**Electromagnetic Theory**
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**Lecture - 37**
**Waveguides**

We have been discussing the properties of waveguides. As we have seen that waveguides are essentially conduits through which electromagnetic energy is transmitted from one point to another. So, we had earlier discussed a waveguide which was basically two semi infinite metallic planes. Today, we will be discussing initially the rectangular waveguides which we introduced last time.

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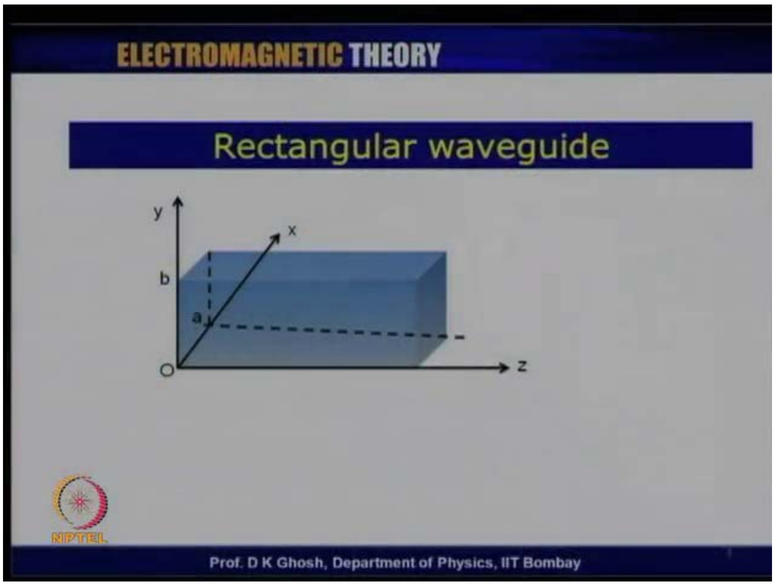


So, it is basically one of the dimensions is of still infinite, but otherwise we have two-dimensions namely it is a rectangular parallelepiped of infinite length, cross section being a by b in the x and y direction, and z direction is the direction in which the wave would propagate.

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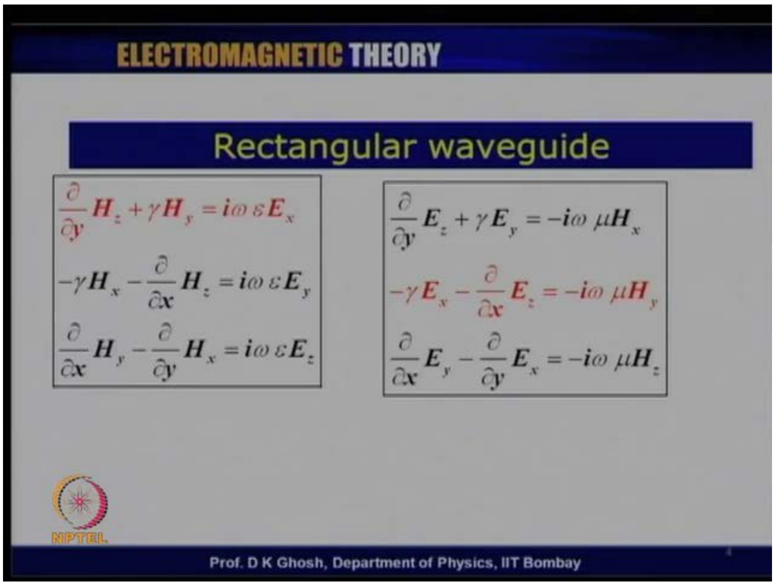


We had seen that the Maxwell's equations which we had I will just illustrate one of them which was essentially...

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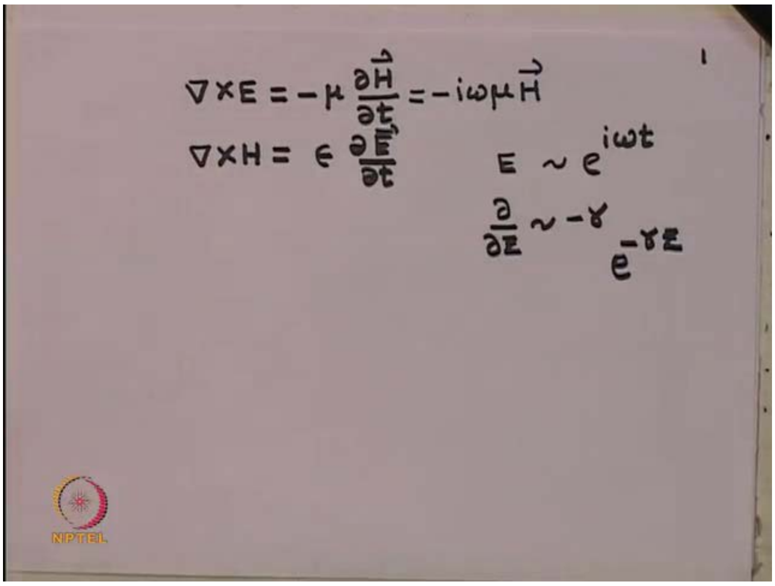


For example Del cross of E equal to minus well d b by d t, but I will write it as minus mu d H by d t. There are no currents or sources. No displacement currents del cross H. Now, as there are no currents so there is no current term here is epsilon d E by d t. And what we are going to do is to take as before the time dependence as E or H they go as e to the power i omega t. As a result this d b by d t that gives you that results in essentially a multiplication by i times omega. So, as a result you can see the first equation here which is essentially this equation, because this became minus i omega times mu times H.

So, if you take its x components it will become del cross E’s x components and del cross E’s x component is as you can see here d by d y of E z minus d by d z of E y, but we have seen that since the propagation direction is z direction. So, d by d z essentially means multiplication by minus gamma, because the propagation goes as e to the power minus gamma z. And therefore, this equation became d by d y of E z plus gamma times E y is this. Similarly, this is our Faraday’s law where well this is, this was Faraday’s law and this is the ampere Maxwell’s law.

So, this is del cross H x component which is d by d y of H z minus d by d z which is same as plus gamma times H y is i omega epsilon E x and I have written down essentially all the six equations, six components. I have three components of magnetic field and three components of the electric field in this transparency. Now, so what we want to do essentially is to realize that these six equations I can cast them such that every component. So, I have six components. So, I I will choose any two components for example, I will choose E z and H z and I can manipulate these equations to write them with each component explicitly in terms of the z component of E and H only. So, we had talked about it last time, but let us go through it again.

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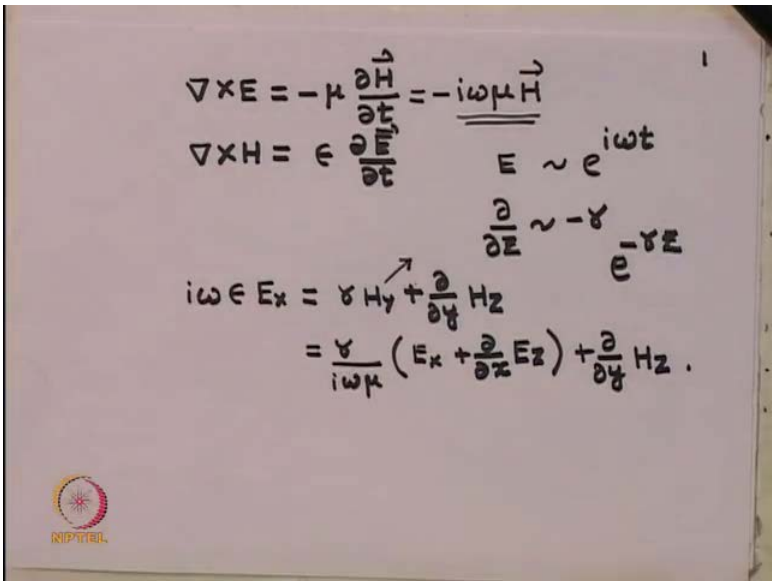

So, this equation which we talked about for example the x component of the electric field i omega epsilon E x, this is this equation and that is del cross H's x component and which is, as we have seen x component meanings d by d y of H z and d by d z minus d by d z which is plus gamma H y and plus d by d y of H z. So, what I do next in this that this H y which can from here that is the y component of this, I will write it. So, I already have a gamma and you can see that if I take y component of this equation, I will essentially get I divide it by 1 over i omega mu and y component means d by d z of E x minus d by d x of E z and therefore, this equation becomes gamma by i omega mu times E x plus d by d x of E z plus d by d y of H z.

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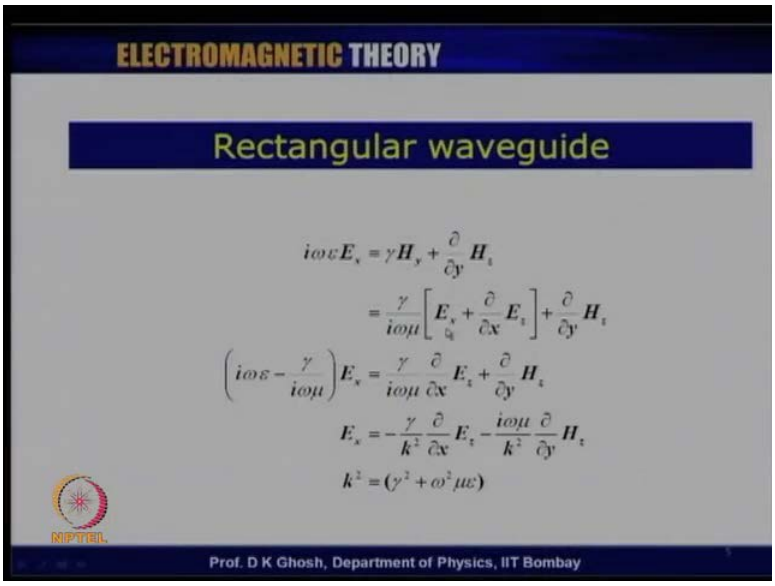


And now what I do is this that there is I have this x E x there and that I write down again by writing down what is this. So, you write notice that I can write this E x in terms of for example, I could eliminate E x from these two equations and write them in terms of the other components. So, if you do all that then it turns out, if you do all that this is a bit of an algebra, but fairly straight forward that you can write E x in terms of the derivative of E z and H z.

So, I have chosen for no particular reason, but because of the fact that the z component z direction is my direction of propagation. I have written E x in terms of derivative of E z and derivative of H z. And if you simplified all these, this is the way the equation works out where k square is gamma square plus omega square mu epsilon. This is and and this you can do for each one of the other components that is u i H x and H y.

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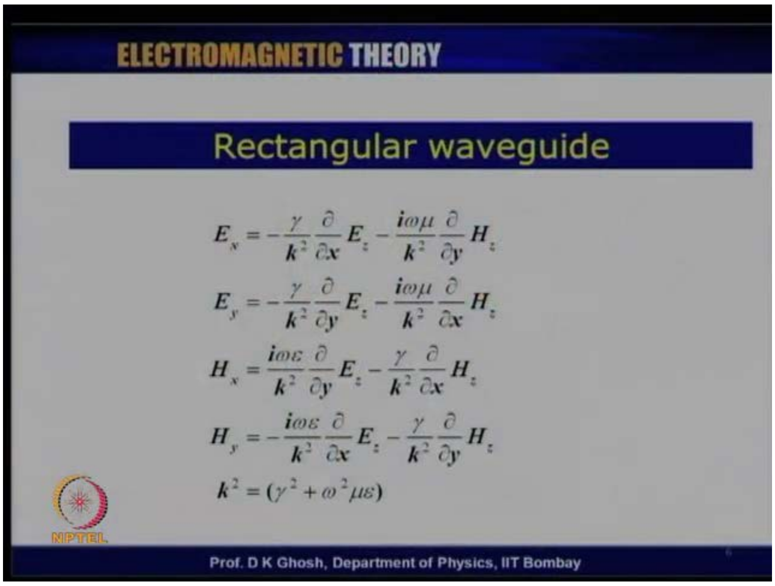


And this gives you a complete list of the equations where E x E y H x and H y are written in terms of the derivatives of E z and H z only. And other than omega epsilon and mu which are of course, there we have one quantity there which is k square which is appearing as gamma square plus omega square mu epsilon. Now, this is the set of equations.

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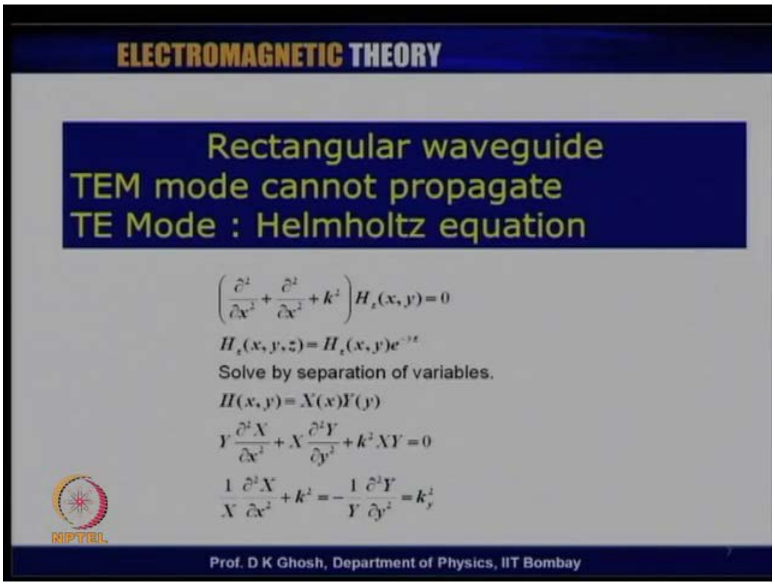

Now, what I will do now is this that now decides the following, that we define what we called modes of propagation. Now, this is done by selecting or rather classifying these solutions in terms of such propagations for which the z component of electric field is equal to 0 that is one group. z component being the direction of propagation if z component of the electric field namely E z becomes equal to 0, it means that the electric field is transverse. So, this is called transverse electric solution T E. We had seen this earlier also and similarly, another group would be T z T m.

Namely transverse magnetic field for which the H z is equal to 0 and E z in not equal to 0, and we had seen earlier that in case of the parallel plate waveguides there are also very special solutions for which both E z and H z were equal to 0 and this we called as the transverse electromagnetic or T E M mode. But we will see that was a rather special case in case of parallel plate wave guides, but in this case we do not have such a situation.

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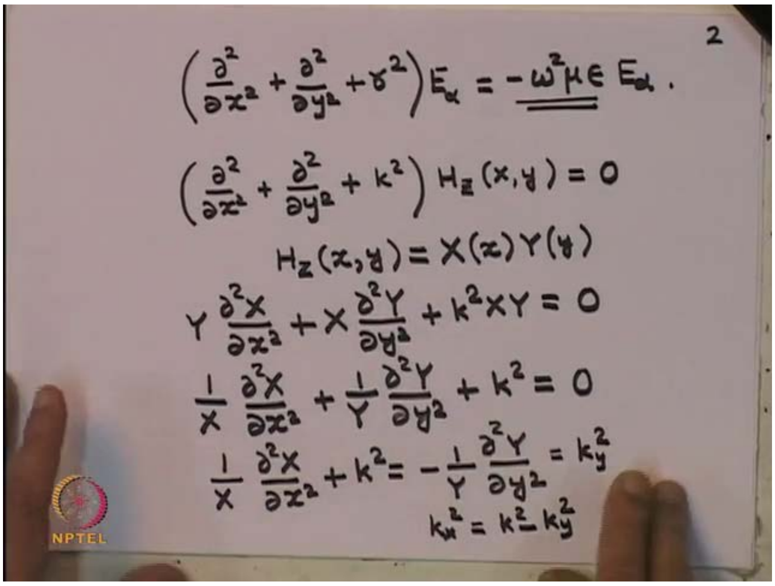


So, coming back to our equation, the equation that we had for the components of the electric field that is the Helmholtz equation was basically d square by d x square plus d square by d y square plus d square by d z square which is nothing but gamma square times your any component of the electric field. Actually it is minus gamma square, because E to the power. Let us write it as gamma square because E to the power minus gamma z we have taken, but if this is propagating actually this will be i beta. Therefore, it will come to that side and that any component let us say E alpha that is equal to minus omega square mu epsilon times the same component. This was the equation that we had.

So, as a result what you are getting is this that if you bring this to that side you find that I get d square by d x square plus d u square by d s d y square plus gamma square plus omega square mu epsilon. If you recall this we had defined as k square. Therefore, this is k square times any component of electric field, but we are interested in let us say we are talking about T E mode for which E z is equal to 0.

So, every solution can be written in terms of H z only. So, this quantity is equal to 0 and H z because z direction is basically infinite. So, the only dependence it can have is on x y and that quantity is equal to 0. So, this is the equation for which we need a solution. So, what we said is that you can solve this equation by what is known as separation of variable. So, H z of x y you write it as some quantity X which depends upon x only and some quantity Y which depends upon y only. So, you plug this in because this depends only on x. So, what you find is Y times d square X by d x square plus X times d square Y by d x d y square plus k square times X Y that is equal to 0.

Now, if you divide this equation by X Y all through you get 1 over X d square X by d x square plus 1 over Y d square Y by d y square plus k square is equal to 0. Now, we are going to do this technique inferentially is known as separation of variable. So, you notice one thing that I can write this equation as equal to 1 over X d square X by d x square plus k square is equal to minus 1 over Y d square Y by d y square. Now, notice one thing this quantity depends upon x only, this quantity depends upon y only and you want for whatever X and Y these two must always be equal.

Now, obviously this is possible if each one of them is a constant and let’s call that constant as equal to k y square. So, this plus, this is equal to that is equal to k y square. Now, so as a result you notice that these equations are well known equations to us. For instance this equation is familiar to us in simple harmonic motion because this tells me d square Y by d y square plus k y square Y is equal to 0, so as a result my solution for X and Y. Now, and the same thing is true for this quantity also. What you could do is you could define a quantity which is k x square which is such that it is equal to k square minus k y square.

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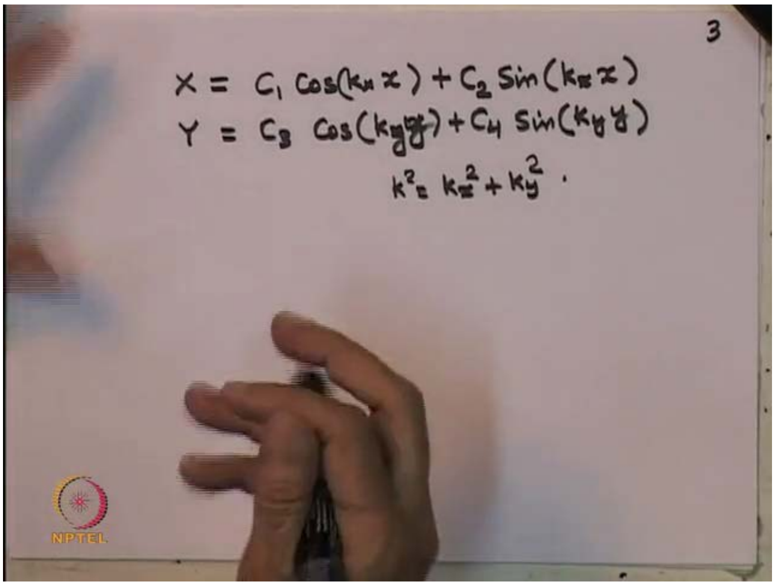


So, as a result both of them have the same structure and my solution to these equations then become X is some constant C 1 cosine k x x plus C 2 sin k x x. And likewise capital Y is C 3 cosine k x x plus C 4 sin sorry yeah this is k y y and this is sin k y y. So, my total solution is the product of this with this. In other words this is a product of terms like cosine cosine, cosine sin, sin cosine and sin sin. So, this is this is what I have got. Now, and as we have seen that k square is equal to k x square plus k y square. So, once I have got this equation. So, I am writing down solution to my H z, H z because we have taken T E mode so E z is equal to 0. So, H z is essentially a function of this into this.

Now, let us look at how to determine some of these things. The thing that you have to realize is this that if you take for example, go back to the picture of the rectangular parallelepiped. So, notice this is the z direction is infinite and so here I have got the x y plane for example, x is equal to 0 here. Now, these planes namely x is equal to 0 x is equal to a, y is equal to 0 y is equal to a on these surfaces my tangential component of the electric field must be equal to 0. And notice this that my tangential components will be related to or will be equal to the E x or E y, because I have got these surfaces. For example this surface is in the x y plane, and therefore this is at z equal to 0. So, let us look at what does it give me.

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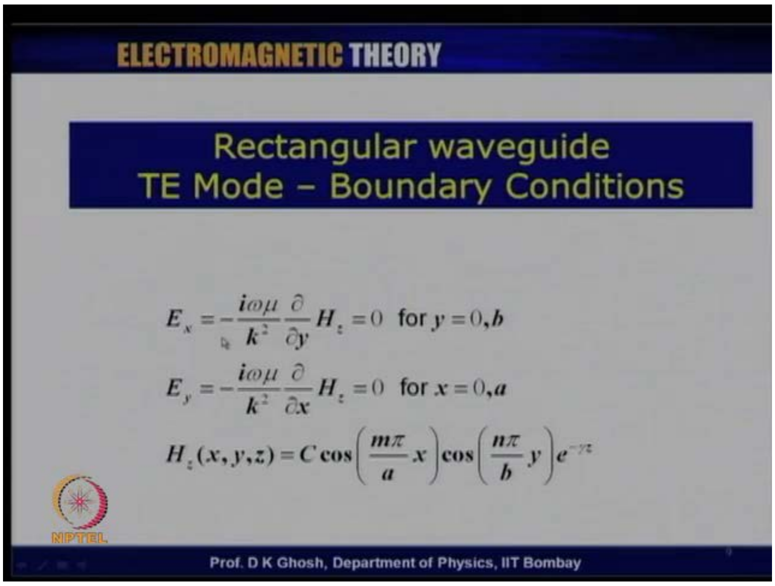


So, take for example, E x. Now, E x is the electric field which is essentially the tangential component of the electric field at y is equal to 0 and at y is equal to b. Now, if we looked up the set of equations that we had written down, the E x for the T E mode namely if E z is equal to 0 works out to some constant d by d y of H z. Now, remember this quantity has to be equal to 0 for y is equal to 0 as well as y is equal to b. What it implies is at y equal to 0 and y is equal to b my derivative of H z with respect to y must be equal to 0. Remember, that my H z is a function of, consists of four terms cosine k x x sin cosine k y y cosine k x x into sin k y y etcetera. Now, if you want the derivative of H z with respect to y to become equal to 0. So, it goes back here.

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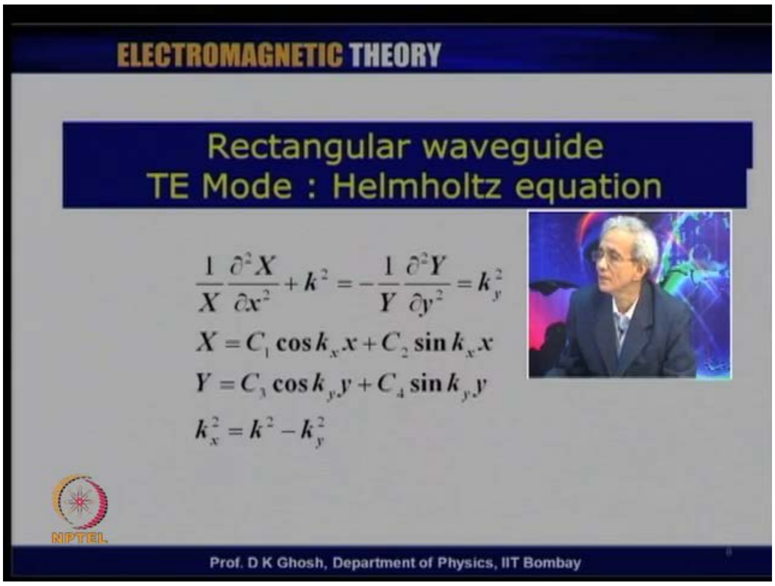


So, my let me write down what is x into y which is my H z.

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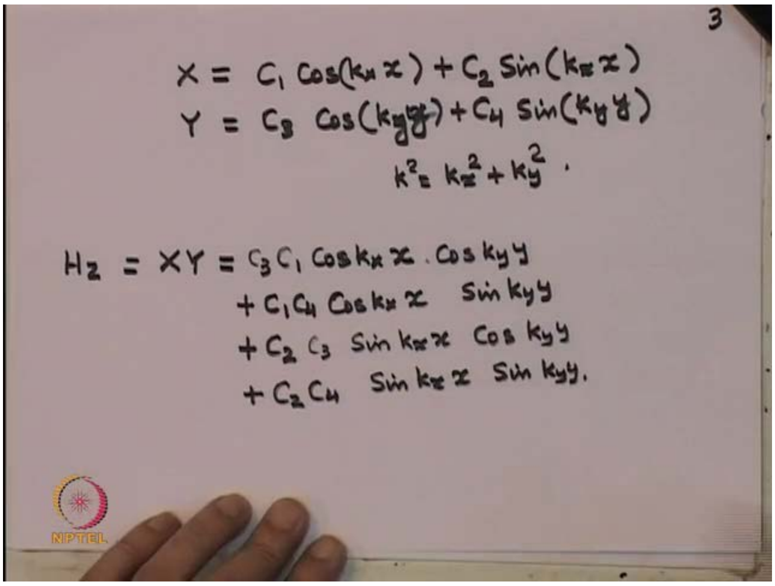


This is C 1 cosine k x x into C 3 cosine k y y plus C 1 cosine k x x C 4 sin k y y plus C 2 into C 3 sin k x x cosine k y y plus C 2 into C 4 sin k x x into sin k y y. Now, let us look at what are we trying to say. We are saying that derivative of this derivative of this with respect to y should be 0. Now, if you are differentiating with respect to y this term for instance will give you k y sin k y y. This term will give you k y cos k y y and like this.

Now, you want that for y is equal to 0 this term should give me 0. So, that now this has become sin k y y. So, I do not have a problem with this term becoming equal to 0.

Now, so what am I actually left with? You notice that the the term that I must have, this will tell me that I must have C 3 into C 1 this term must become equal to 0 etcetera. Now, if you do that than ultimately you are left with because these are the four conditions to be satisfied. Ultimately you are left with that I can have H z only given in terms of this type of an equation. That is C times cosine m pi by a times x times cosine n pi by b times y and remember my boundary conditions are that the tangential component of the electric field which comes as derivatives of H z. And the normal component of the magnetic field they will be 0 on the plates.

So, that that tells me that H z is just a product of cosine into cosine. And these facts that the k x and k y becoming equal to m pi by a and n pi by b is because I want these electric fields to vanish at x is equal to a as well as y is equal to b.

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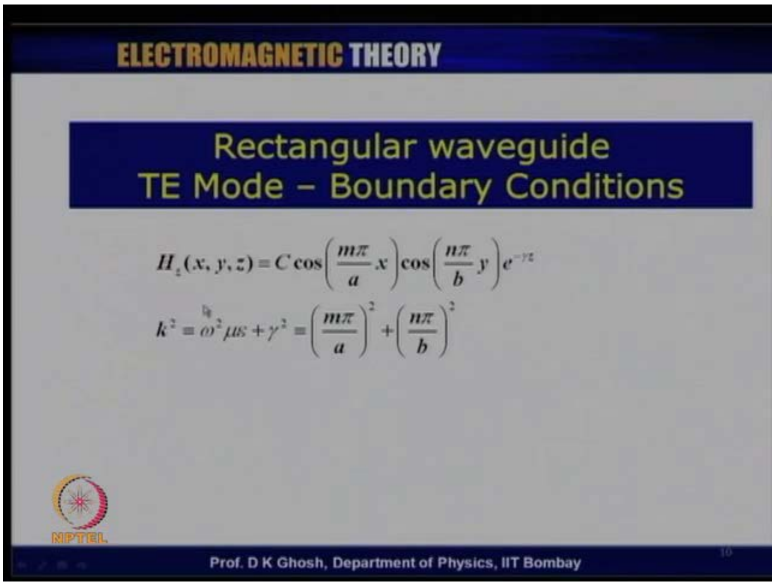


So, as a result if you now want to write down your full equations H z which is a function of x y z is this solution which we have just now obtained. Times e to the power minus gamma z and k square which we had seen is omega square mu epsilon plus gamma square k square is k x square which is m pi by a whole square and plus k y square which is n pi by b whole square. So, these these are my solutions for H z and E z is equal to 0.

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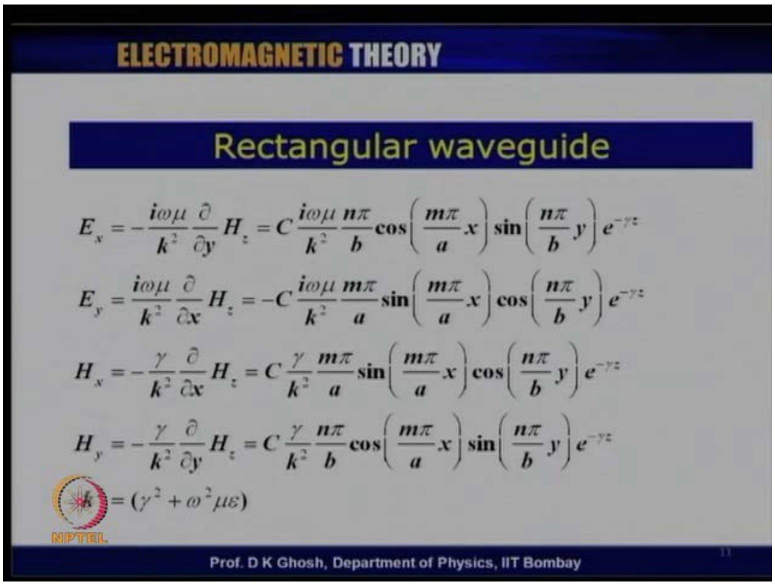


So, I will just leave this transparency for some time, this tells you that once you have got H z of z you can obtain E x E y H x and H y purely in terms of the H z. So, these are the things which are there.

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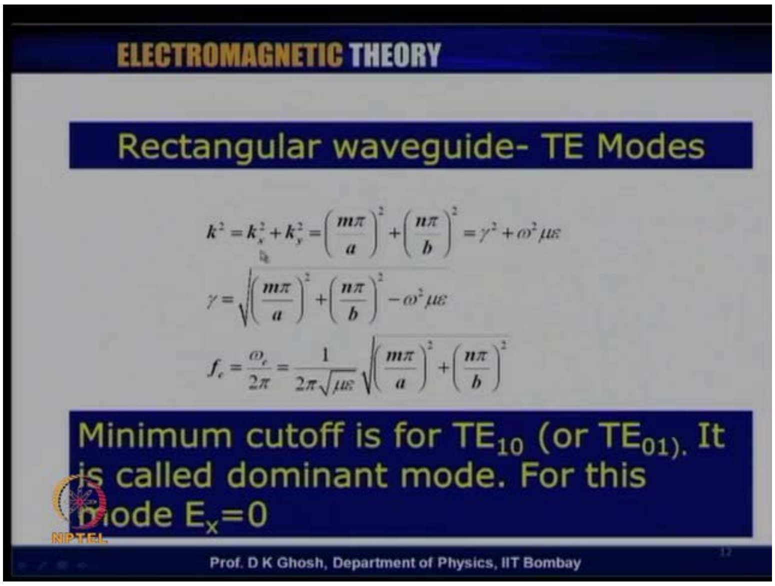


So, as we have seen k square is this quantity and this is what it is. Now, if you look at gamma now. Gamma is given by m pi by a whole square plus m pi by b whole square minus omega square mu epsilon. Now, this tells me now you remember that my propagation went as e to the power minus gamma z.

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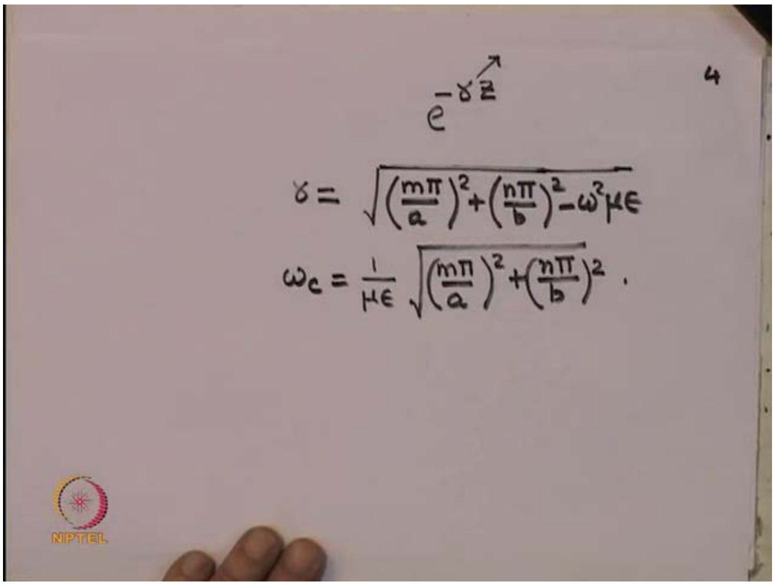


And we have seen that gamma is given by root of m pi by a whole square plus n pi by b whole square minus omega square mu epsilon. Now, in order that this may be a propagating solution I want gamma to be imaginary. Otherwise it is an attenuating solution. Now, in order that gamma is imaginary it tells me there is a minimum frequency omega c given by well you want omega square mu epsilon to be greater than this. So, omega c is given by 1 over mu epsilon times square root of m pi by a whole square plus n pi by b whole square. And of course, so this if you like is my cut off frequency.

What I have written down here is the, this was of course, your angular frequency and this is the normal f c which is omega c by 2 pi. So, this is a minimum cut off, this is a cut off frequency for T E M n mode. Now, notice that if you take for instance T E 1 0 mode. So, m is equal to 1 n is equal to 0 and this depending upon which one you called to what, this is what is known as a dominant mode, because this is this is the mode for which the frequency is minimum whether it is T 1 0 or it could be T 0 1. It would depend upon which one of these dimensions is smaller or bigger and therefore, that is the cut off frequency.

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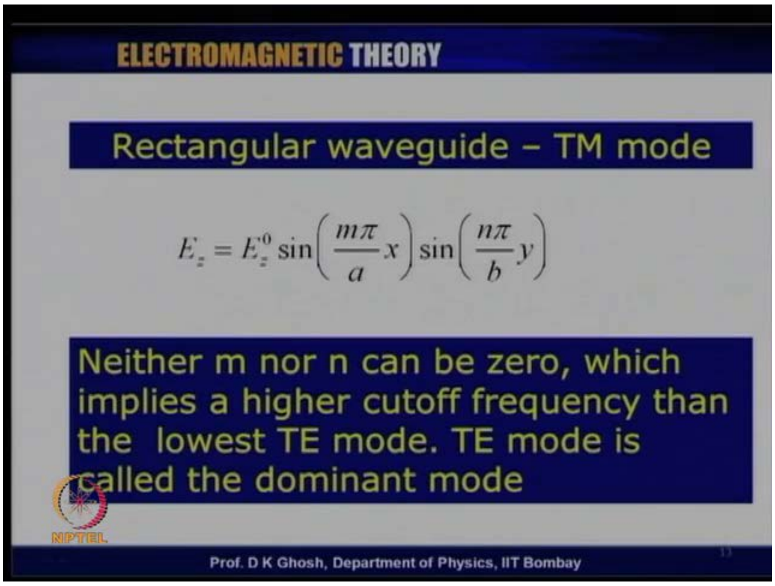


And likewise you could work with E z that is H z equal to 0 transverse magnetic mode and E z is not equal to 0 and if you did exactly the same algebra as we talked about earlier you find that for this T M mode E E z works out to be a product of sin into sin. Now, notice because this is product of two sin function unlike the T E mode which was the product of two cosine function, I cannot have m or n to be equal to 0 because if it is then E z will be equal to 0. Now, I did not have that problem with the cosine function.

There all that I required is both m and n should not be 0 because if they were then that would be a constant, but in this case if either m or n becomes equal to 0 E z vanishes and once E z vanishes since H z is equal to 0 all my components would trivially vanish. So, because of that the T M mode has a higher cut off frequency than that of T E mode and hence T E mode is known as the dominant mode.

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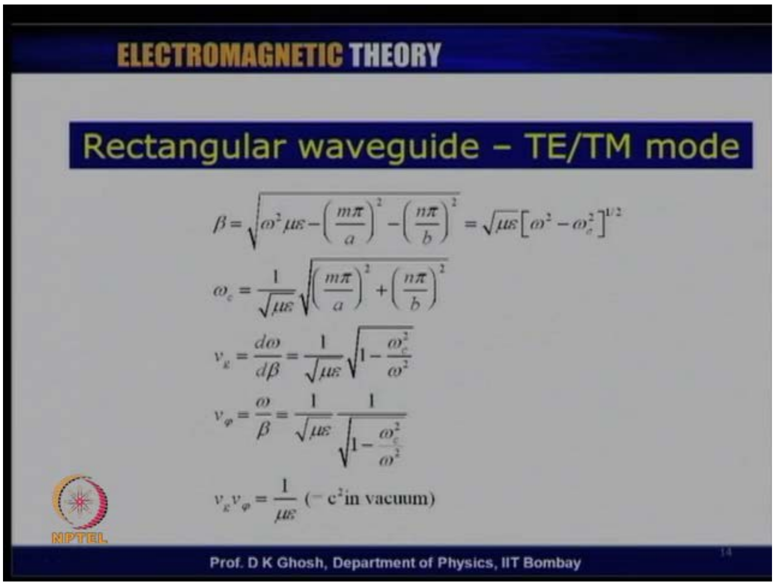


So, let us look at the some minute properties. Supposing there is propagation. Again we are back to both it could this will be valid for both T E as well as T M mode. So, when there is a cut off when the frequency is above the cut off the propagation vector beta which is the same as i beta being equal to gamma is simply square root of omega square mu epsilon minus m pi by a whole square minus n pi by b whole square. And we define omega c as 1 over root of mu epsilon by into square root of this quantity. So, that beta can be written as root mu epsilon omega square minus omega c square raise to the power half and omega is greater than omega c for propagation in order that propagation may take place.

Now, if you have this. So, beta is your propagation vector. Now, remember that the group velocity of the wave is given by d omega by d beta where I have already seen what is the relationship between omega and beta which is gives me 1 over root mu epsilon. Incidentally this quantity is nothing but the velocity of light if it is in vacuum multiplied by 1 minus omega square by c square, but the phase velocity this of course, is less than the speed of light in vacuum as it ought to be. But the phase velocity if it is simply omega by beta is given by this expression namely c by square root of 1 minus omega c square omega square. Now, if you take the product of the group velocity with the phase velocity that gives you 1 over mu epsilon. And if you are dealing with vacuum between the plates then it is just equal to c square.

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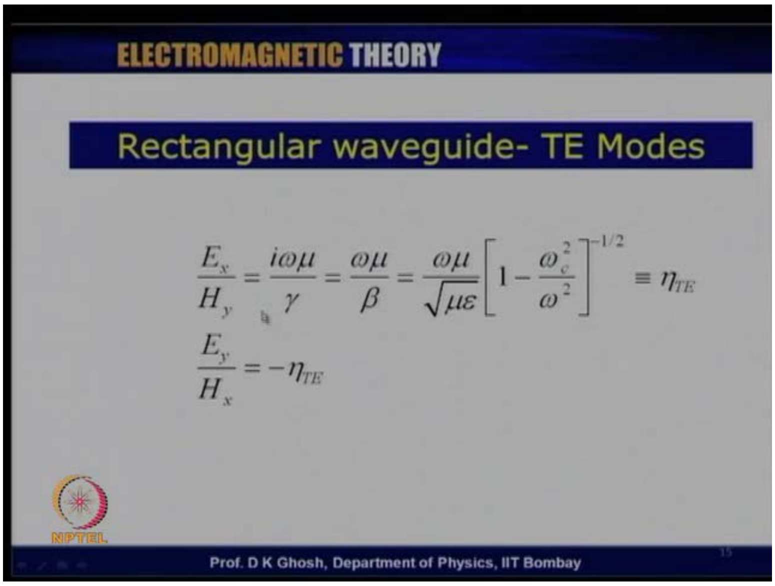


Notice, one thing that in case of a T E mode if you took the ratio of E x to H y, if you recall when we talked about the propagation of uniform waves in free space we had seen the ratio of E x to H y was nothing but what we called as the characteristic impedance. Now, here also something very similar works out E x by H y is i omega mu by gamma and we have seen gamma is i beta. So, it is omega mu by beta and by substituting for beta you get this is equal to omega mu by root mu epsilon into this factor here. And this is what you denote as eta T E which means it is the characteristic impedance corresponding to the T E mode. So, E x by H y works out to eta T E and because of the directions E y by H x also works out in magnitude be eta T E, but this is given by minus eta T E.

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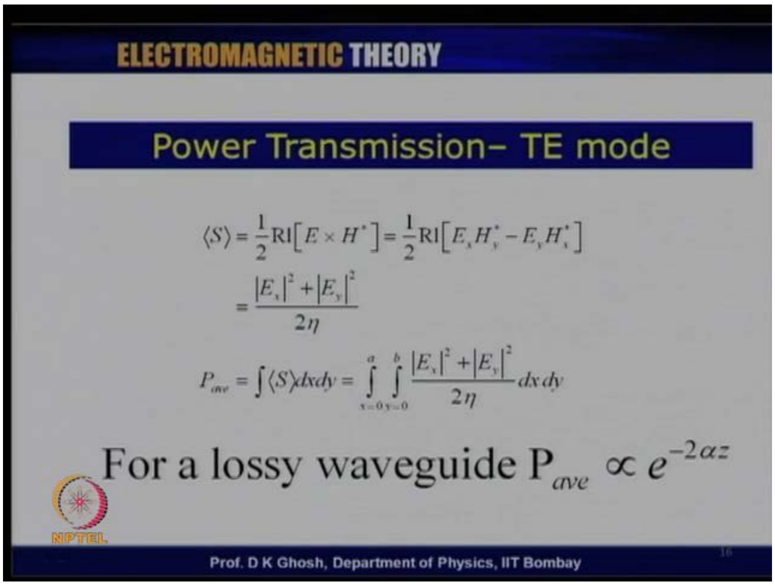


Now, let us look at the power translation amount. So, we have seen that what we are looking for is transmission of power which is designated by the pointing vector. Pointing vector in terms of complex electric and real field magnetic field is given by half of real part of E cross H star. And just now we have seen that E x and H y’s are related by this eta factor. Therefore, I can write the H as or E so here what I have got E cross H star real part and you remember I am in T E mode. So, my z component is 0. So, as a result the remaining components are given by E x H y star minus E y H x star.

What I have not written here is that because the direction of propagation is along the z direction. So, I have essentially taken the z component of E cross H star and if you now write down that H y by E x is eta, eta T E. So, you get E x square plus E y square divided by 2 eta. So, this is my average pointing vector. And the power that is transmitted is since the z direction is the direction of propagation it flows through at any point of time through x y plane. Therefore, you have to take the pointing vector and average pointing vector and integrate it from d x d y x going from 0 to a and y going from 0 to b. And if you do that this is the expression because you have to substitute the expressions for E x and E y and do this integration.

Supposing, we had a lossy wave guide. Now, if you had a lossy waveguide then we have seen that gamma will not become imaginary. But gamma will become real and by designating its real part as alpha, we get this quantity to be proportional because there is an E and there is an H to e to the power minus 2 alpha z. So, that is the rate at which the power would attenuate. We will come back to this point a little later.

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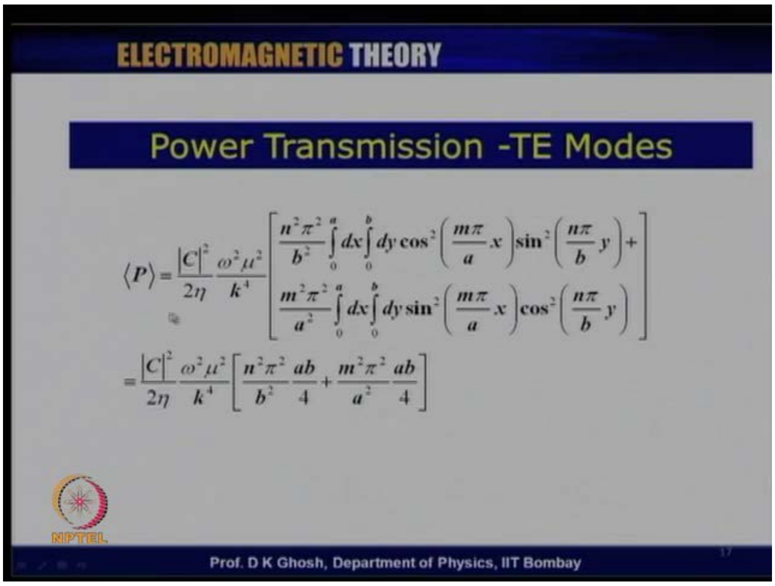


So, if you look at what we said just now the power transmission is given by this expression, the you have to these are the two terms and you can do this integrations because they are fairly straight forward cosine square and sin square terms. Just if you do the integration each one on them will give you a by this will give you a by 2 this will give you b by 2. Therefore, this is the way they will work out. So, this is the expression for the power transmission. Having talked about rectangular parallelepiped wave guide I am going to now close the remaining sides as well. That is what I have got is what is known as a rectangular cavity.

So, I have got electromagnetic waves which are confined within the boundaries of a rectangular parallelepiped, I have taken the dimensions to be a b and d. I have not taken a b and c because c is the velocity of light, I do not want any confusion. So, I have taken this is what is known as a resonating cavity the, this is very similar to for example, like an L C circuit. We will see just as L C circuit is used to store electrical energy. A resonating cavity is used to store electromagnetic energy the typical resonating circuits however have a great advantage over the L C circuits because these have frequencies up ward of a few hundred mega hertz.

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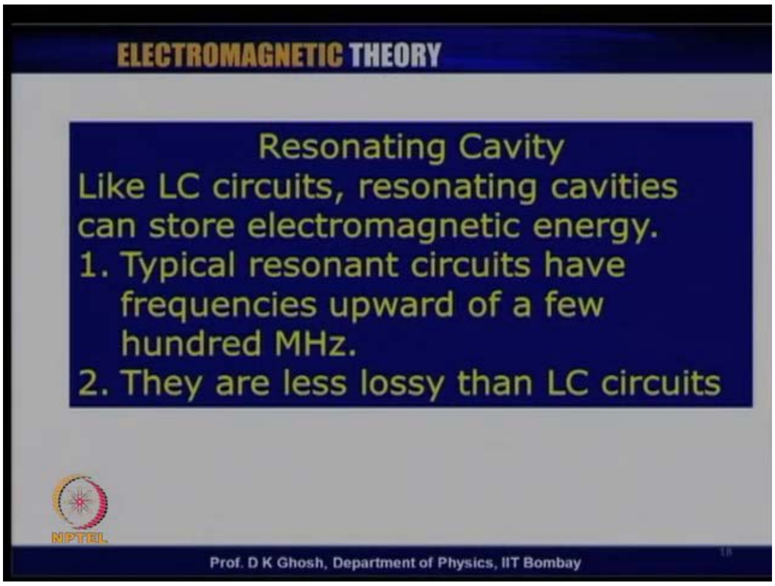


And they turn out to be less lossy than L C circuit. Now, remember when we talked about the parallelepiped rectangular waveguides we had not talked about losses at all, but we know that losses are facts of life. So therefore, we have to worry about losses you can minimize it, but we will be talking about it as we go along. The the technique of discussing the resonant cavity is very similar to what we did. So, let us first look at the wave equation.

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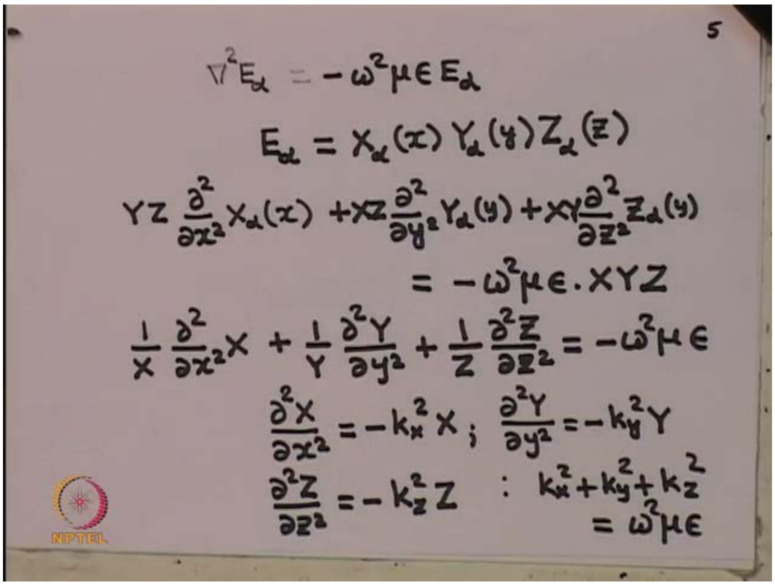

Remember, we had said del square E any component E alpha x y or z is given by minus omega square mu epsilon E alpha. So, what you do is this. Now, this is the equation which you need to solve del square is d x d by d x square plus d by d y square plus d by d z square. Now, I will use the same technique namely the technique of separation of variable by assuming that this E alpha remember my alpha can be x y or z component whichever fancies me. This is written as a product of capital X which is a function of x only, capital Y which is a function of y only and capital Z which is a function of z only.

And what I do is exactly what we did earlier so if you write down this is d square by d x square of E alpha which is x y z. So, this gives me X of x multiplied by Y Z plus d square by d y square of Y multiplied by X Z plus d square by d z square Z multiplied by X Y. This is equal to minus omega square mu epsilon into X into Y into Z. So, if you divide this equation all through by X Y Z you will find this thing that 1 over X d square X by d x square plus 1 over Y d square Y by d y square plus 1 over Z d square Z by d z square is equal to minus omega square mu epsilon. So, what we do is this. Now, we we exactly work the same way as we did earlier.

This is a function of X only, this is a function of Y only, this is a function of Z only. These three things must be added in such a way that for all values of X Y and Z it must give me a constant. Now, obviously that is a tall order unless this is the constant, this is a constant and this is a constant and these three constants added together gives me the third constant. Now, so what I require is d square X by d x square that is equal to a constant. So, let me write it as minus k x square times X, then d square Y d y square equal to minus k y square times Y, d square Z by d z square is equal to minus k z square Z.

So, if I do that this term becomes minus k x square, this terms becomes minus k y square, this term becomes minus k z square so that this equation will be satisfied if I have k x square plus k y square plus k z square is equal to omega square mu epsilon. This is the technique of separation of variable. The advantage is if you look at any one of these equations, these are equations which are known to us in solution of simple harmonic motion.

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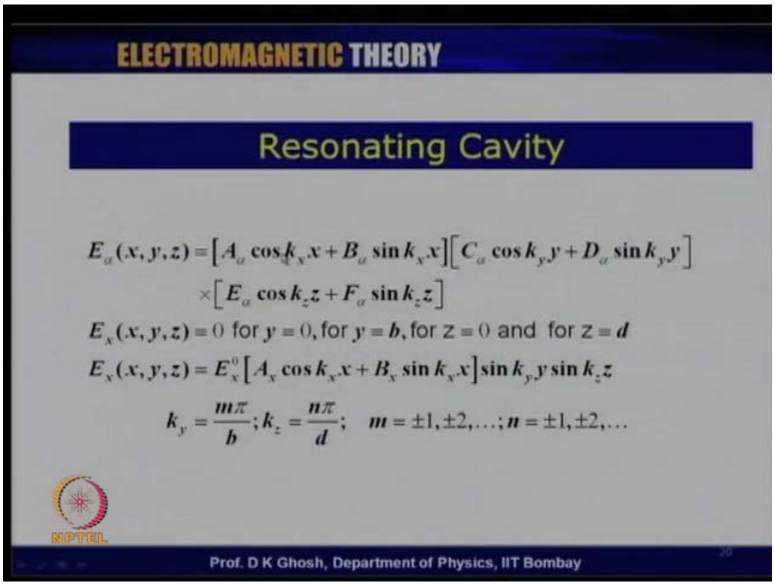


So, that they all give me linear combination of the sum of x and cos k x x sin k x x etcetera that is supposing I am writing E alpha. So, I have got this term which is my capital X which is A alpha cos k x x and B alpha sin k x x and similarly y and similarly z. Now, what you do from here? The, what I do now is this. I now know that I have a rectangular parallelepiped. Therefore, my x component for instance will be equal to 0 either when y is 0 or d because that is one plane on which my x component of x direction is the tangential direction or z is equal to 0 and z is equal to d. So, E x of x y z let me illustrate this here is equal to 0 for y is equal to 0 y is equal to b z is equal to 0 and z is equal to d.

Now, look at what it means. If I want this quantity to be equal to 0 for y is equal to 0. Now, substitute y is equal to 0 here. So, this term automatically goes away, this term automatically goes away and I am left with A x cos k x so C alpha cos k y y remains. Now, the, so this term will become 0 and I will have C alpha cos k y y. Now, I want that to be 0 for all values, all for y is equal to b as well. Now, if I do that then that tells me that my k y must take values m pi by b because then only this term become equal to 0. I cannot have both C is equal to D 0, D is equal to 0 simultaneously.

Because if I did it then the entire E alpha will go away and likewise you substitute that this field must become equal to 0 for z is equal to 0 which will make this this term go away. As a result my E alpha term must be equal to 0 and then for z is equal to d. So, if you put that then I get E alpha this term remains and from these two only the sin k y y and sin k z z remains and the factor D alpha into F alpha, I have combined it into one constant which is E x 0. So, this is my structure of E x with m becoming equal to plus minus 1 plus minus 2 etcetera, etcetera. So, that is the E x x.

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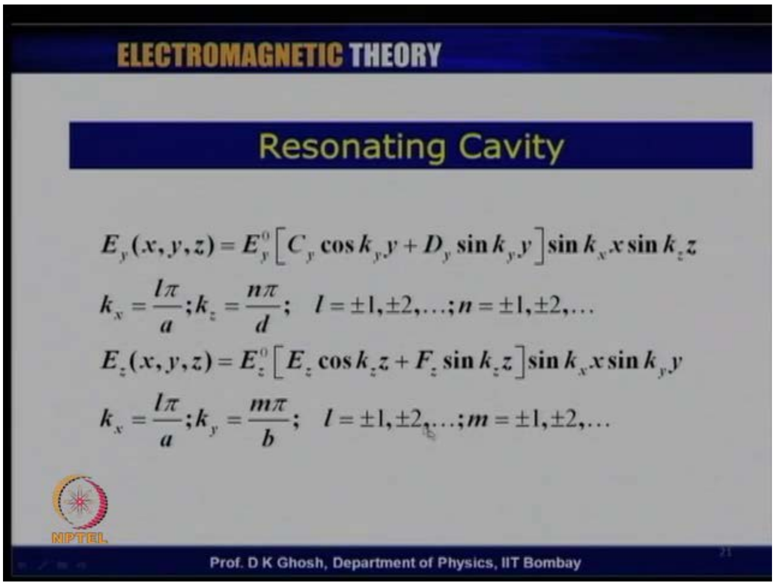


Now, likewise I go to E y and well the argument is essentially the same. The just as you got when you add E x you got sin k y y and sin k z z and the x part had both the terms. Here, also I will get since it is E y the y part will have cosine and sin, the x parts will have only sins and k x will be l pi by a k z will be equal to n pi by d and likewise for E z as well.

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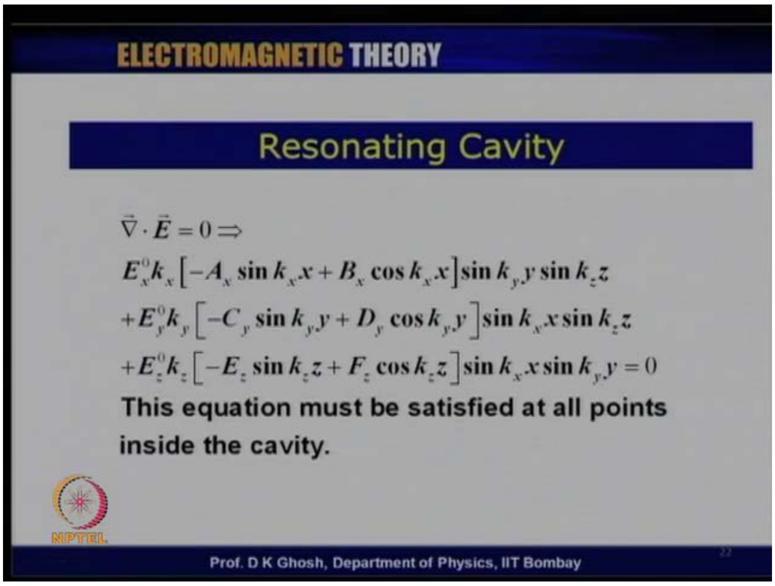


Now, how do you proceed from here? Now, notice that irrespective of my solutions my del dot of p must be equal to 0. del dot of p is d by d x of E x plus d by d y of E y plus d by d z of E z. Since, I have already obtained E x E y and E z what I require is that simply differentiate. So, when you differentiate cos k x x you get minus A x sin k x x. sin k x x give me B x. So, this is what you get, this is first term is obtained by differentiating with respect to x, the second term differentiating with respect to y, the third term differentiating with respect to z.

Now, unlike E which must be equal to 0 the tangential component must be equal to 0 at the boundaries my del dot of E must be equal to 0 anywhere, for any value of x y z my del dot of E must be equal to 0. Now, so what you do is for instance look at this equation. Try to satisfy this del dot of E equal to 0 at for instance try to satisfy it at x is equal to 0. Now, if your x is equal to 0 let me first dispose off the last two terms. If x is equal to 0 these two terms go away. Remember, I have said this relationship is valid at all values of x y z in particular for x equal to 0 and whatever y z you prefer. So, when I put x equal to 0 these two terms go away, but if I put x equal to 0 I have left with B x cos k x x sin k y y sin k z z, d x is a constant which can come here.

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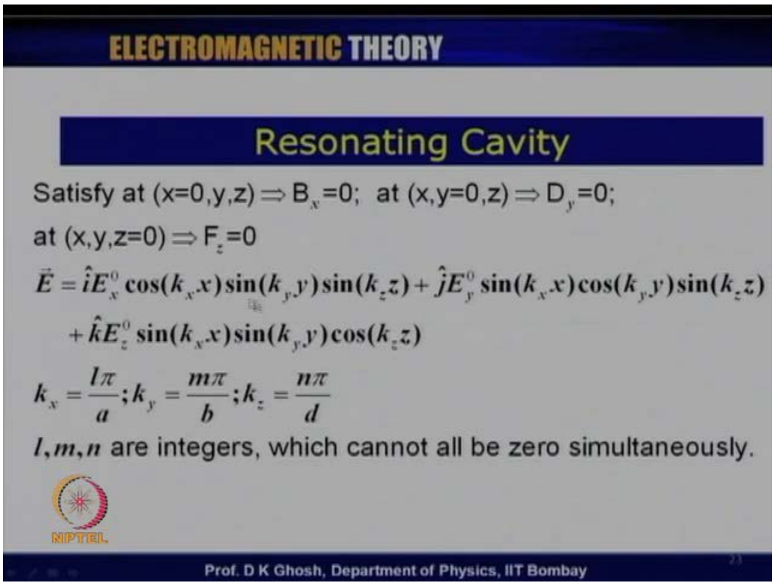


So, this is what I am left with. Now, identically what you do is this for example, so that get me a D y equal to 0 and you can again do this for y is equal to 0 x and z arbitrary, z is equal to 0 x and y arbitrary. If you do that one by one, one of these terms constants will go away and you will be left with terms of this type x component cos k x x, the two others are sin y components cos k y y, the two other things are sin z components cos k z z, two other things are sin and k x is l pi by a k y is m pi by b k z is n pi by d. Notice that I cannot have l m and n becoming simultaneously 0 that is not permitted because if I did then this E will trivially go out it could become 0.

Because at x equal to b for example, y equal to b this will make sin k y y equal to 0. So, these integers cannot be simultaneously 0. So, I write down del dot of E is equal to 0 since all of them are of sin because this is my E. So, when I do del dot of E I get d by d x will give me a sin.

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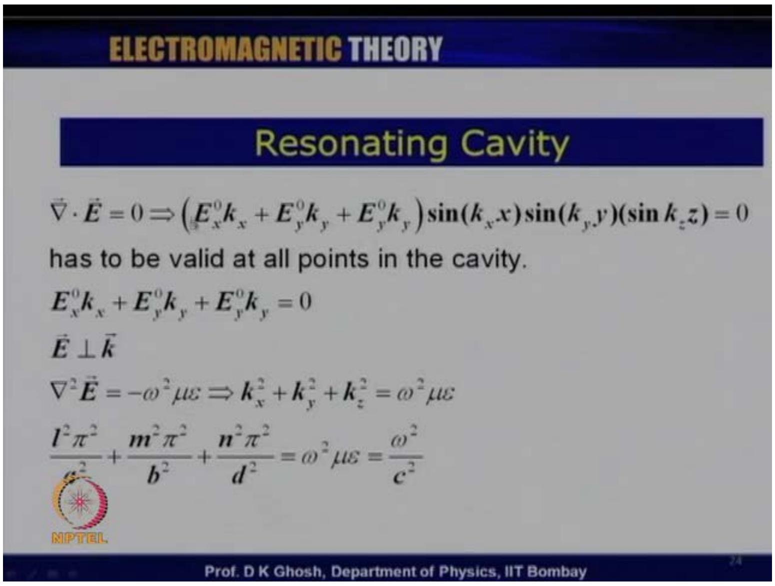


So, this is the way it will be that all the sin are common. I get E x 0 k x, E y 0 k y, E y 0 k k z this is wrongly written. Now, this equation has to be valid for all values of x y and z inside the cavity. As a result this tells me E x k x plus E y k y plus E z k z, this is to be corrected is equal to 0. So, let me write it down since that equation is slightly wrongly written.

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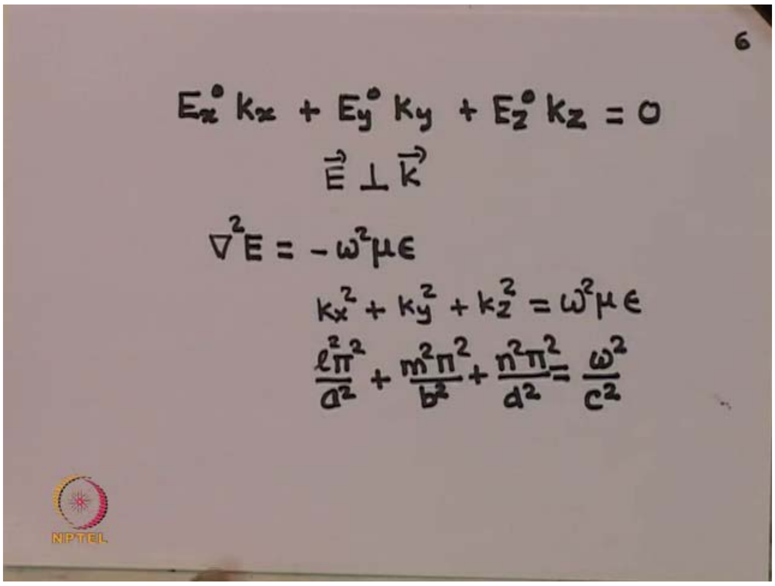


So, I must have E x 0 k x plus E y 0 k y plus E z 0 k z. Since, E x, E y, E z 0s are E x 0 E y 0 E z zeroes or the magnitudes of the components of the electric vector, k x k y and k z are component of the propagation vector. This equation simply tells me that the electric field direction is perpendicular to the direction of the propagation vector. Returning back to the wave equation at del square of E. So, del square is d square by d x square plus b square by d y square plus b square by d z square, this quantity is equal to minus omega square mu epsilon.

Remember, in each of the components take any component. I have to differentiate only the cosine part twice. So, I get k x square plus k y square plus k z square equal to omega square mu epsilon and of course, I know that k x is equal to l pi by a. So, this l square pi square by a square plus m square pi square by b square plus n square pi square by d square is equal to omega square supposing I am in vacuum then omega square by c square.

So, this is what I have got. Now, like we classified the waves in a rectangular waveguide or in a infinite semi infinite waveguide it is possible to talk about the classification in terms of T E and T M mode as well. Generally, conventionally the longer direction will be considered to be the propagation direction, but of course, it really is not particularly important here.

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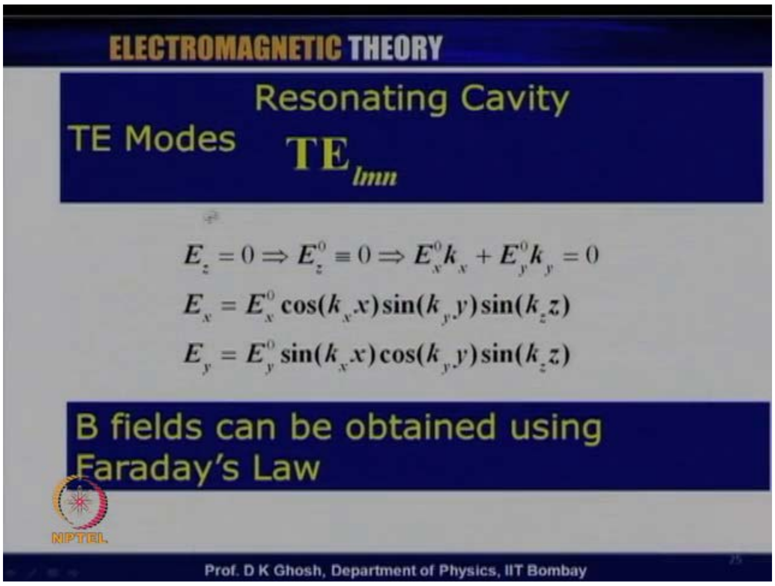


So, if I take E z is equal to 0 which is my T E mode. So, the, I will label the modes by T E l m n. Now, let in this case since it is T E my E z is equal to 0 which implies that E z 0 is equal to 0 and this implies that E x 0 k x plus E y 0 k y is equal to 0. This is because E dot k is equal to 0. So, I am left with now, E x and E y. So, E x I have written down. E x 0 cos k x x E y E 0 i 0 cos k y y, the other two components are sin.

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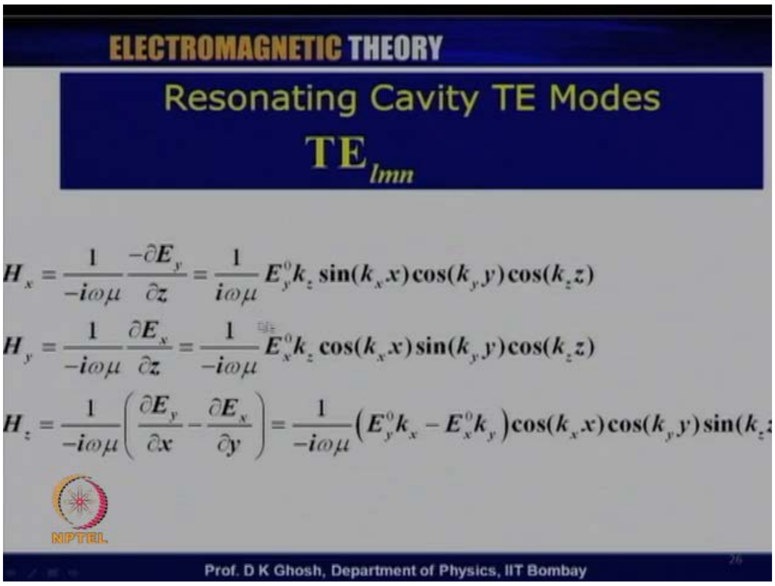


And you can now determine the corresponding H components, the components of the magnetic field H x. This is once your E z is 0 you can write them down and these are the expressions for the H x H y and H z. So, this is fairly standard just a differentiation and you can just look at it and work it out yourself.

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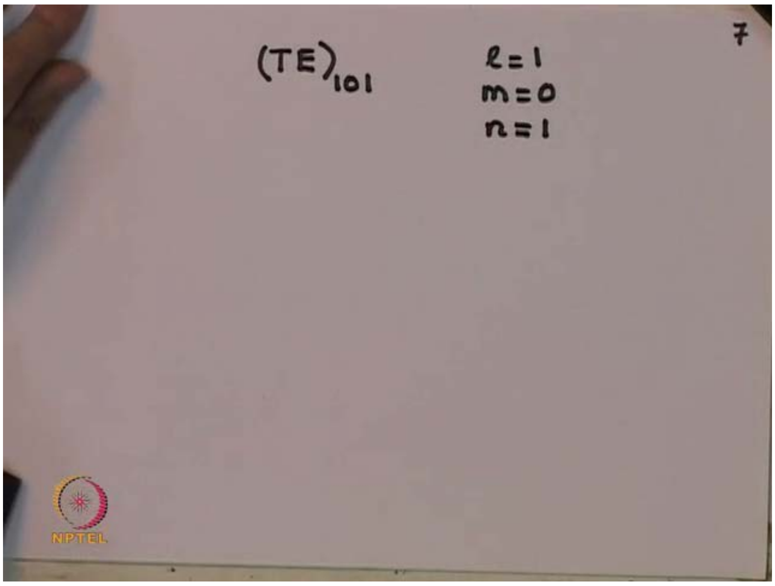

Let me let me now specialize it to a particular mode supposing I am working out to the T E 1 0 1 mode that is l is equal to 1, m is equal to 0 and n is equal to 1. Just going back a little bit you notice that if m is equal to 0 then by E x becomes equal to 0. So, the only component of electric field that survives is E y which will be given by E y 0 sin k x x. This term is 1 and into sin k z z. So, I have written that down that you have got E x is equal to 0. And normally it is conventional to write the T E 1 0 1 mode in terms of the H z 0 that is the amplitude of H z which is the one which determines the T E 1 0 1 mode.

So, for example, H z is this quantity here. So, this quantity here, remember k y is equal to 0. So, this quantity here E y 0 k x by minus i omega mu will be designated as H z 0 so this is what I have done here. Now, if you plug this in then I will write H z as equal to H z 0 cos k x x sin k z z, the y thing is not there. And using my standard equations relating the E y and H x to H z I can write E y and H x like this, H y which again has a proportional to sin k y y is equal to 0 because k y is equal to 0. So, this gives me the modes, the fields for 1 0 1.

Now, the next thing that we need to do is to find out an expression for the loss in case of the resonating waveguides. This is called resonating because you notice that because of this relationship that we had that omega square mu epsilon is given by this. The omega of that are permitted, are very specified frequencies. See, in the rectangular parallelepiped wave guides that we had where z direction was the direction of propagation, we had seen that there was a minimum frequency about which the propagation took took place.

In this case however there are specified frequencies at which the propagation will take place depending upon the mode that we have got, because omega square mu epsilon is l square pi square by a square plus m square pi square by b square plus n square pi square by d square. And once you are fixed l m and n the omega can take only specified values. Because of that we call it resonating modes, because unlike the other two wave guides that we have talked about in this case there is no cut off frequency above which the propagation takes place, there are frequencies at which the propagation takes place.

So, let us look at this. So, as I said in the beginning the purpose of a resonating cavity is to provide a mechanism for storing electromagnetic energy. However, in principle though we have said that my walls are perfectly conducting the there would be losses in the dielectric. So, with every cycle by cycle I mean supposing my z direction is taken as a propagation direction. So, my wave would proceed towards z is equal to d come back to z is equal to 0. So, with every cycle a certain amount of loss will be there. Now, what I will do is this. I define the quality factor which is also known as the q factor of a cavity as the ratio of the energy that is stored in the cavity to the amount of energy that is lost through the walls of the cavity, every cycle. So, that is that is defined as the q.

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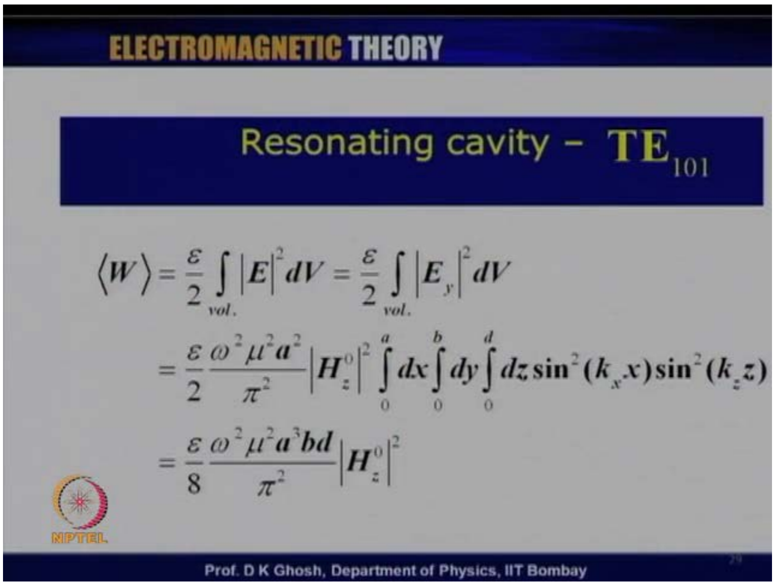


Now, how do I calculate this q? I will just begin the calculation today and we will try to complete it next time.

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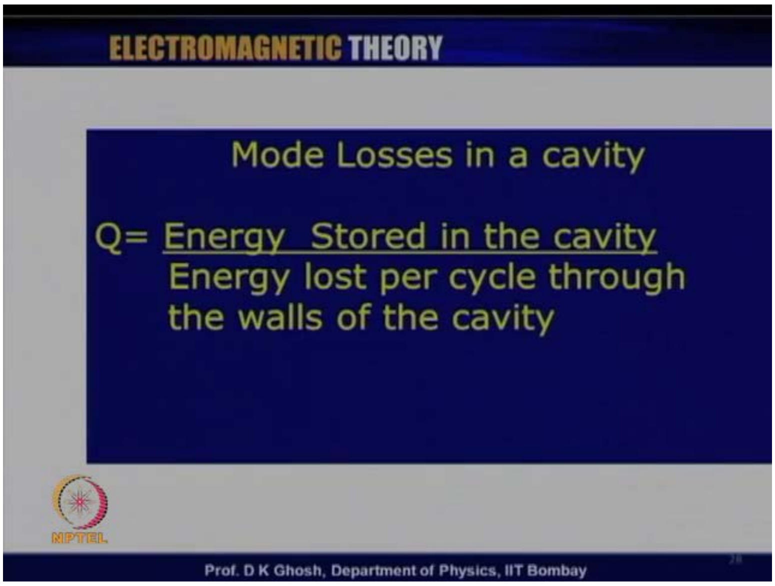


So, first is you notice the numerator. The numerator is the amount of energy that is stored in the cavity. Now, we had seen that the average energy that is stored in a electromagnetic field is given by epsilon by 2 E square, there is an additional 1 over 2 has come in because of my time factor that sin square omega t cos square omega t average. So, it is epsilon by 2 and integrate over the volume of E square d V. Now, I am in T E mode, E z is equal to 0, E x was automatically equal to 0.

Therefore, I have got simply E y square to take care of, if you put in the expression for E y square which is written in terms of H z 0 you will notice that E y is product of sin k x x and sin k z z. So, I have got sin square k x x sin square k z z. So, the d y integration give me a b. And integrations you can easily do by writing sin square in terms of cosine 2 sin sin square theta in terms of cosine 2 theta and integrating this out. Now, if you integrate that out you get the amount of energy that is stored to be given by this.

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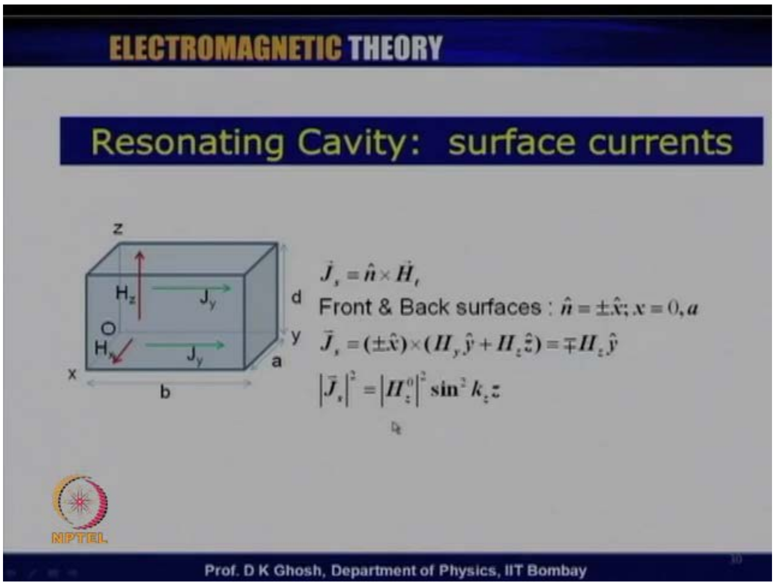


The next question is what does one do to calculate the amount of loss. So, first thing that we realize is the losses must take place through the walls of the cavity. Now, I assume that the walls are made of good conductors. Now, if the walls are made of good conductors then the skin depth that is the depth to which the fields penetrate will be small, the skin depth is low. Now, if this skin depth is small I can assume the field in these is the same as the field on the surfaces of these conductors, conducting boundaries.

So, I also know that if I have the tangential component of the magnetic field that gives me a surface current. It is this surface current which I will need to calculate on each of the six phases. And once I know what is the resistance of each one of these phases, I simply calculate how much is the joule loss on each of these phases. This is a calculation which I will leave for the next lecture.