

Electromagnetic Theory
Prof. D. K. Ghosh
Department of Physics
Indian Institute of Technology, Bombay

Lecture - 40
Radiation


We have come to the end of this course, and last time we were talking about radiation theory. So, we will bring that to some conclusion today. Then we will sort of summarize whatever we have learnt in this course. So, if you recall this course has been primarily for understanding the physical implications, and the mathematical importance of electromagnetic theory.

(Refer Slide Time: 01:01)

ELECTROMAGNETIC THEORY

Localized oscillating source

$$A(\vec{x}) = \frac{\mu_0}{4\pi} \int d^3x' \frac{e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|} J(\vec{x}')$$
$$V(\vec{x}) = \frac{1}{4\pi\epsilon_0} \int d^3x' \frac{e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|} \rho(\vec{x}')$$

 **Prof. D K Ghosh, Department of Physics, IIT Bombay**

We have introduced the concept of a vector and a scalar potential. So, last time what we have doing was to talk about a localized oscillating source. We had seen that the these are the general expression for the vector potential, and the scalar potential corresponding to a current distribution and a charge distribution.

(Refer Slide Time: 01:26)

ELECTROMAGNETIC THEORY

Localized ($\lambda \gg d$) oscillating source

Three Regions : $R = |\vec{x} - \vec{x}'|; \lambda = \frac{2\pi c}{\omega}$

1. Near Field : $d \ll r \ll \lambda$
2. Intermediate Zone : $d \ll r \approx \lambda$
3. Radiation Zone : $d \ll \lambda \ll r$

NPTEL

Prof. D K Ghosh, Department of Physics, IIT Bombay

So, what we said is that the, if you have oscillating source, we have essentially fields going as $e^{-i\omega t}$. We divide our region of interest into three. The first one is what we call as the near field, where the dimensions of the source which is given by d here is much less than the distance from the source where we make the observation, which is still less than the wave length of the radiating field. There is an intermediate zone where we do not do much, because it requires lot more rigorous solutions. Where this r is of the order of λ and our main interest as we said is on the radiation zone, where the dimensions of the source is much less than the wave length, which in turn is much less than the distances at which we make our observation.

(Refer Slide Time: 02:36)

ELECTROMAGNETIC THEORY

Near Field is Quasi-Stationary

$$R = |\vec{x} - \vec{x}'|; \lambda = \frac{2\pi c}{\omega}$$

$$d \ll r \ll \lambda$$

$$kR = \frac{\omega R}{c} = \frac{2\pi R}{\lambda} \ll 1 \Rightarrow e^{ikR} \approx 1$$

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi} \int d^3x' \frac{\vec{J}(\vec{x}')}{|\vec{x} - \vec{x}'|}$$

NIPTEIL

Prof. D K Ghosh, Department of Physics, IIT Bombay

We have seen that the near field is quasi stationary. This statement is understood in the following way that if you look at the field other than for the time variation like $e^{i\omega t}$ to the power $i\omega t$, the solutions will be the same, as what you get in case of static sources or steady currents. So, this is our near field and as we said we are interested primarily in the radiation field for which d is much less than λ which in turn is much less than r .

(Refer Slide Time: 02:58)

ELECTROMAGNETIC THEORY

Radiation Field

$$d \ll \lambda \ll r$$

Let $\vec{x} = \hat{n} r \approx \hat{n} r$

$$|\vec{x} - \vec{x}'| = (r^2 + x'^2 - 2r\hat{n} \cdot \vec{x}')^{1/2}$$

$$\approx r \left[1 - 2\frac{\hat{n} \cdot \vec{x}'}{r} + \frac{x'^2}{r^2} + \dots \right]^{1/2}$$

$$= r - \hat{n} \cdot \vec{x}' + \dots$$

$$\frac{1}{|\vec{x} - \vec{x}'|} = (r^2 + x'^2 - 2r\hat{n} \cdot \vec{x}')^{-1/2}$$

NIPTEIL

Prof. D K Ghosh, Department of Physics, IIT Bombay

So, in other words r is the largest scale in the problem. So, let us say we have been using vector \vec{x} to represent x, y, z .

(Refer Slide Time: 03:20)

Handwritten mathematical derivation on a whiteboard:

$$\vec{x} = \hat{n} x = \hat{n} r$$

$$|\vec{x} - \vec{x}'| = (r^2 + x'^2 - 2r\hat{n} \cdot \vec{x}')^{1/2}$$

$$\approx r - \hat{n} \cdot \vec{x}'$$

$$\frac{1}{|\vec{x} - \vec{x}'|} = \frac{1}{r} + \frac{\hat{n} \cdot \vec{x}'}{r^2} + \frac{1}{2} \frac{1}{r^3} (3(\hat{n} \cdot \vec{x}')^2 - x'^2) + \dots$$

$$\frac{e^{i\vec{k} \cdot \vec{x}'}}{|\vec{x} - \vec{x}'|} \approx \frac{e^{i\vec{k} \cdot \vec{x}'}}{r} \left[1 + \left(\frac{1}{r} - ik \right) (\hat{n} \cdot \vec{x}') + \dots \right]$$

A small logo for NPTEL is visible in the bottom left corner of the whiteboard image.

Let the unit vector corresponding to that be \hat{n} and the magnitude is x , which we will also indicate as r . Occasionally that is the magnitude of the vector x , will be taken as r . Now, if you do that, then x minus x' modulus which we have seen can be written as x square which is r square plus x' square. Remember prime is the variable corresponding to the source and minus 2 times r times, vector \hat{n} unit vector \hat{n} multiplied with x' and that rise to the power half.

So, if you expand this out we had seen that this gave me r minus, sorry. This is not a vector r minus the unit vector \hat{n} dotted with x' , I also require this will, as we know will appear as the exponent of the exponential function. I have a 1 over x minus x' , there which is essentially the same quantity, but with rise to the power minus half and what we had seen is that this is given by 1 over r plus $\hat{n} \cdot x'$ divided by r square plus there are other terms like this. But we will not really be interested in these terms, these are higher order terms which are there, but just for to indicate.

So, we write this term. Now, last time we had sort of seen what was the electric dipole approximation. So, just to recall for you, what we did is to put this quantity as in the exponent. So, I have e to the power x minus x' divided by x minus x' . So, this was written as equal to or above e to the power of $i k r$ by r . Then I have a 1 plus 1 by r minus $i k$, times $\hat{n} \cdot x$ $\hat{n} \cdot x'$ and plus terms which we have said we will neglect.

Now, what we did last time is to ignore this term and replace e to the power x minus x prime by x minus x prime as e to the power $i k r$ by r . That gave us the dipole approximation to the problem the terms, that we have here. We will take this up and these will see give us two things. One is what is called as the magnetic dipole approximation. The next one which we will not have really have much of a time to do, is essentially electric quadrupole approximation. So, let us look at how does it go? So, the radiation field that I have got is.

(Refer Slide Time: 07:01)

ELECTROMAGNETIC THEORY

Radiation Field

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \int \vec{J}(\vec{x}') d^3x'$$

$$\vec{\nabla} \cdot (x_i \vec{J}) = (\nabla x_i) \cdot \vec{J} + x_i \nabla \cdot \vec{J}$$

$$= (\nabla x_i) \cdot \vec{J} - x_i \frac{\partial \rho}{\partial t}$$

$$= (\nabla x_i) \cdot \vec{J} + x_i i\omega \rho = J_i + i\omega \rho x_i$$

$$\int J_i(\vec{x}') d^3x' = \int \vec{\nabla} \cdot (x_i \vec{J}) d^3x' - i\omega \int \rho x_i d^3x'$$

$$= -i\omega p_i$$

Prof. D K Ghosh, Department of Physics, IIT Bombay

So, first I will look at how this works out?

(Refer Slide Time: 07:05)

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \int \vec{J}(\vec{x}') d^3x'$$

$$\vec{\nabla} \cdot (x_i \vec{J}) = (\nabla x_i) \cdot \vec{J} + x_i \vec{\nabla} \cdot \vec{J}$$

$$= (\nabla x_i) \cdot \vec{J} - x_i \frac{\partial \rho}{\partial t}$$

$$= (\nabla x_i) \cdot \vec{J} + i\omega x_i \rho$$

$$= J_i + i\omega x_i \rho$$

$$\int \vec{J}_i(\vec{x}') d^3x_i = \int \nabla \cdot (x_i \vec{J}) d^3x' - i\omega \int \rho x_i d^3x'$$

$$= -i\omega \vec{P}_i$$

So, my radiation field the vector potential a of r or x use it interchangeably μ_0 over 4π e to the power $i k r$ by r. This is just the first approximation integral J of x prime d cube x prime. Now, this is the lowest order approximation that we have got. Now, so what we need to do is, this that notice one thing that if I now, calculate del dot of x i times J. Now, we know that this is gradient of the scalar dotted with the vector. So, this is del x i as to why I am doing it will become obviously.

Second del x i dotted with J vector vector J. So, this is of course, a vector x i is the i th component of the vector x plus x i times del dot of J. Now, by equation of continuity the second term I have got del dot of J. Therefore, this is same as d rho by d t. Since I have said that the variation with respect to time is e to the power minus i omega t. So, I get this as del gradient of x i dotted with J. Minus minus, plus i omega x i times rho. So, notice this this is gradient of x i.

So, since this is gradient of x i so this is essentially i depending upon what i is this is just a unit vector in which ever direction i is. When it is dotted with J i simply, get it j i plus i omega x i times rho. So, the reason why I am doing this is that if I look at this integral I have integral J of x prime d cube x prime. Now, what I do is let us look at the i th component of this. So, this i th component of this this is a d cube x. So, I will write this down as equal to del dot of x i prime J d cube x prime minus i omega integral rho. So,

basically what I am doing is, I am replacing for this $\nabla \cdot \mathbf{x} = \nabla \cdot \mathbf{r} = \frac{3}{r}$ minus $i\omega \times \mathbf{r}$. So, this is $\frac{3}{r} - i\omega \times \mathbf{r}$.

Now, so this is this quantity of course, as expected by divergence theorem can be converted into a surface term. Since, I know my current is confined within a small region the as if I take this surface go to infinity, this term will drop out. What I will be left with minus $i\omega \times \mathbf{r}$. If you recall this is nothing but the definition of the dipole moment. Well actually $i\omega \times \mathbf{p}$ component on the dipole moment. So, this is what I have got from this $\nabla \cdot \mathbf{J}$ integration.

(Refer Slide Time: 11:17)

The image shows handwritten mathematical derivations on a whiteboard. The equations are as follows:

$$\vec{A}(\vec{r}) = -\frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} i\omega \vec{p}$$

$$= -\frac{1}{4\pi\epsilon_0} \frac{ik}{c} \frac{e^{ikr}}{r} \vec{p}$$

$$\vec{B} = \nabla \times \vec{A} = -\frac{1}{4\pi\epsilon_0} \frac{ik}{c} \nabla \left(\frac{e^{ikr}}{r} \right) \times \vec{p}$$

$$= -\frac{1}{4\pi\epsilon_0} \frac{ik}{c} \left(ik - \frac{1}{r} \right) \frac{e^{ikr}}{r} \vec{r} \times \vec{p}$$

$$\vec{H} = \frac{ck^2}{4\pi} \frac{e^{ikr}}{r} \left(1 + \frac{i}{kr} \right) \vec{r} \times \vec{p}$$

Therefore if I substitute this into the expression for A, I get A of x is equal to μ_0 by 4π as before e^{ikr} by r , which was there times minus $i\omega \times \mathbf{p}$. So, minus $i\omega \times \mathbf{p}$ so this is what I get. If we could sort of use that $\omega/c = k$ and this can be written as $1/4\pi\epsilon_0$ times ik/c . Well basically ω is being written as Ck and then $\mu_0\epsilon_0 = 1/C^2$. So, this just works out, then I have got e^{ikr} by r times \mathbf{p} . Now, this is incidentally just the dipole approximation having what the vector potential A, I calculate the magnetic field B by simply working out. What is $\nabla \times$ of this quantity? So, which is equal to minus $1/4\pi\epsilon_0$. Let us keep these terms ik/c . Which are common, which is constant times $\nabla \times \mathbf{p}$ times \mathbf{p} . But \mathbf{p} is a dipole moment which is a fixed vector.

Therefore, what I get is gradient of e^{-ikr} to the power ikr over r cross \vec{p} . Now, this gradient is easily calculated so it is $-\frac{1}{4\pi\epsilon_0} \frac{ik}{r^2} C$. So, calculate this gradient e^{-ikr} to the power ikr gives you $-\frac{ik}{r^2}$. Then I have got, I have to differentiate $1/r$ so that I get $1/r^2$ actually minus $1/r^2$. So, let me just take out e^{-ikr} to the power ikr by r outside. This times a unit vector \hat{r} cross \vec{p} . This unit vector came from the gradient so this is my expression for the magnetic field \vec{B} and the corresponding \vec{H} field which is of course, simply obtained by dividing \vec{B} by μ_0 and realizing that $1/\mu_0\epsilon_0$ is C^2 .

So, what I get is $c^2 k^2$ by $4\pi\epsilon_0$ e^{-ikr} by r times $1 + ikr$. I have just changed the order of these two terms to take care of this minus sign i by kr times \hat{r} cross \vec{p} . So, this is the expression for the magnetic field. Now, I can obtain the the electric field by realizing that, electric field is nothing but $\epsilon_0 \frac{\partial \vec{E}}{\partial t}$ sorry the $\epsilon_0 \frac{\partial \vec{E}}{\partial t}$ is equal to $\nabla \times \vec{H}$. So, I can calculate the $\nabla \times \vec{H}$ of this quantity and $\frac{\partial}{\partial t}$ is minus $i\omega$.

(Refer Slide Time: 14:56)

ELECTROMAGNETIC THEORY

Radiation Field

$$\vec{H} = \frac{ck^2}{4\pi} \frac{e^{ikr}}{r} \left(1 + \frac{i}{kr}\right) \hat{r} \times \vec{p}$$

$$\nabla \times \vec{H} = \epsilon_0 \frac{\partial \vec{E}}{\partial t} = -i\omega\epsilon_0 \vec{E} \Rightarrow \vec{E} = \frac{ic\mu}{k} \nabla \times \vec{H}$$

$$\vec{E} = \frac{ic\mu}{k} \frac{ck^2}{4\pi} \nabla \times \left[(\vec{r} \times \vec{p}) \frac{e^{ikr}}{r^2} \left(1 + \frac{i}{kr}\right) \right]$$

$$= \frac{ik}{4\pi\epsilon_0} \left[\nabla \left(\frac{e^{ikr}}{r^2} \left(1 + \frac{i}{kr}\right) \right) \times (\vec{r} \times \vec{p}) + \left(\frac{e^{ikr}}{r^2} \left(1 + \frac{i}{kr}\right) \right) \nabla \times (\vec{r} \times \vec{p}) \right]$$

Prof. D K Ghosh, Department of Physics, IIT Bombay

Therefore, using this I can get an expression for the electric field as well. So, that is a rather long expression, but here it is on the screen. This is there, is really nothing that you need to do. What we realize is \vec{E} is basically $iC\mu$ by k $\nabla \times$ of \vec{H} and this vector. So, this is little complicated because it has a scalar it has vectors only \vec{p} is a

constant vector. So, you have to have the gradient of the scalar cross and things like that. So, if you do that you get a huge expression like this.

(Refer Slide Time: 15:31)

ELECTROMAGNETIC THEORY

Radiation Field

$$\begin{aligned} \vec{E} &= \frac{ik}{4\pi\epsilon_0} \left[\vec{\nabla} \left(\frac{e^{ikr}}{r^2} \left(1 + \frac{i}{kr} \right) \right) \times (\vec{r} \times \vec{p}) + \left(\frac{e^{ikr}}{r^2} \left(1 + \frac{i}{kr} \right) \right) \vec{\nabla} \times (\vec{r} \times \vec{p}) \right] \\ &= \frac{ik}{4\pi\epsilon_0} \left[ike^{ikr} \left(\frac{1}{r^2} + \frac{i}{kr^3} \right) + e^{ikr} \left(-\frac{2}{r^3} - \frac{3i}{kr^4} \right) \right] \hat{r} \times (\vec{r} \times \vec{p}) \\ &\quad + \frac{ik}{4\pi\epsilon_0} \left(\frac{e^{ikr}}{r^2} \left(1 + \frac{i}{kr} \right) \right) \left[-\vec{p}(\vec{\nabla} \cdot \vec{r}) + (\vec{p} \cdot \vec{\nabla})\vec{r} \right] \\ &= \frac{e^{ikr}}{4\pi\epsilon_0 r} \left[k^2 \hat{r} \times (\vec{p} \times \hat{r}) - \frac{ik}{r} \left(1 + \frac{i}{kr} \right) (3(\hat{r} \cdot \vec{p})\hat{r} - \vec{p}) \right] \end{aligned}$$

Prof. D K Ghosh, Department of Physics, IIT Bombay

Now at this stage let us look at for example, what is the radiation field like? Now, I have this is the electric field.

(Refer Slide Time: 15:44)

ELECTROMAGNETIC THEORY

Radiation Field

$$\begin{aligned} \vec{H} &= \frac{ck^2}{4\pi} \frac{e^{ikr}}{r} \left(1 + \frac{i}{kr} \right) \hat{r} \times \vec{p} \\ \vec{E} &= \frac{e^{ikr}}{4\pi\epsilon_0 r} \left[k^2 \hat{r} \times (\vec{p} \times \hat{r}) - \frac{ik}{r} \left(1 + \frac{i}{kr} \right) (3(\hat{r} \cdot \vec{p})\hat{r} - \vec{p}) \right] \end{aligned}$$

Prof. D K Ghosh, Department of Physics, IIT Bombay

I am sort of looking at so these are the final expressions for the electric and the magnetic field. I will just put this screen for sometime, so that you can have a look at how the

algebra works out. I have actually sort of in detail worked out the whole thing. So, look at that.

(Refer Slide Time: 16:10)

4

$$\vec{H} = \frac{ck^2}{4\pi} \frac{e^{ikr}}{r} \left(1 + \frac{i}{kr}\right) \hat{r} \times \vec{p}$$

$$\vec{E} = \frac{e^{ikr}}{4\pi\epsilon_0 r} \left[k^2 \hat{r} \times (\vec{p} \times \hat{r}) - \frac{ik}{r} \left(1 + \frac{i}{kr}\right) (3(\hat{r} \cdot \vec{p}) \hat{r} - \vec{p}) \right]$$

Radiation approximation

$$\vec{H} = \frac{ck^2}{4\pi} \frac{e^{ikr}}{r} \hat{r} \times \vec{p}$$

$$\vec{E} = c\mu \vec{H} \times \hat{r}$$

NPTEL

So, the final expression as we have seen is given by let me write it down again, is H equal to $C k^2$ by 4π e to the power $i k r$ by r times $1 + i$ by $k r$ times unit vector r cross p . The electric field expression finally, works out to which I have not worked out here, but I have shown it how to do it. e to the power $i k r$ by r 4π ϵ_0 r . If you do it correctly it will become k^2 unit vector r cross p cross r minus $i k$ by r $1 + i$ by $k r$ into 3 times r dot p . Actually it is a unit vector r dot p minus r dot p times unit vector r minus p .

So, these are the two expressions that I have. Now, let us look at what are the points that are coming out of this expression. Firstly you notice that the magnetic field is transverse to the direction of r perpendicular to the direction of r , but electric field has a longitudinal components because this is r cross r cross p or r cross p cross r . Both these fields E and B at large distances very large r go as 1 over r . This is the one times this this is the major field expression and this is also true of the electric field here. There is a $i k$ by r . The at small distances however, the magnetic field is dominated by 1 over r square which is this term, where as the electric field is dominated by r cube term and this is this term there is a 1 over r there $i k$ by r there and i by $k r$ there. So, this is 1 over r cube. This is what dominates the electric field.

Therefore, we write down for the radiation zone where r is very large, I neglect this term which gives me a rather simple expression for the magnetic field. So, in the radiation zone I get magnetic field \vec{H} is equal to $C k^2 \mu \sin \theta / r$ by $\hat{r} \times \vec{p}$ and if you look at the same approximation, you will find that the electric field can be written in terms of the magnetic field as $\vec{E} = c \mu \vec{H} \times \hat{r}$. So, this also tells you that the electric field is perpendicular to the direction of the magnetic field. The magnetic field is of course, perpendicular to direction of the unit vector \hat{r} . The once we had written these down, this is incidentally true only in the radiation approximation. Now, that is $1/r$ is the dominating factor for both electric field. The magnetic field the what we are interested in is how does the emitted power go?

(Refer Slide Time: 19:57)

ELECTROMAGNETIC THEORY

Radiated Power

$$\vec{H} = \frac{\omega k}{4\pi r} e^{ikr} \hat{r} \times \vec{p}; \quad \vec{E} = c\mu \vec{H} \times \hat{r}$$

$$\vec{S} = \frac{1}{2} \text{Re}(\vec{E} \times \vec{H}^*)$$

$$= \frac{\omega kc\mu}{8\pi} \text{Re}[(\vec{H} \times \hat{r}) \times \vec{H}^*]$$

$$= \hat{r} \frac{\omega kc\mu}{8\pi} |\vec{H}|^2 = \hat{r} \frac{(\omega k)^2 c\mu}{32\pi^2} \frac{1}{r^2} (\hat{r} \times \vec{p})^2$$

$$= \hat{r} \frac{(\omega k)^2 c\mu}{32\pi^2} \frac{1}{r^2} p^2 \sin^2 \theta$$

Prof. D K Ghosh, Department of Physics, IIT Bombay

If you remember the the emitted power is given by your average value of the pointing vector.

(Refer Slide Time: 20:07)

$$\begin{aligned}
 \vec{S} &= \frac{1}{2} \text{Re}(\vec{E} \times \vec{H}^*) \\
 &= \frac{\omega k c \mu}{8\pi} \text{Re}[(\vec{H} \times \hat{r}) \times \vec{H}^*] \\
 &= \hat{r} \frac{\omega k c \mu}{8\pi} |\vec{H}|^2 \\
 &= \hat{r} \frac{(\omega k)^2 c \mu}{32\pi^2} \frac{1}{r^2} (\vec{r} \times \vec{p})^2 \\
 &= \hat{r} \frac{(\omega k)^2 c \mu}{32\pi^2} \frac{1}{r^2} p^2 \sin^2 \theta
 \end{aligned}$$

So, since we are using complex quantities so it is half of real part of E cross H star and by plugging in these two expression remember that E is also written in terms of H therefore, E cross H star I will have 2 H there. So, I have to simply square these so if you plug this in you are going to get omega k c mu by 8 pi. This times real part of well E is given by H cross r and of course, cross H star. So, this quantity is, well I have made probably some slight error. There what I have done is to write these numbers down earlier so does matter anyway.

So, this clearly because we have said the we are the magnetic field is perpendicular direction of r. Therefore, this would be in the direction of r itself. So, this quantity will be r omega k c mu by 8 pi times H square and that is equal to well r I have got a here, I have got c k square there so I write this term. So, omega k whole square c mu divided by there was a 4 pi there, there is a 8 pi there. So, i got 32 pi square, 1 over r and 1 over r gives me 1 over r square and of course, r cross p whole square which is there in the structure of the magnetic field. So, if you look at this you find that this is given by unit vector r omega k square c mu over 32 pi square 1 over r square r cross p. Therefore, this gives me p times p square because this is square there times sin square theta.

So, notice that the pointing vector is proportional to sin square theta and what we can do is we can find out how much is the power flowing through a unit solid angle in the

direction of theta pi. So, which is like a doing simply find out how much d P by d omega. This is the power is given by well r square r dot the pointing vector.

(Refer Slide Time: 23:15)

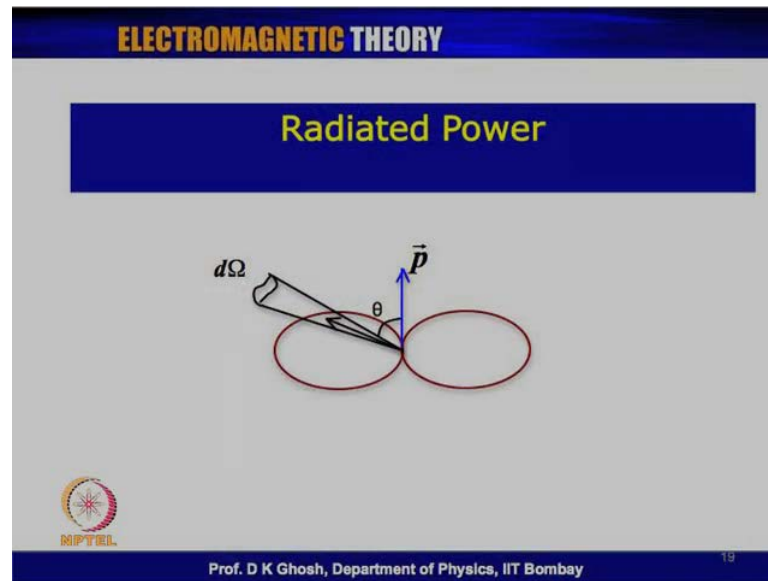
The image shows a whiteboard with handwritten mathematical equations. A hand is pointing to the equations. The equations are:

$$\frac{d\vec{P}}{d\Omega} = r^2 \hat{r} \cdot \langle \vec{S} \rangle$$
$$= \frac{\omega^4 \mu}{32\pi^2 c} p^2 \sin^2 \theta$$
$$P = \frac{\omega^4 \mu}{12\pi c} p^2$$

There is an NPTEL logo in the bottom left corner of the whiteboard.

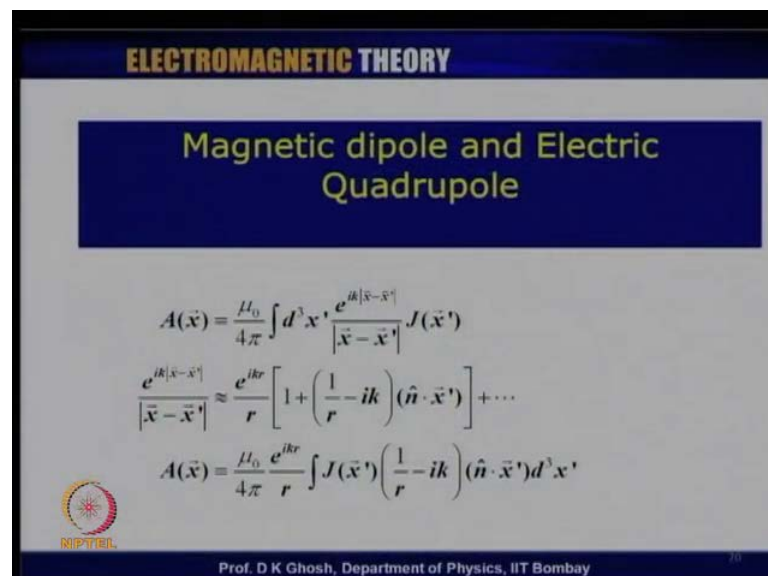
If you calculate this, you get omega raise to the power 4 times mu divided by 32 pi square c times p square sin square theta. If you want to calculate the total power that is emitted you can simply integrate, this over theta. Remember that there is no azimuthal dependence. So, you get a 2 pi there we have a sin square theta sin theta d theta which you can integrate out, sin cube theta d theta. So, you get omega fourth mu divided by 12 pi c times p square. This is sin square theta has been integrated out. So, if you look at now these two structures, you notice that this is the vector P the dipole moment vector P and the the where the power vector s is being radiated.

(Refer Slide Time: 24:35)



That is symmetric with respect to the direction of P and it is this is where we are showing that at an angle theta, the you know there is solid angle d omega. The length of if you join from here to the circumference, then that would give you the magnitude of s. So, this is the way the radiation pattern looks like for a dipole. It is perfectly symmetric about the direction of P. So, that was dipole approximation where we took the simplest possible thing. So, let us look at the next approximation.

(Refer Slide Time: 25:15)



The in the next approximation there are basically two terms. So, this is I recall back A_x is given by this expression and it is e to the power $i k x$ minus x prime by x minus x prime. I had shown to leading order this gives you these terms. So, as a result what we did till now is to take only ignore this factor and take only e to the power $i k r$ by r and there that give us the electric dipole approximation. So, that was the dipole radiation field. Now, what we now want to do is to go over and take account of this term. I am not writing the full A_x now, I am only writing the term in A which corresponds to this.

So, this is A_x equal to μ_0 by $4\pi\epsilon_0$ to the power of $i k r$ by r these are all there. I have 1 over r minus $i k$. Well I have also while, the unit vector \hat{n} is same as the magnitude of x . I have a occasionally I have been writing it as unit vector \hat{r} small r . So, this is the term which is in the my next order of approximation. The the first term I have not written that is the electric dipole term. So, let us let us write it down.

(Refer Slide Time: 26:40)

$$\begin{aligned} \vec{A}(\vec{x}) &= \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \int \vec{J}(\vec{x}') \left(\frac{1}{r} - ik \right) (\hat{n} \cdot \vec{x}') d^3x' \\ &= \left[\frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \left(\frac{1}{r} - ik \right) \right] \int \vec{J}(\vec{x}') (\hat{n} \cdot \vec{x}') d^3x' \\ &= \left[* \right] \frac{1}{r} \int \vec{J}(\vec{x}') (\vec{x} \cdot \vec{x}') d^3x' \\ &= \sum_{j=1}^3 \int \vec{J}(\vec{x}') x_j x'_j d^3x' \end{aligned}$$

So, I got A of x is equal to μ_0 by $4\pi\epsilon_0$ to the power $i k r$ by r integral J_x prime 1 over r minus $i k$ $\hat{n} \cdot \vec{x}'$ d^3x' . I will now do a bit of a complicated algebra because this is the rather complicated expression. But but nevertheless let us try to work this out. Notice the this term does not come in to the integration at all because it is independent of x prime this is equal to μ_0 by $4\pi\epsilon_0$ to the power $i k r$ by r 1 over r minus $i k$ integral of J of x prime $\hat{n} \cdot \vec{x}'$ d^3x' . Notice this left hand side

is a vector and the vector character comes from because this is scalar. So, vector character comes from J.

So, let us look at the component wise. What what do we get from this integration in other words? Let me look at let us say the you know I mean what do I get. For example, from the i th component of x like that so let us write down this expression here. Well I will for the moment skip this constant term. So, let me just plug it in here like this. n dot x prime, what I can do is this, that let me let me evaluate instead of n dot x prime. Let me take out a 1 over r there. This is this part so that let me call this is star. So, this is the star there.

On the screen you can see the full expression, 1 over r integral J of x prime x dot x prime d cube x prime. Remind you once again, that vector x is the position at which we are evaluating, the vector potential and x prime is of course, an integration medium. So, what we do is this, that since we are interested in this part only. This part of the expression is given by since it is x dot x prime, which is summation over three components. So, let us call this sum over J equal to 1 to 3 integral J of x prime.

Well x J times x prime of J d cube x prime. So, this is this is what this part looks like. This is what we are interested. In the remaining part at this moment, so for as the calculation of A of x is concerned, the remaining part remains unaltered. Therefore, we will need it while calculating the magnetic field which will bring them at that time. So, let me then do this try to evaluate this quantity.

(Refer Slide Time: 30:35)

ELECTROMAGNETIC THEORY

Magnetic dipole and Electric Quadrupole

$$A(\vec{x}) = \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \left(\frac{1}{r} - ik \right) \int \vec{J}(\vec{x}') (\hat{n} \cdot \vec{x}') d^3x'$$

$$\int \vec{J}(\vec{x}') (\hat{n} \cdot \vec{x}') d^3x' = \sum_{j=1}^3 \int \vec{J}(\vec{x}') x_j x_j' d^3x'$$

$$\int \vec{\nabla} \cdot (x_i' x_j' \vec{J}) d^3x' = 0$$

$$\int \left[\vec{\nabla} (x_i' x_j') \cdot \vec{J} + x_i' x_j' \vec{\nabla} \cdot \vec{J} \right] d^3x' = 0$$

$$\vec{\nabla} (x_i' x_j') \cdot \vec{J} = x_j' (\nabla x_i' \cdot \vec{J}) + x_i' (\nabla x_j' \cdot \vec{J}) = x_j' J_i + x_i' J_j$$

$$\int (x_j' J_i + x_i' J_j + i\omega \rho' x_i' x_j') d^3x' = 0$$

Prof. D K Ghosh, Department of Physics, IIT Bombay

So let us look at what we are trying to do here.

(Refer Slide Time: 30:41)

$$\sum_{j=1}^3 \int \vec{J}(\vec{x}') x_j x_j' d^3 x'$$

$$\int \vec{\nabla} \cdot (x_j x_j' \vec{J}) d^3 x' = 0$$

$$\int [\nabla(x_j x_j') \cdot \vec{J} + x_j x_j' \vec{\nabla} \cdot \vec{J}] = 0$$

$$\nabla(x_j x_j') =$$

So, we are trying to evaluate sum over J equal to 1 to 3 integral J of x prime, remember J is a source quantity, x j x j prime d cube x prime. Now, before I do this, I want to do an subsidiary algebra. We know that if we took the divergence of the whole thing suppose say, I wrote del dot of this quantity which is written there x j x j prime times capital J d cube x prime using divergence theorem and extending the surface to infinity. Since J is localized, I get this quantity to give me 0. Now, what we are trying to say is this, if this is 0 this is del dot of a scalar times of a vector. So, let us look at what it gives me?

So, this tells me, gradient of a scalar so let us put gradient of x j x j prime dotted with J the vector plus the scalar times x j x j prime times del dot of J. So, this quantity is equal to 0. Now, let us look at what does this give me? Now, remember that this is gradient of a product of two things. Therefore, the this gradient of x j x j prime is given by, well I did did do a bit of a mess up, but let me sort of redo this. What I want to do is the following, this is what I want to calculate. But instead what I want to say is this, that let me pull it aside, let me pull it aside. So, this is a quantity which we are interested in calculating.

(Refer Slide Time: 33:09)

Handwritten mathematical derivation on a whiteboard:

$$\sum_{j=1}^3 \int \vec{J}(x') x_j x_j' d^3x'$$

$$\int \nabla \cdot (x_i' x_j' \vec{J}) d^3x' = 0$$

$$\int [\nabla(x_i' x_j') \cdot \vec{J} + x_i' x_j' \nabla \cdot \vec{J}] d^3x' = 0$$

$$\nabla(x_i' x_j') \cdot \vec{J} = x_j' J_i + x_i' J_j$$

$$\int (x_j' J_i + x_i' J_j + i\omega \rho' x_i' x_j') d^3x' = 0$$

The whiteboard also features a small logo in the bottom left corner with the text "NIPTEEL" and a page number "9" in the top right corner.

j equal to 1 to 3 integral J of x prime $x_j x_j$ prime d cube x prime. So, what I am saying is this, that consider divergence of x_i prime x_j prime J , this is equal to 0 for because this is divergence means I can make it a surface integral and J will vanish on the surface. This del dot I have, I know can be written as gradient of well gradient of x_i prime x_j prime dotted with J plus x_i prime x_j prime del dot of J d cube x prime is equal to 0, now this gradient that I have got.

I got gradient of x_i prime x_j prime is equal to dot with J . So, let us look at what does it give me. Firstly, I can take x_j prime this is just a chain rule. So, I can take x_j prime out, I get gradient of x_i prime dotted with J which is nothing but J_i , so this is x_j prime J_i . Like that when I take the x_i prime out, gradient of x_j prime dotted with J gives me J_j . So, I get x_i prime J_j . So, if I now substitute it here, I get integral this is x_i prime. What I have got is this. I get x_i prime rather x_j prime $J_j x_j$ prime J_i plus x_i prime J_j .

If you remember that I had this term $i\omega \rho'$ the second which I was not bothered about. x_i prime, this is this comes from the fact that I have got del dot of J which is equal to minus $d\rho$ by dt and which goes as $i\omega \rho'$. Therefore, I get $i\omega \rho' x_i$ prime x_j prime d cube x prime is equal to 0. Now, this is the expression which I have to evaluate $\sum_{j=1}^3 x_j$ prime x_j prime sum over j equal to 1 to 3. Now once I have said that this quantity is equal to 0.

(Refer Slide Time: 36:22)

The whiteboard shows the following derivation:

$$\sum_{j=1}^3 \int J_j x_j x_j' = \sum_{j=1}^3 x_j \int J_j x_j' d^3x'$$

$$= -\sum_{j=1}^3 x_j \int (J_j x_i' + i\omega \rho' x_i' x_j') d^3x'$$

$$= \frac{1}{2} \sum_{j=1}^3 x_j \int (J_j x_j' - J_j x_i' - i\omega \rho' x_i' x_j') d^3x'$$

The number '10' is written in the top right corner of the whiteboard. A logo for 'NIPTTEL' is visible in the bottom left corner.

What I can do is to rewrite, sum over j equal to 1 to 3 integral. Suppose I have, I am talking about the i th component. So, I have got $x_i J_j x_j x_j'$, this what I want to talk about and that is equal to sum over j equal to 1 to 3. Now I bring the x_j out and I have got $J_j x_j x_j' d^3x'$. This is where I used this identity that I had obtained. So, I had said $x_j x_j' J_j$ and so this is what I have got $x_j x_j' J_j$ so I will write this is as negative of these two terms, which will go to the other side. So, this quantity will then be equal to minus sum over j is equal to 1 to 3 x_j and integral of J_j rather $J_j x_i x_j'$ plus $i\omega \rho' x_i x_j'$ d^3x' .


This is the 1, which I will replace with that. What I would now do is, this the, this is this is equal to this. So, I am going to do take half of this expression and half of that expression. So, that I write it in a slightly symmetric fashion. So, it is half of sum over j is equal to 1 to 3. I get x_j integral of $J_j x_j x_j' - J_j x_i x_j' - i\omega \rho' x_i x_j'$ d^3x' . That is the prime which was missing here d^3x' . So, my expression is this and if I now want to integrate this out if you look at the screen.

(Refer Slide Time: 39:01)

ELECTROMAGNETIC THEORY

Magnetic dipole and Electric Quadrupole

$$\begin{aligned}
 \int \vec{J}(\vec{x}')(\vec{x} \cdot \vec{x}') d^3x' &= \sum_{j=1}^3 \int \vec{J}(\vec{x}') x_j x_j' d^3x' \\
 \sum_{j=1}^3 \int J_i x_j x_j' d^3x' &= \frac{1}{2} \sum_{j=1}^3 x_j \int (J_i x_j' - J_j x_i' - i\omega \rho' x_i' x_j') d^3x' \\
 &= \frac{1}{2} \sum_{j=1}^3 x_j \int (-\epsilon_{ijk} (\vec{x}' \times \vec{J})_k - i\omega \rho' x_i' x_j') d^3x' \\
 &= -\frac{1}{2} \int \left[(\vec{x}' \times (\vec{x}' \times \vec{J}))_i + i\omega \rho' x_i' (\vec{x}' \cdot \vec{x}') \right] d^3x'
 \end{aligned}$$



Prof. D.K Ghosh, Department of Physics, IIT Bombay

This is what I had written it down. Now, notice this term that I have got here $J_i x_j x_j'$ minus $J_j x_i x_i'$, you notice this is nothing the k th component of \vec{x}' cross \vec{j} . Of course, I need a epsilon $i j k$ because I do not know what it whether $i j k$. What is the natural order? So, this will be minus epsilon $i j k$ \vec{x}' cross \vec{J} component k and the other one I have not done anything about it. Now, if you now plug this sum over j is equal to 1 to 3 x_j and bring it inside and you notice that there is a epsilon $i j k$.

There with a minus sign, there minus sign I can bring it out. This is nothing but \vec{x}' cross \vec{x}' cross \vec{J} i component because this is j th component, this is k th component with epsilon $i j k$. Therefore, it is \vec{x}' cross \vec{x}' cross \vec{J} i th component and the other term other than this minus sign which is there outside, now I have taken down this is remained exactly the same.

(Refer Slide Time: 40:21)


ELECTROMAGNETIC THEORY

Magnetic dipole and Electric Quadrupole

$$\int \vec{J}(\vec{x}')(\vec{x} \cdot \vec{x}') d^3x' = \sum_{j=1}^3 \int \vec{J}(\vec{x}') x_j x_j' d^3x'$$

$$\sum_{j=1}^3 \int J_j x_j x_j' d^3x' = -\frac{1}{2} \int \left[(\vec{x} \times (\vec{x}' \times \vec{J}))_i + i\omega \rho' x_i' (\vec{x} \cdot \vec{x}') \right] d^3x'$$

$$\int \vec{J}(\vec{x}')(\vec{x} \cdot \vec{x}') d^3x' = -\frac{1}{2} \int \left[(\vec{x} \times (\vec{x}' \times \vec{J})) + i\omega \rho' \vec{x}' (\vec{x} \cdot \vec{x}') \right] d^3x'$$


24

Prof. D K Ghosh, Department of Physics, IIT Bombay

So, this is what we have proved so far. This is what we have proved so far because this what I had. I just now showed that this sum over j is equal to 1 to 3. This is what we have proved there. Therefore, if you write this J vector J as i j i etcetera, etcetera, then this is what you will get. This was i th component therefore, this is a vector relations. Now, it is this term it is this term which is my magnetic dipole term. Let us see why.

(Refer Slide Time: 40:59)

ELECTROMAGNETIC THEORY

Magnetic dipole and Electric Quadrupole


$$\int \vec{J}(\vec{x}')(\vec{x} \cdot \vec{x}') d^3x' = -\frac{1}{2} \int \left[(\vec{x} \times (\vec{x}' \times \vec{J})) + i\omega \rho' \vec{x}' (\vec{x} \cdot \vec{x}') \right] d^3x'$$

$$\vec{m} = I\vec{A} = I \frac{1}{2} \oint \vec{r} \times d\vec{l}$$

If current exists in as medium rather than being in a wire,

$$I d\vec{l} \rightarrow \vec{J} dV \Rightarrow \vec{m} = \frac{1}{2} \int \vec{r} \times \vec{J} dV$$

$$\int \vec{J}(\vec{x}')(\vec{x} \cdot \vec{x}') d^3x' = -\vec{x} \times \vec{m} - \frac{1}{2} \int i\omega \rho' \vec{x}' (\vec{x} \cdot \vec{x}') d^3x'$$


25

Prof. D K Ghosh, Department of Physics, IIT Bombay

So, this is what we have proved. Now, notice that the magnetic moment m which we know is the current times the area vector and the area vector can be written as half of

loop integral of $\mathbf{r} \times d\mathbf{l}$. If I take I times $d\mathbf{l}$, remember I is a current. So, I times $d\mathbf{l}$ is goes to if the current exists in a medium rather than in a wire. Then I times $d\mathbf{l}$ is essentially $\mathbf{J} d\mathbf{v}$ because I is $\mathbf{J} \cdot d\mathbf{s}$ and this is $d\mathbf{l}$. Therefore, this definition of magnetic moment which is given by this can also be written as equal to half of $\mathbf{r} \times \mathbf{J} d\mathbf{v}$. So, once I have done that that this is $\mathbf{r} \times \mathbf{r} \times \mathbf{J}$.

So, you notice this that, what we are trying to say is here I got $\mathbf{x} \times \mathbf{x}' \times \mathbf{J}$ and I have the integration of course, the remember the integration variables could be prime. So, this term gives me minus $\mathbf{x} \times \mathbf{m}$ the this is the magnetic momentum and this is the term which will give me the quadruple momentum. But that we will see little later.

(Refer Slide Time: 42:21)

The slide contains the following text and equations:

ELECTROMAGNETIC THEORY

Magnetic dipole

$$\int \bar{\mathbf{J}}(\bar{\mathbf{x}}')(\bar{\mathbf{x}} \cdot \bar{\mathbf{x}}') d^3x' = -\bar{\mathbf{x}} \times \bar{\mathbf{m}} - \frac{1}{2} \int i\omega \rho'(\bar{\mathbf{x}} \cdot \bar{\mathbf{x}}') d^3x'$$

$$A_m(\bar{\mathbf{x}}) = \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \left(\frac{1}{r} - ik \right) \int \bar{\mathbf{J}}(\bar{\mathbf{x}}')(\hat{\mathbf{n}} \cdot \bar{\mathbf{x}}') d^3x'$$

$$= \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} ik \left(1 - \frac{1}{ikr} \right) \hat{\mathbf{n}} \times \bar{\mathbf{m}}$$

NIPTEEL

Prof. D K Ghosh, Department of Physics, IIT Bombay

Therefore, if I want to now simply restrict myself to the magnetic field, to the magnetic dipole, then I need to only worry about this term. I bring back the constants that were there in the problem. μ_0 by 4π e^{ikr} by r $(1/r - ik)$ and integral $\mathbf{J} \cdot \hat{\mathbf{n}} d^3x'$ and I write this because I have shown this integral to be given by $\hat{\mathbf{n}} \times \mathbf{m}$. So, this is the complete expression for the magnetic field for the magnetic vector potential. Now, from this my job is exactly the same as before. So, what I do is I write down calculate that $\nabla \times$ of this A_m . A_m is magnetic dipole term, I have neglected all other terms.

(Refer Slide Time: 43:12)

ELECTROMAGNETIC THEORY

Magnetic dipole

$$A_m(\vec{x}) = \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} ik \left(1 - \frac{1}{ikr}\right) \hat{n} \times \vec{m}$$

$$\vec{H}_m = \frac{1}{\mu_0} \vec{\nabla} \times A_m(\vec{x}) = \frac{1}{4\pi} \left[\vec{\nabla} \left(e^{ikr} \left(\frac{ik}{r^2} - \frac{1}{r^3} \right) \right) \right] (\vec{r} \times \vec{m})$$

$$+ \frac{1}{4\pi} e^{ikr} \left(\frac{ik}{r^2} - \frac{1}{r^3} \right) \vec{\nabla} \times (\vec{r} \times \vec{m})$$

$$= \frac{1}{4\pi} \left[ik e^{ikr} \left(\frac{ik}{r^2} - \frac{1}{r^3} \right) + e^{ikr} \left(\frac{-2ik}{r^3} + \frac{3}{r^4} \right) \right] \hat{n} \times (\vec{r} \times \vec{m})$$

$$+ \frac{1}{4\pi} e^{ikr} \left(\frac{ik}{r^2} - \frac{1}{r^3} \right) \vec{\nabla} \times (\vec{r} \times \vec{m})$$

Prof. D K Ghosh, Department of Physics, IIT Bombay

So, I calculate del cross m this is bit of algebra, but let us go through this. I have got μ_0 by 4π this numbers $\hat{n} \times \vec{m}$ I want del cross of this quantity. So, first thing that I do is realize that this is all scalar ik . I will take out because it does not really depend on anything. So, this is a scalar so I have got instead dealing with $\hat{n} \times \vec{m}$, let me divide it by r and write it as $\vec{r} \times \vec{m}$ and because I have divide it by r and there is another r there I have written as gradient of e to the power ikr , ik by r square. This r and that extra one over r that I have pulled out minus 1 over r cube into $\vec{r} \times \vec{m}$. This is what I have got so I got gradient of this quantity times this vector, because I am calculating del cross.

Therefore, well actually it is del of this quantity cross $\vec{r} \times \vec{m}$. There is a cross missing here and of course, this quantity which is a scalar itself times del cross of $\vec{r} \times \vec{m}$. So, these are the two terms, there is a cross missing here. This gradient can be easily calculated so you can see it that e to the power of ikr take for example, this term when you take the gradient of e to the power of ikr , I get ik so this is ike to the power ikr . Of course, the same thing plus keep e to the power ikr differentiate this this gives me minus 2 by r cube because this is 1 over r square. This gives me minus 3 by r fourth, but there is a minus here. So, it is plus 3 by r fourth times this $\hat{n} \times \vec{r} \times \vec{m}$ because it should be along the unit vector.

This other term I have not disturbed so this is this is the full expression. But then I should be able to simplify them. Just add them up properly because there are similar terms

coming out of here. There is a 1 over r cube term here there is a 1 over r cube term here, add them up properly. This gives you a 3. So, you notice that I would get this. So, here I had got i k into i k is minus k square.

(Refer Slide Time: 45:36)

ELECTROMAGNETIC THEORY

Magnetic dipole

$$\vec{H}_m = \frac{e^{ikr}}{4\pi} \left[-\frac{k^2}{r} - \frac{3ik}{r^2} + \frac{3}{r^3} \right] \hat{n} \times (\hat{n} \times \vec{m})$$

$$+ \frac{1}{4\pi} e^{ikr} \left(\frac{ik}{r^2} - \frac{1}{r^3} \right) \vec{\nabla} \times (\vec{r} \times \vec{m})$$

$$\vec{\nabla} \times (\vec{r} \times \vec{m}) = (\vec{\nabla} \cdot \vec{m})\vec{r} - (\vec{\nabla} \cdot \vec{r})\vec{m} + (\vec{m} \cdot \vec{\nabla})\vec{r} - (\vec{r} \cdot \vec{\nabla})\vec{m}$$

$$= 0 - 3\vec{m} + \vec{m} - 0 = -2\vec{m}$$

Prof. D K Ghosh, Department of Physics, IIT Bombay

So, I got minus k square by r minus 3 i k by r square and plus 3 by r cube, which came from there from this term. And and this term which is said is is also there. Now, what I do is, this it is this second term which I have simplify del cross r cross m is del dot m r. This is the standard vector algebra, when you have del cross of two vectors, you get del dot the second vector r minus del dot r m etc. etc Notice that m is a constant vector so this term is 0. Similarly, this term is also 0 because it is gradient of m. So, i am left with these two del dot of r is known to be equal to 3. This is m dot del r, you know what does it mean it.


Means you have to take the gradient of each one of the components and then dot it with the i times one component plus J times the other component. So, that gives you just m because gradient of x. For example is 1, so this gives me minus 3 m, this gives me plus m. So, I get a minus 2 m. So, instead of del cross r cross m put in a minus 2 m there and simplify these things further.

(Refer Slide Time: 46:56)

ELECTROMAGNETIC THEORY

Magnetic dipole

$$\begin{aligned}\vec{H}_m &= \frac{e^{ikr}}{4\pi r} k^2 \hat{n} \times (\vec{m} \times \hat{n}) + \frac{1}{4\pi} e^{ikr} \left[-\frac{3ik}{r^2} + \frac{3}{r^3} \right] (\hat{n}(\hat{n} \cdot \vec{m}) - \vec{m}) \\ &+ \frac{1}{4\pi} e^{ikr} \left(\frac{ik}{r^2} - \frac{1}{r^3} \right) (-2\vec{m}) \\ &= \frac{e^{ikr}}{4\pi r} k^2 \hat{n} \times (\vec{m} \times \hat{n}) + \frac{1}{4\pi} e^{ikr} \left[-\frac{ik}{r^2} + \frac{1}{r^3} \right] (3\hat{n}(\hat{n} \cdot \vec{m}) - \vec{m})\end{aligned}$$

 Prof. D K Ghosh, Department of Physics, IIT Bombay

So, if you do that what you find is like this. This is a \vec{H}_m and we have written this down. This is already $\hat{n} \times (\vec{m} \times \hat{n})$, simplify this. We have rewritten what these things are. What I have done is this. I have simplified $\hat{n} \times (\vec{m} \times \hat{n})$ using a cross product formula. Namely $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$ and I have written it in this fashion. This is that term which I just now showed to be equal to $3\hat{n}(\hat{n} \cdot \vec{m}) - \vec{m}$. If you plug in all these things you get $e^{ikr} k^2 / (4\pi r) \hat{n} \times (\vec{m} \times \hat{n})$ plus this thing. The reason I am not really concentrating so much on these is, I am mostly interested in the radiation zone, where only $1/r$ terms become.


(Refer Slide Time: 47:53)

ELECTROMAGNETIC THEORY

Magnetic dipole

$$\begin{aligned}\vec{H}_m &= \frac{e^{ikr}}{4\pi r} k^2 \hat{n} \times (\vec{m} \times \hat{n}) + \frac{1}{4\pi} e^{ikr} \left[-\frac{ik}{r^2} + \frac{1}{r^3} \right] (3\hat{n}(\hat{n} \cdot \vec{m}) - \vec{m}) \\ &\rightarrow \frac{e^{ikr}}{4\pi r} k^2 \hat{n} \times (\vec{m} \times \hat{n}) \quad (\text{radiation zone})\end{aligned}$$

$$\begin{aligned}\vec{E}_m &= -\frac{\partial \vec{A}}{\partial t} - \nabla V \equiv -\frac{\partial \vec{A}}{\partial t} \\ &= \frac{\mu_0 c k^2}{4\pi} (\vec{m} \times \hat{n}) \frac{e^{ikr}}{r} \left(1 - \frac{1}{ikr} \right) \\ &\rightarrow \frac{\mu_0 c k^2}{4\pi} (\vec{m} \times \hat{n}) \frac{e^{ikr}}{r} \quad (\text{radiation zone})\end{aligned}$$

 Prof. D K Ghosh, Department of Physics, IIT Bombay

Therefore, in the radiation approximation I am only interested in this term which goes as e^{ikr}/r . The power is $4\pi k^2 r^2 \sin^2\theta$. This is the radiation zone the magnetic field can be found by directly calculating $\nabla \times \mathbf{h}$ or by showing that this is equal to $-\dot{\mathbf{A}}$. But basically the radiation zone, in the radiation zone this is the expression for the electric field. These are the only the magnetic dipole term. Now, we do exactly what we did earlier, namely we calculate the pointing vector and calculate the total radiated power.

(Refer Slide Time: 48:34)

The slide contains the following text and equations:

ELECTROMAGNETIC THEORY

Magnetic dipole

$$\vec{H}_m = \frac{e^{ikr}}{4\pi r} k^2 \hat{n} \times (\vec{m} \times \hat{n}) \quad (\text{radiation zone})$$

$$\vec{E}_m = \frac{\mu_0 c k^2}{4\pi} (\vec{m} \times \hat{n}) \frac{e^{ikr}}{r} \quad (\text{radiation zone})$$

$$P = \frac{\mu_0 c k^4}{12\pi} |\vec{m}|^2$$

NIPTIIL

Prof. D K Ghosh, Department of Physics, IIT Bombay

So, if I do that, I find that this is the radiation zone and this is the electric field. In the radiation zone and the corresponding power expression works out to $\mu_0 c k^4$ to the power fourth divided by 12π . Notice this ω to the power 4 dependence is very very common because k to the power fourth is also proportional to the ω to the power fourth this times m^2 and being. I am not drawing the power pattern, but power pattern is very similar to the electric dipole pattern.

(Refer Slide Time: 49:13)

ELECTROMAGNETIC THEORY

Electric Quadrupole

$$\int \vec{J}(\vec{x}')(\vec{x} \cdot \vec{x}') d^3x' = -\vec{x} \times \vec{m} - \frac{1}{2} \int i\omega \rho' \vec{x}'(\vec{x} \cdot \vec{x}') d^3x'$$

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \left(\frac{1}{r} - ik \right) \int \vec{J}(\vec{x}')(\hat{n} \cdot \vec{x}') d^3x'$$

$$\vec{H}_Q = -\frac{1}{3} \frac{ick^3}{8\pi} \frac{e^{ikr}}{r} \hat{n} \times (\vec{Q} \cdot \hat{n})$$

$$Q_{ij} = \int d^3x' (3x_i'x_j' - x'^2 \delta_{ij})$$

Prof. D K Ghosh, Department of Physics, IIT Bombay

I will not be talking about the electric quadrupole moment and that comes from this term, that comes from this term and you can see I am not yet worked it out that the correspondingly the magnetic field expression can be written like this where Q_{ij} is the quadrupole moment tensor and this given like this.

(Refer Slide Time: 43:31)

ELECTROMAGNETIC THEORY

Quadrupole Radiation Pattern

Prof. D K Ghosh, Department of Physics, IIT Bombay

The quadrupole radiation pattern shows the maximum at angle equal to $\pi/4$ and this is the way the quadrupole moment looks like this is something, which you will you will do well to take it as an exercise. So, this is basically what we want to talk about in the

radiation basically talking about how the power you know is transported? The radiation zone is $1/r$ and how much is the power which is sent from the the transmitting antenna, which is later on received by a receiver antenna.