Electromagnetic Theory
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Lecture - 5
Laplacian

In the last lecture we had talked about the curl of a vector field. Before that we introduced the ideas about a divergence of a vector field as well as the gradient of a scalar field. Today what we want to do is to introduce another operator which goes by the name Laplacian or dell square operator.

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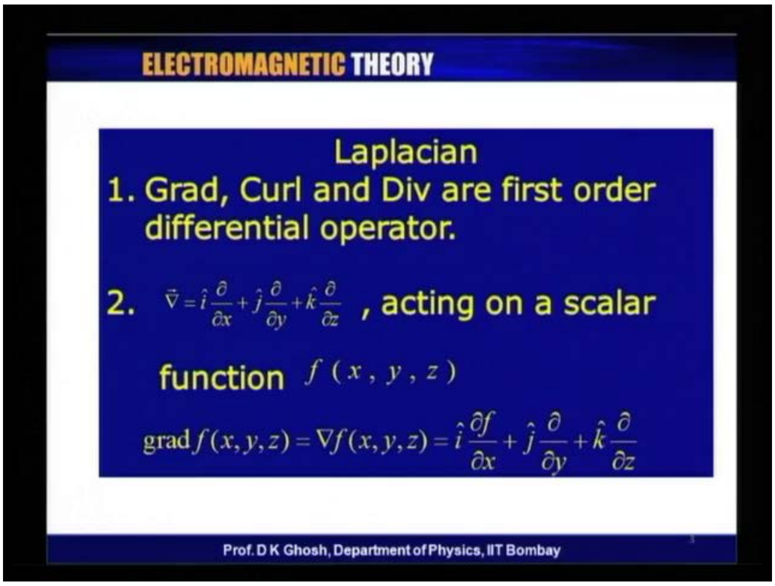

The grad, curl and divergences, they are essentially first order differential operator. For instance the gradient which is given by i d by d x plus j d by d y plus k d by d z as we talk said earlier acts on a scalar function f of x, y, z and gradient of f is a vector given by unit vector i times the partial f with respect to x plus unit vector j times partial f with respect to y and unit vector k times partial f with respect to z.

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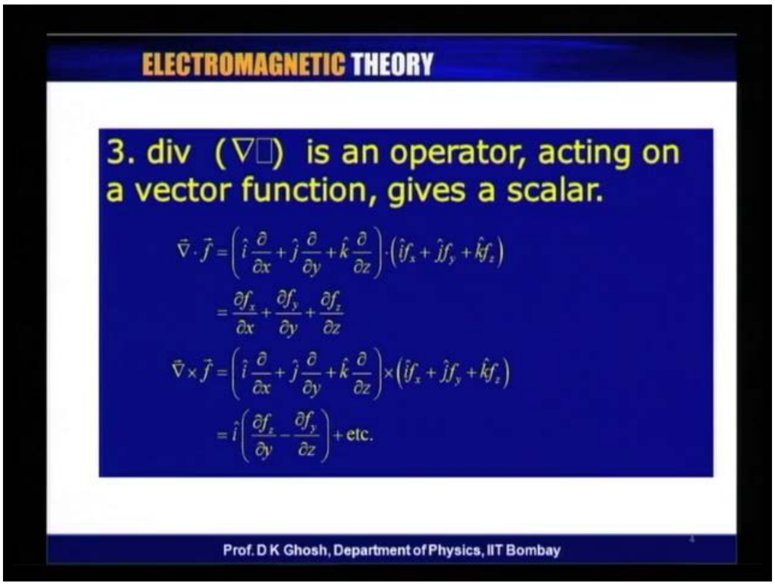

Divergence or del dot on the other hand is an operator which acts on a vector function giving a scalar. The the expression for divergence is almost similar to that of the gradient accepting that after i d by d x plus j d by d y plus k d by d z, there is a dot. So that on the right hand side of the dot I expect to see a vector function, and if vector f has components f x, f y, f z; then del dot of f or the divergence of f gives me partial f x with respect to x plus partial f y with respect to y plus partial f z with respect to z.

The curl is again a vector operator acting on a vector it gives me yet another vector. So, what we have seen that how one can get a Cartesian expression for the curl last time and for example, the x component of the curl of f is d by d y of f z minus d by d z of f y etcetera, etcetera. Now, this del square which is also called nabla square sometime, but is also it has a name Laplacian operator which is basically a del dot del.

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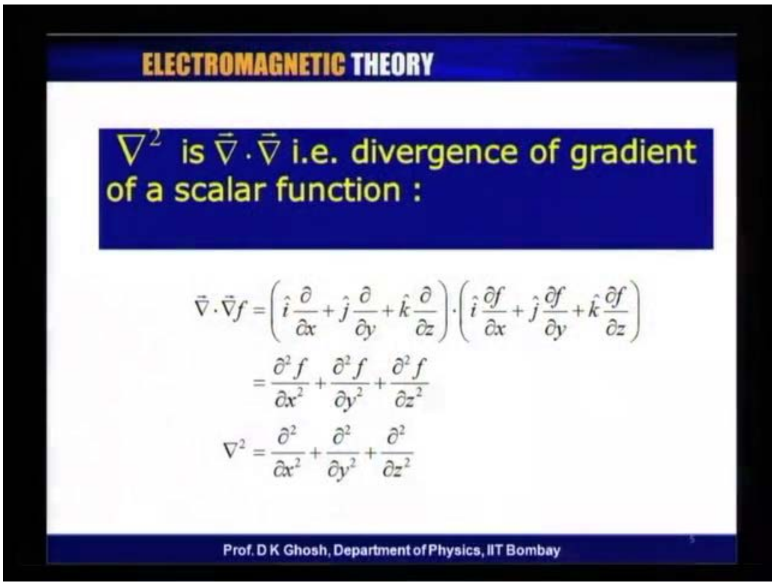

Now, what is del dot del? Now, del dot del means the second del is essentially a gradient. So, in other words I am expecting a scalar function to appear to the extreme right. Therefore, del dot del let me sort of work it out.

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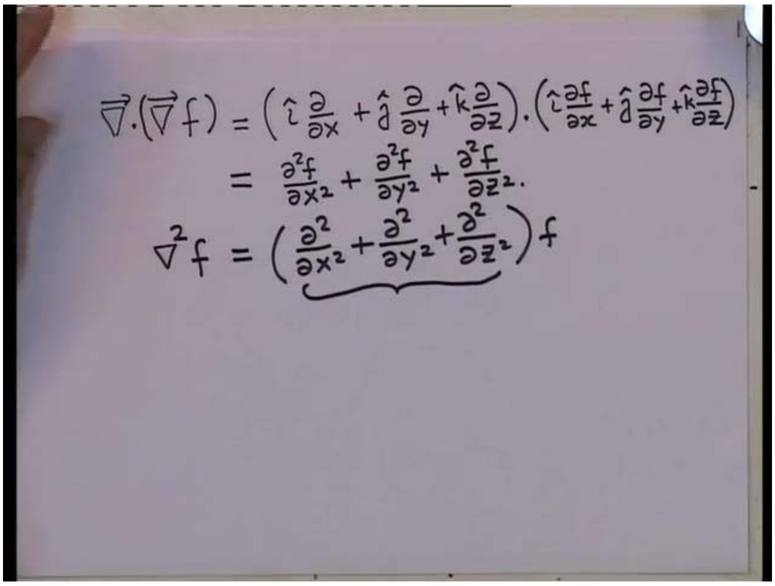

So, del dot del means divergence of the gradient of the function f. So, let us keep the write down the Cartesian expression for each one of them i d by d x plus j d by d y plus k d by d z dotted with we have seen the expression for the gradient and that is yet another vector which is i d f by d x plus j d f by d y plus k d f by d z.

Now, the way to look at this is that the vectors on this side is to be dotted with the vectors on that side. So, I get i dot i which is equal to 1 and d by d x of d f by d x which is nothing but second differentiation of f, second partial differentiation of f with respect to x. So, I get d square f by d x square. Now, i dot j is 0 and i dot k is 0. From the second j dot j I will get d square f by d y square and from the third one I get d square f b y d z square. So, in other words the del square operating on a scalar function f is same as d square by d x square plus d square by d y square plus d square by d z square operating on f. So, this is the expression for the Laplacian operator or del square operator.

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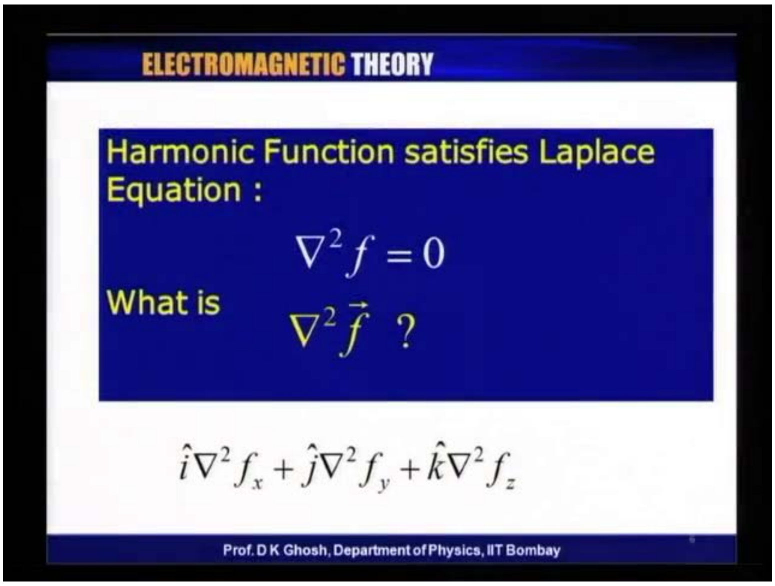

Now, there is a class of function known as harmonic function, and they are characterized by the fact that the Laplacian of such functions is equal to 0.

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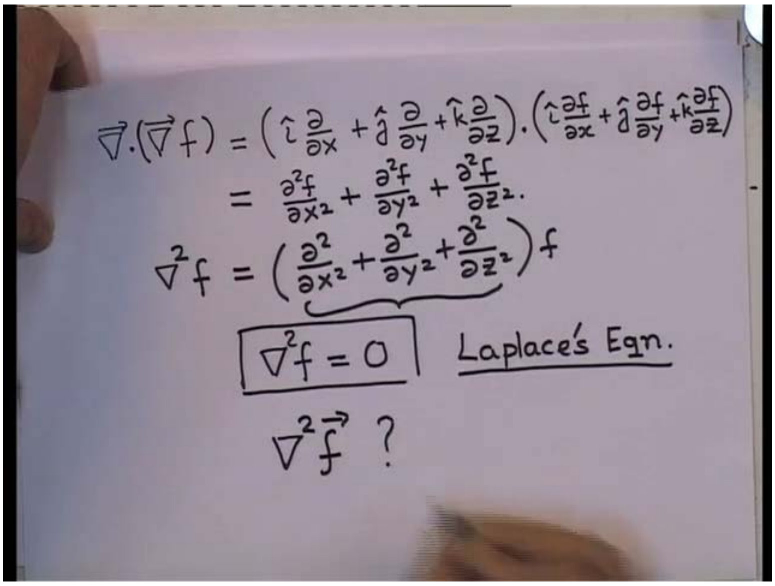

In fact this equation is known as Laplace’s equation. This equation is known as Laplace’s equation and we will see that in electrostatics this equation is of great importance. So, we have seen what is del square of a scalar function. Now, is it meaningful to write del square of a vector function, is it meaningful? This is really a some sort of a notation if you like. Say, del square is del dot del therefore, obviously a gradient of a scalar make sense but when we write del square of f.

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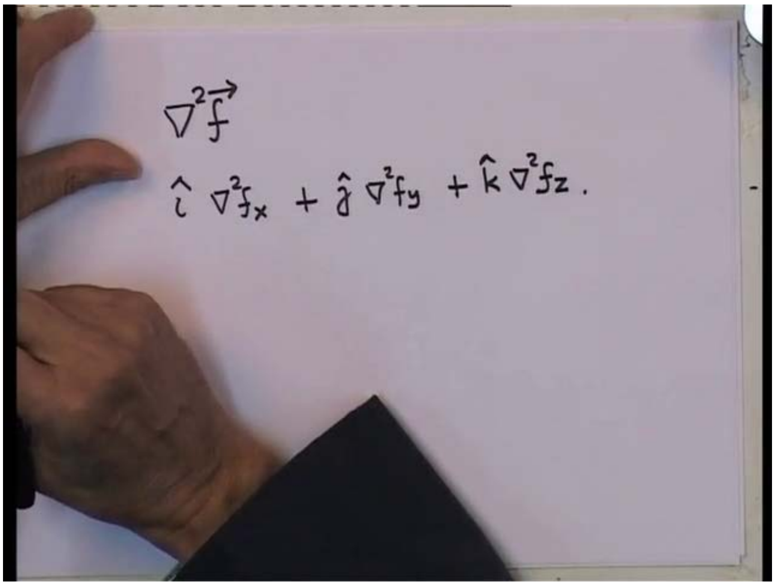

This is f as a vector. This is a shorthand notation for three quantities. These are basically three terms there. So, in another words I have, it is a vector giving me unit vector i times del square f x plus unit vector j times del square f y plus unit vector k times del square f z. Notice, f x f y and f z are scalar function, so del square of each one of them is defined as per our previous discussion. In this I have listed a few vector identities.

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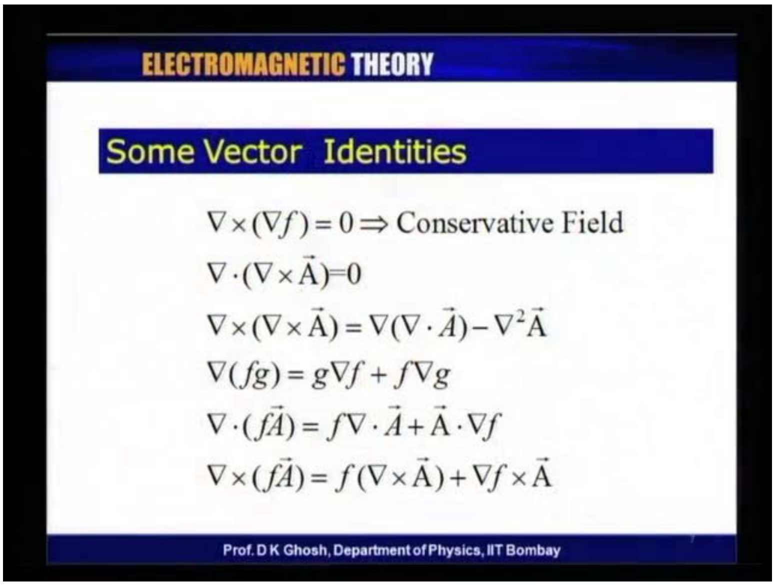

I will not be deriving them, but will expect you to try to work out some of them. But notice some of them are rather interesting. The first equation says curl of a gradient is equal to 0. We have discussed it earlier, we have said that if a force field can be written as a gradient of something then that is a conservative field. Say, if it is a conservative field curl of such a force field is 0. This is actually a test of conservative force field so del cross del f is equal to 0.

Now, I know that del cross A is a vector, but it can be shown that the divergence of the curl of a vector function is equal to 0. So, remember we had said earlier that if the curl of a vector field is 0 the field is said to be irrotational. If the divergence of a vector field is 0 it is said to be solenoidal. So, what we are trying to say if I have a field del cross A then such a field vector field has to be a solenoidal field. The third equation that I have written down this is an identity you must have seen in case of ordinary vectors, vector triple products like A cross B cross C.

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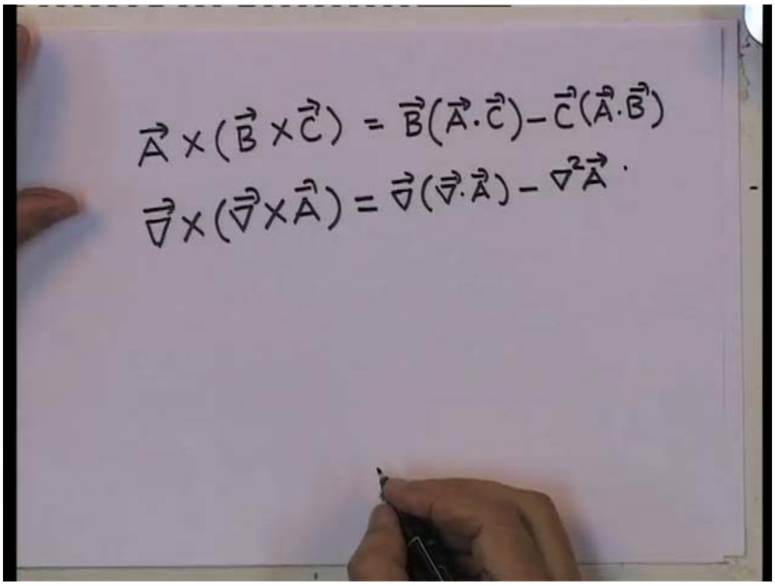

For instance we are aware of this that A cross B cross C. Now, you have to put in a bracket because otherwise we cannot be sure whether you are first taking A cross B and then taking a cross product with C or the way I have written down. And this turns out to be given by the famous occasionally called the back minus cap formula B A dot C minus C A dot B.

Now, when we write del cross del cross of A, again the brackets are important. The difference between this expression and this is the fact that these are differential operator. So, one is to write them. When you write them they act to the quantities that appear to their right. So, you cannot simply reproduce this, you have to rework it out. Now, it turns out to be the great del del cross del cross of A which has to be a vector because this is a vector and this is a curl of a vector. It is gradient of del dot of A, it looks very similar to the first term excepting that you have to know that this is my B, but this is a differential operator this is a differential operator, del of del dot of B A minus del square of A.

The the next equation that I have written down is what is gradient of f g? Now, this is like a chain rule. It is f times gradient of g times gradient of f plus f times gradient of g. What is divergence of a scalar times a vector function? Very interesting, so it also works like a chain rule, but you have scalar times divergence of the vector function then the dot product of this vector with a gradient of the scalar function. So, these are some of the identities which you will have to work out and we will be probably using them as we go along.

We have been talking about a vector field and we have introduced we have introduced two properties or two types of operators which can act on a vector field and they are the divergence of a vector field and the curl of a vector field. The question is what makes a vector field unique? Given what quantity you can say this uniquely describes the vector function.

Now, turns out that if you are, you provide the curl and the divergence of the vector, of a vector and the normal to the surface in which this field is defined. Then these three properties uniquely determine a vector field. This is a very important property, what makes vector field unique? So, let me let me try to establish this.

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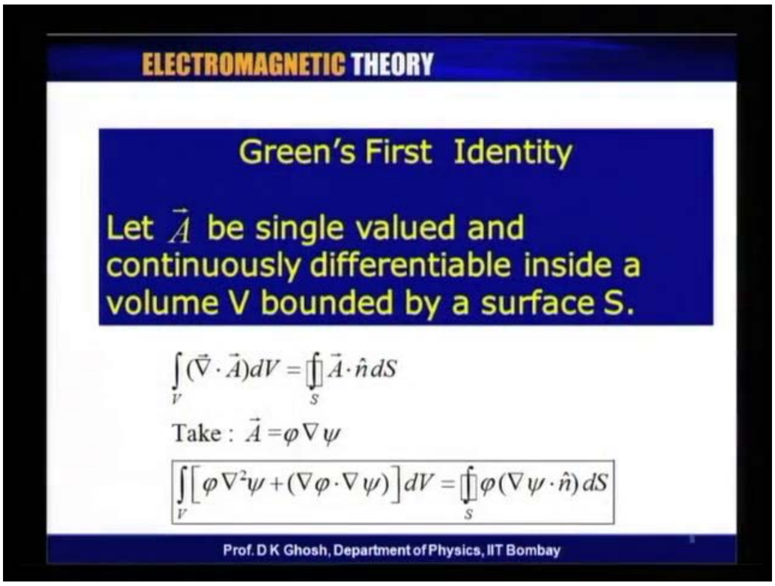

In trying to establish this we use an identity which is known as Green's identity. There are two of them, the first Green's identity and the second Green's identity. The second one is also known as Green's theorem. They are rather straight forward, but let us look at what they imply. Supposing, A is a single valued and continuously differentiable vector function inside a volume V which is bounded by a region S. So, I know the vector A in that.

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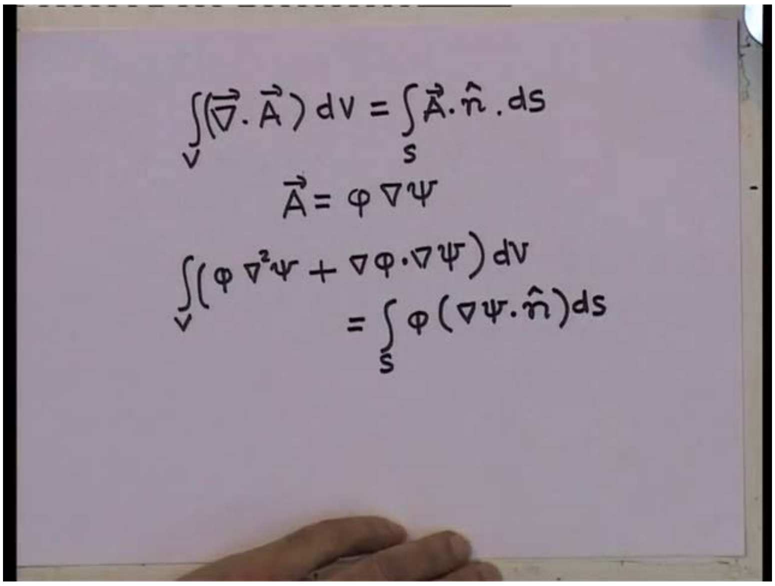

Now, we had seen earlier that if a vector A is defined in a region which has a volume V and is bounded by the surfaces, then the divergence of the vector del dot A, its volume integral is given by the surface integral of the vector A taken over the surface which binds this volume, this was known as the divergence theorem. Now, what you are going to do is this. Let me take the vector A to be given by a scalar function phi times the gradient of a second scalar function phi phi, substitute this into this equation.

So, let us see what we get? We, get del dot of A. We have just now said what is del dot of a scalar times a vector, we had shown an identity. So, remember what you said is the del dot of scalar times a vector is this scalar times del dot the vector which is del dot del psi. So, scalar is phi times del square psi plus dot product of the gradient of this with the second vector which is gradient phi dot with gradient psi and this whole thing volume integral being taken and that is equal to the surface integral of phi is a scalar, so I just write like this times gradient of psi which is a vector dotted with n d S.

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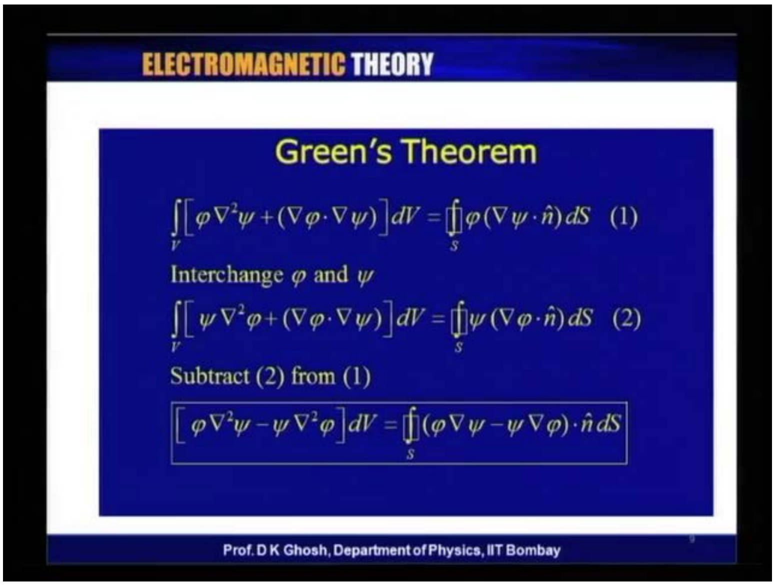

This is known as the Green's first identity. So, phi del square psi plus del phi plus del psi d V is phi del psi dot n d S. Now, phi and psi are arbitrary scalar function. So, what I do is, I simply interchange them.

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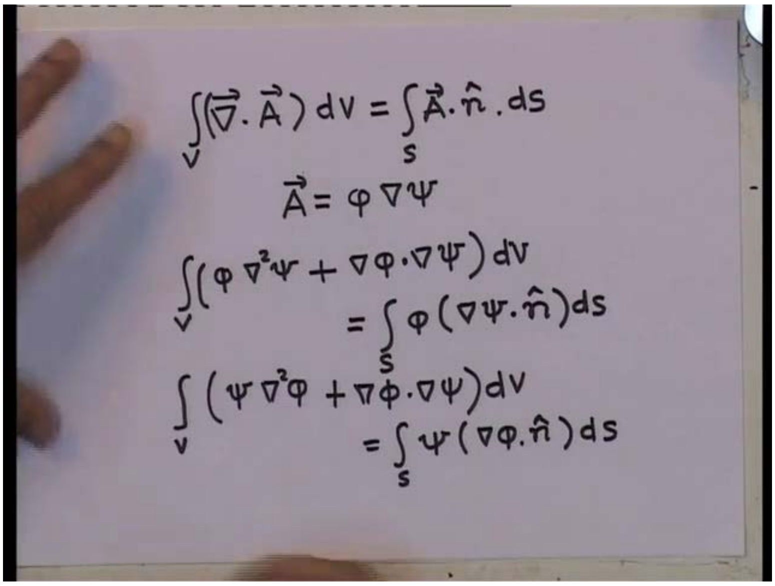

Now, if I interchange them I would get wherever there is a phi I write a psi, wherever there is a psi I write a phi. I will get a psi del square phi plus del psi dot del phi which is the same as del phi dot del psi integrated over the volume is equal to integral over psi del phi dot n d S.

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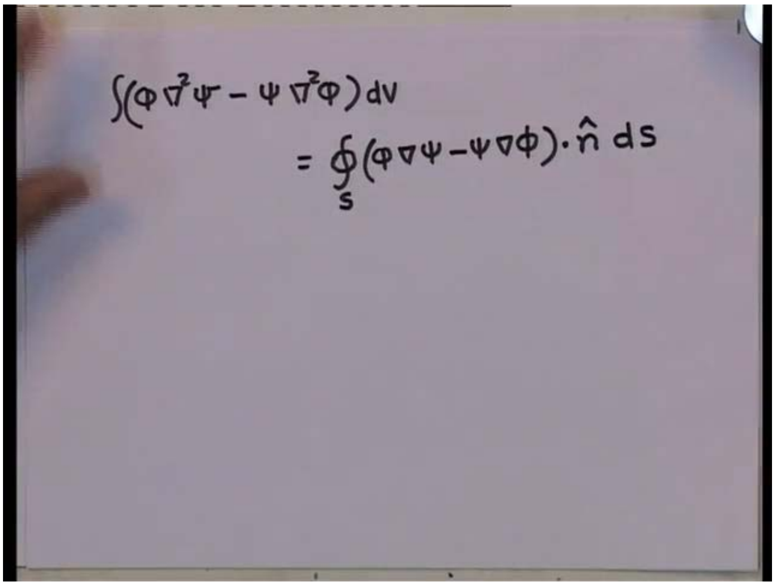

Now, when you subtract 2 from 1 you notice this del phi dot del psi term cancels out and I will be left with then integral of phi del square psi minus psi del square phi d V that is equal to the closed surface defining the volume phi del psi minus psi del phi dot n d S, this is known as the Green's theorem. Let me come back to the theorem that we talked about.

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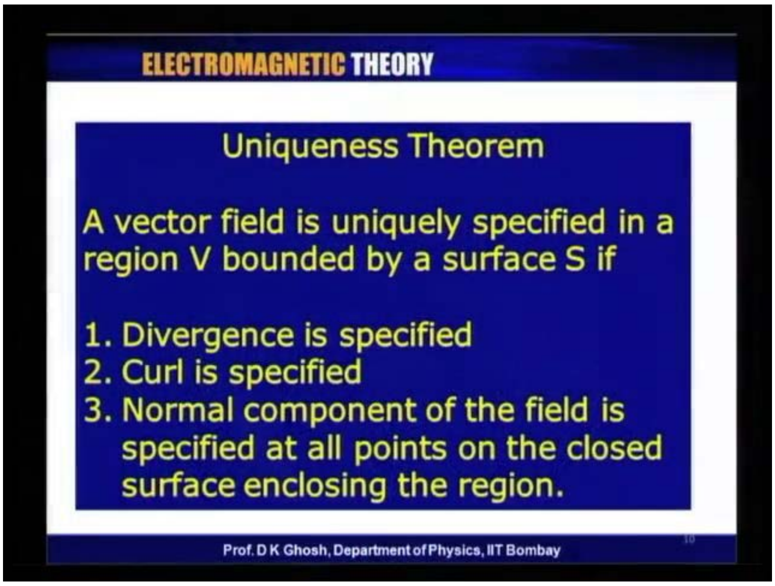

We said that a vector field is uniquely specified in a region V which is bounded by the surface S provided its divergence is known at every point, its curl is known at every point and the normal component of the field is specified at all points on the surface which encloses the volume, the three conditions which are necessary to make a vector field unique. Let us see how does it work?

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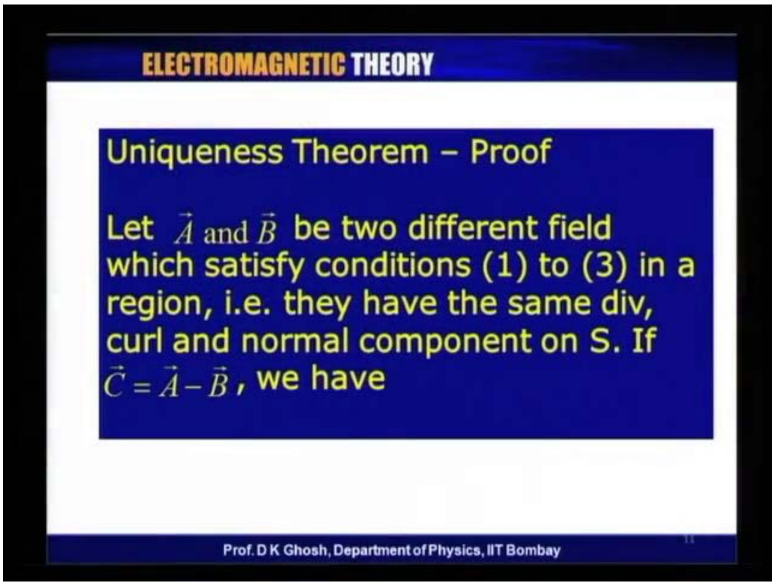

Now, what do you do is this. We will prove it by what is known as the reductio ad absurdum namely let us assume that these three things are given and a vector field A satisfies it and a second vector field B also satisfies the same three conditions. In other words what we are saying is del dot of A is the same as del dot of B, curl of A is the same as curl of B and the normal component of A on the surface is same as the normal component of B on the surface.

Now, what you do is this. Now, if this is true that A and B are different then if I define a vector C equal to A minus B then C should be a non 0 vector, but let us see whether that can be satisfied or not.

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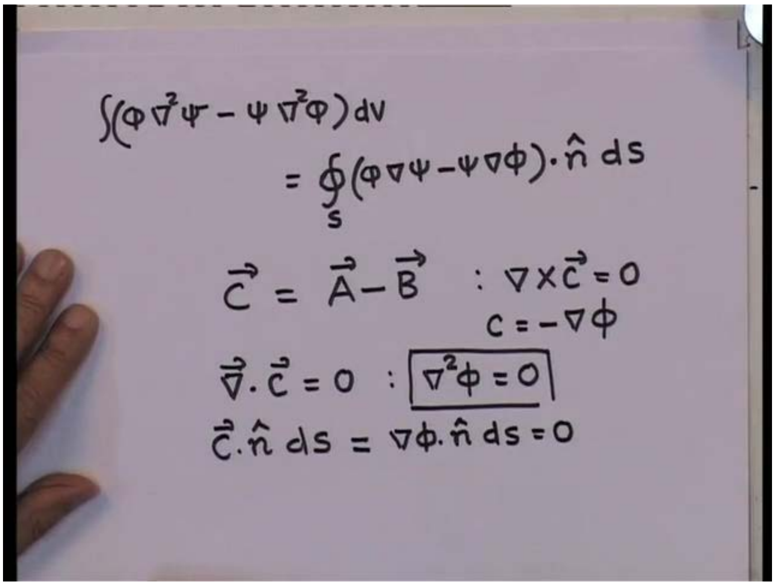

So, I have defined C is equal to A minus B. Now, if C is equal to A minus B and if del cross of A is equal to del cross of B, it tells me that del cross of C must be 0. In other words C must be a conservative field. So, I can write C as I take a traditional minus sign times the gradient of phi, a scalar function phi.

Secondly, we know that divergence of A is the same as the divergence of B. So, would that tells me that C is such that del dot of C must be equal to 0, but we have just now seen that C is gradient of a scalar function. So, what it tells me is C is such that del dot del phi so del square phi must be equal to 0 everywhere in that volume. The third thing that we have given is A dot n is the same as B dot n at every point on the surface. So, that tells me that C dot n d S which because C is gradient of phi which is del phi dot n d S must be equal to 0 because C is A minus B.

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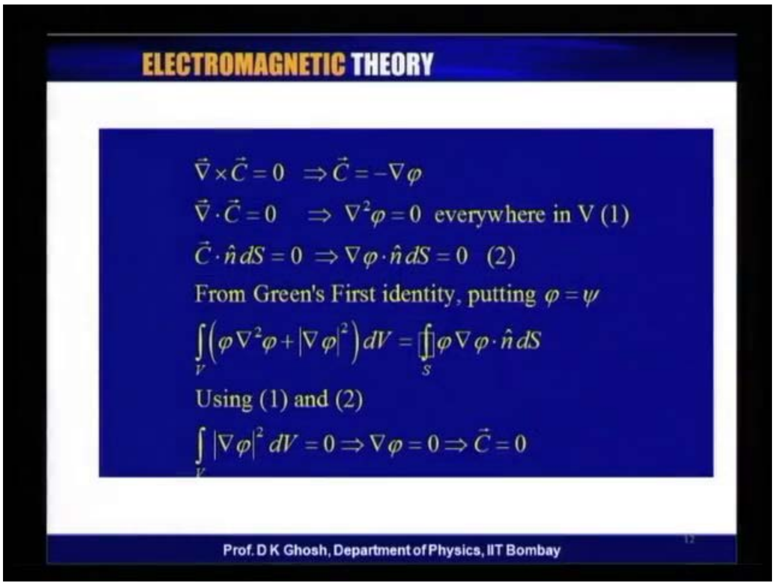

Now, let us do the following. We had worked out an identity known as the Green's first identity. If you recall Green's first identity was let us go back a little bit.

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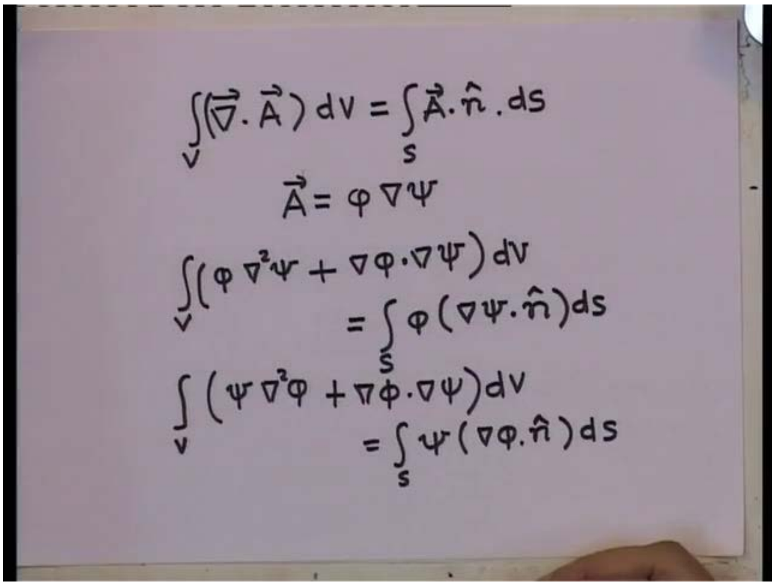

The green's first identity was phi del square psi plus del phi dot del psi integrated over a volume is the same as the surface integral of phi del psi dot n d S. Now, let us suppose phi and psi are arbitrary scalar functions. Let me take phi to be equal to psi in the Green's first identity. Then this equation becomes phi del square phi plus del phi square absolute square integrated over a volume is equal to integral of phi del phi dotted with n d S.

Now, notice right hand side of this should be equal to 0. We had further seen that del square phi is also equal to 0. That tells me that the integral of del phi absolute square d V equal to 0. Now, del phi absolute square is an integral over positive terms. Now, it cannot be 0 unless gradient of phi is 0. This implies that gradient of phi must be 0, but what is gradient of phi? It is nothing but minus C therefore, C is equal to 0. So, that proves that C defined as A minus B cannot be a distinct vector function. So, what is I have said so far is that if in a given region I need to specify a vector field uniquely I must know three things about it.

Its divergence must be known at every point, its curl must be known at every point and at each point on the surface which is which bound provides a boundary for the volume, the value of the normal component of the vector field must be known at every point on the surface. Given three conditions the vector field is uniquely determined. This essentially brings us to an end of discussion on various vector operators which are useful in electromagnetism or electromagnetic theory that we will be doing in the rest of the course. However, in the remaining time that we have I would like to bring in two more mathematical notes which are also useful. The first one is called Dirac's delta function.

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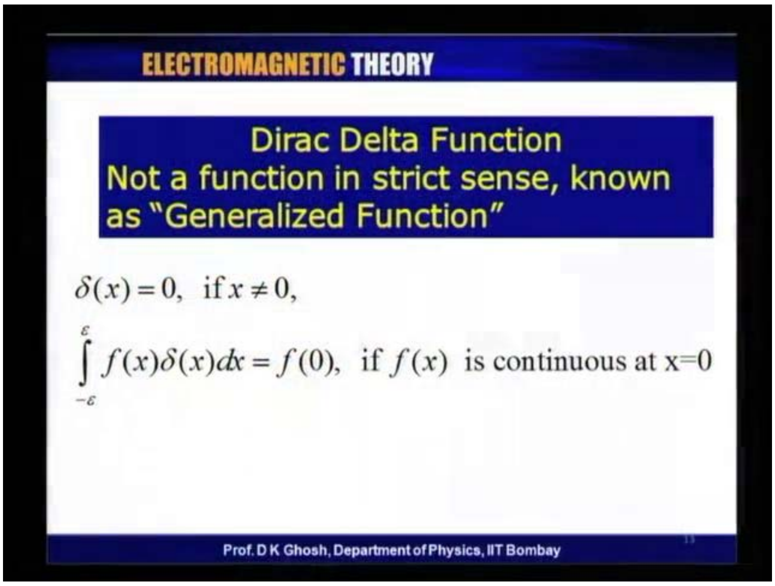

Dirac as you probably know was a very well known physicist who had actually shared a Nobel prize with Schrodinger for his contribution to quantum mechanics. Now, Dirac's delta function is a funny function. In fact in strict sense it is not a function at all. It is either called a generalized function or a distribution, but we will not go into the mathematical final points of a delta function, but let us look at how does one define delta function. So, the definition of delta function is interesting. It says it is a function which is 0 everywhere excepting at a point.

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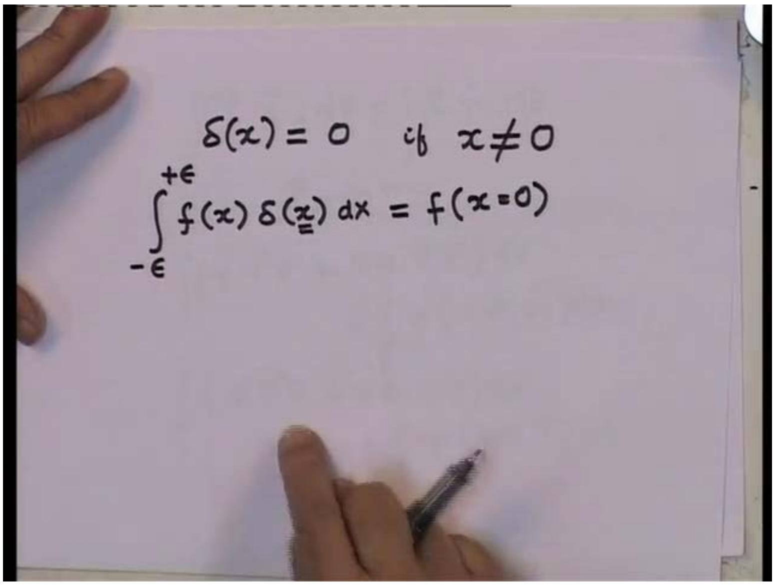

So, delta of x is equal to 0 delta of x is equal to 0 if the argument is not equal to 0, if x is not equal to 0. Now, what happens at x equal to 0? Well we do not quite define it by giving a value, but you can already see so what do we do is this. We say that suppose you take an integral of a test function f x multiplied with this delta of x and if this interval if this interval includes the point which provides the argument of the delta function. In this case the argument of the delta function 0 of the delta function is at x equals to 0 and not 0, but the argument of the delta function is x. And therefore, the integration has to be done over a region which includes the point x equals to 0. So, let me let me just take it from some minus epsilon to plus epsilon, this is guaranteed to include the point x equals to 0.

You could take it from minus a to plus a, minus infinity to plus infinity as you desire. What is required is that wherever the 0, the argument of the delta function as a 0 the integral limits must include that point and this is equal to function value of the function at the point x equal to 0. In fact if you wrote integral f x delta x minus a d x then it will be f of a. So, this is at f at 0. Now, this is of course, f x has to continuous function.

Now, notice already why delta function is not a function? We are all familiar with Riemann integration. Now, if I have a function which is 0 everywhere excepting at a point. No matter what could may be the value of the, might be the value of the function at that point. The integral of f x is essentially the area of the curve below that point.

Now, since the width is 0 the area of such a curve is 0. So, the Riemann integration should give you 0. Now, we are saying that the Riemann integral gives you a 0. So, obviously this is not strictly a function. So, how do you understand this? Derive delta function is best understood in terms of a limit of a sequence of functions.

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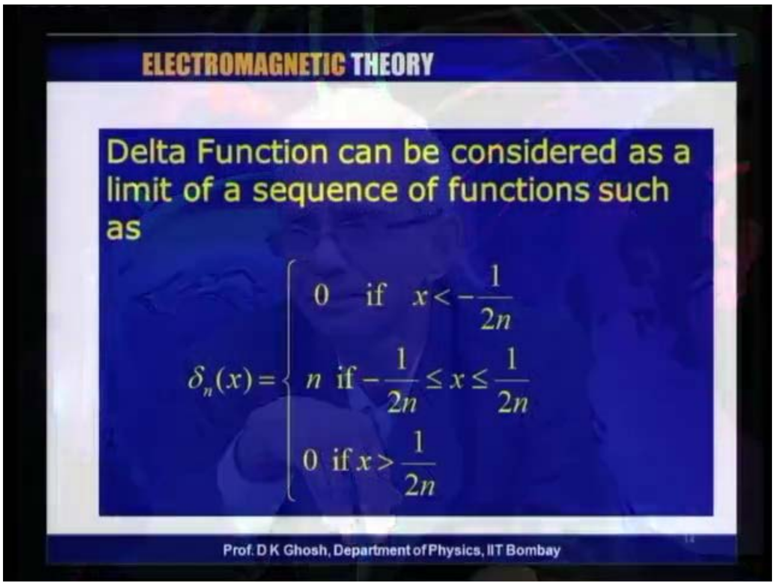

For instance this is a sequence delta n x which is 0 if x is less than minus 1 by n. n in a small interval from minus 1 by 2 n 2 plus 1 by 2 n and is again 0. In other words it is a rectangle it is a rectangle which has a spread only from minus 1 by 2 and 2 plus 1 by 2 n.

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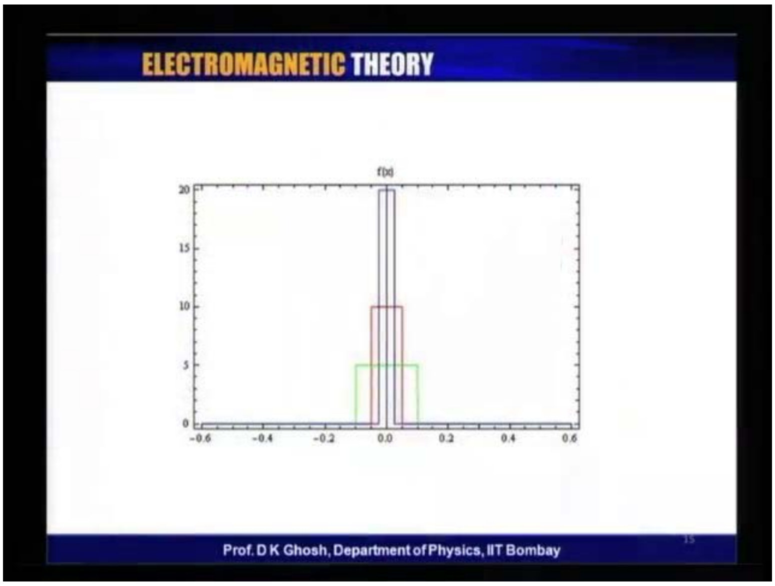

Now, notice let me draw this function. I have plotted this function here for various values of n, you can check that the green one for instance is for a smaller value of n, there, this next one is red, I increase n further increase n. Now, as I go on increasing n you notice what is happening is that the width of this function is going on decreasing, but the height keeps on increasing in such a way that the width times the height is nothing but the area remains constant.

Now, as n goes to infinity then this represents a delta function because as n goes to infinity its width will essentially become 0, but the area under the curve namely the area of the rectangle is finite because it remains finite because its height times the width.

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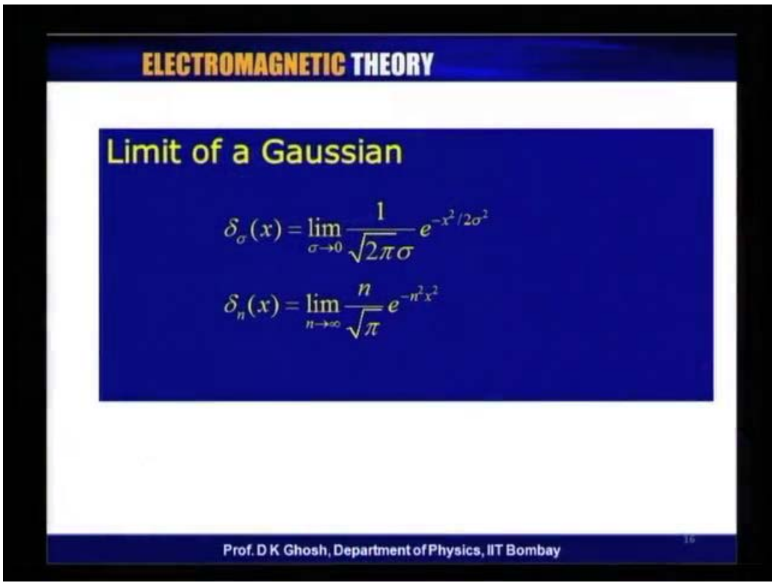

Other examples would be as a limit of a Gaussian. Limit sigma going to 0 1 over root 2 pi sigma e the power minus x square by 2 sigma square, this is a Gaussian function, as the limit of sigma going to 0.

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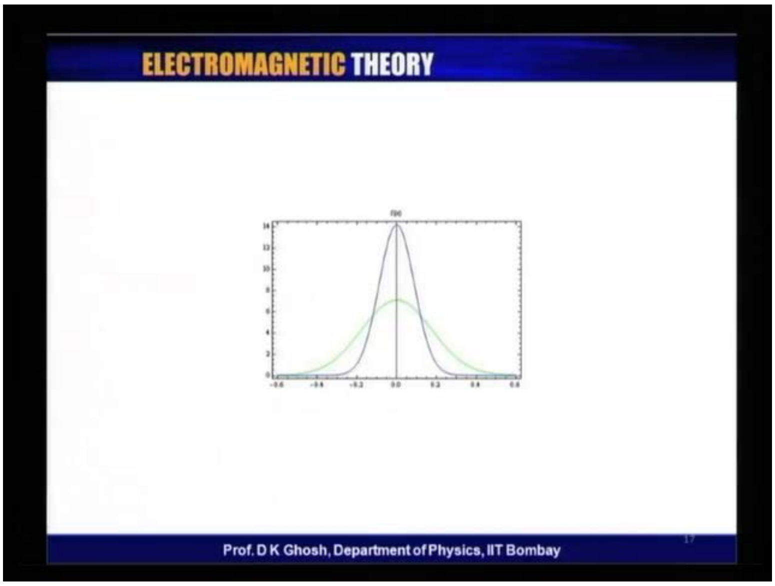

So, this is again a plot of that. Go on changing the parameters and you find that the width goes on becoming smaller, and smaller and the height keeps on rising such that the area under the curve remains the same.

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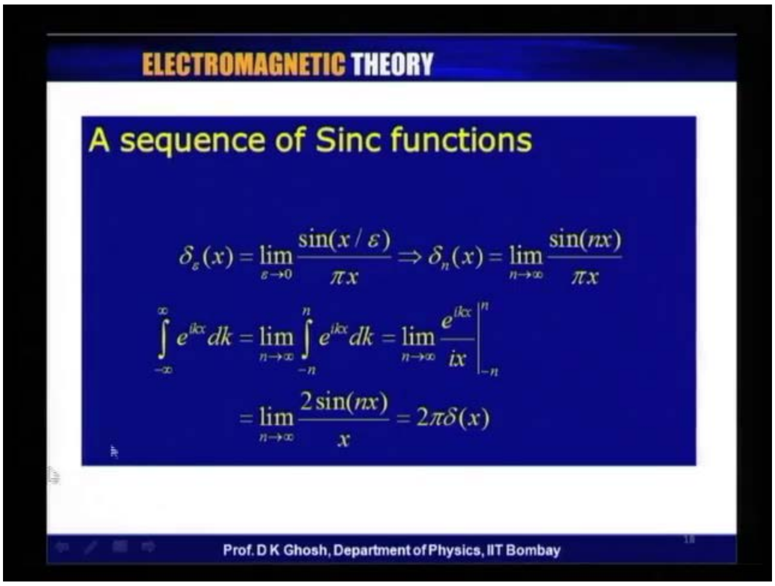

Well, there are other options as well for instance as a sequence of a Sinc function. Sinc function is basically sin x by x function, but there are some normalization factors we have done. Now, notice if you again I am not showing you, but if you again plot this function sin n x by pi x as limit n goes to infinity. It is oscillating function, but because of that 1 over x as distance increases the amplitude of oscillation decreases with distance and the height of the central peak increases.

This is a function which is familiar to you while you did, you discussed refraction phenomenon in optics. Now, notice one interesting thing that let us look at and what is known as an integral representation of a delta function.

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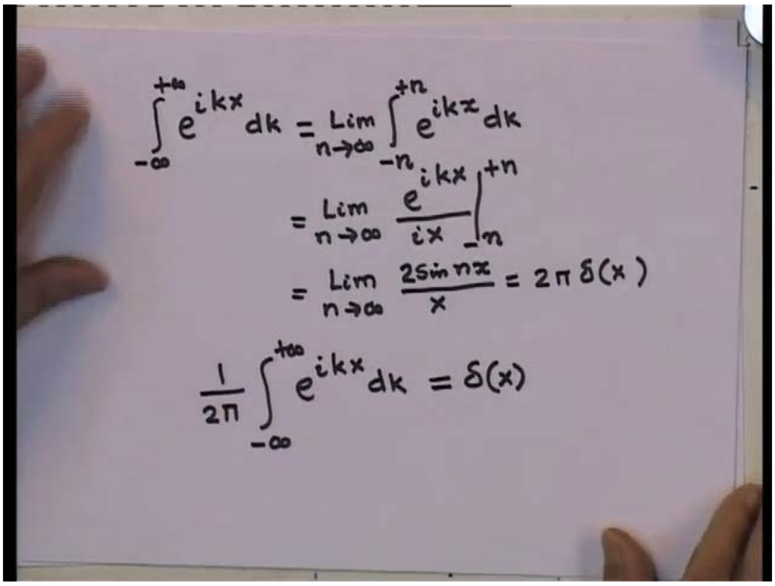

So, for instance look at a simple function like e to the power i k x integrated over k and the value of k is from minus infinity to plus infinity. Now, you would not be able to directly evaluate it. The reason is e to the power i k x integrated over k gives you e to the power i k x divided by i x and if you evaluate it at x equal to plus or minus infinity I mean you do not know how to find the values, but let us look at it this way. Instead of integrating it from minus infinity to plus infinity let us integrate it from some minus n to plus n and make this n as large as possible.

Then of course, I know how to integrate this because this is limit n going to infinity e to the power i k x divided by i x n is equal to from minus n to plus n substitute you get e to the power i n x minus e to the power minus i n x by i if I had another i too on the denominator I will get this as a sin n x, but because by definition of sin x is e to the power i x minus e to the power minus i x by 2 i. So, I need to multiply it to the 2 and divide it by x.

But this is precisely was our definition of the delta function. So, this is nothing but 2 pi times the delta function. So, notice 1 over 2 pi integral from minus infinity to plus infinity e to the power i k x d k. This is what is known as an integral representation of the delta function. Well, we could go on.

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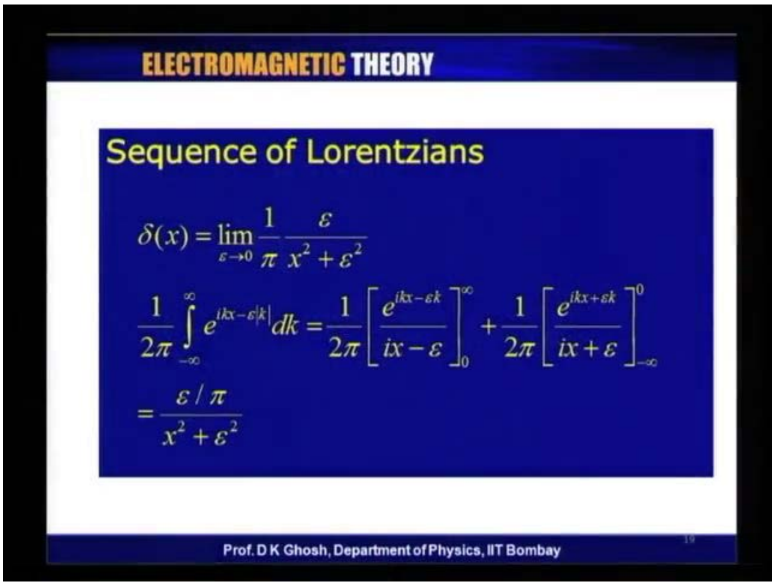

For instance a delta function can also be viewed as a sequence of the Lorentzian function. This this function is called a Lorentzian function, epsilon by x square plus epsilon square. Let this epsilon go to 0 you can again find another representation, integral representations of the delta function here.

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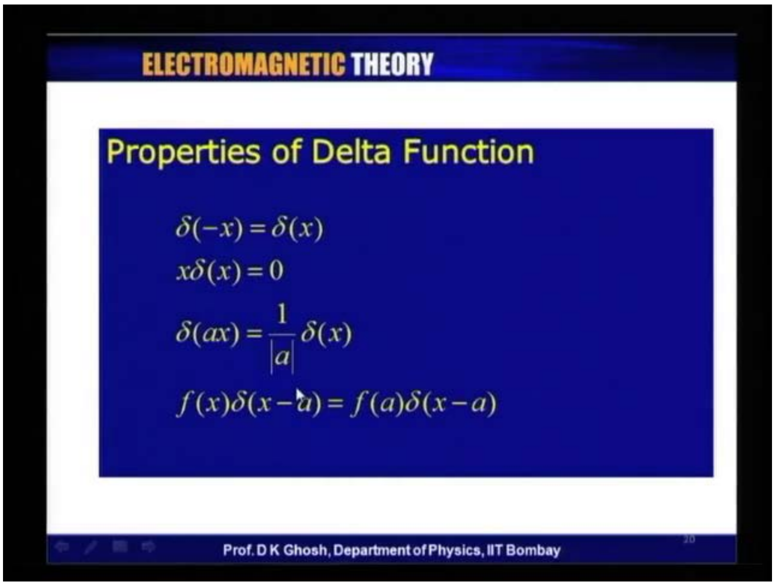

Delta function has some interesting properties, delta of minus x equal to delta of x which should be obvious because if delta of x equals to 0 at x not equal to 0. If x is not equal to

0 minus x is also not equal to 0. x times delta x is 0 and some of the other properties that we have written down which you can yourself prove fairly trivially.

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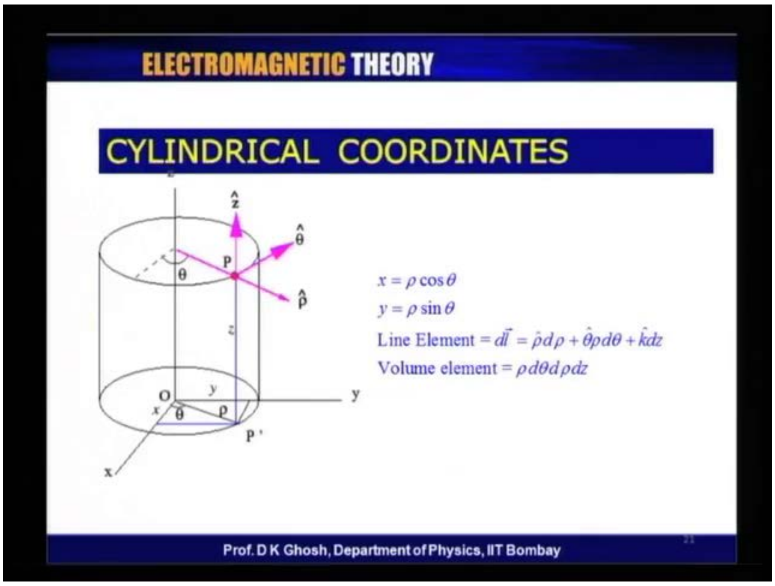

Having discussed the delta function what remains in building up a mathematical machinery for the discussion of electrostatics, magnetostatics etcetera that we are going to be doing is to discuss some different coordinate system. So, far excepting occasionally we have been mostly confining ourselves to Cartesian coordinate system. However, in some problems it is more natural to have a different coordinate system.

One of them is known as a cylindrical coordinate system. Now, a cylindrical coordinate system as the name suggests is useful if this system that you are discussed describing has what is known as a cylindrical symmetry. The cylindrical symmetry means that the object has a symmetry axis which we call as the z axis and if we go around the z axis the object looks exactly the same. Now, cylindrical coordinate system is extremely easy to understand because is its z axis is identical to the z axis or the z direction of the Cartesian coordinate system. So, I am only left to describe the x y coordinates and in the x y coordinate system.

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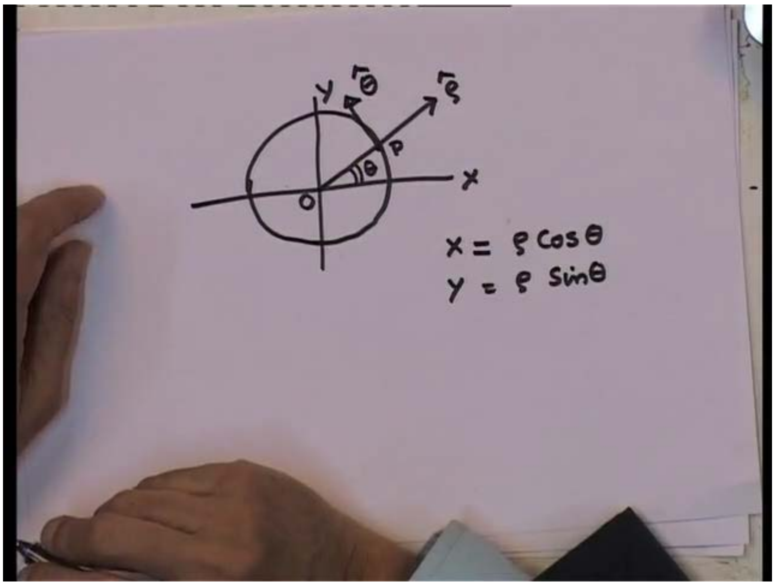

So, supposing I am in the x y plane. So, this is x this is y and the z direction is the same as that of the Cartesian system, it is as if on this circle there is a cylinder. Now we parameterize in the following way. The, instead of taking x and y. Supposing, I am looking at a point P. If I am looking at a point P the direction from the origin to the point P that is origin of the axis, this base to the point P, this I call as the radial direction. I will represent it by rho so that you do not get confused with the radial direction of the spherical polar system that we will be talking about.

This angle is theta therefore, the x coordinate is given by rho cos theta, rho is the distance from origin and y coordinate is rho sin theta. This is the radial direction. Actually, polar two dimensional polar radial direction and if you take a perpendicular to this in the direction of increasing theta, that is my angular direction, transverse direction. Now, look at what happens. How how does one describe a volume element in a cylindrical system? The length elements are rho unit vector rho d rho along the theta direction rho theta because theta if you remember does not have any dimension. So, I need since I am talking about a length I need a length and it is an arc. So, arc is nothing but r theta or rho in this case, rho theta, so rho theta along theta direction and of course, z direction the same k d z. So, volume element there is given by rho d theta d rho d z. The more interesting and slightly more complicated coordinate system is what is known as the spherical polar coordinate system.

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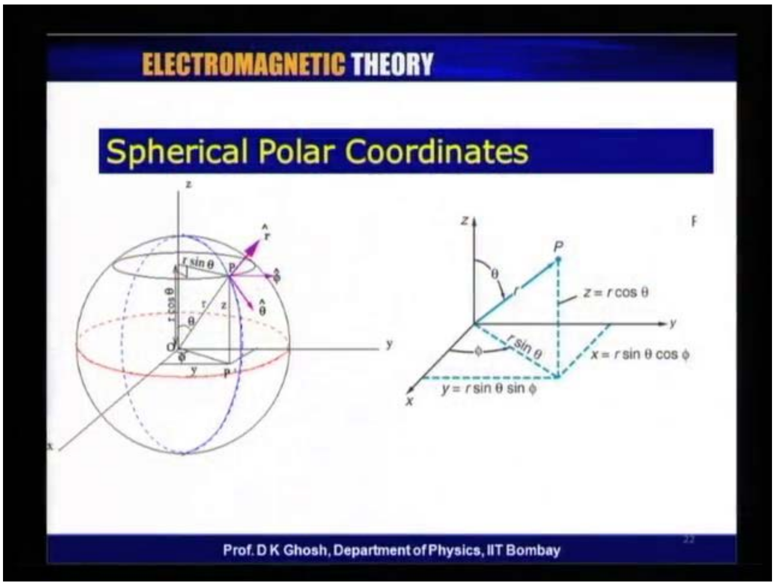

In fact this is a very natural coordinate system to us because our earth itself is a spherical polar coordinate system. So, the way you do it is this. Imagine a sphere imagine a sphere and take the center, origin at the center of this sphere. Now, notice there is an equatorial plane. This equatorial plane is what I call as the x y plane. I identify the equatorial plane with the x y plane and the axis of the, if you like the rotation of the earth, suppose the earth is rotating about an axis joining the center of the earth with the pole. So, that is perpendicular to the x y plane. I am of course, assuming earth is a perfect sphere which as you know, it is not.

So, what happens is this. Now, how do I describe a point P, what are the coordinates? So, the point P the first coordinate is very simple. Now, remember I do not have to be on the surface of the earth. It is anywhere in that sphere. So, first coordinate is simply join the point P with the origin and the distance r is the radial distance, this is called the radial coordinate. And the direction from the origin to the point P along the radial direction along that will be my unit vector r, r cap, now other point P. Now, what I do is this. This radial vector O P is making an angle theta.

Here is the picture which describes what I am doing, making an angle theta with the z axis. Now, what I do is this now this angle theta is another coordinate of my spherical polar coordinate system, from P drop a perpendicular on to the x y plane. Now, notice that and then join the origin to the foot of the perpendicular so this is the P drop a perpendicular on to the x y plane join O with this. So, whatever angle this makes with the x axis that is called an azimuthal angle. So, I have got r theta and phi. Notice, that this distance here that is the distance from the origin to the foot of the perpendicular is r sin theta and the perpendicular distance from P onto the x y plane is r cos theta where theta is the angle which O P makes with z.

Now, you can immediately see what are the relationship between the x y and z coordinates of the Cartesian system with my spherical polar coordinates. So, one of them is obvious which is z is equal to r cos theta. Now, let us look at what is x? Now, so I may put up the perpendicular here so x is nothing but this distance parallel to x axis. And since this distance is r sin theta the x is nothing but r sin theta cos phi and y is r sin theta sin phi.

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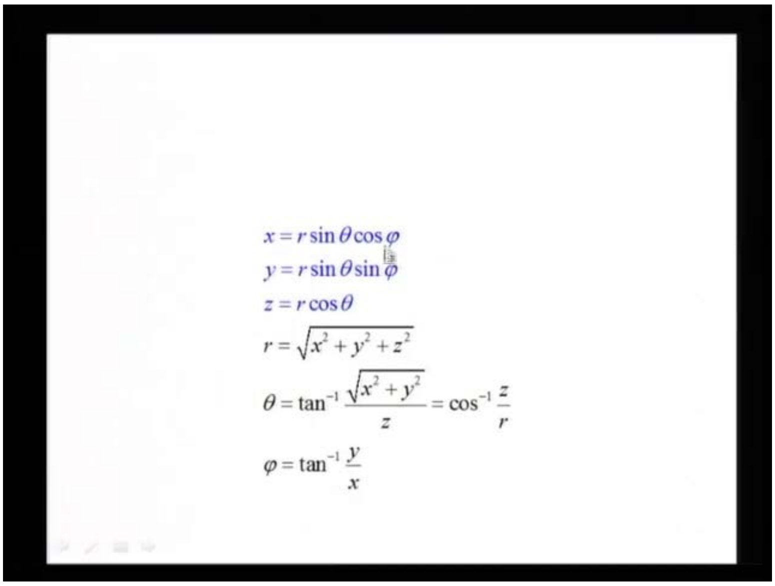

So, this is what we have written down there. Elementary trigonometry tells me r is nothing but square root of x square plus y square plus z square which obviously is the distance of the point O to the point P and from those relation you can find out what is theta, what is theta and what is phi? So, these are you can see immediately that cos theta is z by r. So, if cos theta is z by r, I know what is sin theta? x square plus y square plus z square is r square and all these relationship become clear.

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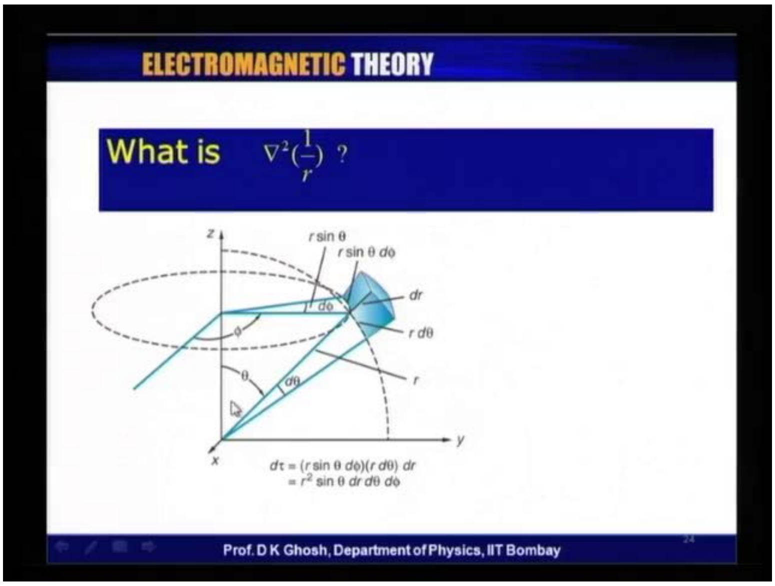

The other thing that I want to point out in this is what is the volume element in this spherical polar coordinate system? Now, look at this picture. Now, here I had a point P and I am sort of looking at how much is the length element in various directions. Length element in the radial direction is clearly d r. Now, in the theta direction it is r d theta very much like what happens in the polar case and from the picture it becomes clear that in the azimuthal direction a length element is given by r sin theta d phi. Therefore, the volume element which I have written here as d tau is r square sin theta d r d theta d phi.

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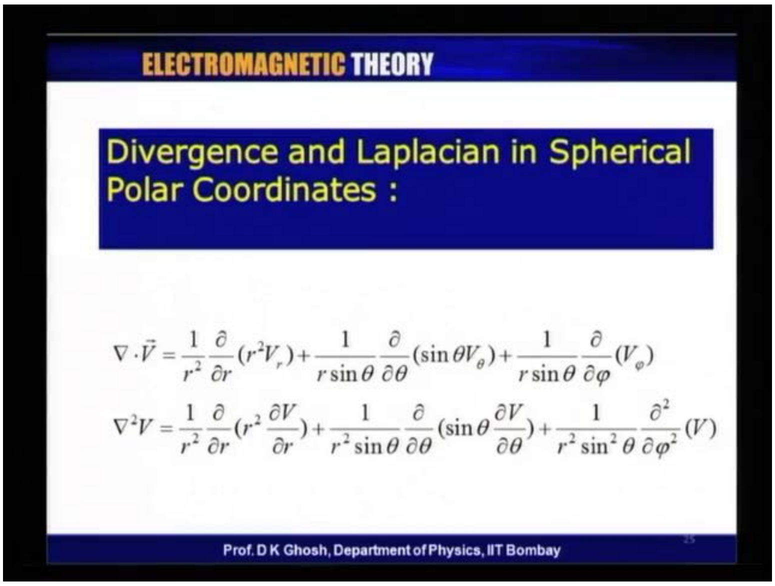

The last algebra that I want to do is to find out what is del square of 1 over r.

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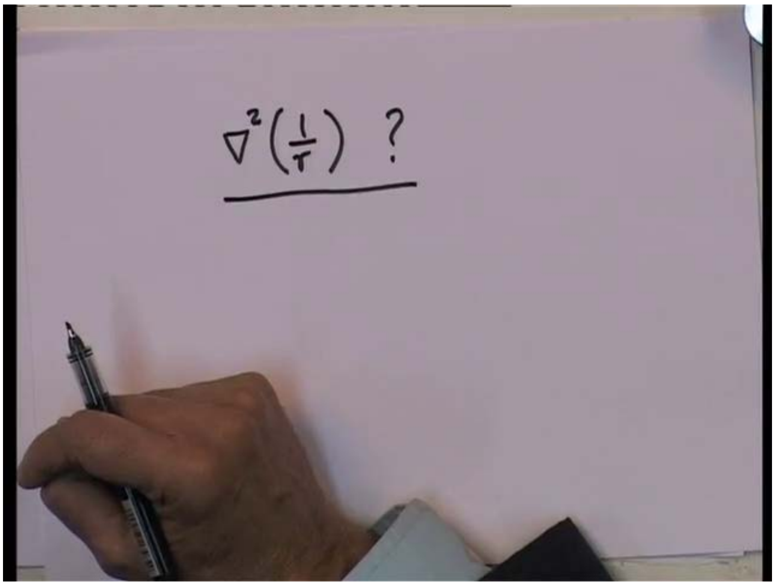

This is very important because 1 over r happens to be the potential, the coulomb potential with which will be very much you know involved in the next few lectures. Now, in order to understand what is it, I need the expressions of the divergence and Laplacian del square operator acting on so what does divergence of a vector V in spherical polar system is a involved algebra, but on the other hand straight forward because we have already given the relationship between the Cartesian coordinates and the spherical coordinates and you can sort of see that these are the type of expressions that you get.

However, since I am dealing with 1 over r this function does not have any dependence on theta or phi. Therefore, if my function V, vector function V here or scalar function V does not have any theta or phi dependence then only points that I am involved with is the radial component there the radial component there.

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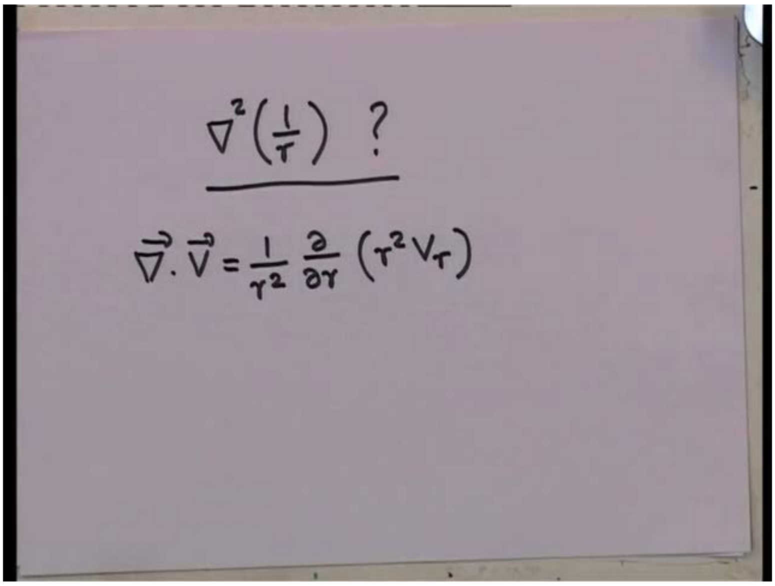

So, let us look at what is it. So, we said del dot V is 1 over r square d by d r of r square times the radial component of r because my function V, vector function V does not have any theta or phi components.

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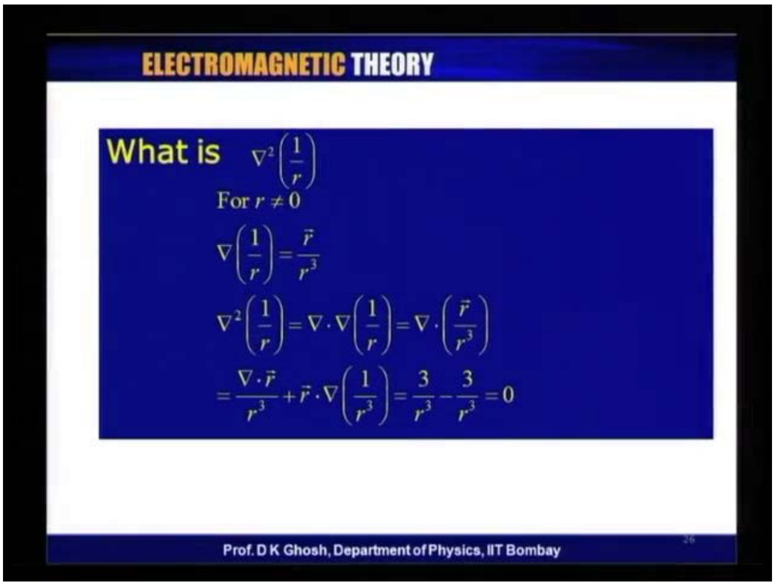

Now, I am interested in finding out what is del square of 1 over r.

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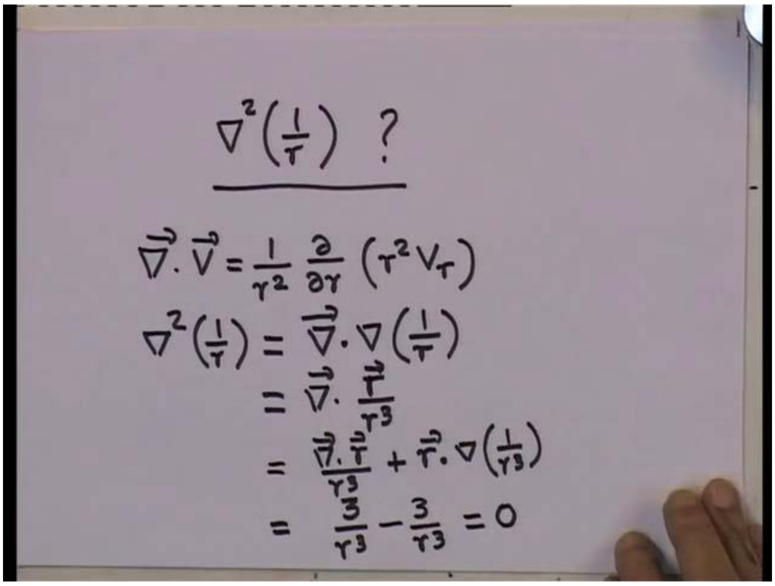

Now, I have told you earlier that del square is nothing but del dot gradient of 1 over r. The gradient of 1 over r you can easily calculate because there is nothing so V r is just 1 over r. So, there is a r square there. So, I get just d by d r of r so which is nothing but 1 so I get del of 1 over r is given by. So, this is a vector which is 1 over r square, but along the unit vector r. So, which can be written as vector r by r cube.

Now, you know that this is divergence of a vector by a times scalar. So, this is equal to del dot r by r cube plus vector r dotted with gradient of 1 over r cube. del dot of r we have seen is 3 so this is 3 by r cube, this is r dot del upon 1 over r cube and you can easily see that this is also 3 by r cube, but with a minus sign and hence it is equal to 0. However, this is true only if r is not equal to 0. What happens if r is equal to 0?

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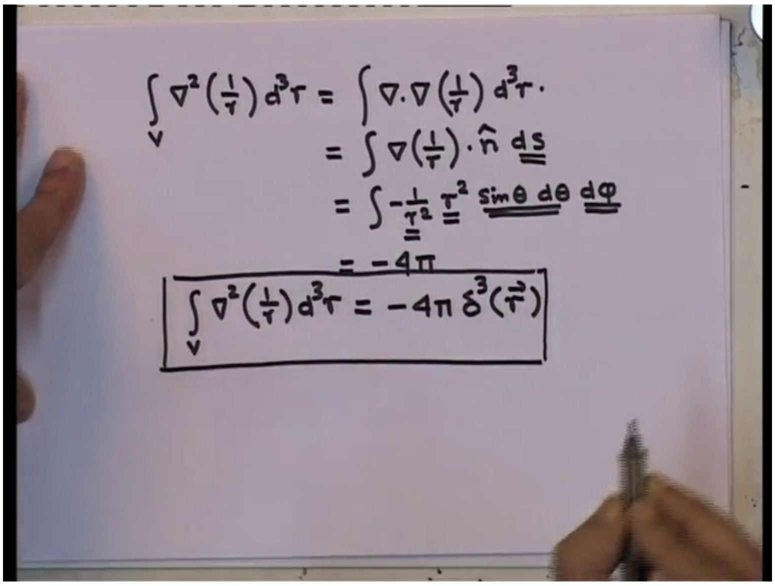

So, let us look at what happens to the integral of del square of 1 over r over the volume. I know del square is del dot del of 1 over r d cube r and using divergence theorem, this is simply the gradient of 1 over r dotted with the unit vector n d S. I know that the gradient of 1 over r is minus 1 over r square times the unit vector r. So, as a result this gives me this 1 over r square and there is a d S there. So, this is minus 1 over r square r square sign theta d theta d phi.

Now, notice that there is no integral over r that is because it is on a surface where r is fixed. So, this is equal to minus the integral. Now, azimuthal angle gives me 2 pi and sin theta d theta that gives me a factor of 2. So, I am left with minus 4 pi. Now, this is the definition of a delta function that r not equal to 0 the function vanishes, but at r equal to 0 the integral does not vanish. Therefore, I can, what we have shown is del square of 1 over r d cube r is equal to minus 4 pi delta function. Since, it is a three dimensional delta function I occasionally write delta cube of r. With this we have completed the mathematical introduction to the electromagnetic theory and from the next module onwards, we will start discussion of electrostatic phenomena.