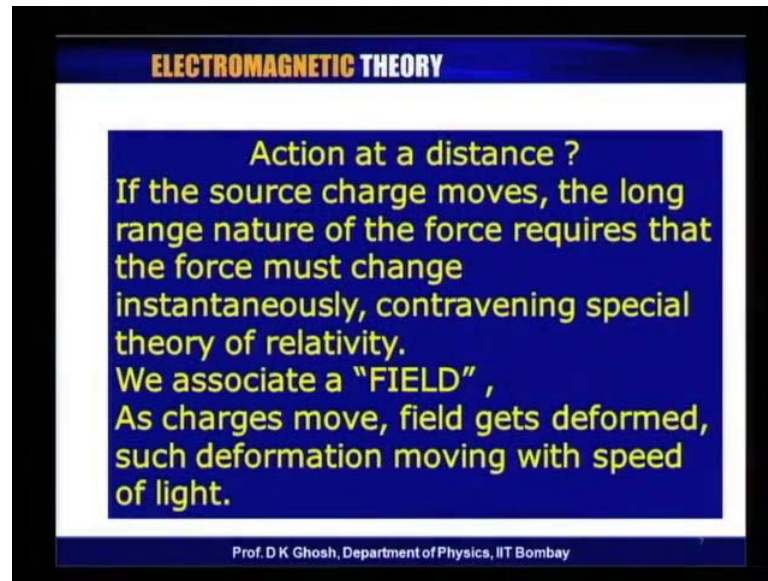
For our purpose the electromagnetic force is a long range force. So, therefore between two charged objects there is always an exchange of a mass less particles or mass less Boson's. the name of this mass less Boson is a photon photons are actually quantum of light, but again we will be probably talking about it in another course. So, and the photon is a neutral objects without any mass. So, basically our picture our picture of the force between to objects is that they are continuously exchanging photons and normally photons are represented by the letters, letter gamma.

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So, continuously they are exchanging photons. Now, another problem that I would like to point out in this context is that the existence of this inverse square law force. In long range force that is perfectly as long as as long as the charges are static. The however, supposing the charge, one of the charge let us say moves. Now, imagine these two charges are located at great distance. How does the force between them instantaneously change?

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Now, this thing is called action at a distance. In physics action at a distance means that the force if one of the objects changes its position let us say, the force on the second one because the distance has changed it changes instantaneously. Now, this is very difficult to understand because according to special theory of relativity propounded by Einstein no signal, no information can ever travel with a speed greater than the speed of light in vacuum which is 3×10^8 meter per second. Therefore, how does it work that as when an object moves this information is essentially instantaneously transmitted to the object on which it is exerting a force.

Now, it is in this context that one introduces the concept of a field associated with a charged object. We have discussed in detail the fields scalar and the vector fields, but think of it very crudely in the following manner. Imagine it is a crude picture. Let say that I am talking about two objects with each of which I associate a medium and this medium let us say, is tightly bound to these two objects. Now when I move one of the particles it deforms that medium, it deforms that medium, because the charged body object is tightly bound to this medium which we are talking about.

I am not talking about a material medium on this moment I am just this is just to fix your ideas. Therefore, this deformation can propagate and ultimately be communicated to other objects on which this one is exerting a force. Now, instead of talking about a medium we say that with every object a field is associated. This field is the

electromagnetic field and when an object moves the field associated with that object changes and this information this information or deformation of the field if you like progresses with the speed of light. The electromagnetic theory is completely consistent with the special theory of relativity.

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ELECTROMAGNETIC THEORY

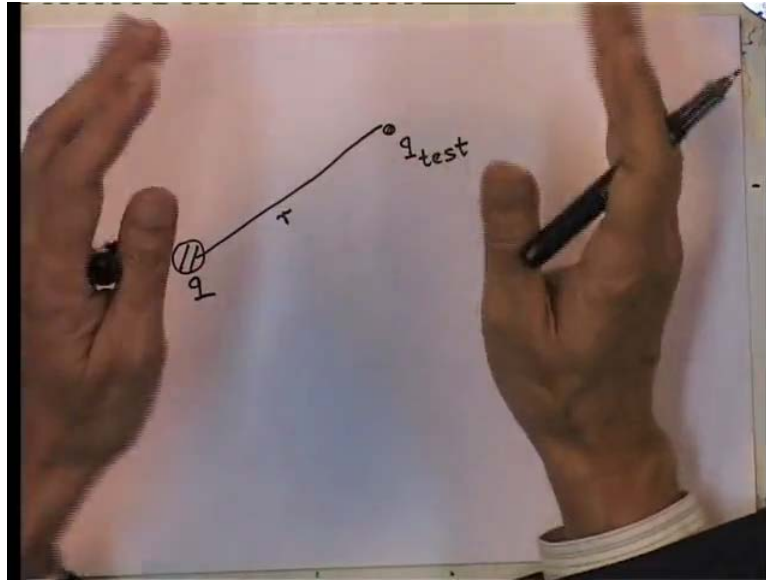
Electric Field
Concept of a test charge : small enough so that it does not significantly modify the field of the source.
Electric field of a source is the force exerted on a unit test charge

$$\vec{E} = \lim_{q_t \rightarrow 0} \frac{\vec{F}}{q_t}$$

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So, let us processed and define the electric field. That, this is something which we have discussed earlier and a, the the field that is associated with a charged object is a vector field. In this case we are talking about an electric field. Now, in order to fix to our ideas about what is an electric field consider a small charge a a minute charge I will call it test charge. I have a charge, let us suppose this charge is q.

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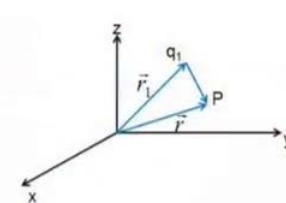
I do not care, how bigger small that charge is and around surrounding this I have my electric field of this charge q . Now, what I do is I put a small charge, let us call it q_{test} at a distance let us say r . Now, this charge as I told you is infinitesimally small. This is this is the required. It is the required because there is field due to the charge q and if you bring in a test charge, this test charge will have its own field which will result in changing the field of this charge q itself and this process will go on. Now, if you bring in a infinitesimally small charge, we assume that its field is not strong enough to significantly change the electric field due to this charge q .

Now, the we define the field due to charge q as the ratio of the force that is felt by this test charge q . In the, a divided by the magnitude of the test charge, in the limit of this test charge going to 0, the magnitude of test charge going to 0. So, the electric field of the source charge which I called as q . There is the force exerted on a unit test charge the formal definition is E is equal to limit q_{test} going to 0 of force f on the q_{test} divided by the q_{test} . Now, obviously such a definition does not depend upon the magnitude of the test charge q itself, q_{t} itself.

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ELECTROMAGNETIC THEORY

Electric Field
If the electric field at a point P is \vec{E}
a charge q at P experiences a force $q\vec{E}$
If a charge q_1 is at \vec{r}_1 the field at P(\vec{r})

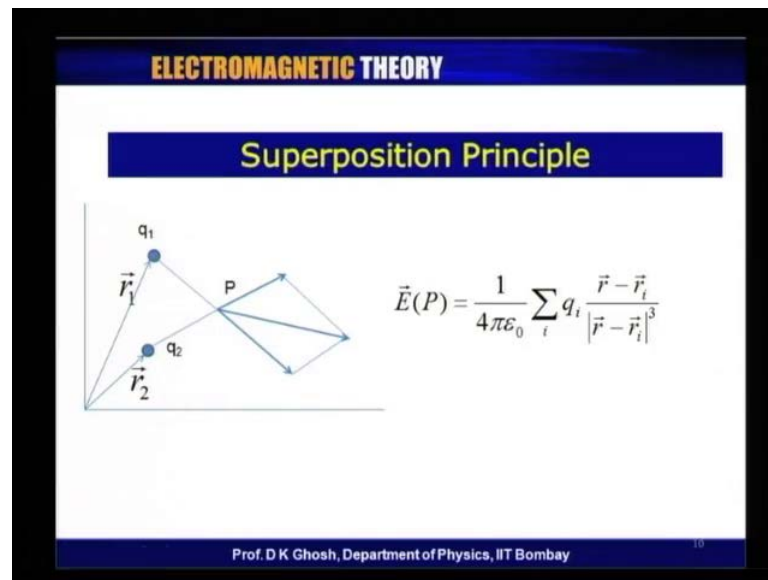

$$\vec{E}(P) = \frac{1}{4\pi\epsilon_0} q_1 \frac{\vec{r} - \vec{r}_1}{|\vec{r} - \vec{r}_1|^3}$$

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Now, so if according to this definition if the electric field at the point P happens to be E then if you put a charge. Now, this electric field is due to some source or sources. I am not specifying now what has given rise to this electric field. Now, suppose you put a charge q at this point P, then a charge q at that point will experience a force given by q times E . Now, let us suppose let us suppose the electric field is generated by a q_1 which is at the position r_1 then the electric field at the point P. Remember that the, if there is a charge q at P the force between q_1 and q_2 is given by $q_1 q_2$ divided by the distance square, the square of this distance and for the electric field I divide by the charge test charge I am putting at P.

So, therefore the electric field at the point P due to the charge q_1 located at r_1 is given by $\frac{1}{4\pi\epsilon_0} q_1 \frac{\vec{r} - \vec{r}_1}{|\vec{r} - \vec{r}_1|^3}$ and the point P is at the position \vec{r} . This is that term with permittivity q_1 and $\vec{r} - \vec{r}_1$ that is the direction of this vector divided by $|\vec{r} - \vec{r}_1|^3$. You notice I have written $\vec{r} - \vec{r}_1$ vector at the top and the q_1 at the bottom. So, that the dimension actually is 1 over r^2 .

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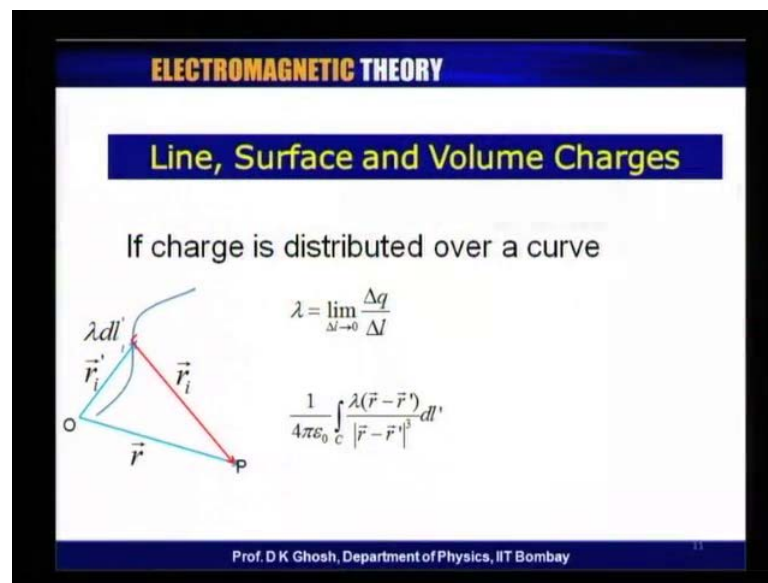
Now, I come to what is known as a super position principle. Now super position principle is that is the source of the electric field is due to multiple charges many charges, supposing I have got a charge q_1 at r_1 q_2 at r_2 etcetera etcetera. Then so this picture for instance talks about only two charges, but this I can generalize it to any number of charges. I will charge q_1 at the position r_1 q_2 at the position r_2 and I am still looking at what is the field at the point P . Now, notice that electric field is a vector therefore the charge the q_1 gives rise to a force the field, which is directed like this along the line joining P and q_1 and it has some magnitude.

Similarly, the force the charge q_2 at P gives rising the electric field which is directed like this. You do a vector addition so the result field that P due to q_1 at r_1 and q_2 at r_2 is the vector sum of the forces, exerted on a test charge on a unit test charge kept at the point P . Now, if I generalize it to multiple charges then this becomes E the electric field at the point p is equal to 1 over π epsilon 0 some over i q_i which is the i x charge located at the position r_i . Vector r which is, vector r is the position vector of the point P with respect to our origin minus r_i by r minus r_i cube. Now, this is known as the super position principle.

Now, we assume that the super position principle is valid for the electric field that is the effect of multiple charges is simply the linear sound of the effects due to individual charges. Fair enough. So, we have talked about what happens to the electric field due to a

single point charge or a multiple charges located at discrete points in space. Now, I can generalize it to include continuous charge distribution. So, let us look at what is meant by continuous charge distribution, supposing I take a curve and in this arbitrarily shaped curve.

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There is a charge which is uniformly distributed. The uniformly distributed charge is has a charge, density linear charge density lambda. In principle this lambda could depend upon the position on that curve, but we have assumed that the charges are uniformly distributed. So, if you take a charge element, a length element delta l, along that curve then the amount of charge in that line element delta l is delta q which is equal to lambda times the length delta l. So, one defines the charge density lambda, linear charge density lambda as limit of delta l going to zero delta q by delta l. Now, remember that in principle this lambda could vary from point to point, but I have taken for simplicity that lambda is a constant.

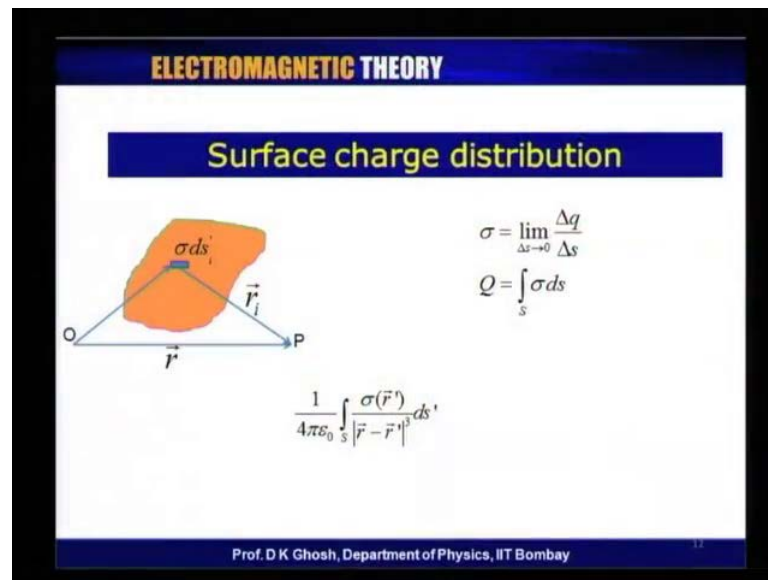
This is not implied by this definition. Now, let us look at how does one express the electric field the electric field due to a linear charge distribution. We have talked about super position principle. Super position principle says that if I have many charges then the electric field at a point due to collection of charges is equal to the sum of the electric field due to the constituent charges. Now, if I have a continues charge distribution, the summation goes over to an integration. So, what I have is this if you refer to this picture.

This is some arbitrarily chosen origin depend upon origin. So, take for example, a small length element Δl . This is the curve, Δl along the curve.

The amount of charge in this is given by this, the length element I am calling it Δl and the prime will be my index on the curve so Δl prime. Therefore, the charge is λ times Δl prime of this little infinitesimally small length element. Now, I am interested in calculating the electric field at a point P located at the position r . So, I connect this. This is the vector r I, which means this gives me the vector position of point P with respect to the i f element charge length element and then I simply add out or in this case I do an integration. Add a 1 over $4\pi\epsilon_0$ λ Δl λ times $d l$ λ times $d l$ prime is the amount of the charge there. That replaces the q_i in our earlier expression and the strength of the field which is vector r minus r prime which is just the r i prime vector divided by r minus r prime cube and the integration is over the entire length there.

Now, remember that even if you take a small enough, even if you take a small enough length. They are literally very large number of charges there, because they discreet charges that I have in a material are actually electrons which give rise to the electric field, but however whatever I am talking about should not be done at the atomic level because there are very few electrons and this continuous limit that I am talking about is absolutely not applicable there. Now I can I can extend this, I can extend this to a distribution of charge on a surface which is in two dimension.

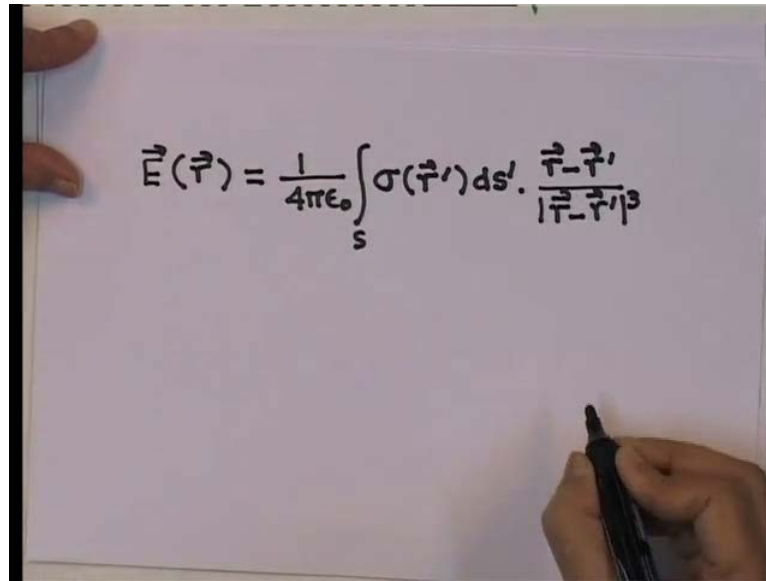
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So, here I have given you shown you an irregular surface. There is an irregular surface and and there is charge distributed there. Once again, how do I define a surface charge density, which I will indicate by sigma. So, what I do is I look at a small area element that small area element, I will call it as ds_i again because prime is the index which I am using for coordinates on the surface. So, if the amount of charge in that area element happens to be Δq , then I define a surface charge density sigma as the limit of ΔQ by Δs as the surface element Δs goes to 0. So, the total charge would be integration of sigma ds over the entire surface.

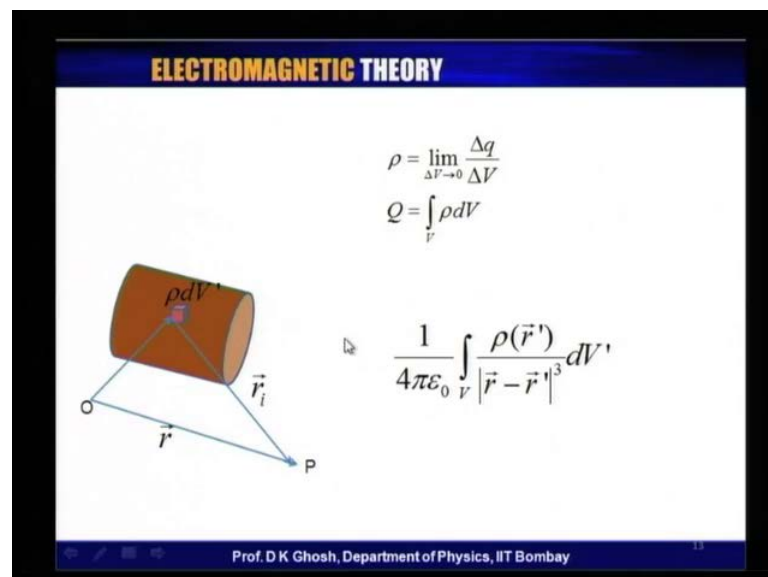
Now, to find the electric due to such a surface at a point P, which is located at the position vector r is exactly we proceed the same way. Suppose, this element ds_i is located located at this position here, position some r' . Now, what I am interested is this vector and therefore, the field at this point is given by there is a small error in the expression for the electric field given in this slide. So, let me write it down.

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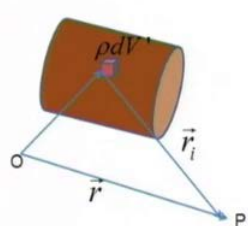

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_S \sigma(\vec{r}') ds' \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3}$$

The electric field at the position r is given by 1 over 4 pi epsilon 0 integral over the surface. Now, sigma which in principle could depend upon the position r prime d s prime, so that is your charge element and the electric field we know varies as inverse square. So, vector r minus r prime divided by r minus r prime cube. So, this is the surface charge distribution. What about the volume charges?

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ELECTROMAGNETIC THEORY

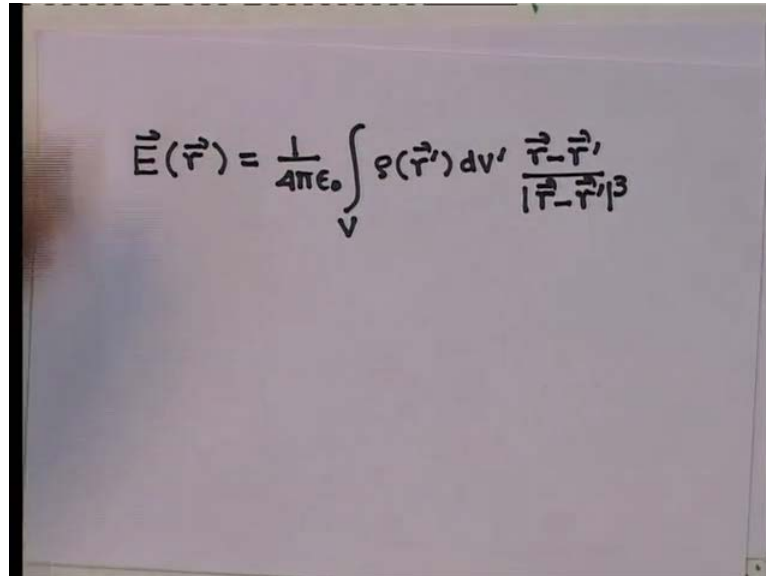
$$\rho = \lim_{\Delta V \rightarrow 0} \frac{\Delta q}{\Delta V}$$
$$Q = \int_V \rho dV$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|^3} dV'$$

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Now, once again the volume charges are this. So, this is a rho d V. I take a small volume element d V and the once again I am interested in calculating the field at the point P,

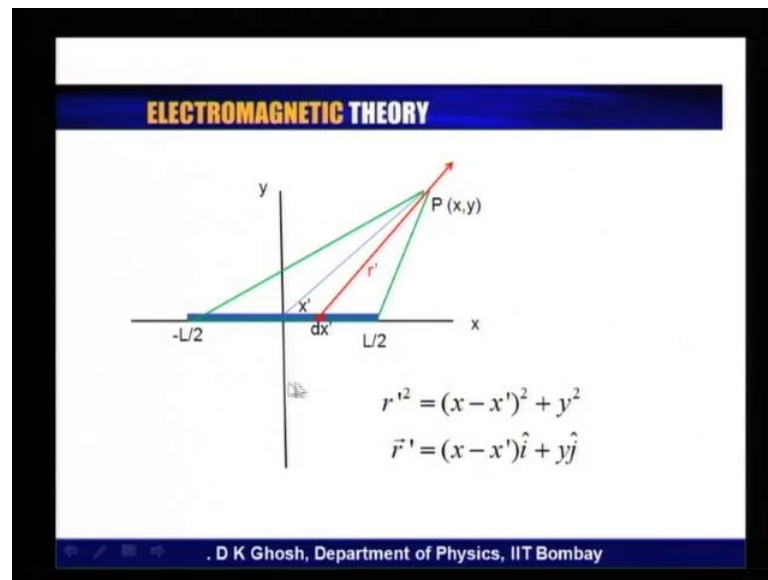
therefore the amount of charge that is contained in this is ρdV prime. dV prime is the volume element there and once again I used the same expression.

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$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_V \rho(\vec{r}') dV' \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3}$$

The electric field at the point r is given by 1 over $4\pi\epsilon_0$ integral over the volume. ρr prime that is the volume density dV prime which is the element of charge in that element of volume, times vector r minus r prime divided by r minus r prime cube. So, that gives me the electric field due to what I call as the volume charged distribution. The volume charge density ρ is defined as limit of ΔV going to 0 ΔQ by ΔV where ΔQ is the amount of charge that is contained in that volume element. So, the total charge Q is you have to integrate over that entire volume is integral of ρdV .

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Let me illustrate some of this with couple of examples of calculating the electric field. I have taken a line charge of length L and I have sort of placed it such that the origin is taken at the center. It is a simple line and I am interested in calculating what is the field, electric field at the point x, y point P whose coordinates are x, y . The line is along the x direction and of course, the y axis is perpendicular to it. So, let us look at this how does one do this. Consider a charge element at a distance x' and having length dx' . Now, what I do is this, that this red rho indicates the direction of the force at the point P , that electric field at the point P due to the charges contained in this element dx' which is of course, $\lambda dx'$.

Now, what is this distance r' . Now, notice r'^2 is given by, you can complete a, you can complete a right angle triangle here. r'^2 is given $(x - x')^2 + y^2$. x is the position x coordinate of P . plus y^2 is the y coordinate of the P . So, x and y are fixed where as the x' will change depending upon where am I taking this length element dx' . Now, and the vector \vec{r}' is given by $(x - x')\hat{i} + y\hat{j}$.

(Refer Slide Time: 36:16)

ELECTROMAGNETIC THEORY

Field due to a charged line

The field at P(x,y) due to the element dx' at (x',0)

$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{\lambda dx'}{[(x-x')^2 + y^2]^{3/2}} [(x-x')\hat{i} + y\hat{j}]$$

The net electric field at P(x,y) is

$$E_x = \frac{\lambda}{4\pi\epsilon_0} \int_{-L/2}^{L/2} dx' \frac{(x-x')dx'}{[(x-x')^2 + y^2]^{3/2}}$$

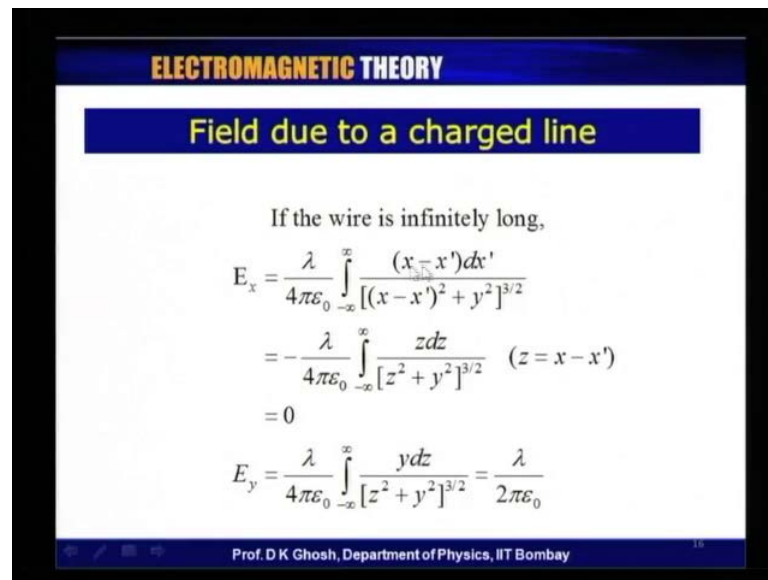
$$E_y = \frac{\lambda}{4\pi\epsilon_0} \int_{-L/2}^{L/2} dx' \frac{y dx'}{[(x-x')^2 + y^2]^{3/2}}$$

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So, therefore the field at the point x y due to such an element d x prime, which is located at x prime 0; 0 is the y coordinate is simply given by our familiar expression, which is 1 over 4 pi epsilon 0. The amount of charge that is contained in d x prime namely lambda d x prime that distance Q remember. The square of the distance was x minus x prime whole square plus y square. So, distance cube is x minus x prime whole square plus y square rise to the power 3 by 2 times the position vector here. That is x minus x prime i plus y z. Now, so what I have done here is to write down the x and the y component of this electric field that is simply writing this term and this term separate. Now, for an arbitrary position x y, this integration cannot be done in a close form and one normally you can however, go to computer and sort of plot what are the e x and e y.

However, let us look at a particularly important case, let us look at a infinite charge. Now, if I have an infinite charge the instead of wire being located from minus L by 2 to plus L by 2. It is from minus infinity to plus infinity. In this limit these integrals which we talked about here. Look at the e x prime integral e x the e x so if it is from minus L by 2 to plus L by 2 this is a difficult integral. Now, suppose it is a minus infinity to plus infinity, I can reduce this integration very simply by replacing x minus x prime with some other variable. Let us say z then the integral that I get is this.

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ELECTROMAGNETIC THEORY

Field due to a charged line

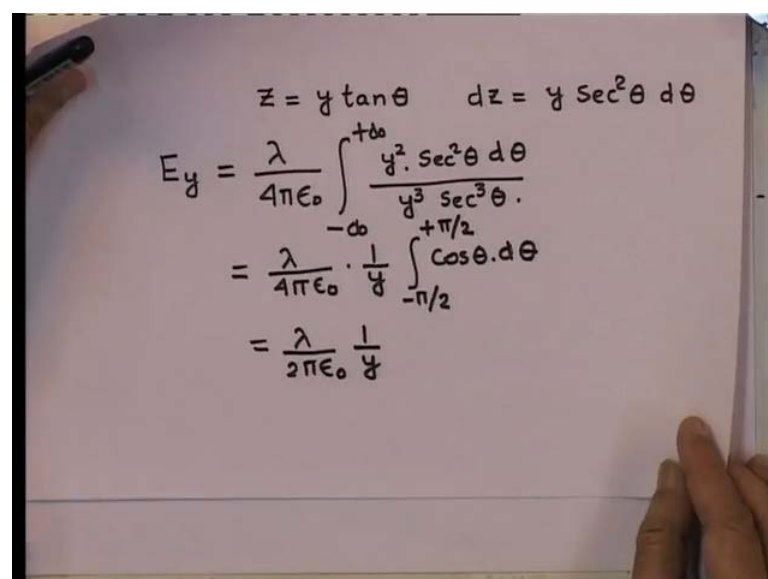
If the wire is infinitely long,

$$E_x = \frac{\lambda}{4\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{(x-x') dx'}{[(x-x')^2 + y^2]^{3/2}}$$
$$= -\frac{\lambda}{4\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{z dz}{[z^2 + y^2]^{3/2}} \quad (z = x - x')$$
$$= 0$$
$$E_y = \frac{\lambda}{4\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{y dz}{[z^2 + y^2]^{3/2}} = \frac{\lambda}{2\pi\epsilon_0 y}$$

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That suppose, I say x minus x prime is equal to z , then dx prime is minus dz . So, this integral becomes integral $z dz$ square plus y square to the power 3 by 2 from minus infinity to plus infinity and this integral is 0 because this is an odd integral and however therefore, the x component vanish. The y component however, does not vanish. Remember, that the y component was the so I will go back a little bit y component is given by this. If you have the same substitution x minus x prime equal to z you get $y dz$ by z square plus y square. So, this is what I have written down here to the power 3 by 2 this integral you can do this integral.

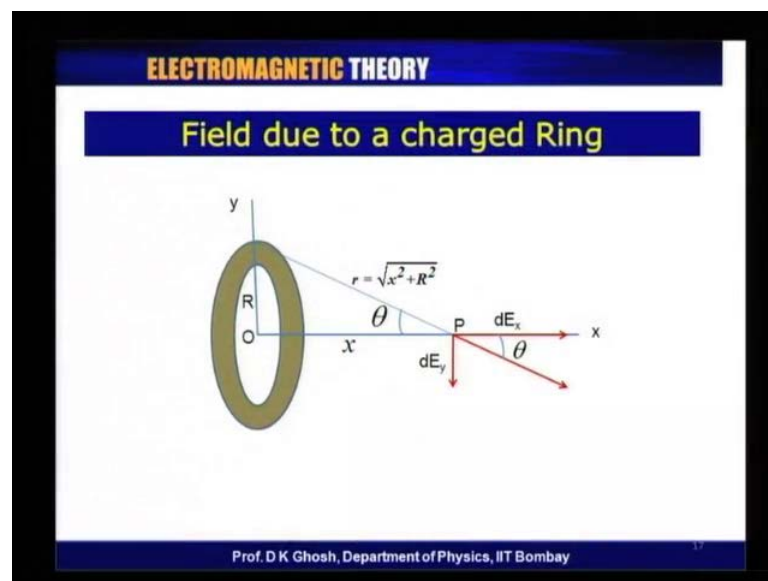
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$$z = y \tan \theta \quad dz = y \sec^2 \theta d\theta$$
$$E_y = \frac{\lambda}{4\pi\epsilon_0} \int_{-\infty}^{+\infty} \frac{y^2 \sec^2 \theta d\theta}{y^3 \sec^3 \theta}$$
$$= \frac{\lambda}{4\pi\epsilon_0} \cdot \frac{1}{y} \int_{-\pi/2}^{+\pi/2} \cos \theta d\theta$$
$$= \frac{\lambda}{2\pi\epsilon_0} \frac{1}{y}$$

You can do very easily so by substituting z is equal to $y \tan \theta$. So, E_y is equal to λ by $4 \pi \epsilon_0$. Integral from minus infinity to plus infinity. $y \, dz$. So, y is of course, fixed coordinate so I cannot change it dz . So, therefore if z is equal to $y \tan \theta$ dz becomes $y \sec^2 \theta \, d\theta$. So, let us write down dz is another y so that gives me $y^2 \sec^2 \theta \, d\theta$ divided by I have got $z^2 + y^2$ and z is $y \tan \theta$. So, I get y^2 into $1 + \tan^2 \theta$ inside so what I get is $y^3 \frac{1 + \tan^2 \theta}{1 + \tan^2 \theta}$ is $\sec^2 \theta$ so I get $\sec^3 \theta$.

So, therefore, what I get here is λ by $4 \pi \epsilon_0$ and I have got a 1 over y there and integral of 1 over $\sec^3 \theta$ which is $\cos \theta \, d\theta$. Now, notice that I said z is equal to $y \tan \theta$. So, if z is going from minus infinity to plus infinity, then my $\tan \theta$ must go from minus $\pi/2$ to plus $\pi/2$ and integral of $\cos \theta \, d\theta$ is $\sin \theta$ which if I put the limit I get 2 from this integration. So, I will be left with λ over $2 \pi \epsilon_0 y$.

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This is the electric field due to a an infinite line charge. As a second example let us look at the field of on the axis of a charged ring. This is the ring of radius R , the ring of radius R . This distance I am looking at the field along the axis which I have taken as the x axis. the point P is at a distance x which is the fixed distance from the origin and this is this is where I am interested in calculating the field. Now, let us look at what happen, this this incidentally is cuts the x y plane perpendicular to that. So, if I now take again as we did

earlier a length element here, ignore the width that is shown that is shown for clarity. If I take a length element here then this is at a distance R and the field at the point P is along this. The direction is shown by this red arrow. This distance is equal to for any element is equal to square root of x square plus r square.

Let us suppose this is at an angle theta, this element is at an angle theta with respect the geometry that has been shown. Now, this is the direction of the electric field due to this charge element and I can resolve it into an x component and a y component.

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ELECTROMAGNETIC THEORY

Field due to a charged ring

Field due to an element λdl along the axis
(along line joining the element to P : \vec{r})

$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{\lambda dl}{x^2 + R^2}$$

$$dE_x = \frac{1}{4\pi\epsilon_0} \frac{\lambda dl}{x^2 + R^2} \cos \theta = \frac{1}{4\pi\epsilon_0} \frac{x \lambda dl}{(x^2 + R^2)^{3/2}}$$

$$E_x = \frac{\lambda}{4\pi\epsilon_0} \frac{x}{(x^2 + R^2)^{3/2}} \int_c dl = \frac{Q}{4\pi\epsilon_0} \frac{x}{(x^2 + R^2)^{3/2}}$$

For $x \gg R \Rightarrow \frac{1}{4\pi\epsilon_0} \frac{1}{x^2}$

Prof. D K Ghosh, Department of Physics, IIT Bombay

Now, notice the direction of the field d E is along the line joining the element to the point P and the magnitude is 1 over 4 pi z epsilon 0 Lambda d l which is the charge, x square plus r square. So, remember all the elements because this is a circle which is perpendicular to the x y plane. I mean which is cutting the x y plane perpendicular to the x axis. x axis is the axis of that ring so all the elements are located at the same distance from the point p and that distance is square root of x square plus y square. So, what we do we get now is the x so this is the direction and if this angle is theta if this angle is theta then the x component of the field is d E cosine theta and the y component of the field is d E sin theta.

So, this is what I have written down lambda d l by x square plus R square this is this is a constant because x is the fixed number R is the radius cosine theta and now, if you look at what is cosine theta. The cosine theta is nothing but x divided by this distance again.

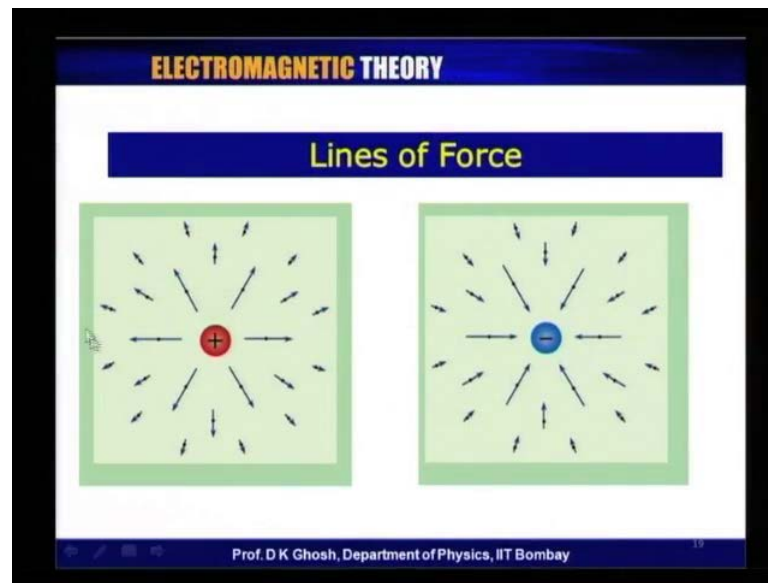
So, therefore what I get is $\frac{1}{4\pi\epsilon_0} \frac{Qx}{x^2 + R^2} dx$. Now, all the numbers here are constant. They do not depend upon what is the position of x .

So, if you now have to integrate I am talking about x component at the moment. If you now integrate all those being constant come out, and you have to integrate over the length $2R$ which is obviously equal to $2\pi R$, the circumference. So, the x component that you get of the electric field is $\frac{Q}{4\pi\epsilon_0} \frac{x}{x^2 + R^2}$. What happened to the y component? Look at this, that while the x component added up while the y component will cancel because from symmetrically placed element. Supposing I look at this section the force will be directed like this. Now, if it is directed like this the x component will still be along the x axis but the y component will be in the reverse direction.

So, the y component of the electric field cancels by symmetry and the field is directed along the axis, the field is directed along the axis. So, therefore the electric field due to a charged ring on its axis, is given by $\frac{Q}{4\pi\epsilon_0} \frac{x}{x^2 + R^2}$. There is an interesting consequence there of supposing the distance at which you are interested in measuring the field, is much larger than the radius. What would you expect? See if you go far far away. A small range looks to you like a point charge. So, we must get back in this limit, the electric field due to point charge and you can see what actually happens.

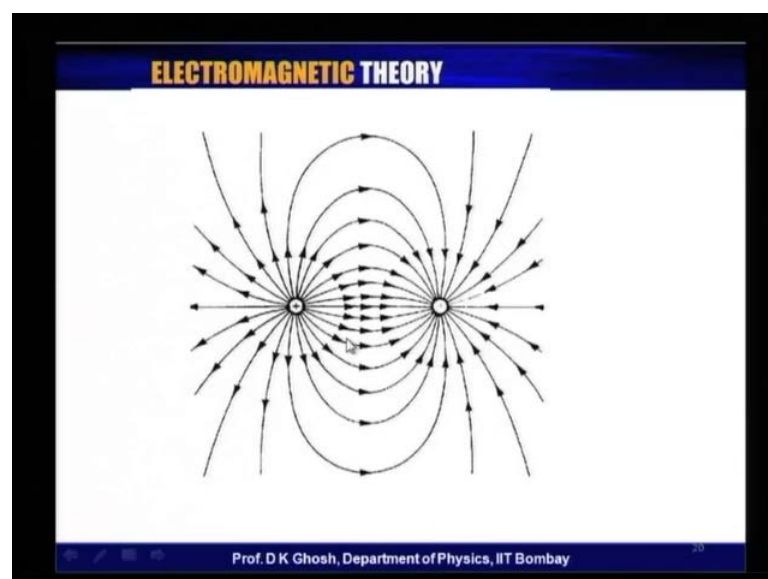
See if x is much larger than R then I can neglect this R^2 in the denominator and $\frac{x}{x^2 + R^2}$ will give me $\frac{1}{x^2}$ and I am left with $\frac{1}{x^2}$ which is simply equal to $\frac{1}{x^2}$ which is nothing but my Coulomb's law. So, we have been talking about the electric field due to point charge to begin with. A distribution of point charges at discrete points. We talked about superposition principle. Let us recall back. The vector field concept which we had discussed a few lectures back.

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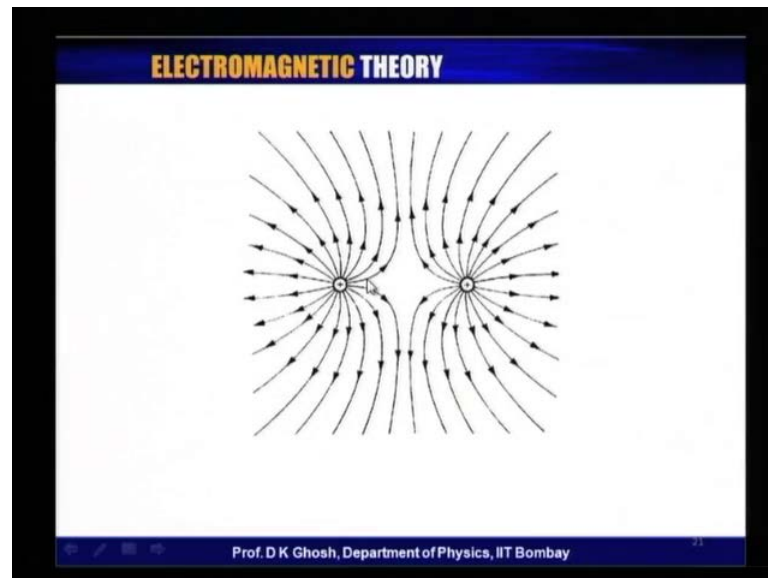
Supposing, I have a point charge positive charge look at this picture. Now, your field electric field due to a positive charge, remember my test charge is always taken as positive. So, a test charge will be repelled in this field and the closer the test charge comes to the source charge which is positive here, stronger will be the electric field. Now, clearly since it is a point charge I expect a perfect symmetry between the forces. So, this is the way the electric lines of force, the vector field looks like for a positive charge. Reverse is the situation for the negative charge because they should all point towards the charge.

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What happens if I have a positive charge and the negative charge? The lines of force must start the forces directed such that the lines of force are those where if you put a test charge there it experiences a force along that tangent and it must get away from a positive charge go towards a negative charge. So, that was the force lines of force due to a positive and a negative charge.

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And if I have two positive charges, it will repel. And therefore, you notice that the now two lines of force can never intersect, because at the point of intersection then the field will be ill defined. So, what we have done today, what we have done today is to discuss the concept of an electric field. Next time we will be discussing the nature of the electric field and talk about concepts such as potential and consequences, thereof.