Electromagnetic Theory

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Module - 2
Electrostatics
Lecture - 20
Dielectrics

So far we have been discussing the electrostatics of a conductor. As we have seen that conductors are objects, which have free charges meaning there by that if you apply an electric field, these charges can move around freely within the material. And as a result, since we have free charges, we have seen it is necessary that under equilibrium conditions, the electric field inside a conductor must be equal to 0. Of course, if you connect a conductor end of a conductor to a battery, which is not an equilibrium condition of course, a current in flow, but that is a different part which we will not be talking about now. So, in in under electrostatic conditions, the field inside the conductor is equal to 0 and charges if they exist, they exist only on the surface of the conductor.

We had seen that in source free region, the potential is satisfies what is known as the Laplace's equation, and last several lectures we have been talking about various methods of solving such Laplace's equation. What do we wish to do now, is to talk about a different class of material, where the electric field has a totally different effect, and these are these materials as oppose to conductors are known as insulators in common language or dielectrics. What is the different between an insulator and a conductor. Firstly of course, we realize that in an insulator or in a dielectric the charges which by which we mean the charges which exist in the atoms of the molecules, they are tightly bound to their parental atoms on the molecules, and when you apply the electric field, the effect is not quiet, what happens when you apply electric field on a conductor.

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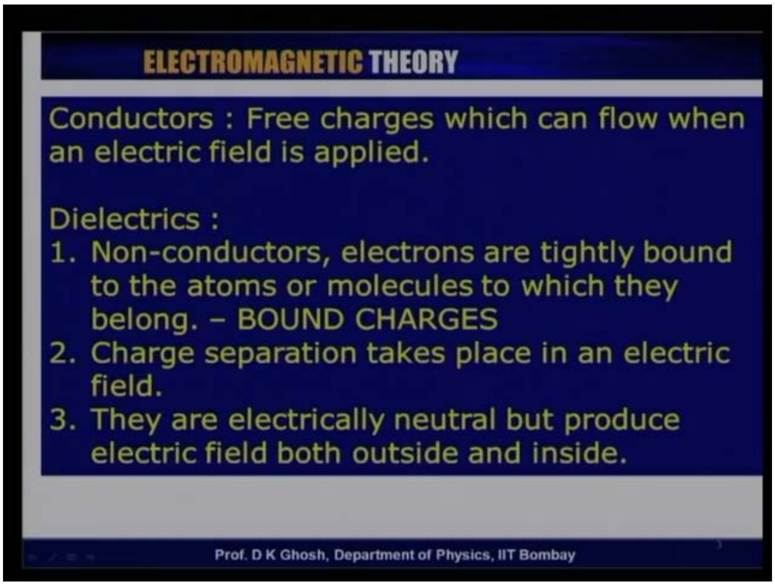

So, basically the charges are very tightly bound, and when you apply an electric field unlike the conductors the charges do not move around. However, there is still some interesting situation that takes place, when you apply an electric field on these insulators or the dielectrics. Firstly, that there are different types of dielectrics. So, there are dielectrics in which the there is what is known as a charge separation and the charge separation meaning that the charge center of the l positive charges and those of the negative charges, they do not coincide. Now, as a result such a material or such a molecule has what is known as a permanent dipole moment. If you recall that had defined a dipole as separation between two equal charges positive and negative, by distance d and the direction the dipole moment was a vector dipole was vector and the direction of the dipole moment vector was from the negative charge to a positive charge.

There is another class of material which are non polar material, where the positive and the negative charge centers coincide. However, when you apply an electric field these charges get separated by a small amount. There by creating and if you like an induced dipole or induced dipole moment. So, charge separation in such molecules take place only in the presence of an electric field. Now, recall that the material as a whole is electrically neutral because they consist of neutral atoms and molecules. However, because of this effect that the positive and the negative charge centers do not coincide either presence of the magnetic electric field or naturally.

This results in that there able to produce electric field both outside and inside. Now, this is a rather a curious phenomena and what we are planning to do today is to look at a this effect as to what are this dielectrics. How do they produce electric field outside and inside and how do they modify, what we have learnt so for as the Maxwell's equation corresponding to the electrostatic phenomena. We are still within the realm of electrostatics and so let us look at what is happening.

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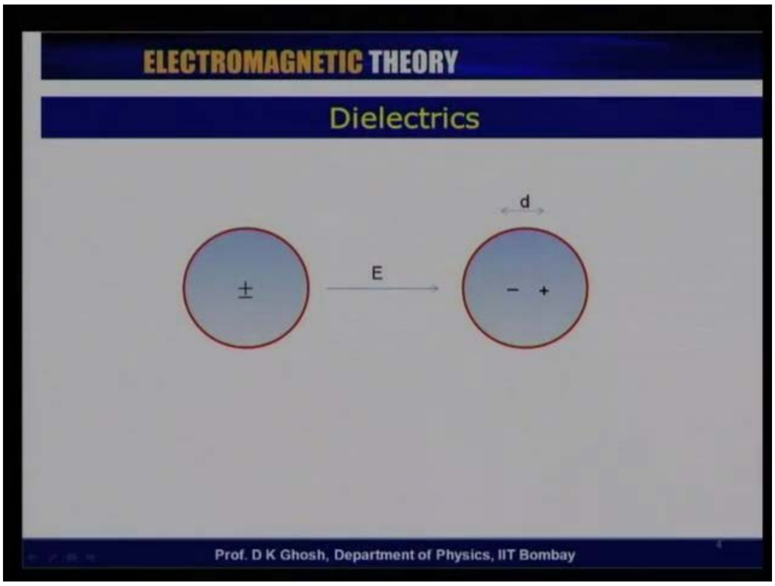

So, here you see a picture of an atom, where the let say it is a spherical atom. With the nuclei's which have a net positive charge there at the center and the negatively charged electron cloud suppose it is spherical, such that the center of this negatively charged electron cloud is coincides which this center a charge center of the nuclei. Now, this would mean that the net dipole moment is equal to 0, because if you recall dipole moment is q times the distance and so this is equal to 0, however when you apply an electric field. So, what happens is that the positive charges are attracted in the direction of the electric field and negative charges move away from the direction of the electric field and as a result a small charge separation that takes place. Small charge separation that is d takes place in the presence of an electric field.

Now, the result of this is to create a dipole moment whose magnitude is q times this distance and the direction as we have learnt is from the position of the negative charge to the position of the positive charge. Now, this is this is what happens and creates an induced dipole moment.

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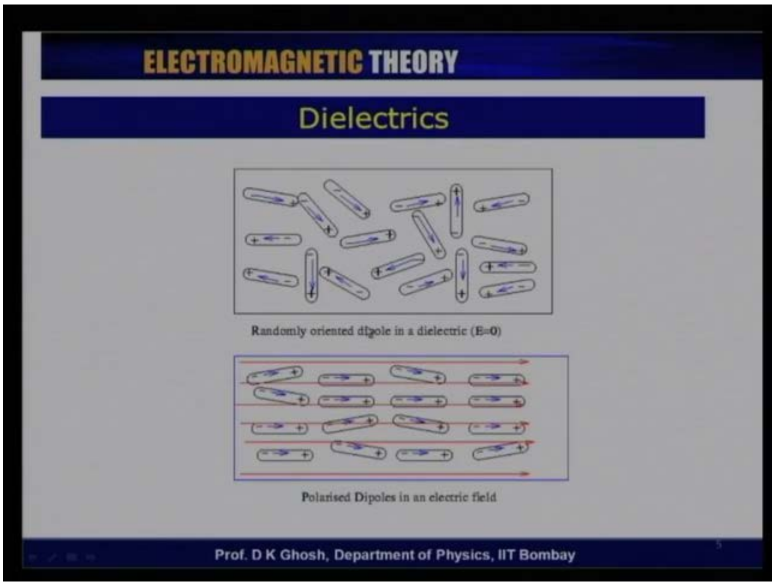

Now, let us look at what happens in bulk. Now, in bulk material even when there is a a permanent dipole moment. The the molecules are sort of randomly oriented, which means if you look at a collection of these dipoles which are randomly oriented, even though at a very microscopic level. If you come close to the location of a dipole you will find there is a positive and the negative charge separation, but there haphazardly located randomly located and the net dipole moment which is the vector some of these individual dipole moments works out to be 0. Therefore, in the absence of an electric field a bulk material does not have a dipole moment.

Now, if on the other hand, if you apply an electric field. Now, if you apply an electric field as we have just now stated, that the positive charge centers are pulled attracted towards the direction of the electric field and the negative charge centers go away from the direction of the electric field, and these dipole movements which were randomly located, they start now sort of aligning l, such that the this is negative positive negative positive etcetera etcetera. And therefore, if you add up these vectors you find that the material has developed a net dipole moment and and so this is what we are going to be looking at that is we are looking at materials which have developed some dipole moment.

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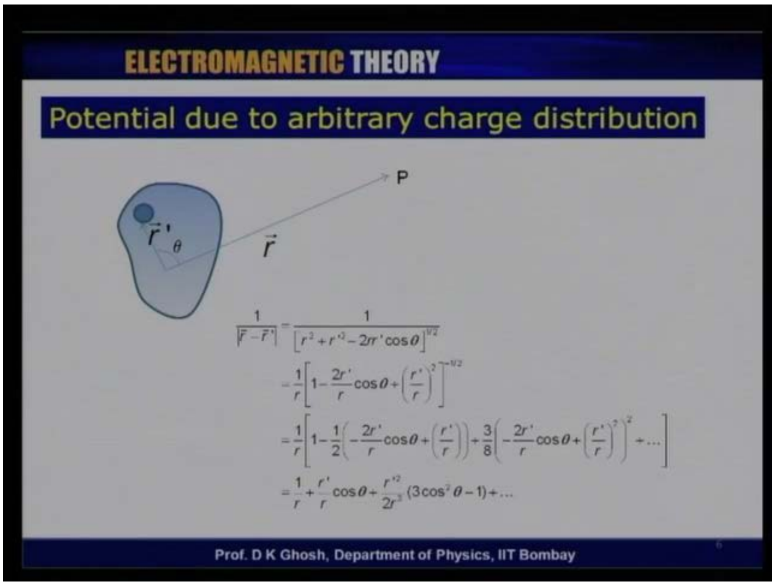

Now, what do you want to do is to look at such a material and find out what is the for instance the potential that is generated due to this. But before that let us look at, that suppose I have this is a material. Let us suppose I have a sort of a small volume in which I have got some charge distribution rho. How do I write down the electric field due to such a charge distribution at an arbitrary point P, let us say outside this material. Now, notice with respect to an arbitrary origin chosen here, if the position of the charge distribution is r prime and we are interested in finding out the potential at a point P which is at a vector the distance r from the origin and we take the angle between r and r prime to be equal to theta. Now, we can write down an expression for which is useful in calculating the potential and you know that this is a Coulomb's potential.

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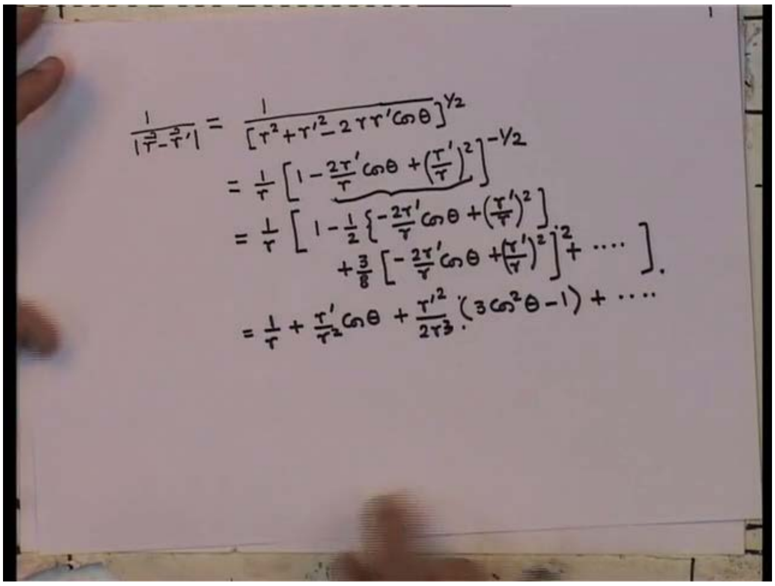

So, let me write down one over r minus r prime which is the, which is appears in the expression for the Coulomb's potential and when the reference that I have given here in this picture the angle between r and r prime is taken to be equal to theta. So, what you can do is to write this down as 1 over square root of r square plus r prime square minus 2 r r prime cos theta, raise to the power half. Now, what I do is this that, let me assume for a moment that this r this r is greater than this r prime in magnitude. So, that what I can do is to take this 1 over r outside and write this 1 as 1. What I mean is to expand this in a binomial series. So, this is equal to 1 minus 2 r prime by r cos theta and plus r prime by r whole square and all these raise to the power minus half.

So, this gives me one over r. Just let us do an expansion. So, I get one. So, I take this entire amount here as some x and I expand in 1 plus x to the power minus half. So, that gives you 1 minus half into minus 2 r prime by r cos theta plus r prime by r. Actually this r prime by r whole square and then n into n plus 1 by factorial 2 that is 3 by 8 and minus 2 r prime by r cos theta plus r prime by r whole square and this whole is squared plus dot, dot. Now, what I am going to do is to pick out or make this expansion in powers of cosine theta and let us look at what does it give me.

I have got a course 1 over r, which comes from this term 1. I pick up terms of cos theta and that comes from here, so I get 1 plus r prime by r cos theta. Now, plus I get r prime square by 2 r square into 3 cos square theta. Actually it is r cube because there is already

1 over r there and 3 cos square theta minus 1. Now, I am assumed that r is greater than r prime so that I can arrange them as r prime by r etcetera, etcetera This is you actually it should be r square, because the dimensionally r into r. So, that, so this is what we get as an expansion. So, writing this you notice that what you get here.

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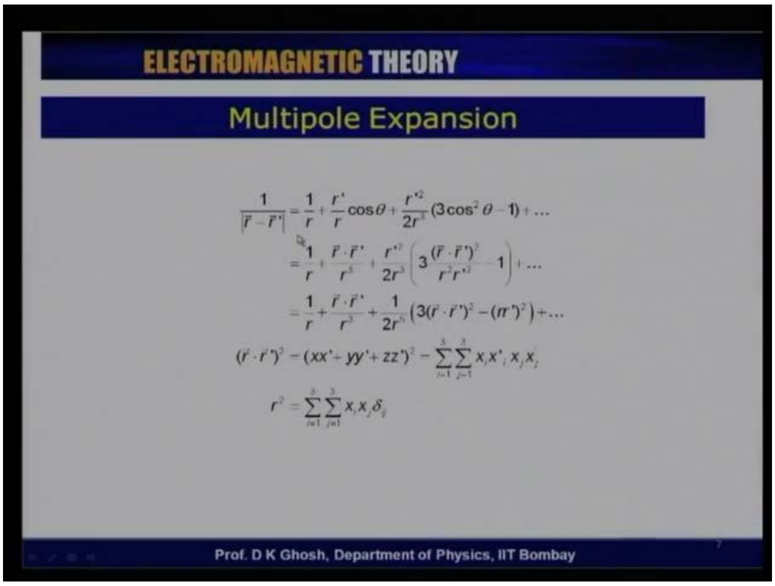

So, 1 over 1 r minus r prime is 1 over r and I have written this as r prime by r square cos theta and what you have done is to write this since, angle between r and r prime is theta. I have written this as r dot r prime by r cube and here r prime square by 2 r cube and simi, similarly, cos square theta has been written as r dot r prime square divided by r into r prime. So, this is the way this whole thing is written down and if you rearrange them it sort of rearranges like this 1 over r r dot r prime by r cube plus 1 over 2 r to the power 5 you notice r cube and r square there 3 r dot r whole square minus r r prime whole square etcetera, etcetera Now, let us look at this structure of this equation notice that r dot r prime r dot r prime square.

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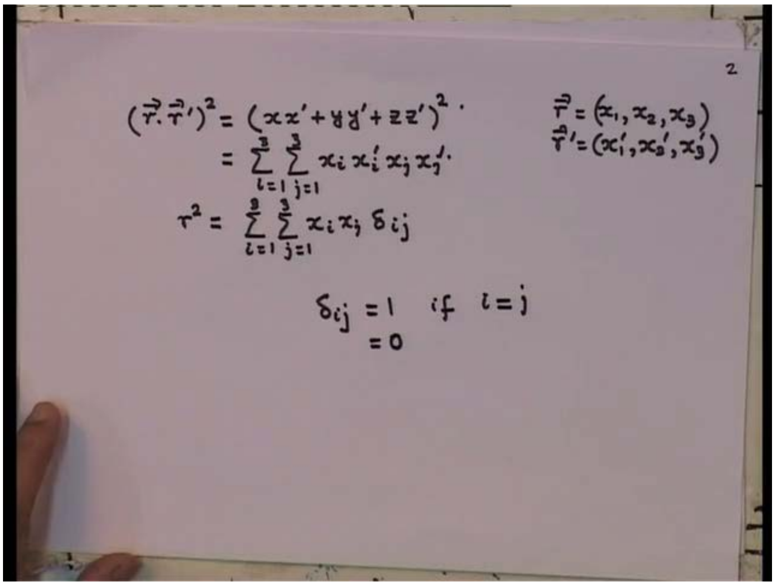

So, let me write down this in Cartesian coordinate, r dot r prime square. So, let us assume that the components of r is x y z and that of r prime is x prime y prime z prime. So, i get x x prime plus y y prime plus z z prime whole square. Now, I can expand this whole square out and you notice I will get various combinations of x x prime etcetera. So, let me write this down as sum over i and j. So, i goes from 1 2 3 and j also goes from 1 2 3. I will assume that a vector r's components will be written as x 1 x 2 x 3 and those of r prime will be written as x 1 prime x 2 prime x 3 prime.

So, if I do that I find that this is written as x i x i prime and x j x j prime and if you want to find out what is r square, that is of course, very simple because it is x square plus y square plus z square. So, what I can do that I can write it in the same form. We can write it as i equal to 1 2 3 sum over j equal to 1 2 3. This is for cemetery I will write down this as x i x j and with a chronical delta delta i j. Recall that delta i j is equal to 1 if i is equal to j and is equal to 0 otherwise. So, that allows me to write this in this format.

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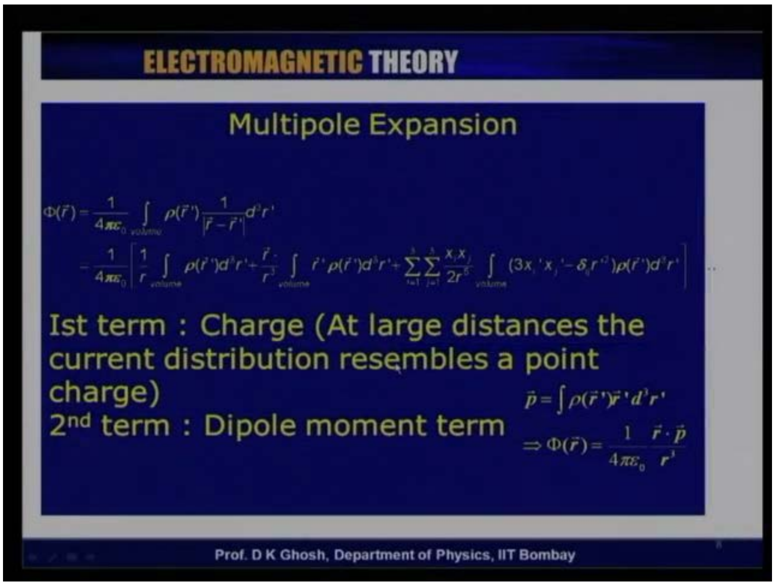

So, having expanded 1 over r minus r prime in this way, I will now write down a general expression for the potential in terms of this expansion that we have given.

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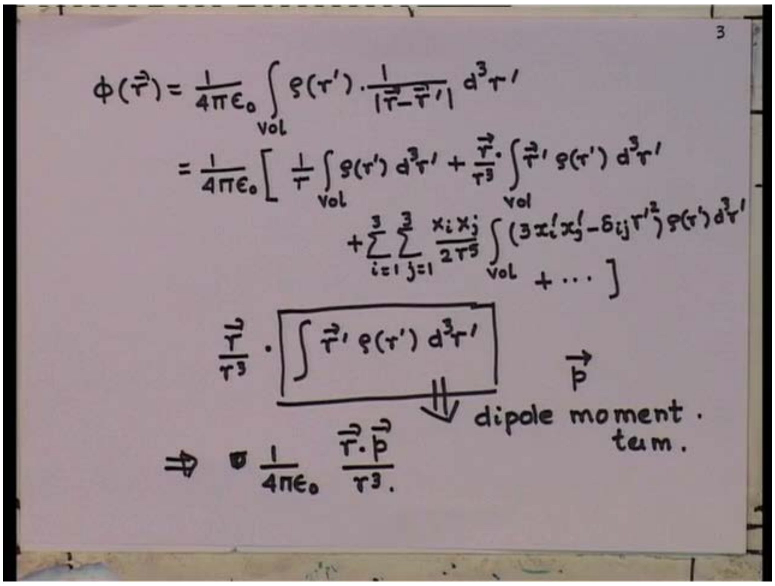

So, recall that the potential phi at the point r is given by 1 over 4 pi epsilon 0. Now I am integrating over the whole volume and the index inside that volume is given by r prime. So, this is the integration over the volume. I get rho of r prime 1 over r minus r prime. This is the Coulomb potential term and d cube r prime. Now, what I am going to do now is this we had just now seen how to expand this 1 over r minus r prime. In terms of vector r and r prime, just to recall just look at the first term here you notice that this term for example, the leading term is 1 over r the next term is r dot r prime by r cube r dot r prime by r cube. So, let me write that as equal to 1 over 4 pi epsilon 0 and inside I have got 1 over r 1 over r is independent of the integration variable. So, I am simply left with from the leading term the over volume rho r prime d cube r prime the first term.

Now, in the second term we notice that we had a dot product we said it is r dot r prime but r prime is integration variable so you have to go inside the integration and we had r dot r prime divided by r cube. So, let me take the r cube here and put a dot and integration over the volume of r prime vector rho r prime d r prime. This is the second term and let me write down the third term, which we have just now written in a very symmetric fashion. We said I equal to 1 2 3 sum over j equal to 1 2 3 x i x j divided by 2 r to the power 5 and integral over the volume. Now, these are variables which depend upon the point of observation and we have 3 x i prime x j prime minus delta i j of r prime square and rho r prime d cube r prime. l plus terms that, we have not bothered about. Now, it turns out that the terms we have neglected, really are not all that important it can be safely neglected and these are the three terms which one always concentrates on.

So, let me let us look at what are these terms and what are what is their meaning and if you look at the first term you notice, that the integration is simply a charge density integrated, which means this is nothing but the charge q. Now, if it is charge q the first term simply gives you other than 1 over 4 pi epsilon 0 q over r. Where q is the total amount of charge contained in that volume. Now, this if you recall is nothing but the potential due to a point charge at a an external point. Now, what does it actually mean? Remember that we have expanded these in powers of r prime over r assuming r to be very large. So, therefore if r is very large the rest of the terms can be safely neglected and I will be only left with the first term. But any volume from a very long distance large distance resembles a point charge.

That is if you look at a finite body from an extremely large distance you cannot distinguish it from a point charge. So, it is only reasonable that if the distances at which you are calculating the potential is very large. The potential will resemble that due to a point charge namely the Coulomb potential due to a point charge, which is nothing but 1 over 4 pi epsilon 0 q by r. So, the first term stands for the point charge only. Now, suppose that distances are large but not that large now. One of course, I have to go to my next turn. Now, let us look at what is this next term. Now, the next term if you notice is nothing but I have got r by r cube dotted with a quantity I have got an integration r prime rho r prime d cube r prime. Now, this quantity has distance and the charge density there and you can immediately see that the the dimension of this quantity is that of a distance times a charge, which is the same dimension as that of a dipole moment.

The. So, we identify recall the dipole moment p for a point dipole was given by the distance of separation between the charges and this quantity we identity as the electric electric dipole moment corresponding to a continuous charge distribution. r prime is the distance of that point charge where I look that is basically what I am doing is this I have located a small volume element found out how much is the charge there, which is obviously rho r prime d cube r prime and multiplied it with the distance from the origin and integrated over.

So, this quantity is my dipole momentum. At least at this moment it appears like having a right dimension. Now, if you look at that supposing I identified it with dipole moment p for a continuous charge distribution. Then I can write this term for the potential as 1 4 pi epsilon 0 r dotted with p divided y r cube. If you recall this is the nature of the dipole potential that we had seen for a point dipole as well the third term is a little more complicated.

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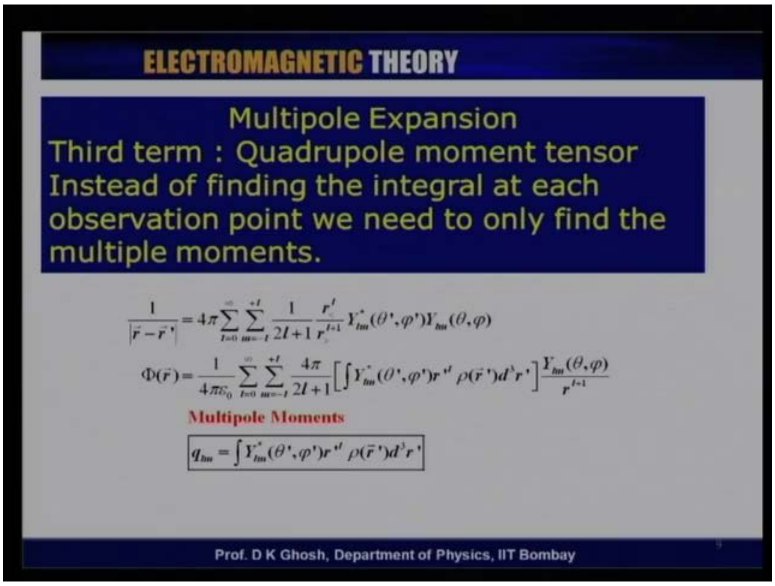

We have not come cross it yet, but let us go back and look at it for a moment notice what is there in this this is x i x j divided by 1 over r to the power 5 and I have got a term here, where the charge density is multiplied with a quantity whose dimension is that of square of a distance and of course, integrator of it. Now, this is called a quadruple term. Something which we have not done, but you could do it very easily. Supposing, you are to take 4 charges 2 positive 2 negative alternately situated on the the edges of a square. This combination of 4 charges form what is known as a quadruple. This is the most elementally quadruple that you can have you could of course, arrange this charge as a different methods and get different configurations. This term that we have at the end which I will be talking about little more is what is known as the quadruple momentum.

Now, incidentally you must have realized that this calculation is going to become very messy and the reason why it is messy is the following that we need to calculate the potential when we are calculating potential at different points because the expression is involved. We need to make a separate integration at every point where we are making the observation. This is this is what the potential term would be that 1 over r minus r prime. This expansion has an advantage that the integration is being done only once and that is the integration variables are only with respect to the within the material of the object and as a result there is no mix mix up between the variable r and the variable prime.

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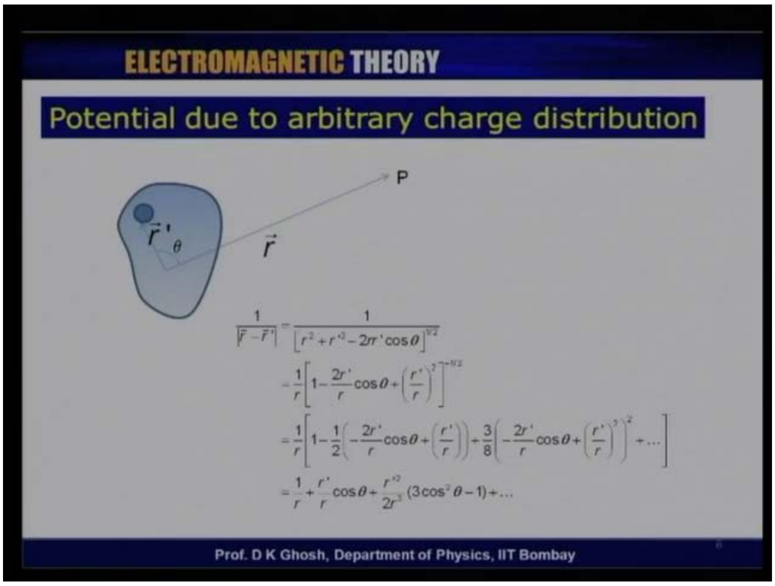

So, let us look at a little more formal way of doing it. Now, remember that when we worked out this expression the we had shown this geometry and in this geometry. We had chosen the angle theta to be that between the location r prime and the point P. Now, what we could do instead is to look at a general situation where the coordinate of a volume element if you like, is given by r prime theta prime phi prime and that of the point of observation is given as r theta. Now, in such a case as well, we can do an expansion and this expansion of 1 over r minus r prime is usually written down in terms of what are known as spherical harmonics. We have come across spherical harmonics in the past and at least if you take your time out and compare the first few terms and identify them with the spherical harmonics that you have learnt earlier.

You will see that we are talking about is the same as before. So, what we are saying is this that 1 over r minus r prime. Remember vector r has components in spherical polar as r theta phi and r prime has has r prime theta prime phi prime. This is the written as 4 pi sum over l and l. l going from 0 to infinity and sum over m going from minus l to plus l, corresponding to a given l 1 over 12 plus 1. Now, this is r less to the power l by r greater to the power l plus 1. Where r greater and r less or lesser, r to be identified with r or r prime whichever happens to be greater or lesser of the two. y l m start, theta theta prime phi prime theta prime and phi prime are coordinates of the volume element inside the object and y l m theta phi. Now, this is a general expansion

Now, we what I have done now is to write down the potential phi of r in terms of this expansion. So, if you remember that potential has other than 1 over 4 pi epsilon 0. I need simply a rho r prime factor and what I have done in this expression is to combine those terms which depend upon the integration variable which gives me y l m star theta prime phi prime r prime r prime times rho of r prime d cube r prime and y l m theta phi and this is actually r prime to the power l and here r to the power l plus 1. Once again I have assumed that r greater is r, r less is r prime. Now, what is the advantage this advantage is precisely what we talked about. I have to do one integration only of this quantity.

Let me identify this quantity to be sum q l m. It has a name it is called multiple moment. So, q l m is y l m star theta prime phi prime, r prime to the power l rho r prime d cube r prime and if you want to calculate this phi of r you need to only do this integration once. Having done this, plug it in and then of course, the rest of the thing will follow.

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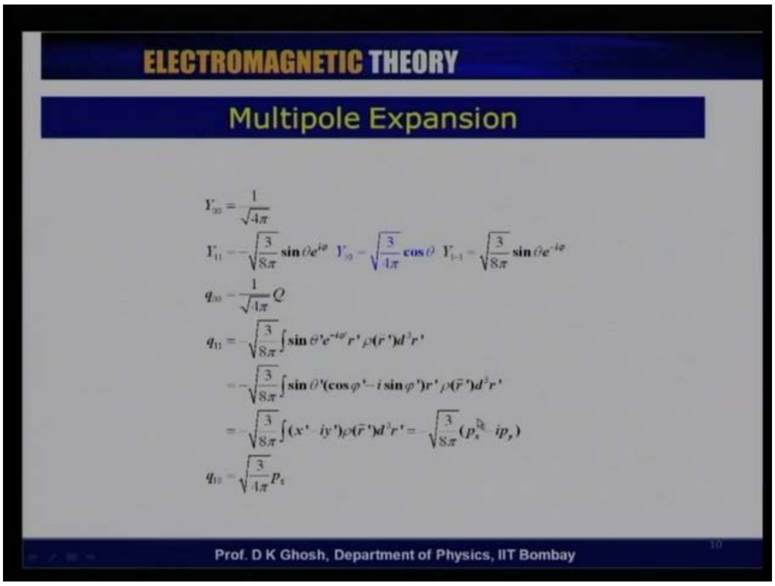

Just to remind you that we had seen that y l m at least the first three is are what we require. y l m 0 0 is 1 over square root of 4 pi y 1 1, remember that y l m and y l minus m they are related by minus 1 to the power m times the complex conjugate of each other. So, you get one 1 1 a square root of 3 by 8 pi with a minus sign sin theta e to the power i phi but y 1 minus 1 will be minus 1 to the power 1, which is plus and e to the power i phi will become e to the power minus i phi. y 1 0 is this and you can we had earlier seen how y two’s five components are written down. Now, let us look at what it implies

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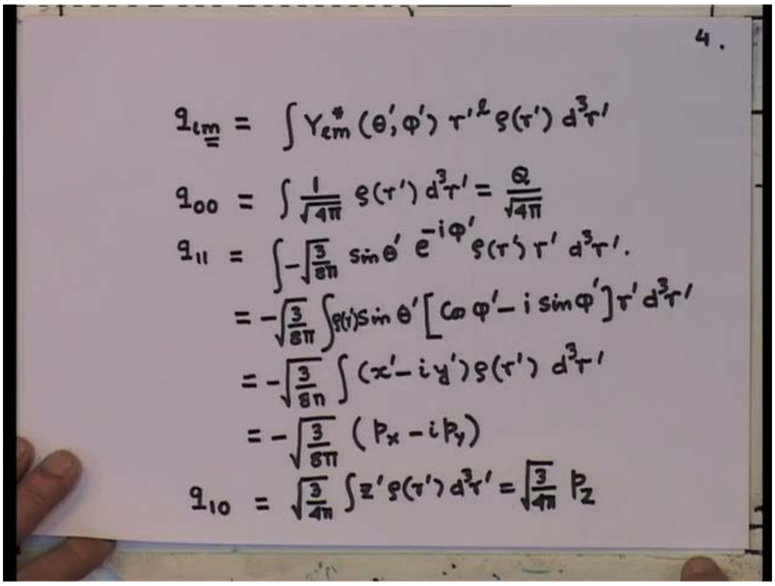

So, we had written down q l m which is the multiple moment, as been given by y l m star theta prime phi prime. r prime to the power l rho r prime and d cube r prime. Remember that the value of m corresponding to that of l it goes from minus l to plus l. So, q 0 0 this is the lowest order of multiple moment, is y 0 0 star and y 0 0 is a number 1 over square root of 4 pi r prime to the power 0 is 1. So, I have simply left with rho r prime d cube r prime, but rho r prime d cube r prime integration is the charge Q. So, I got the charge that is contained inside the body divided by square root of 4 pi. So, in other words the multiple moment q 0 0 is the related to the total charge content within the body, by a constant factor.

Let me do one more calculation. Let see what is q 1 1. So, once again for q 1 1, I need y 1 1 star and y 1 1 is minus 3 by minus root of 3 by 8 pi and I have got a sin theta and because it is complex conjugate I pick up a e to the minus i phi prime of course, all of them, times rho r prime. Now, I need a r prime to the power l. l is equal to 1 so I have got r prime d cube r prime. Let me write it in a slightly different fashion. This is 3 by 8 pi integral sin theta prime. Now, let me write e to the power minus i phi prime as cos phi prime minus i sin phi prime. r prime d cube r prime.

Now, notice interesting thing. r sin theta cos theta is nothing but an expression for x. In this case there is a prime therefore, it is x prime. So, I get 3 by 8 pi integral of x prime minus i times r prime sin theta prime sin phi prime, which is nothing but y prime. This times r has been already taking care of. So, I get rho r prime which has been missed in the first term so let me put it back d cube r prime. Look at this expression, but for this constant factor 3 by 8 pi. What I have got is x prime rho r prime remember I defined for a continuous body the dipole moment as distance the position times the charge density. So, this is nothing but p x prime or p x if you like. So, this is p x minus i p y. I would recommend you calculate what is q 1 minus 1 or and if you look at q 1 0 that requires y 1 0, which has a square root of 3 by 4 pi.

There is only a cos theta here, there no e to the power i phi there and r cos theta is nothing but z therefore, this will be nothing but integral z prime rho r prime d cube r prime which other than for this constant factor that we are carrying, is nothing but the z component of the dipole moment. Likewise, we will see that, the if you consider the higher order terms there will be connected with the quadruple moments and this moment we will skip that and look at slightly different situation. We have seen that a because of the charge separation. If you look at a small volume element, the small volume element can have dipole moment and that if you look at a microscopic body then I can still have a net dipole moment. We define what is called a polarization vector as a vector sum of the dipole moments in that body.

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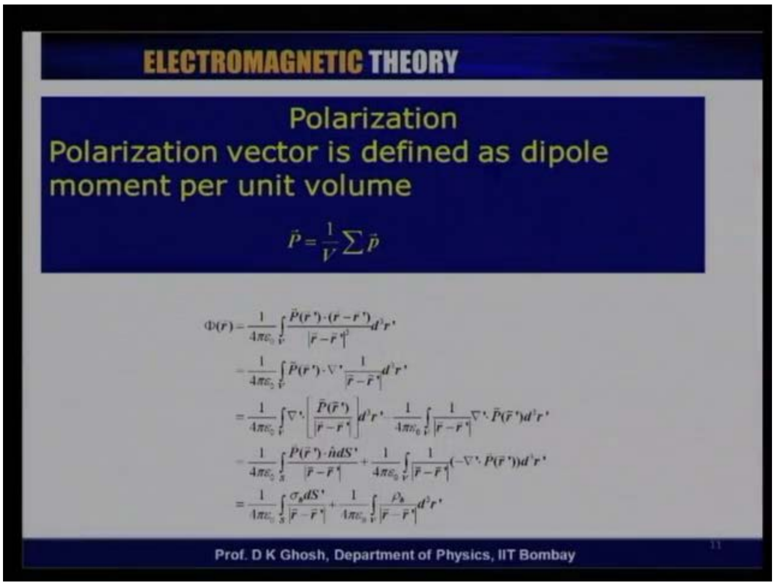

So, vector p is defined as this is per unit volume, we take 1 over V sum over the all the dipole moments. Now, let me then concentrate on the first term as we have seen is the total charge term. It is the second term which is the dipole momentum and today we are going to concentrate only on the polarization or the dipole moment. So, at this moment let us forget about the first term. Let us forget about the quadruple momentum and let us write down what the situation is with respect to the dipole momentum.

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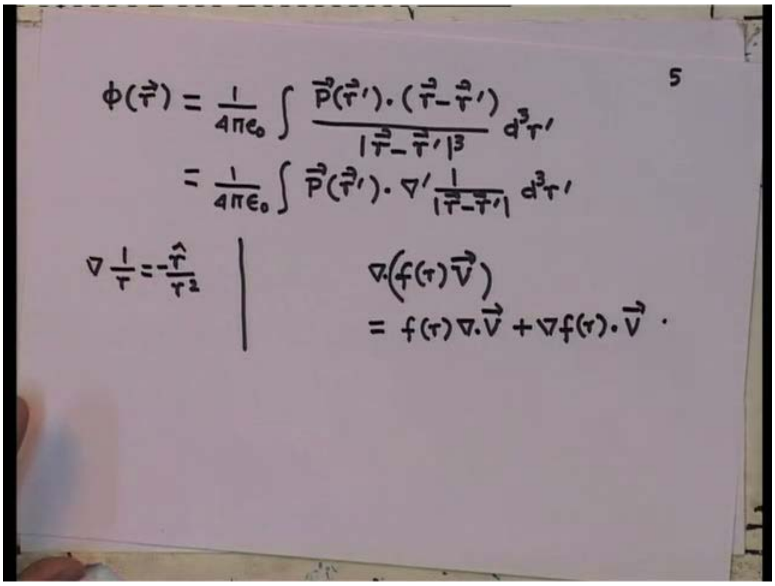

So, phi of r due to the dipole moment, suppose I assume that the body is electrically neutral, so that the first term is 0, because total q is 0. So, I will write this is equal to 1 over 4 pi epsilon 0 integral p of r prime dotted with r minus r prime divided by r minus r prime cube d cube r prime. Now, notice what I have done. I had shown earlier that the potential due to the dipole momentum is given by r dot p by r cube. Now, what is p p is the dipole moment of that unit of that volume. Now, since I have defined the polarization vector as the dipole moment per unit volume. If I consider a volume d cube r prime the dipole moment vector of that volume is given by the polarization p of r prime times the volume element.

So, that is what I have done in this expression I have written as p of r prime d cube r prime that is my dipole momentum and the rest of it is just the way you will wrote down. Now, I want to do some calculus with it. So, notice this thing that I will write this is 1 over 4 pi epsilon 0. Now notice this is r minus r prime by r minus r prime cube now if you recall we know that gradient of one over r is unit vector r divided by r square with a minus sign.

Now, what I am doing here is this? So, therefore this quantity is nothing but the gradient of 1 over r minus r prime. But notice that this gradient is with respect to the variable r and if instead I take the gradient with respect to the variable r prime, and define it denote it by a del prime then I get rid of this minus sign that is written there and as a result I will write this gradient of 1 over r minus r prime. Recall that this gradient is with respect to the primed variable. So, I have got a vector dotted with the gradient. Now, I use an algebra that we have seen several times and this was the following that, we know that if we write down del prime dot actually any del dot.

Supposing, we write down del dot of a scalar times a vector. Let me write a vector V. I know that the result of this is sorry del dot not gradient the divergence of this. So, the result is the scalar function times the divergence of the vector function plus gradient of the scalar function dotted with the vector function. So, you notice I have got exactly the same thing here. I have got a vector function dotted with a gradient of a scalar function, which is my first term. So, I can rewrite this expression as a divergence of 1 over r minus r prime, times p of r prime, just if you observe it here. So, what I have done is to write this vector dotted with gradient as equal to the divergence of the scalar function times the vector minus the scalar times the divergence of the vector function itself. Now, this is what I have used.

Now, the first term which is del dot of P of r prime by r minus r prime integrated over d cube r prime, using the divergence theorem can be converted into a surface integrate. So, look at this expression here. This is divergence of P of r by r minus r prime d cube r prime. So, I am writing this as a surface integral and is the direction of the surface of this vector p of r prime by r minus r prime d s and the second term I am really kept it exactly as it was excepting that this minus sign I have for some reason to follow have brought it inside the integration. Well I have clubbed it with minus del prime dot P r prime. The interest in this expression is the following. That if you examine this expression and compare it with the potential due to a volume charge distribution and a surface charge distribution.

So, this expression for instance tells me that if I had a surface charge distribution given by sigma b surface charge density equal to P dot n. Then this will be the usual expression for the potential due to surface charge distribution. I have put an subscript d for the reason that will become clear in a moment and likewise this expression is nothing but the potential that you expect from a volume charge distribution provided your volume charge density is given by minus del dot P.

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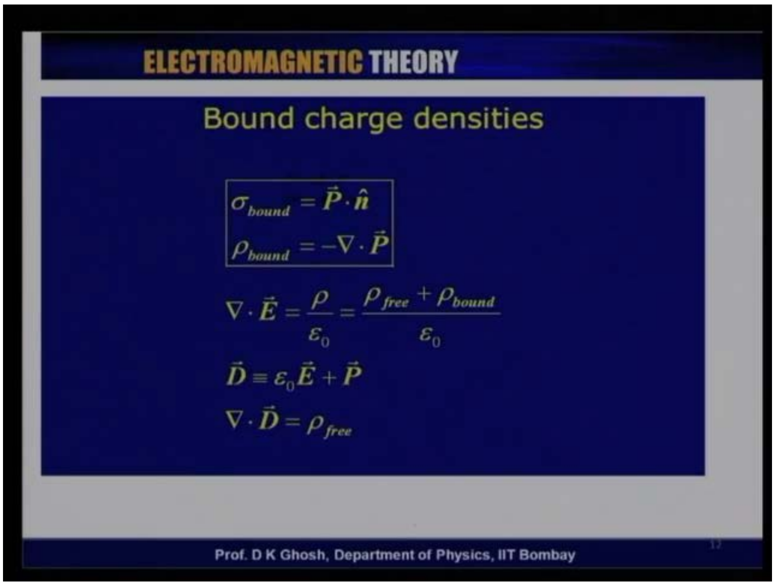

So, what you have found is this. That the effect of the dielectric, the dipole term. Remember I have said that that dielectric is assumed to be electrically neutral. So, my first term is 0 and second term is what I have kept. Third term onwards at this moment I have not done anything. So, we have a replace the original problem by a problem where I have got surface charge density. Since these charges are not free charges, but they are bound to the atoms of the molecules we call them bound charge densities and the surface bound charge density sigma b or sigma bound is just given by the normal component of the polarization vector P and the volume charge density rho b or rho bound is given by minus the divergence of the polarization vector. Remember my electric field E satisfied the Maxwell's equation, which was del dot of P equal to rho by epsilon 0.

So, how what happens to that expression, that expression remains as it is, but for the following foot note. One is that rho which is the charge density which produces the electric field could consist of two parts. One the free charge density rho free, Just I have written it for clarify, big explicit and a volume charge density called rho bound which arises due to the dipoles divided by epsilon 0. Now so this is now now notice this this electric field is the electric field that you actually would measure. If you were to put a test charge at the location where you are measuring the electric field, because that test charge will experience the potential generated by free charges that may be there in the body, and also the bound charges that are there in the body. Now, what one does is to define a vector vector D given by epsilon 0 vector E plus P.

I and you notice that you can this is called displacement vector and you notice that if you rearrange this del dot of D v D becomes equal to rho free. In other words D are the displacement vector, is a vector which we have constituted or we have made out which sort of artificially tries to separate the effect of the free charge density. Now, remember there no physical way of doing it because if I want to calculate the electric field it is the electric field E that is important, but for mathematical reason the using an electric field D might turn out to be convenient. And so as a result the del dot of D is given just by rho free. Remember two differences. There is a dimension difference because D is defined with respect to E with an epsilon 0 term. So, del dot of E D is rho free and not rho free by epsilon 0. Just I will illustrate as to what is happening. Remember we said minus del dot of P gives me the bound charge density now what actually is happening.

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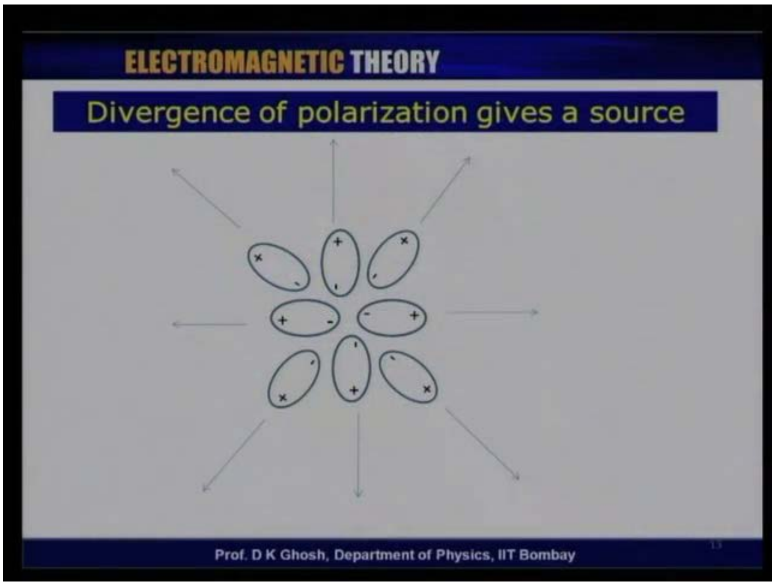

The del dot of P is nothing but a divergence of the polarization vector. So, you notice I am showing here pictorially the divergence of the polarization vector. Now, when when the polarization vectors diverge, you notice that in the central region a negative charge is created, because the vectors have to be aligned like this and this now becomes the source of the electric field.

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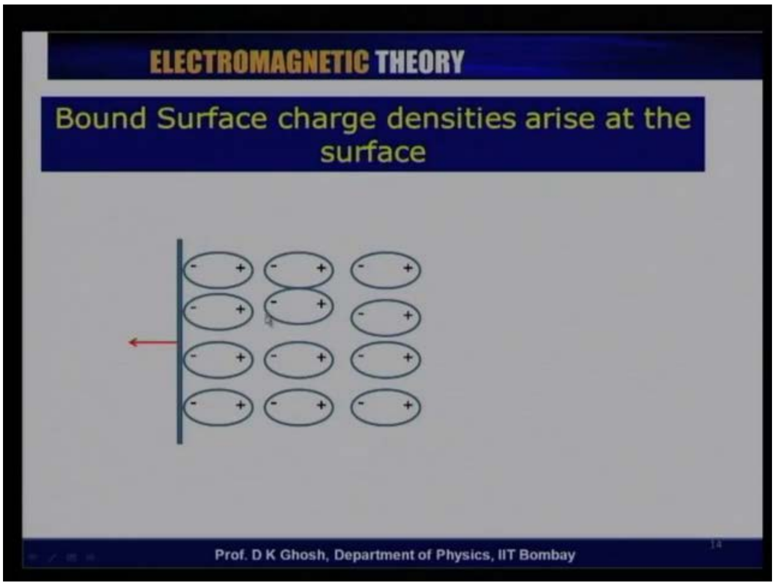

The on other hand, at the surface you notice there is again a bound charge in this space because at the surface inside the material they all cancel out. But at the surface there is again a balance of the charges. So, this is my surface bound charge. So, this these two are some pictorial ways of looking at the surface and the bound charge density. We will continue with the effect that the dielectrics have on the Maxwell's equations and also look at some quadruple effects in the next lecture.