Electromagnetic Theory
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Module - 4
Time Varying Field
Lecture - 34
Electromagnetic Waves

In the last lecture we have talked about propagation of electromagnetic waves in free space. We will continue with that today, but we will be talking about what happens to a harmonic electromagnetic wave, when it comes to interface between two different dielectrics, which is something which we of course know from our schools that is we know that the electromagnetic waves or what we are more familiar with from in our schools for instance the light wave, undergoes a reflection and a refraction at the boundary or the interface between two medium. We will try to establish the well-known laws of electromagnetic waves, well known laws of reflection and refraction from the electromagnetic theory that we have established so far.

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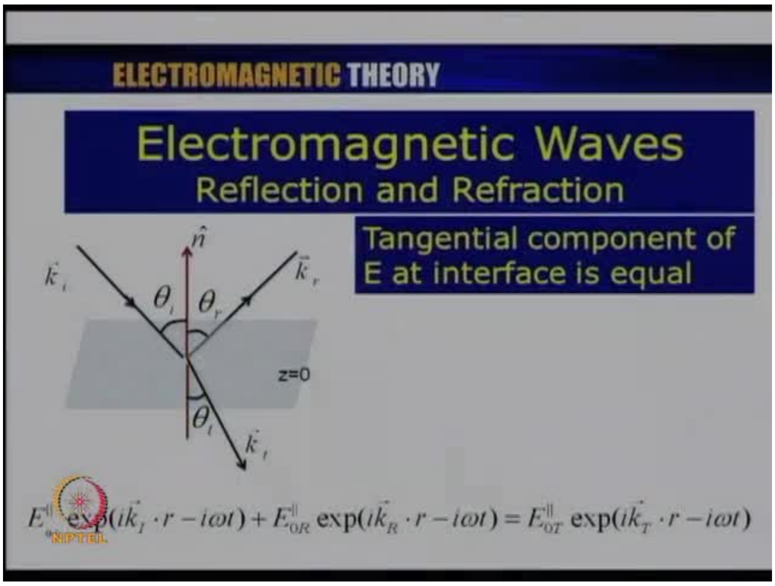

So, let us look at what happens in this picture, what you find is a, this is an interface. So, below this interface is a medium second medium and above is the first medium. So, I will assume that the first medium is air or vacuum and the second medium to be specific, let us say glass. So, I have a an electromagnetic wave incident on this interface, which I am taking as the x y plane namely the z equal to 0 plane, at an angle of incidence of theta i. The angle of incidence is defined the same way as we have been doing always, namely the angle that the incident ray makes with the outward normal to this surface. And of course, we know that there would be a reflected wave and a transmitted wave.

So, I have represented incident as i, the reflected as r and the transmitted with the letter t. So, this is the things, theta i, theta r, the angle of reflection and theta t, the angle of refraction or the transmission. The the principle that is used in obtaining the standard laws of reflection and refraction are the following, that the tangential component of the electric field at the interface between two media must be continues. So, if you look at this picture again, so you notice that the the media or the medium for the incident ray and the reflected ray is the same and the medium for the transmitted ray is different.

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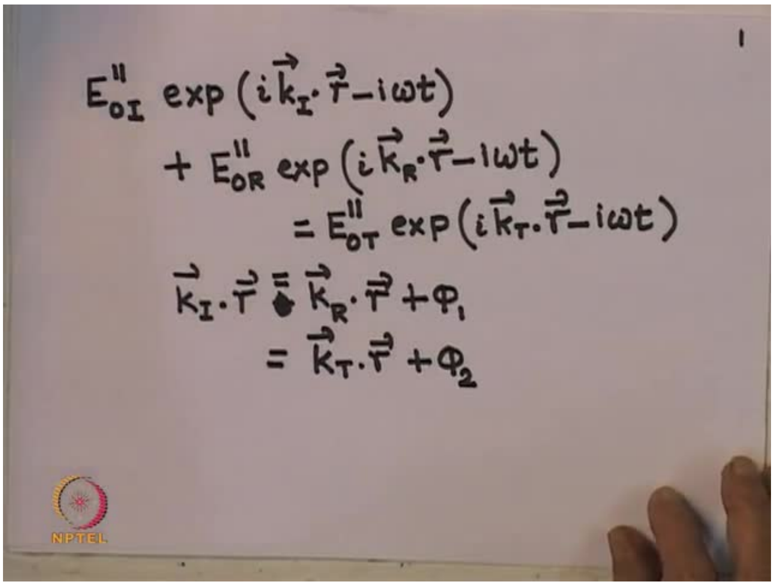

Therefore, what we do is to write down that the tangential component, which I will abbreviate by putting in, so the amplitude, let us say E 0 I parallel because it is a tangential component and I will use the complex notation, but of course one could write down the the harmonic the sine or the cosine wave.

But it is much more convenient to do the algebra in terms of exponential function and later on take the real or the imaginary part as we like. So, this is E to the power i k dot r, so I will write the propagation constant vector propagation vector as k I dot r minus i omega t. This is, I am taking forward moving wave this must be, this plus because in the same medium I have got the reflected ray, so I will write its E 0 R parallel again exponential of E to the power i k R, k r is the propagation vector for the reflected ray dot r minus i omega t.

This must equal the tangential component of that transmitted ray, so which is E 0 T parallel transmitted wave which is E to the power i k T dot r minus i omega t. I have intentionally use the capital letters, so that reflection R, does not confuse with this position vector r and similarly, the transmission coefficient T does not confuse with the time variation t. So, this is the equation which tells me that the tangential component of the these vectors are the same. Now, notice one thing that this type of an equation must be valid at all time and at all points on the interface.

So, interface vector is the vector r and time is arbitrary, so as a result this type of an equation in order that they are valid at all positions are on the interface and at all time t, the exponential factors must be the same. Then we will look at what the other relations should between E 0 I parallel, E 0 R parallel and E 0 T parallel, but unless the exponential factors are the same, I cannot have such an equation satisfied at all time. So, let us write down out, this implies that I must have K I dot r plus K R dot r or rather I must have the all the things are the same.

So, let me say in general it is K R dot r plus let us say phi 1, which is equal to K T dot r plus phi 2. Now of course, I could have an arbitrary phase, constant phase between the three terms because E to the power I phi 1 or E to the power I phi 2 would simply multiply with these complex amplitudes in general. So, so let us look at, what do, what does it imply? So, let us look at the first equation.

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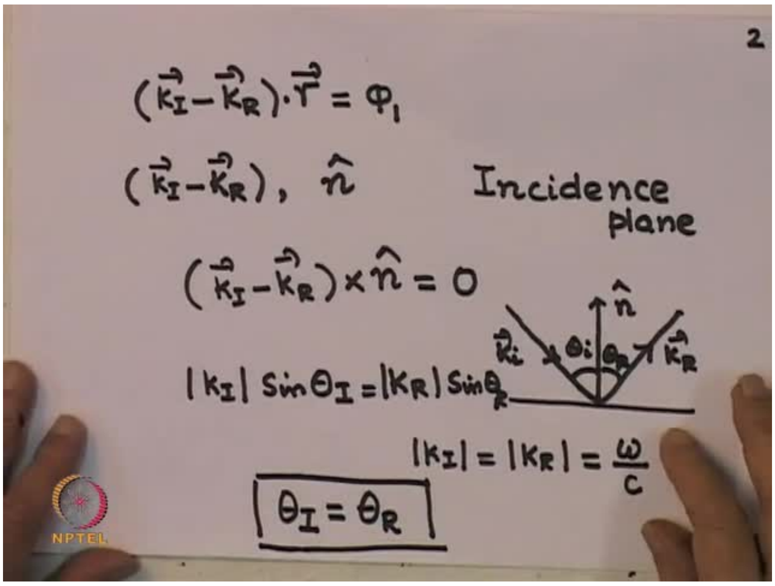

The first equation is telling me that K I, so I will write this down. K I minus K R dotted with r, r as you remember is the position vector on the plane that is equal to phi 1. Now, we are all familiar with vector equation of a surface. We know that if the normal to a plane is given by a vector n, then the n dot r equal to constant defines an equation to a surface. So, in order that this represents the surface the interface between the two medium, so this tells me such a surface is perpendicular to this K I minus K r, just as n dot r equal to constant is the equation pair a vector equation to a surface. Therefore, this tells me this surface is normal to K I minus K R.

Now, so suppose I say, that the K I minus K R and the normal n to the surface. Supposing they lie in x z plane, remember that I have taken x y plane as the interface. Therefore, I have a choice of the x y plane, so let me choose this to be in the x z plane, which is my incident plane. The definition K I minus K R and the normal to the plane of incidence defines my incidence plane. Now, since n and K I minus K R are in the same plane, this tells me that K I minus K R vector cross product with n, must be equal to 0. Alternatively if the angle between the normal as we had shown in the picture, let me repeat that picture again.

So, this is my normal direction this is the direction of K I, this is the direction of K reflected, this is the angle theta i and this is theta R. So, this tells me that K I minus K R cross n is equal to 0, it implies that K I sine theta because sine theta i. So, magnitude of

K I times sine theta i, must be equal to magnitude of K R times sine theta R. Now, since the incident ray and the reflected ray are in the same medium, the magnitude of the propagation vector, their directions are different, but the magnitude of the propagation vector must be the same. So, magnitude of the propagation vector K I is equal K R is equal to omega divided by the velocity in that medium.

Since, I have taken it to be the vacuum, so let the velocity be the velocity of light C. Now, since these two magnitudes are the same, it tells the sine theta i is equal to sine theta R. Alternatively the angle of incidence is equal to angle of reflection, which is of course, very well known lot to us. Now, we will understand it on the basis of the electromagnetic theory, let us try to prove Snell's law. This is almost similarly done because we have said that K I minus K T dot r just as we had proved that K I minus K R dot r.

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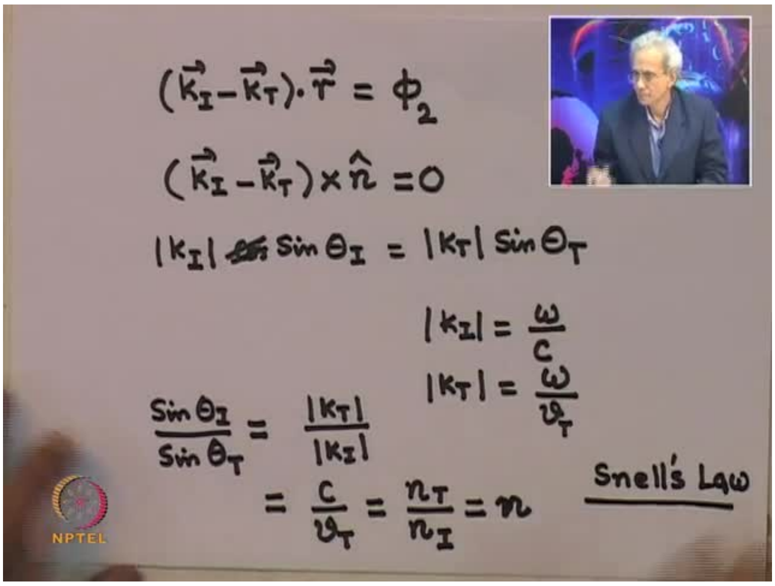

So, we have said K I minus K T dotted with r, that is equal to phi 2, which is another constant and once again this is an equation to A. Surface which is the perpendicular to that surface being K I minus K T vector. So, as a result K I minus K T, just the way we have done it earlier cross n is equal to 0. This tells me that just as we had written it earlier magnitude of K I times cosine of sorry sine of theta i must be equal to magnitude of K T sine times sine theta T. Now, look unlike the case of reflection, the two propagation vectors do not have the same magnitude anymore, and that is because this is in medium one and that is in medium two.

So, I know that the magnitude of the propagation vector is nothing but omega divided by the velocity. So, let us write it K I let us say, is omega divided by let us say, velocity of light it is in medium one and K T magnitude will be omega. Remember that, when a wave is incident at an interface, the frequency does not change the, and so this is equal this divided by the velocity in that medium velocity in that medium T. Therefore, sine theta i by sine theta T, sine of the angle of incidence divided by angle of transmissions is magnitude of K T divided by magnitude of K I, which is equal to, this sine theta i by sine theta T, is K T by K I. K T is omega by v T, therefore this will be C by v T.

If you remember this is nothing but the definition of the refractive index of the medium two, that is the transmitted medium, with respect to the refractive index of the medium, incident medium or this is simply what we call as the refractive index. In common language when it is understood that the medium with respect to which, we are talking about this refractive index happens to be air or the vacuum. Therefore, this is my Snell’s law, sine i by sine r. r being the angle of reflection refraction. So, this is Snell’s law.

So, using electromagnetic theory, and the fact that the tangential component of the electromagnetic field on the interface between the two media, must be the same. We have been able to prove the two simple laws which we have learnt right from our school days. Now, having done that we still have not talked about, what happen to the amplitudes? The E 0 I, what is the relationship between E 0 I, E 0 R and the E 0 T? The exponentials we have taken into account, so our next job will be to point out what these relations are and before we do that, we will be in a, we will require the, what we have learnt earlier.

And the setup equations which will relate the amplitude of the reflected wave with respect to the incident wave or the amplitude of the refracted wave with respect to the incident wave. These set of equations are known as Fresnel’s equations.

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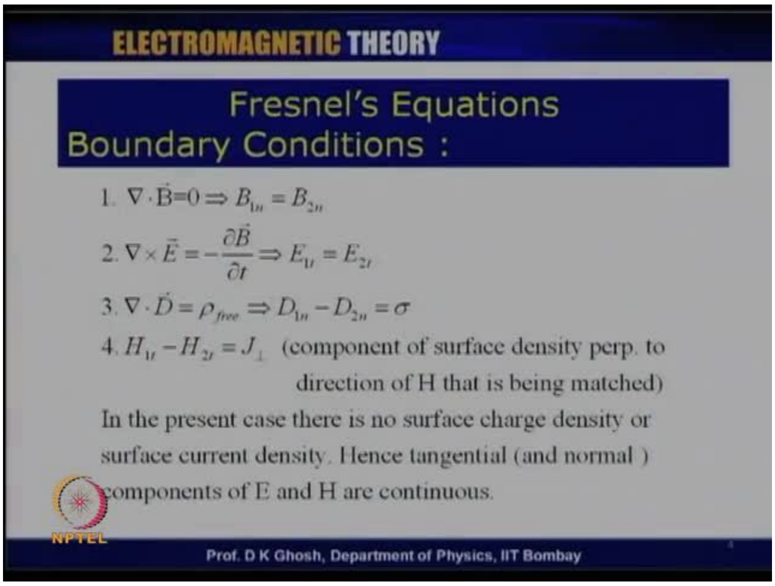

And let us look at how does one do it? In order to do that, we need the other boundary conditions that we have. The firstly, you remember that from my Gaussian law of magnetism, del dot of B is equal to 0, del dot B equal to 0 we have seen by taking a pill box, a Gaussian pill box on the surface I should be able to prove that the normal component of b field is the same. Now, now what we will doing actually, we will assume that both the media they have the same magnetic permeability. We are not discussing the they are different dielectric, but we will assume that the magnetic, they are not magnetic material. So, the magnetic material permeability of both the media will be taken to be equal to mu 0. So, as a result if B 1 n is equal to B 2 n, this is also implies that H 1 n is equal to H 2 n.

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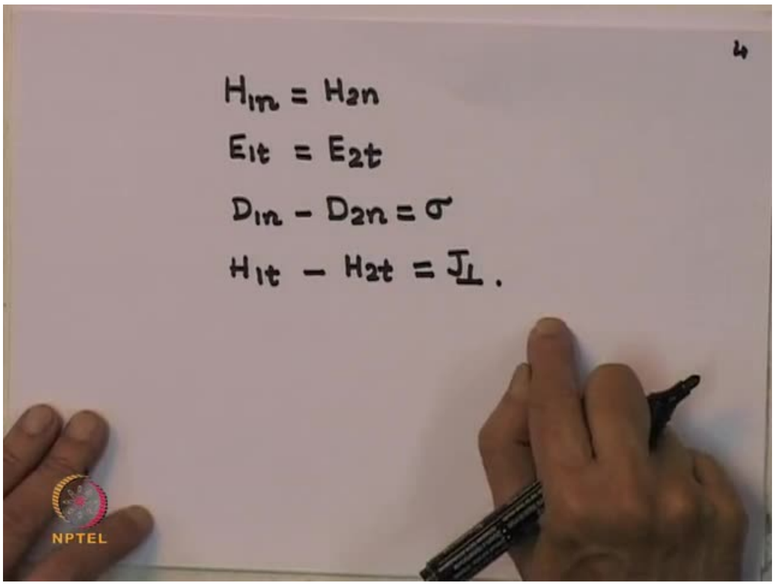

So, normal component of the magnetic field H 1 n must be equal to H 2 n. Now, if you refer back to this again, I have del cross of E equal to minus D B by D t and this is the cross product. So, you remember that this standard mechanism has been whenever I had a del dot divergence, I have taken a Gaussian pill box half in one medium, half in the other medium. Now, whenever we had a cross product, what we did is to take an Amperian loop and then go by a surface integral. A line integral because del cross of E dotted with D s is related to a line integral and this of course, B dot D s will give you the flux and this as we have seen several times has given us E 1 t equal to E 2 t.

Now, del dot of D is equal to rho free, but notice that unlike del dot of B which is equal to 0, my del dot of D is not equal to 0, but is equal to the surface charge density that is there. As a result there is a discontinuity in the normal component of the D field. So, D 1 n minus D 2 n is equal to sigma. So, let me write it down D 1 n minus D 2 n is equal to sigma. In a very similar way is if there is a discontinuity, if the on the surface there are some surface currents then if I look at H 1 t minus H 2 t. Now, this gives me J perpendicular, where J perpendicular is the component of the surface density, which is perpendicular to the direction of H, which is being matched. Remember when I said tangential direction all that I can, I know is it is on a surface.

So, if your matching a particular direction, J perpendicular is perpendicular to those. Now, however what we are going to do, we are going to be talking about perfect dielectrics. So, I do not have charge densities on the surface, I do not have a current density on the surface, as a result both the tangential component and the normal component of the both E field and H field will be taken to be continues. So, these will be the issues that we will be using in our next set of derivations. So, let us look at what it implies?

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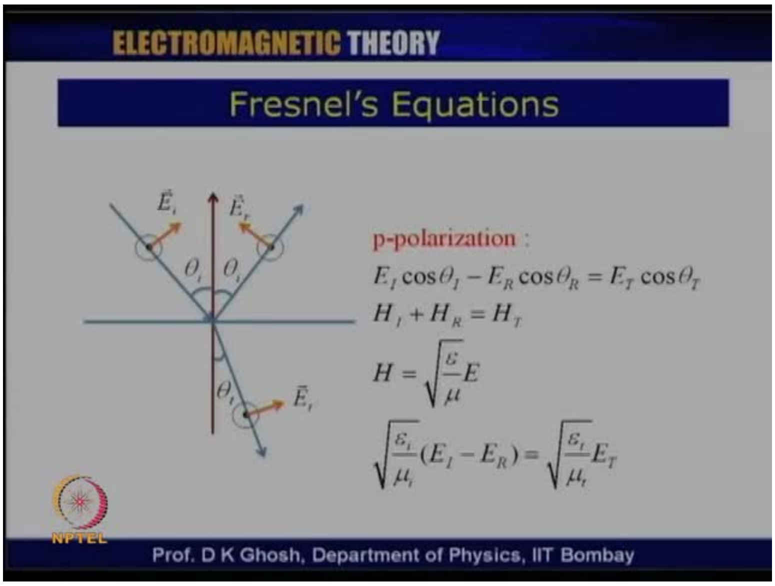

So, if you look at this picture here, so what I have done is this. Now, we will be discussing two different cases. The different cases that we will be doing will be called p polarization, p standing for parallel. It means the direction of the electric field will be taken to be parallel to the plane of incidence. So, this is the picture of the plane of incidence because so the interface actually is perpendicular to the plane of the screen, and so this ray, this ray and that ray, this incident ray, the reflected ray and the transmitted ray, this defines me and of course the normal there are all in one plane. So, this is the picture of that plane

So, what I have done is to draw a direction which is perpendicular to the incident direction and obtusely the magnetic field direction. Then will be perpendicular to both the electric field direction and the propagation direction and this if you look at the fact that electric field magnetic field, and the direction of propagation turns out to be a right handed triad system. Then corresponding to this, the direction of the magnetic field is coming out of the plane of the paper and so what we have done is, actually to define take the direction of the H field coming out of the plane of the paper. This is a convention that we have assumed and then decided what the direction of the corresponding electric fields will be.

This is fairly straight forwards to work out, once you know that this is the direction of the propagation and outward to the plane of the paper is the direction of the magnetic field, and then you can work it out. You can check that if you take this theta to go to 0 then of course, what you find is that the electric field directions will become oppose to each other at normal incidence. And so this is what we have done, so let us look at this p polarization, the other polarization is s polarization, which comes from a German word centrate. The s polarization is squared the electric field is taken perpendicular to the plane of incidence. But let us first talk about the p polarization. p standing for parallel, so let us look at what do I have, let me draw this picture again here.

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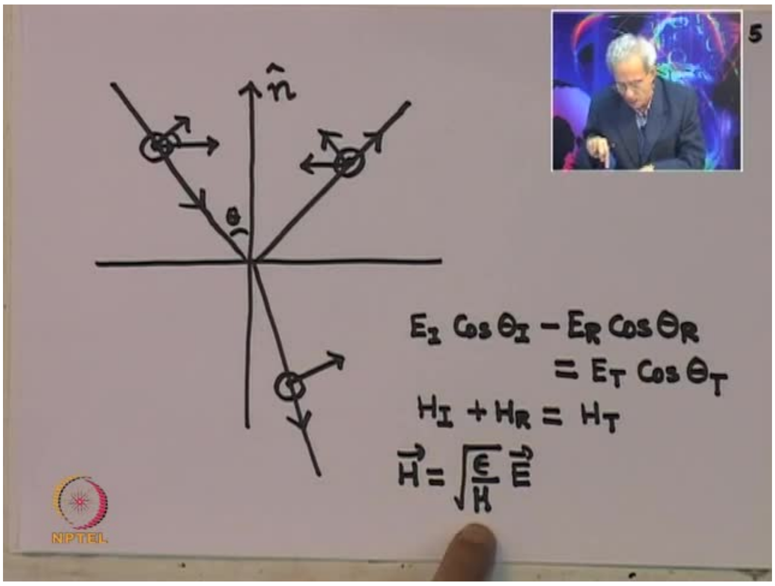

This is my normal, this is the direction of the incident ray, this is the direction of reflected ray and this is the direction of the transmitted ray, and what you have done is to take the magnetic field this way out of the plane, so that the electric field is like this. Now, in this picture I am going to now work out that, what is the equation corresponding to the tangential component of the electric field being the same? So, you notice that this angle is theta, the tangential component is like this. So, if this angle is theta, this angle happens to be the theta and like this that is the same way here.

So, these are in opposite direction, so as a result what I get is E I cos theta i minus E R, R for reflection cos theta R is equal to this is the direction, in which I have subtracted, so is equal to E T cos theta T. Now and similarly, I have got a H I because H is being taken perpendicular to the plane of the incidence. So, I will have H I plus H R, remember the normal components are also continues is equal to H T. Now, I know that I am dealing with propagation in dielectric, so as a result my E and B or E and H have a relationship and that relationship is given by, my H is equal to square root of epsilon by mu 0, actually mu if you like, but I will be taking all mu to be equal to mu 0. So, mu let me write it temporarily with the electric field, now this is this is true in all medium. So, you have to take the corresponding epsilon, corresponding to the medium in which you are in and the correspondingly.

Similarly, mu of course, I have said that I will take the mu is to be the same. So, this is this is fairly straight forward because I know the magnetic field d is related to the electric field by magnetic field d being the electric field divided by the velocity of electromagnetic waves. And the velocity of the electromagnetic waves is 1 over square root of mu times epsilon and B and H are related by B is equal to mu times H. So, if you do that then you find h is equal to square root of E by mu times epsilon. So, if you plug it into this equation, what you find is, so let me write the pair of equations again.

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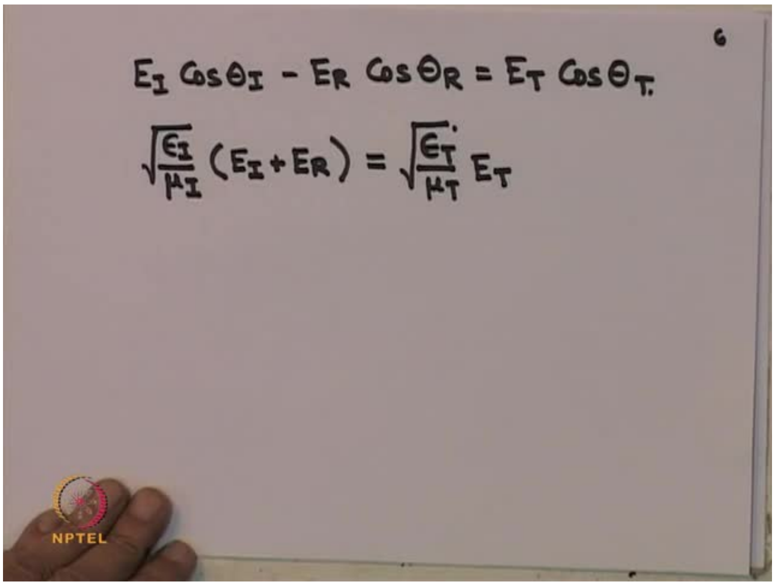

I have bought E I cos theta i minus E R cos theta R is equal to E T cos theta T and I have got in terms therefore, let me write it down. Incident medium, so I have got epsilon I by mu I and reflected medium is the same, so I will write it as E I minus E R and that is equal to square root of epsilon T by mu T times E T. So, this is what we have got and of course, this tells me that I should be able to solve these equations without any problem. So, this is you just divide one by the other this equation should have been a plus because I had a H I plus this. So, just divide on of the equations by other and you find that this, these are whole sort of algebra on the screen.

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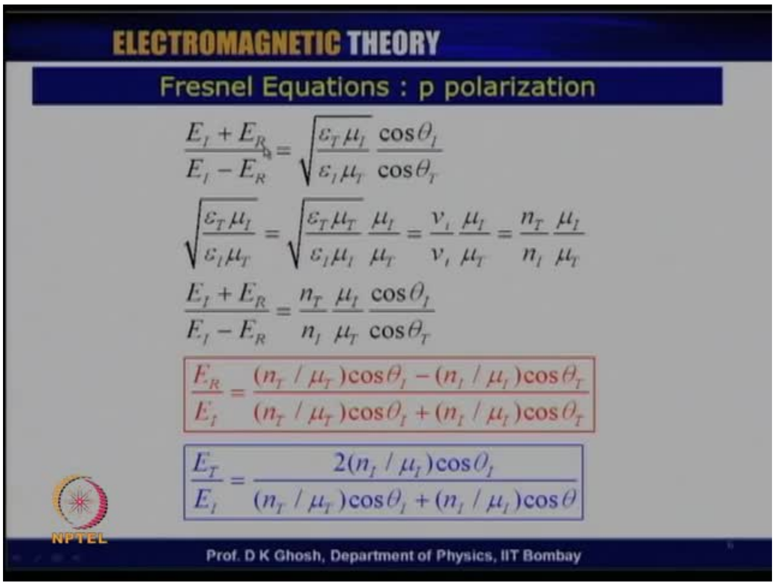

So, I have got E I plus E R divided by E I minus E R, just divide one by the other and sort of work out the algebra you get a rather clumsy set of equations, but that is what it is. E R by E I is given by this and E T by E I is given by this, just nothing, no grade information from here, but just straight forward solutions of these two equations.

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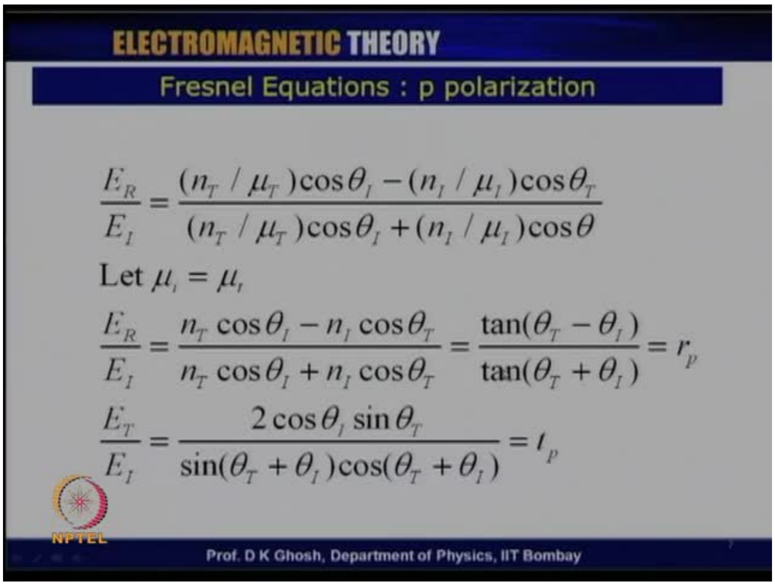

So, let us look at these equations and look at some special cases. This is simply rewriting E Rr by E I, so and I have, I am also going to be assume that the mu's are the same now. Once we take the mu's are the same, you get E R B by E I in terms of the refractive index of the second medium and refractive index of the first medium. And if you remember my Snell's law, that is sine theta i by sine theta R is n T by n I. So, use all that you can prove that E R by E I, can be written as very straight forward algebra. I am not going to do the trigonometry, it happens to be tan theta T minus theta i by tan theta plus theta i.

And that is the amplitude reflection coefficient, I I sort of alert to you, the old reflection coefficient is sometimes used to indicate the square of this quantity, but at this moment I am talking about the amplitude ratio E R by E I. And similarly, E T by E I is given by this expression and that is denoted by t p. So, these are a pair of equations which are going to be useful to us. So, let us just look at what it tells me. First thing you notice is this, that incidentally this is very special to the p polarization, that is parallel thing. So, you notice that E R by E I is given by tan theta T minus tan of theta T minus theta i by tan of theta T plus theta i, so if it happens that theta T plus theta i becomes equal to pi by 2.

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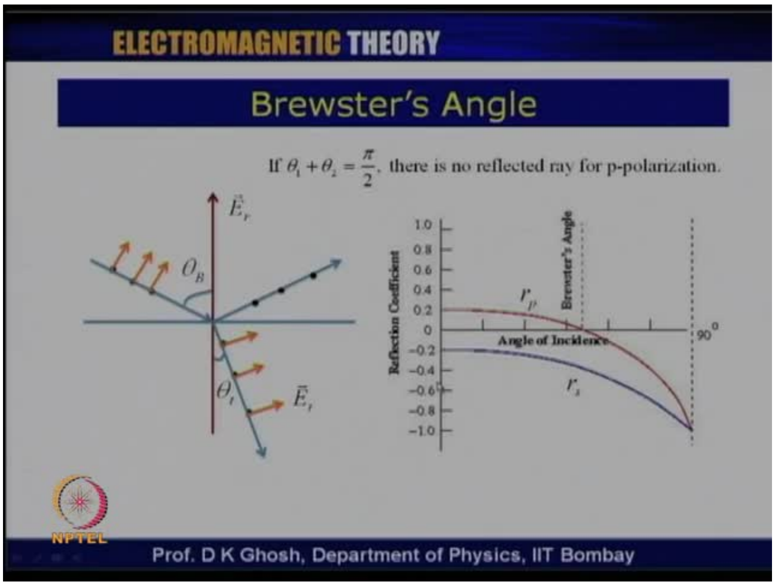

Now, you can look at the picture if this angle plus this angle happens to be pi by 2, elementary geometry will tell you because the angle of reflection is the same as the angle of incidence. The angle between the reflected ray and the transmitted ray is 90 degrees. Now, if that happens, going back to this tan of theta T plus theta i is infinite, so as a result the amplitude of the reflected ray is 0. Now, what I have done in this picture is the following that I told you that I am talking only about p polarization, but this is the picture which is of mix polarization.

That is supposing you take un-polarized light, I can write down un-polarized light has an equal mixture of light, which is polarized in the parallel direction that is parallel to the incident plane and light which is polarized perpendicular to the incident plane. And the way we have shown it is these dots indicate light polarized perpendicular to the incident plane, and these lines arrows in the incident plane. So, if you start with an un-polarized light you have a mixture of both these black dots and the orange arrows, but however at this particular angle, we have seen that there is no component of p polarized wave in the reflected ray, the transmitted ray still as both.

Now, this then tells you that if you take un-polarized light, which is incident at this angle, the angle for which theta 1 that is theta i plus theta transmitted is 90 degrees, this is known as the Brewster's angle. And if you know the refractive index of the medium then of course, you can easily calculate what the Brewster's angle. Therefore, that angle the reflected light does not have any p polarized component. So, this I started with the un-polarized light, the reflected light is plane polarized. Now, this picture here gives you a the reflection coefficient the way the I have defined it namely the ratio of the amplitudes and this is the p polarization.

This is the picture of r s polarization i have not yet taken up, but this will turn a and you can see that this red line crosses the x axis which is the angle at an angle which is the Brewster's angle. So, at that angle the reflected ray is plane polarized, this is of course, something which you have known from our earlier days.

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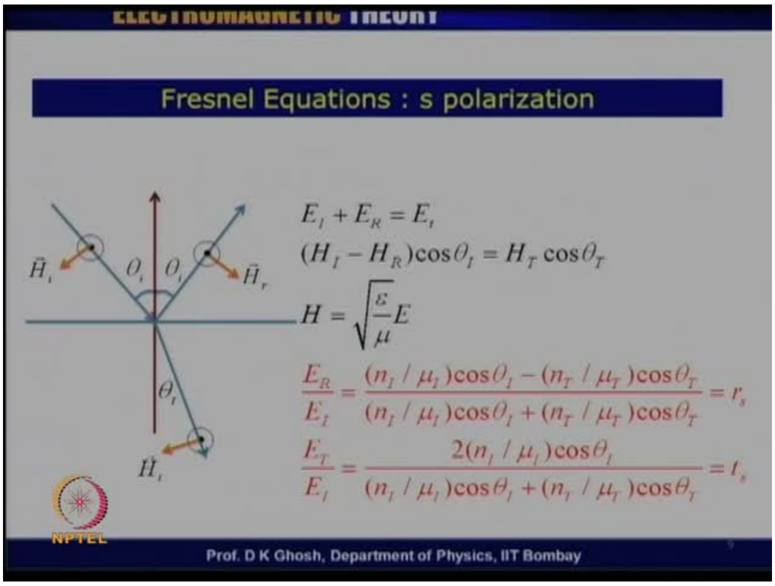

Now, the situation with respect to s polarization is almost identical. So, what I have, I have here is this, that in this case since, the electric field is perpendicular to the plane of incidence, I have taken the electric field to be coming out of the plane of the paper and I have using the fact that E, Hh and the direction of propagation, for my triode I have given the corresponding direction of the H. So, I have got E I plus E R is equal to E T and the same way as I did before H I minus H R cos theta i this is given by this and if you do that you get an expression for r s you get an expression for t s.

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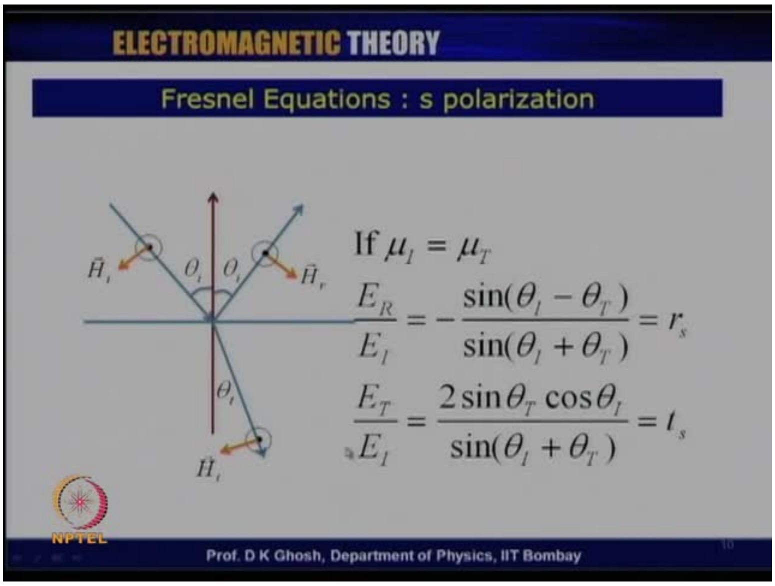

And the, once again use, if you use mu I equal to mu T, you find E R by E I is given by minus sine theta i minus theta t by sine theta i plus theta t, which is my r s and this type of an expression is called my t s.

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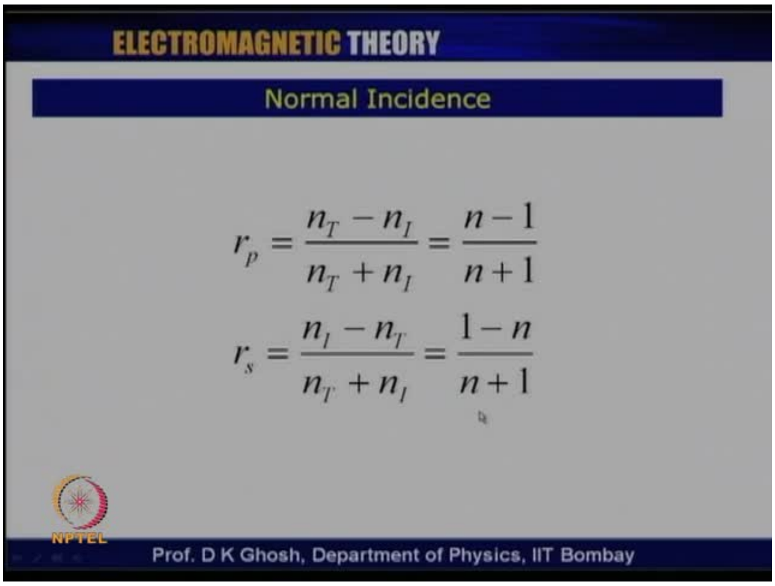

Now, if you take normal incidence, you can take normal incidence means, the angle of incidence is 0, so that if you can work it out that r p is given by n minus 1 by n plus 1 and r s is given by 1 minus n by n plus 1. Now, incidentally you notice that there seems to be a a contradiction here because I have taken normal incidence, both these results must be the same, but here I got n minus 1, here I have got 1 minus n. The reason is connected with the fact that we had adopted two different conventions of of taking the direction of electric field and the magnetic field to be in the same direction, and so this sort of is not very important, but one can sort of take different convention for p polarization and s polarization work it out, but otherwise they they are the same.

So, let us return back to the p polarization again.

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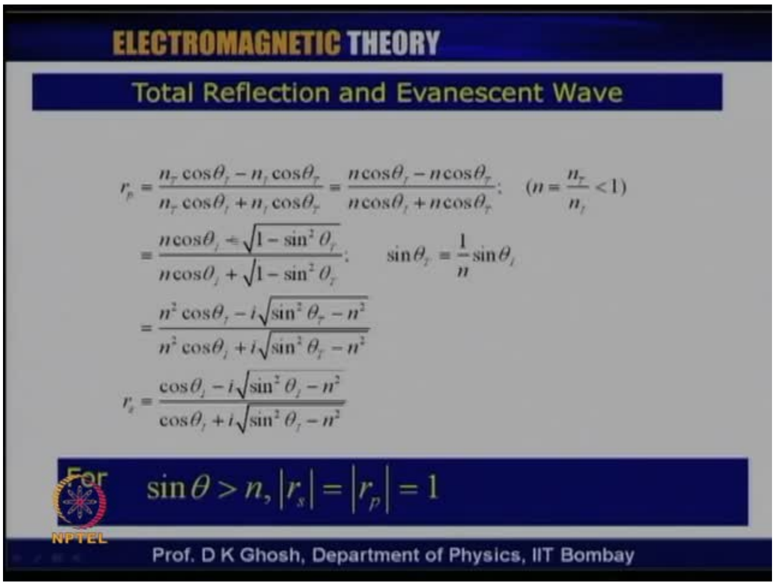

Now, I had shown that the p polarization is written by such an expression. Let me let me write it down.

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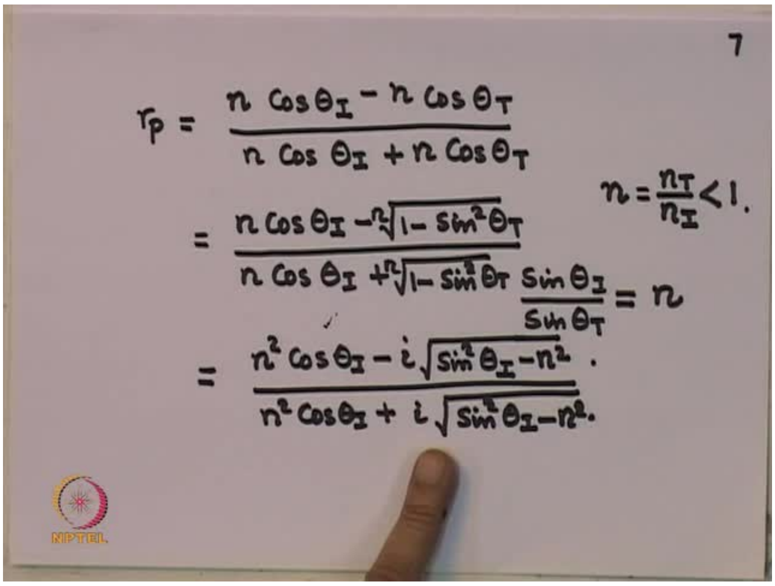

r p is equal to n. n is the refractive index of medium two with respect to medium 1. n cos theta I minus n cos theta T divided by n cos theta I plus n cos theta T. Now, I am going to look for a case for which, the second medium has a lower refractive index then the first medium. So, n T by n I is less than 1. Now, when this happens, I can rewrite this expression, so this is, I will write this as n cos theta I minus I use Snell’s law because Snell’s law gives me sine I by sine theta T is equal to n. So, that sine theta T is sine theta I divided by n, so I can rewrite this as square root of 1 minus sine square theta T, the denominator is identical expression with a plus n cos theta I plus root of 1 minus sine square theta T, now what I am doing is this.

That I am, so this is this is nothing just I have rewritten the cos theta T as root of 1 minus sine square theta T, there should have been an n there, let us put them back. Now, if I use this expression sine theta I by sine theta T equal to n, then I will be able to write this expression in this fashion here. So, this is if you plug it the same and we are going to talk about situation where remember n is less than 1, so I can think of an angle theta T for which sine theta T becomes greater than n, now when that happens this type of an expression gives me an imaginary number.

So, I rewrite this as equal to, if you do a little bit of an algebra it becomes n square cos theta I minus I times root of sine square theta I sine square theta minus n square and divided by the same thing with the plus. And similarly, for r s now what is that if sine theta I exceeds n? If sine theta I exceeds n then notice that, this is a quantity of magnitude 1 because this is a minus I B by A plus I B, then you can show that r s and r p is magnitude will be equal to 1. So, this is true of course, if sine square theta is greater than n square, so that this is a real quantity and so that the numerator and the denominator here are complex conjugate of each other.

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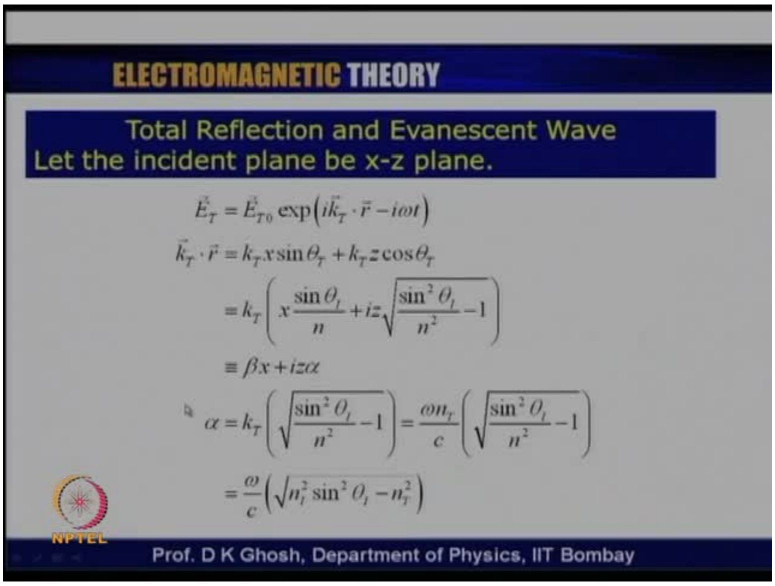

So, r p is magnitude and r s is magnitude works out to be 1, but let us look at what is happening to that wave. So, let us look at what is happening to the transmitted wave. Transmitted wave is E T 0 exponential i K T dot r minus i omega t. Now, as I have taken the incident plane in the x z plane, as I have said earlier my K T dot r is K T times x times sine theta T plus K T times z times cos theta T. Now, I can rewrite that, this is K T now, I have got a x, this sine theta T i simply write as sine theta i by n Snell's law, and exactly the same way cos theta T which will be written this way.

So, this is my K T is equal to x sine theta i by n and I have written this i times z because i am assuming that sine square theta i is greater than n square. So, you notice that K T dot r has two components, one is a real part which is beta times x and what is beta? A beta is simply K T sine theta i by n. And i times z times alpha and this is quantity is my alpha, alpha is k t times root of sine square theta i by n square minus 1 and I write K T as omega n T by C. Remember that omega by the velocity in that medium times the same quantity and I have simply taken this n out so that I write this as omega by C n I square minus this.

So, I have got a part of this, beta is real. So, when I plug this in, you find that I get a propagation vector which is beta which is given by K T sine I by n which you can work out is nothing but K T sine T, which is equal to K I sine i.

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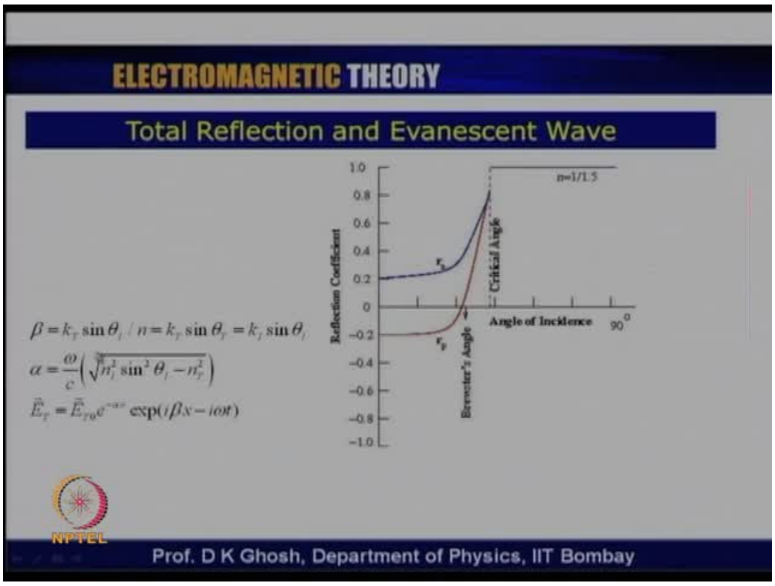

And there is a term which when plugged into the exponential will give you E to the power minus alpha z. So, this is alpha and the transmitted wave becomes E T 0 E to the power minus alpha z times a propagating, let me let me write it clearly, so that one can sort of see what is happening there.

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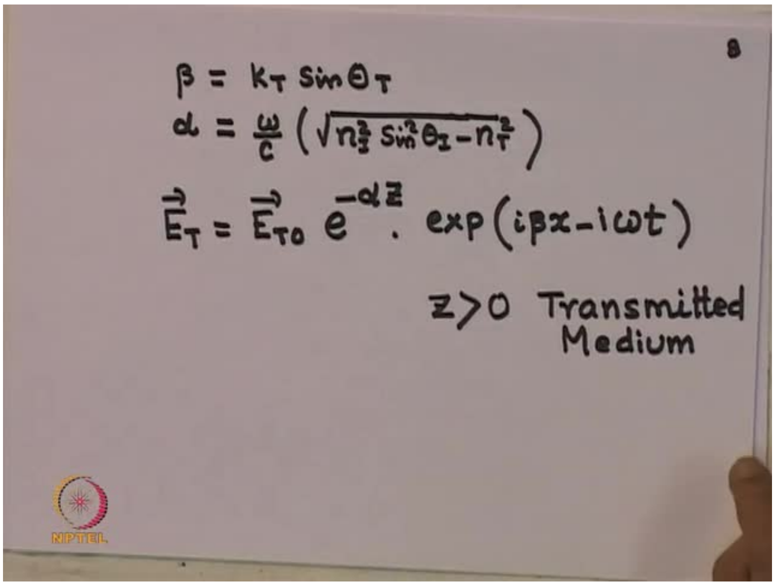

So, we have a propagation vector beta, which is given by K T sine theta T and we have a attenuation factor if you like or an absorption factor alpha and which is given by omega over C root of n I square sine square theta I minus n T square. So, that the E T vector becomes equal to E T 0 times E to the power minus alpha Z times exponential i beta x minus i omega t. Notice what is the picture? The Z direction, Z positive direction is in the second medium, remember that the, my picture now is my second medium has a refractive index, lower than the first medium.

So, Z greater than 0 is the transmitted medium, so as you go into the transmitted medium the amplitude of the wave decreases exponentially with distance Z. There is a propagating term, but if you notice this is propagating along the plane, this is propagating along the plane and we have seen that the amplitudes of the reflection coefficient and the transmission coefficient, they have, they they r p and r s, the reflection coefficient they have become one. So, this is what is known as the total internal reflection, the wave is not 0 in the second medium, but it is a exponentially decaying wave, this is known as an evanescent wave. The propagation takes place on the surface on the Interface.

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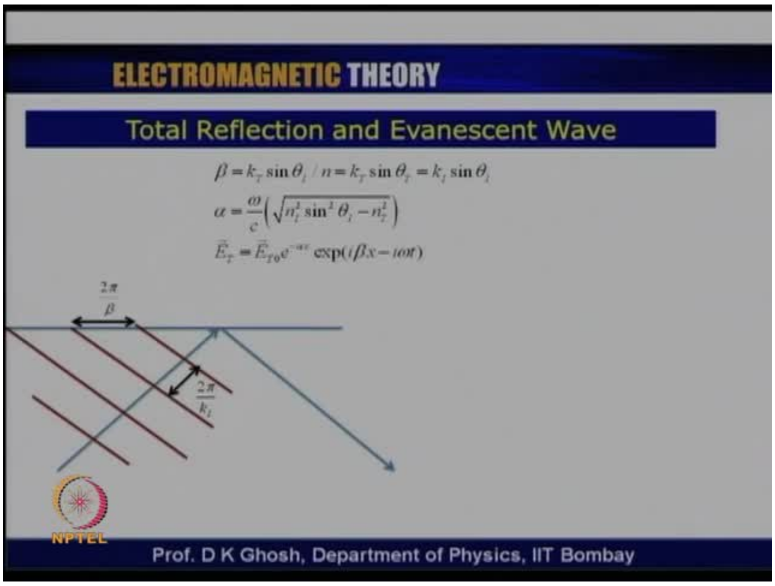

We can do a little better, notice this propagation vector has a very interesting interpretation. The propagation vector we have said is K T sine I by n which is K T sine theta T. Now, here I am showing a for example, this is an incident direction, this is the way which is incident at an interface. I remember that we have talking about a propagation vector, which is 2 pi by the propagation vector is the wavelength. So, suppose I draw perpendicular to the propagation vector direction and I find out the distance between two crests or two troughs that would be my wavelength, and so as a result 2 pi by K I, this is the wavelength here.

But the this is provided the you measure the wavelength in a direction perpendicular to the direction of propagation, incident wave vector, but notice that as it comes to the interface. That is when I go to supposing these were water waves, and this is the surface then as it comes to the shore, the one would be inclined to measure the distance like this. And this is my 2 pi by beta, you can just do an elementary there. Since, these are surface waves propagating along this, the corresponding wavelength is given by 2 pi by beta and not 2 pi by this distance.

So, and and this is fast attenuating wave. Naturally the question arises, is there a total internal reflection 1 for which is there a transfer of energy to the second medium? Now, what will prove is, on an average there is no transfer of medium to the second medium and let us look at it this way.

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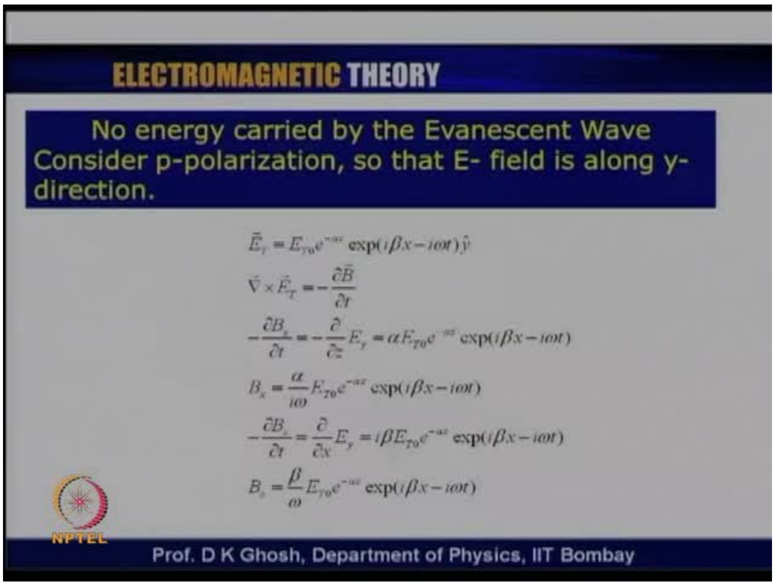

So, let us come back to our expression for the electric field, we have seen that for this case E T is given by E T 0 the amplitude.

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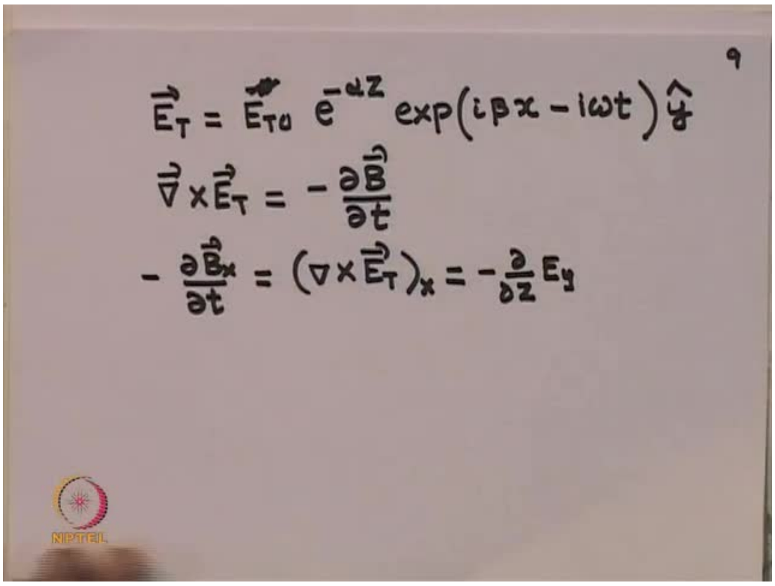

There is a amplitude d k factor E to the power minus alpha z, then a propagation along the surface which is exponential i beta x minus i omega t, this is my E T. Now, what I am going to do is to derive the corresponding magnetic field vectors or H factors from this. And I do not have to, I could have actually gone back to the solution of H and repeated this, but I can do, use Maxwell's equation. So, del cross E that is is my minus d B by d t which is if you recall this is minus mu 0 d H by d t. And remember that when you have an expression like this d B by d t is nothing but multiplying the things by minus I omega.

So, as a result come here, so let me take what is d B x by d t. I am, I am working out the x component of this. So, let me write it down, so minus d B x by d t, I will just work out one, you can work out the remaining. This is equal to del cross E T's x component, so del cross E T x component, which is d by d y of E T z, but I am taking the electric field. Let me let me take this as a number and let me take the electric field because it is p polarized. So, let me take the electric field along the y direction, now if I do, I am doing del cross E T is x component.

So, this is minus d by d z of E y, this is the only thing that survives. So, you can take the differentiation of this quantity with z and from their you can find out what is B x this is a fairly straight forward because d by d t means minus i omega and d by d z is simply minus the alpha. So, as a result i get B x and I get B z, both the components I can work out, B x and Z z.

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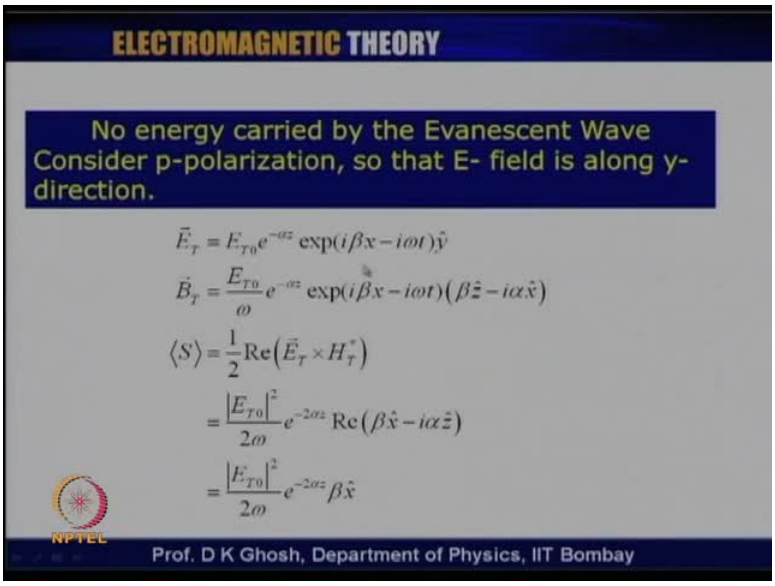

I already had E T, so E T is given by this expression E T 0 E to the power minus alpha z along the y vector and B t is given by this B t has A z component and it has an x component. So, if I calculate now the pointing vector which for complex E and H is half of real part of E T cross H, you can calculate the cross product of these 2 y cross z will give you an x component, y cross x will give you z component, but I need the real part therefore, this is not there, I am left with only this part. But notice my interface is perpendicular to the interface is the z direction.

And this is a vector the s average is vector along the x direction, so as a result when I take the dot product of thus s vector with my z vector, the normal direction there is no average energy flow into the second medium. So, there is a wave which is the evanescent wave, which amplitude of, which exponentially decays the propagates along the interface, but it does not carry any energy to the second medium.