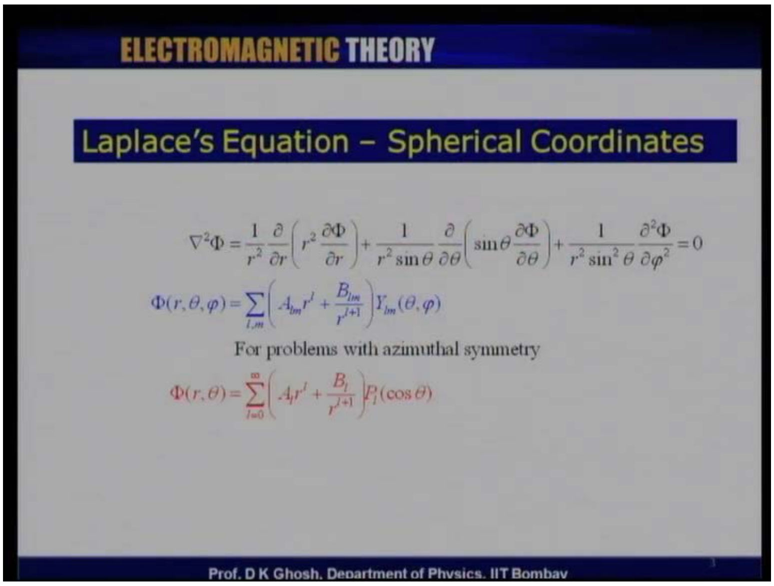
Electromagnetic Theory
Prof. D. K. Ghosh
Department of Physics
Indian Institute of Technology, Bombay

Module - 2
Electrostatics
Lecture - 16
Solutions of Laplace Equation – III

(Refer Slide Time: 00:32)

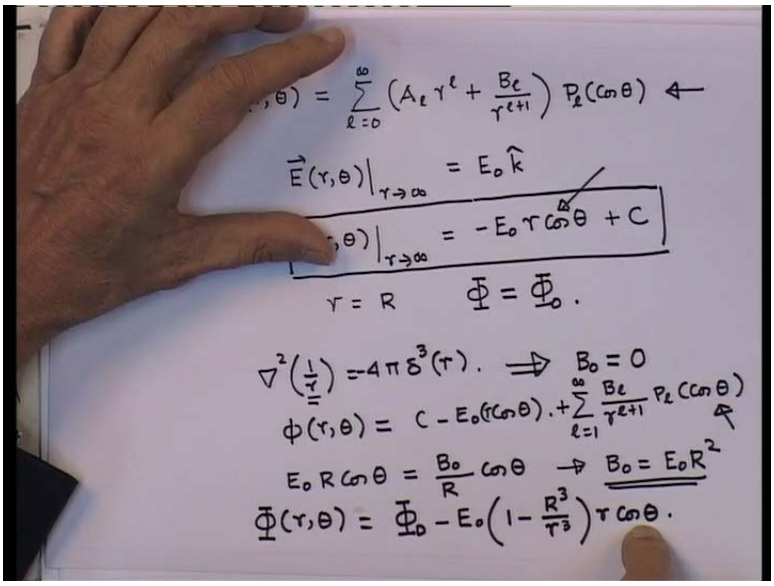


We continue our discussion of solutions of Laplace equations. Last time, we had shown that the Laplace equation in spherical polar can be written by this expression, which you see on your screen. Namely, 1 over R square d by dr of R square d phi by dr plus 1 over R square sin theta d over d theta of sin theta d phi by d theta plus 1 over R square sin square theta d square phi over d phi square equal to 0.

Now, what we did is to show that this equation can be solved by separation of variable technique. The general solution to this equation is given in terms of what are known as spherical harmonics. The radial part of the solution has two parts. One part goes as R to the power l and the second part which goes as 1 over R to the power l plus 1, where l of course, goes from 0 to infinity. This is a general solution.

What we tried to do last time is to specialize to the case of where there is azimuthal symmetry. Azimuthal symmetry simply implies that the solutions do not have phi dependence, azimuthal angle dependence.

(Refer Slide Time: 01:49)



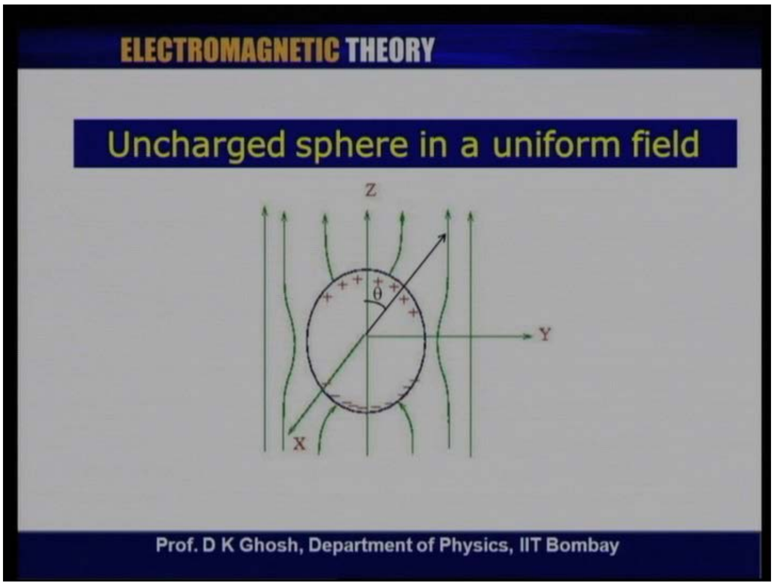
In such a case, the potential phi which only depends upon the radial distance R and the polar angle theta is given by sum over l is equal to 0 to infinity. You have this A l R to the power l plus B l over R to the power l plus 1 times Legendre polynomial pl cos theta. We had also seen that pl cos theta are polynomials in cos theta of degree l or with P 0 being defined to be equal to 1.

So, let us return back to a special application of this. We have a conducting sphere, which have been put in an electric field, which is uniform. The electric field is directed along the z direction. Therefore, very far from this sphere E R theta, when the distance R goes to infinity, is a uniform field in the z direction. So, E 0 times the unit vector k.

Now, what we are interested in knowing is that, because you have now put in a conductor in this electric field, what is the corresponding potential outside the sphere at box distances, which are close to the sphere and far from it. So, we start with a few observations. The first observation is what we just now talked about. That is the electric field act far distances from the sphere is not expected to be very much different from the uniform field with which I started with.

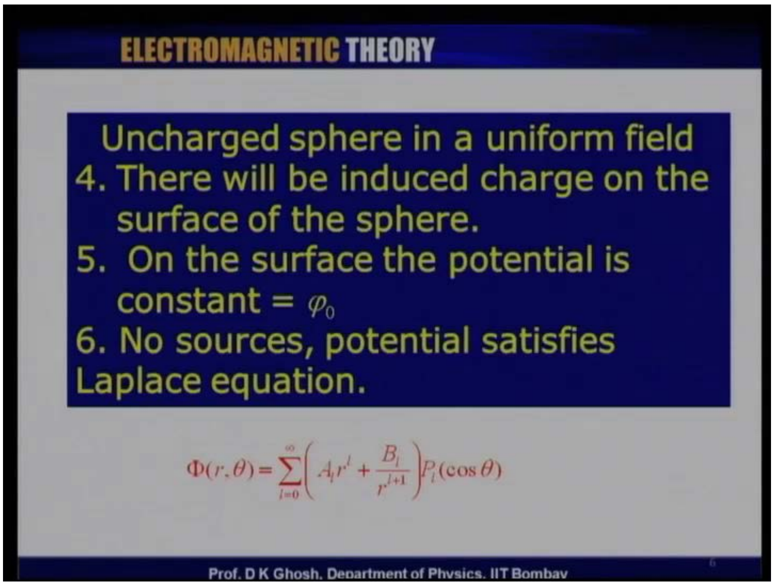
So, as a result E R theta as R goes to infinity is E 0 times k. Correspondingly, if you look at the potential at large distances, can be written as minus E 0 R cos theta, which is nothing but z, because the electric field is along the z direction. So, the potential must be minus E 0 times z plus of course, a constant, which we write as c. There are a few physical observations, which you can make since the conductor has to be an equi-potential.

(Refer Slide Time: 04:26)



The lines of forces have to strike the conductor normally. So, this is what it is. This is the direction of the electric field. So, close to the sphere, the lines of forces must strike it normally and of course, exit from the other end. Of course, the electric field cannot penetrate the sphere. So therefore, there are some bending effects as you closely look at close distances. Now, this edge or this side of the sphere, obviously, picks up a positive charge, induced charge. Positive charges will be there and the rear rend will pick up negative charges. Now, this is required because, we have put the sphere in an electric field and we need to make the electric field inside the sphere 0. So, as a result, there must be an internal electric field, which will be responsible for cancelling the field E 0 that we have started with.

(Refer Slide Time: 05:33)

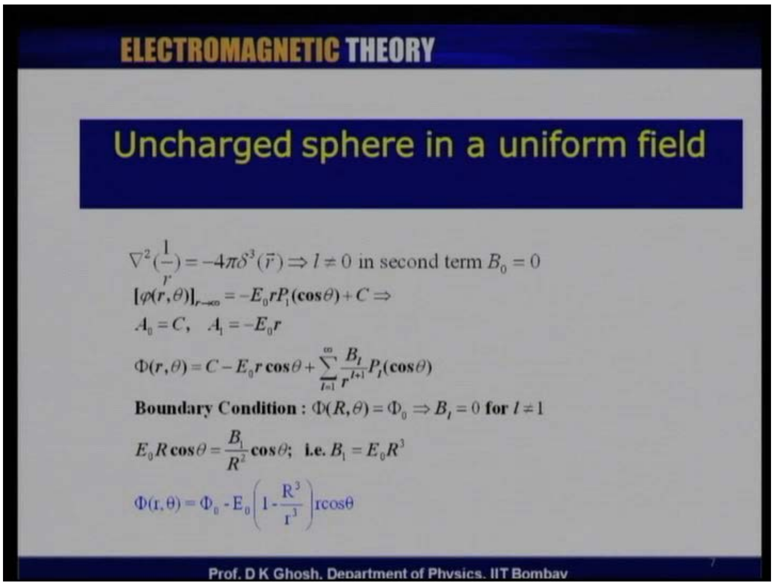


So, let us look at, now we need to try to find out what these constants are. So, let us start with a condition that on the surface of the sphere, that is at R is equal to R, the potential phi is equal to phi 0, let us say. The second thing that we realize is this. That, outside this sphere in the empty space, there are no sources. So, I need to solve a Laplace equation. The general solution to the Laplace equation is what we have written down. Now, let us look at this thing. First thing that we notice is this. This contains terms which go as R to the power l and terms which go as R to the power l plus 1. Now, when l is equal to 0, this second term of the potential is 1 over R potential. Now, you recall that we are aware that del square of 1 over R is minus 4 phi delta 3 dimensional delta function at the origin.

So this, since I do not have a charge at the origin and I have actually the origin surrounded by a sphere, so, I cannot have a term in the potential which goes as 1 over R. What this implies is that, in this summation, l is equal to 0 term will not be there. As a result, B 0 must be identically equal to 0. Now, the second thing that we do is to compare this expression, that is the asymptotic form of the potential and try to compare it with this at large distances. Now, you notice that this has a term other than a constant. It has a term, which is proportional to cos theta. Now, we are aware that p l of cos theta is a polynomial of degree of l in cos theta.

So, if you just have a simple cos theta, the corresponding l value must be equal to 1. So, we start with a form, which is a 1 R plus B 1 by R to the power 1 plus 1, and that is B square B by R square times cos theta. We have said already l is equal to 0 term is not there. Now, so therefore, let us rewrite this equation phi of R theta and this is equal to, well, c l c minus E 0 cos theta is what I have. E 0 times cos theta actually. E 0 times R cos theta plus sum over l is equal to 1 to infinity B l by R to the power l plus 1 p l cos theta. Of course, as we have said just now, the only term that should be there in this is P 1 cos theta term. So, if you compare these two expressions and remember that the potential on the surface that is at small R is equal to capital R, must be constant equal to phi 0.

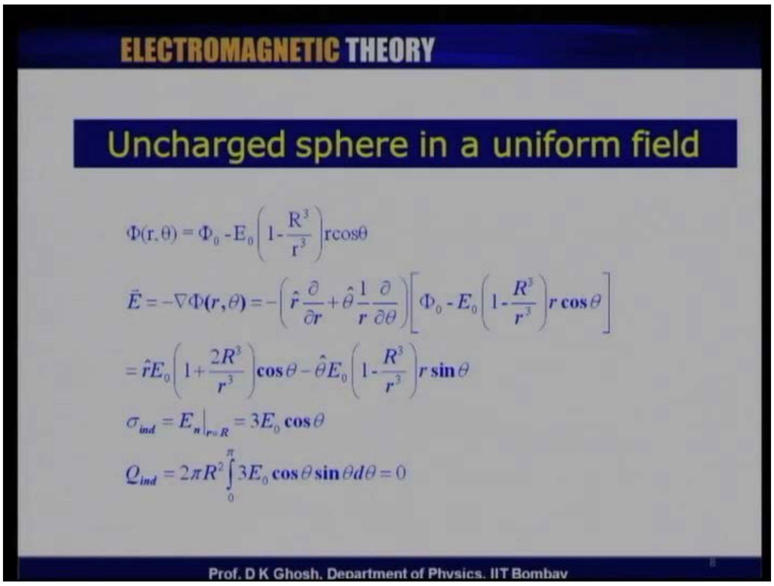
(Refer Slide Time: 09:26)



So therefore, I require, looking at this expression what I require is E 0 times R cos theta must be equal to B 0 divided by R times cos theta. Now, this is required because, at R is equal to capital R, the cos theta dependence from these two terms must cancel out. Now, this immediately tells you that B 0 must be equal to E 0 times R square. Of course, R to the power l for l is equal to 0 gets me a term, which is equal to a 0 and that must be identified with the constant c.

So therefore, combining all these things what I get is, the phi at R theta is given by phi 0 minus E 0 times 1 minus R cube divided by R cube times R cos theta. So, this is actually, there is a E 0 R cos theta term and as expected l is equal to one term, which gives me 1 over R square term there.

(Refer Slide Time: 11:01)

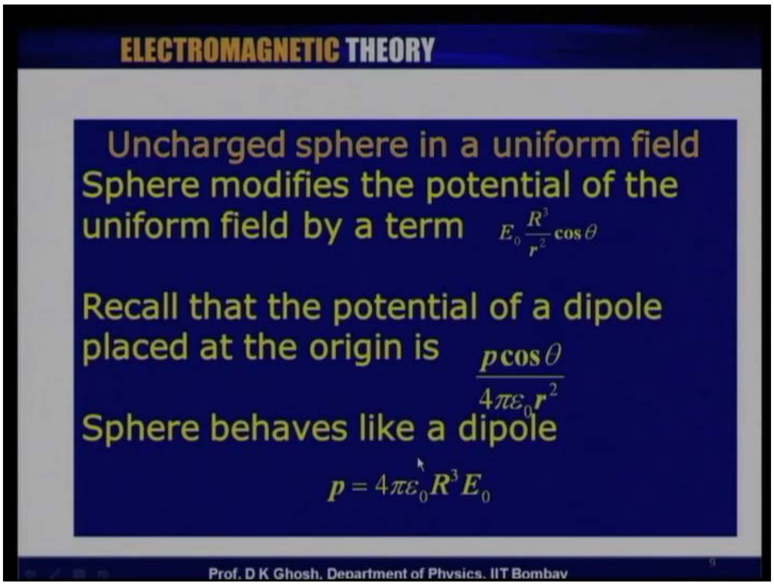


So, this is my expression for the potential. Once I have the expression for the potential, I can find out what the electric field is by simply taking a gradient. Remember that, I do not have any azimuthal dependence. So, the gradient is simply R unit vector R d by d r plus unit vector theta 1 over R d by d theta acting on this potential.

So, fairly straight forward integrations to be done; you can immediately see that the electric field is given by unit vector R times a term proportional to cos theta and an unit vector theta times a term proportional to sin theta. Now, I know that the charges induced on the surface is equal to the normal component of the electric field at R is equal to R. So therefore, take this expression and the normal component is obviously, the radial direction and put small R is equal to capital R. What you get is the normal component, the induced charge, which is equal to the normal component of the electric field and R is equal to R is given by 3 times E 0 times cos theta.

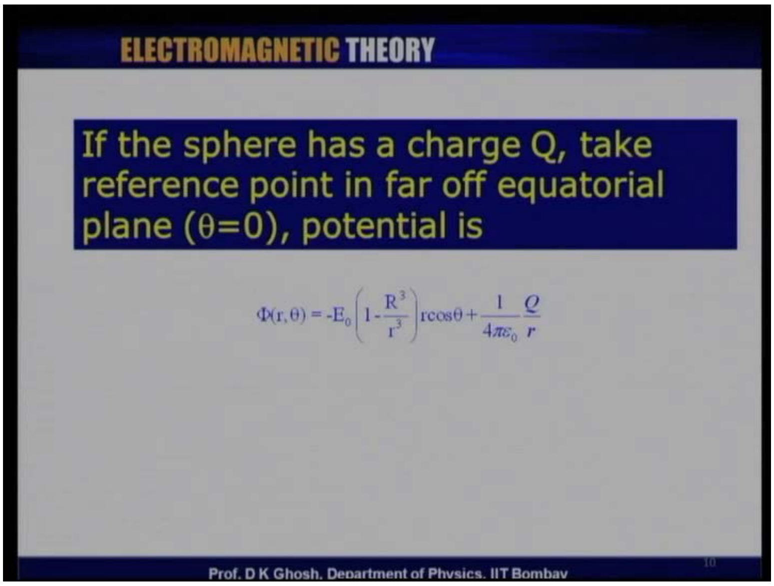
As you can see in the northern hemisphere, the induced charge is positive and in the southern hemisphere, the induced charge is negative. You can actually calculate by integrating over this function, over the angles and show that the total induced charge is equal to 0. This is very straight forward integration. This sin theta comes from the integration, surface integration term and this works out to be 0.

(Refer Slide Time: 12:54)



So, notice what we did. What we said is that, in the presence of the sphere, the potential changes or modifies the potential due to uniform electric field by a term which is equal to E 0 R cube by R square times cos theta; 1 over R square term. Now, if you want to look at an equivalent problem, you recall that the potential of a dipole, which if it is fixed at the origin, is given by P cos theta divided by 4 phi epsilon 0 R square. Now, if you compare these two, it tells you that the sphere is equivalent to or the sphere can equivalently replaced by putting a dipole of dipole moment 4 phi epsilon 0 R cube times E 0 at the origin.

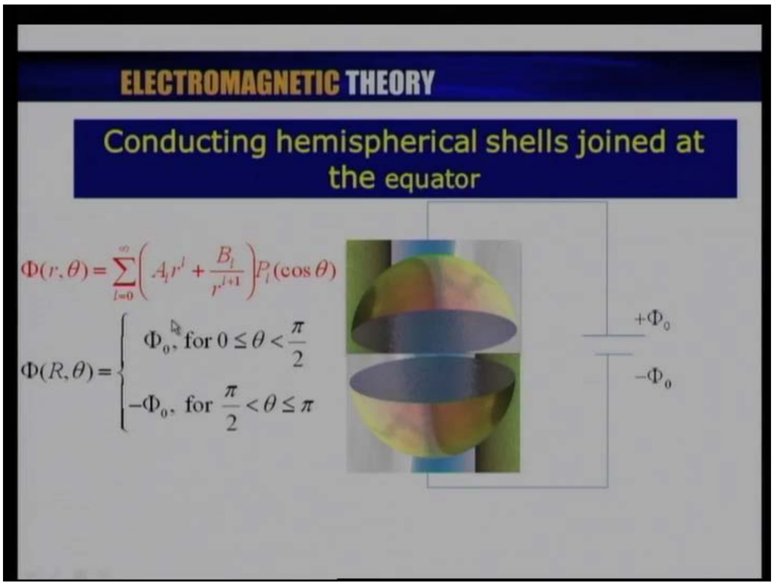
(Refer Slide Time: 14:01)



Now, what happens? This was an uncharged sphere. Now, what happens if this sphere had a charge? Well, this is not a very difficult problem because, all that we need to do is that, at far distances we are aware that if the sphere has a charge, it looks like a point charge at the origin.

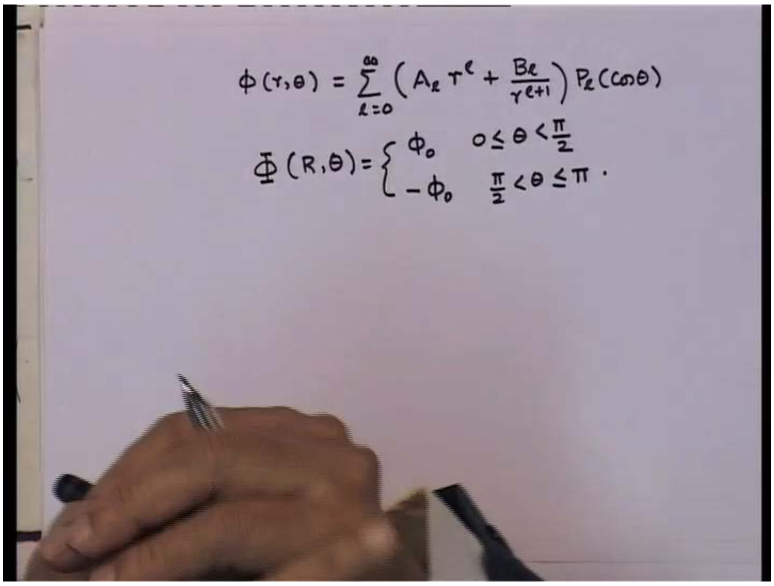
So, with the potential that we have derived in just a few little while back, I must add the potential due to a potential point charge located at the origin. Now, that would be the potential, if this sphere had a charge and is put in an electric field of strength E 0 along this z axis.

(Refer Slide Time: 14:46)



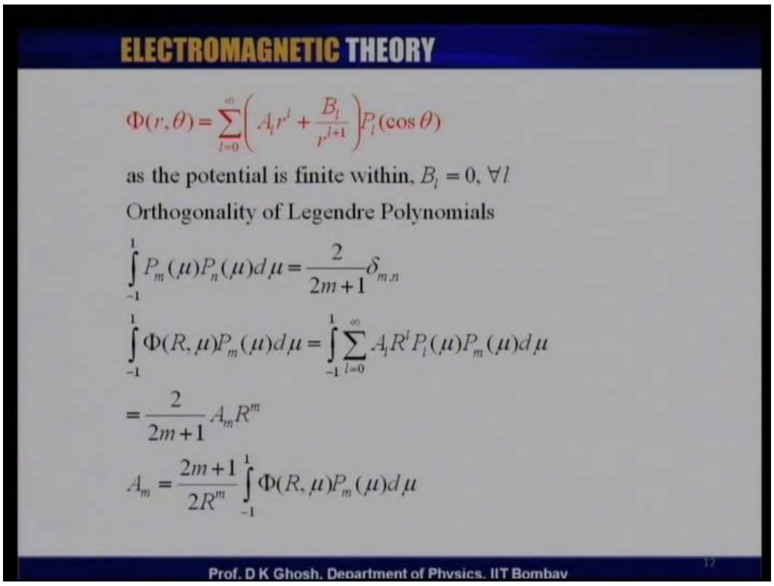
Let me give you another example. In this case, I have two conducting hemispheres. I have just pictorially separated them. The northern hemisphere is connected to a potential phi 0 and while the southern hemisphere is connected to a potential minus phi 0. Now, what we are interested in is, what is the potential inside the sphere? Now, look at this. I have got phi 0, the general expression for phi R theta is what I started with last time.

(Refer Slide Time: 15:30)



What we have is this, phi of R theta that is equal to sum over l is equal to 0 to infinity. This problem also has azimuthal symmetry. So therefore, I have got a l R to the power l plus B l by R to the power l plus 1 times the legendary polynomial P l of cos theta. The boundary condition on the potential R phi at radius R theta and that is equal to phi 0. In the northern hemisphere namely, 0 less than theta less than phi by 2 and is equal to minus phi 0 for phi by 2 less than theta pi. Now, notice that, inside the potential must be finite and therefore, I cannot have a term, which is proportional to either 1 over R or any of its power.

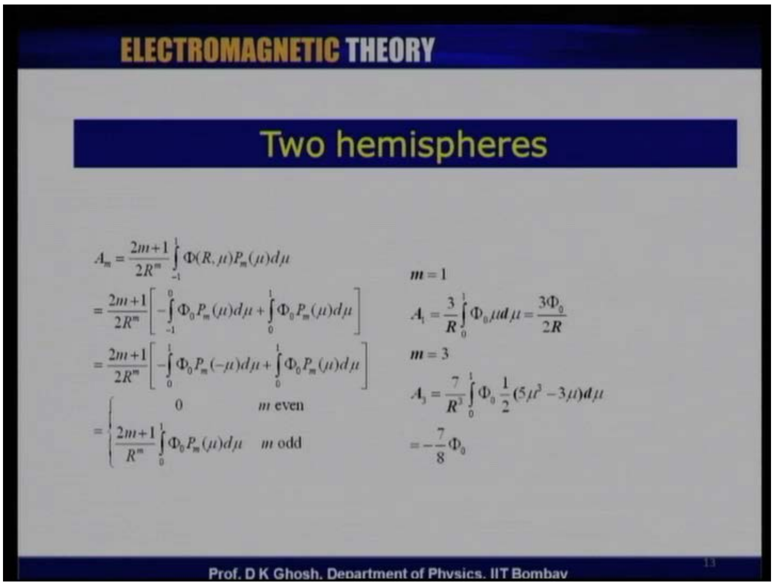
(Refer Slide Time: 16:58)



So, as a result, I do not have actually the B l term at all. All B l are 0. Now, how do I handle this problem? In other words, I need to determine what are various a l's. Now, in order to do that, you recall the Legendre polynomials or Orthogonal polynomials. So, if you take for example, P m mu P n mu integral, unless m is equal to n, it has to be equal to 0. That is the reason I have put this delta chronicle delta m n and the normalization constant, normalization happens to give me 2 by 2 m plus 1. Now, if you use this, you can find out a formal expression for what A l is by simply multiplying both sides of this expression, which you recall, I do not have B l any more by P m of mu and integrate.

Now, if you do that, then by orthogonality of the Legendre polynomial, this expression on the right hand side gives me delta l m into 2 by 2 m plus 1. Of course, since l must be equal to m, I get an a m and R to the power m. So, as a result, the coefficient a m that is there is given by 2 m plus 1 by 2 capital R to the power m and an integral from minus 1 to plus 1 of phi with P m mu. Now, this is the integral that I have to evaluate from minus 1 to plus 1 and you remember that, I know the value of phi in the northern hemisphere and in the southern hemisphere.

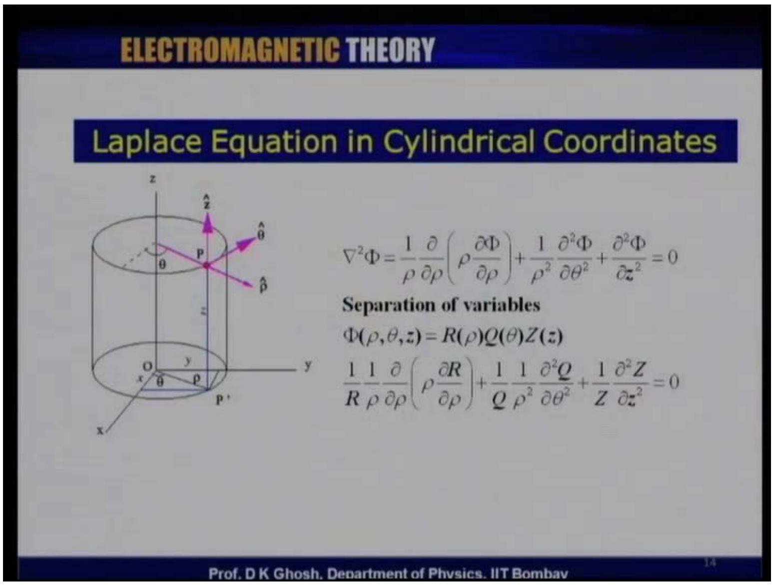
(Refer Slide Time: 18:33)



Now, I could, as an illustration, let me sort of tell how does it work. So, a m is given by this expression and from minus 1 to 0, the potential is minus phi 0. So therefore, phi has been replaced with minus phi 0. From 0 to 1, the potential is just plus phi 0. So, this integral is split into two terms and you can trivially find out what do you get. Now, once you look at and reorganize this and convert both the integrals to 0 to 1, and that can be trivially done by taking mu going to minus mu in this expression. What do you find is, all even a l's must be equal to 0 and the odd a l's survive, which is given by this expression here.

Now, we can, as an illustration, calculate a few values of a l. For instance, if you take a 1, I know that P 1 of cos theta is just cos theta which is mu. So therefore, this is just doing an integration over mu and this gives me a 1 equal to 3 pi 0 over 2 R. If you take m is equal to 2 is absent. Now, if you take m is equal to 3, you have essentially a cubic 5 mu cube minus 3 mu by 2. So, substitute that and once again you can show that a 3 is minus 7 by 8 phi 0 etcetera, etcetera. So, we get this as a series of terms with only polynomials, where Legendre polynomials of odd degrees being there.

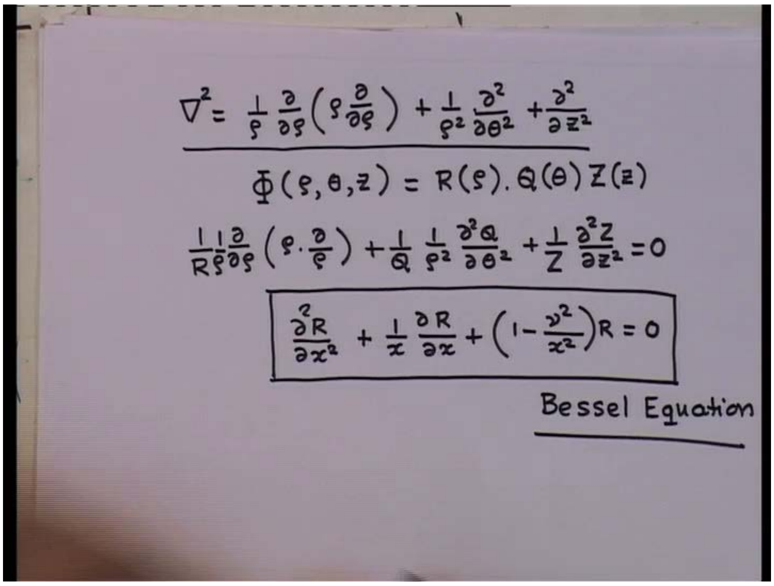
(Refer Slide Time: 20:25)



Now, that brings us to an end of our discussion on these solutions of Laplace equation in spherical coordinates. Now, what do we wish to do now is to look at the solutions of the Laplace equation in cylindrical coordinates. Now, recall the cylindrical coordinate. What we have is the distance, it is basically, recall that cylindrical coordinate is nothing but a polar coordinate in two dimension R theta, which we write as rho theta here and a height, which is same as the Cartesian z direction.

So, this illustrates that where is my R theta phi. So, I have a point P here and the x y plane is here. This point P, from P, I drop a perpendicular on to the x y plane and the foot of the perpendicular is what makes an angle theta, which is the same as this angle theta. The height of that is z and the distance from the origin is given by rho.

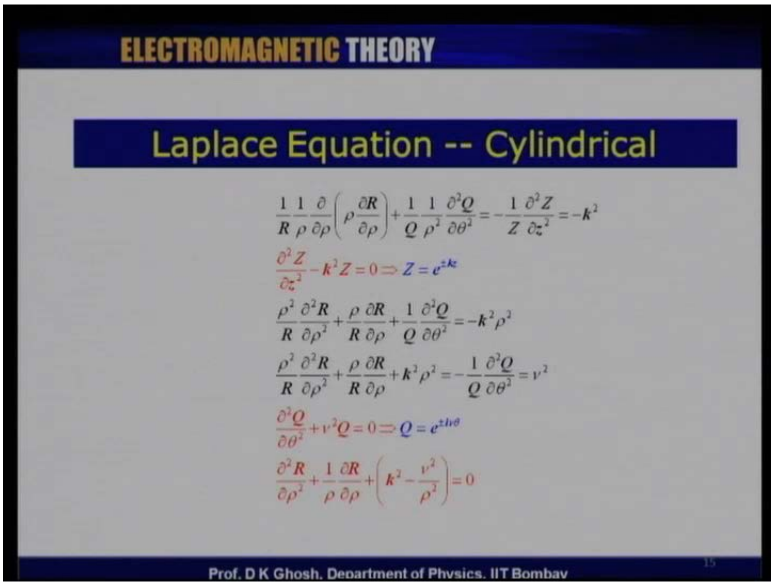
(Refer Slide Time: 21:47)



Now, in cylindrical coordinates, the Laplacian is written as del square equal to 1 over rho d by d rho of rho d by d rho plus 1 over rho square d square over d theta square and of course, the usual d square over d z square, which comes from the z direction. So, if you look at the Laplace equation here, I get del square phi is equal to this quantity is equal to 0.

Now, like the case of the spherical polar, I can solve this problem by first talking about a separation of variables. Remember the variables here are rho, theta and z. So, I write this as a function R of rho, a function q of polar angle theta and a function capital z of the z variable. Now, substitute this and if you substitute this, you can write down 1 over R. Remember what we do. We substitute this into this equation and then, divide the entire equation by R q and z and this gives me this form here. This form is 1 over R d by d rho of rho d by d rho; there is one more R over there and 1 over rho there plus 1 over q 1 by rho square d square q over d theta square plus 1 over z d square z over d z square equal to 0.

(Refer Slide Time: 23:49)



Our principle of solving this is identical. What you do is, you first take the z part to the right hand side and you get minus 1 over z d square z over d z square. Since, the left hand side is a function of rho and theta only, whereas the right hand side is a function of z, for all rho theta and z, in order that such an equation may be satisfied, I must have each one of the term equal to a constant. Let me put that constant to be equal to minus k square.

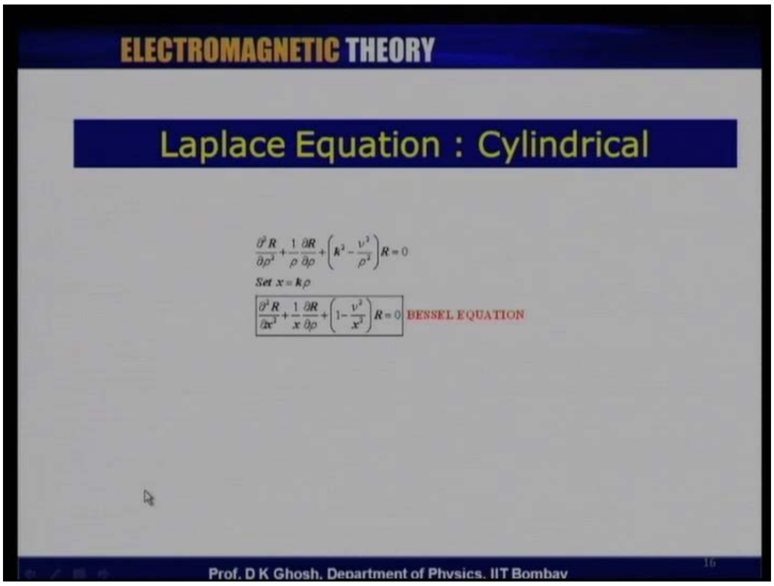
Now, so, if you look at, now the z equation, the z equation then tells me that d square z over d z square minus k square z is equal to 0, which tells me that the solutions of the z equation is simply exponential of plus or minus k z. What is k? We have not said. We have not even said whether k is real or imaginary. If it were to be imaginary, this would of course be the sin cosine functions. Let us rewrite this now. By putting this k square there, write this equation for the rho theta. Now, what I have done is to multiply all over by rho square and this has given me an equation of this type. You notice that, what I have done is to also open up this d by d rho of rho d R by d rho and if you do that, you can get an equation of this type. Now, once again remember that this equation can be split into and a term, which depends only on rho and a term which depends only on theta.

So, bring this to that form. So, I get this as my left hand side; rho square by R d square R by d rho square plus rho by R d R by d rho plus k square rho square equal to this quantity. Once again, I demand that each one of these terms must be equal to a constant.

This time, I put that constant to be equal to some new square. The equation, theta equation, which is d square q by d theta square plus nu square q equal to 0 must give me q equal to plus or minus i nu theta. Now, remember that this I have done intentionally, so that, I get sinusoidal functions or sin cosine functions. The reason is that my problem must be single value as theta changes from 0 to 2 phi. I must return back to the same solution. I cannot do it if I had taken nu, instead of nu square there, if I have taken minus nu square there, then I will not have trigonometric functions, but I would have had hyperbolic function. But in order that the potential is single valued function for theta, so that, potential value at theta plus 2 pi is the same as its value at theta equal to 0, I must choose these nu's is to be real, so that, I get plus or minus i nu theta.

So, that is my solution for q. Now, once I have substituted that into the R equation, my R equation becomes like this; d square R over d rho square plus 1 over rho d R by d rho plus k square minus nu square by rho square is equal to 0.

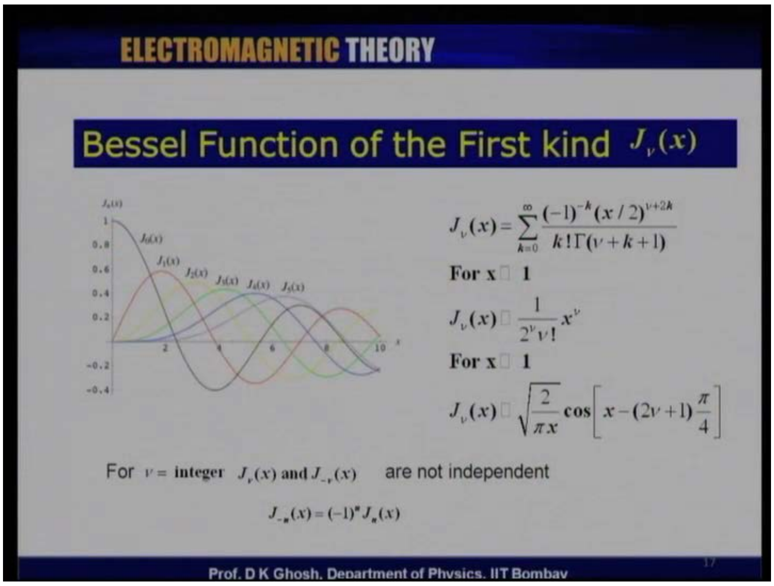
(Refer Slide Time: 27:30)



Now, this equation, I am not going to be actually deriving its solution. But let me first bring it to a more familiar form. Suppose, I check take the variable to be x equal x, which is equal to k times rho. Now, this equation becomes d square R over d x square plus 1 over x d R by d x plus 1 minus mu square by x square R equal to 0. So, this equation is known as the Bessel equation. So, let me rewrite this equation, which is d square R by d x square plus 1 over x d R by d x plus 1 minus mu square by x square times R equal to 0.

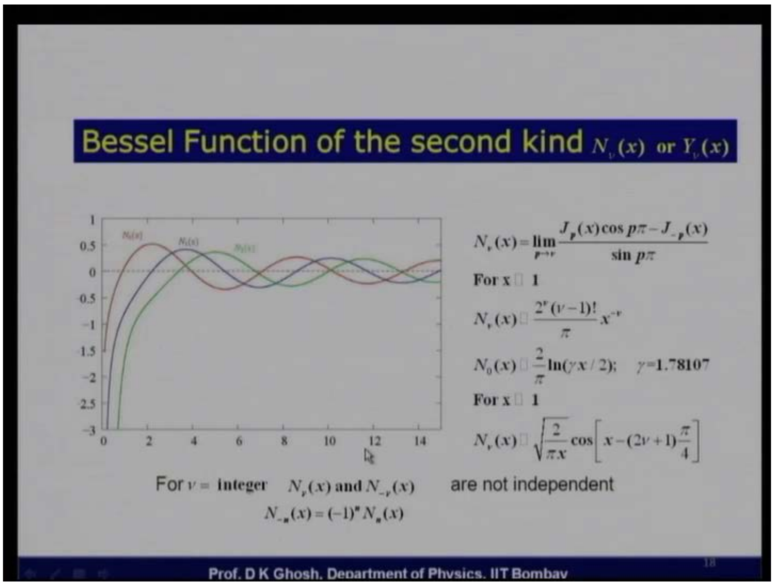
Recall that the single validness of the q requires that a nu must be an integer. Now, this equation is well known in mathematics and you will probably come across it in many other areas as well. This is known as the Bessel equation and the solutions of Bessel's equation are Bessel functions. Now, let me, I will not be able to spend time on these detailed nature of the solution, but let me sort of make some remarks on what the solutions look like.

(Refer Slide Time: 29:22)



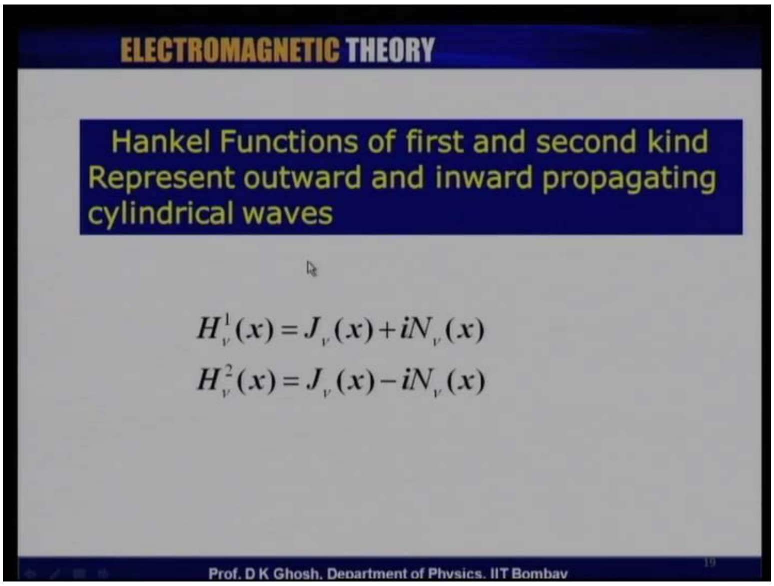
Firstly, there are, this is second order differential equation. So, as a result, there are two solutions. Now, it turns out that when nu is an integer, the solutions are not really independent. The solutions are Bessel's functions. Now, the Bessel function for x, this is a general expansion for the Bessel function. It is a power series on x and J nu of x behaves like this. Now, for x much less than 1, that is very close to the origin J, mu of x goes as x to the power nu. That is, other than for nu equal to 0, where x to the power 0 is equal to 1, all other Bessel functions at origin, they become 0. Now, for large distances, they are oscillating function, cosine functions and as you can see it, all these functions oscillate.

(Refer Slide Time: 30:26)



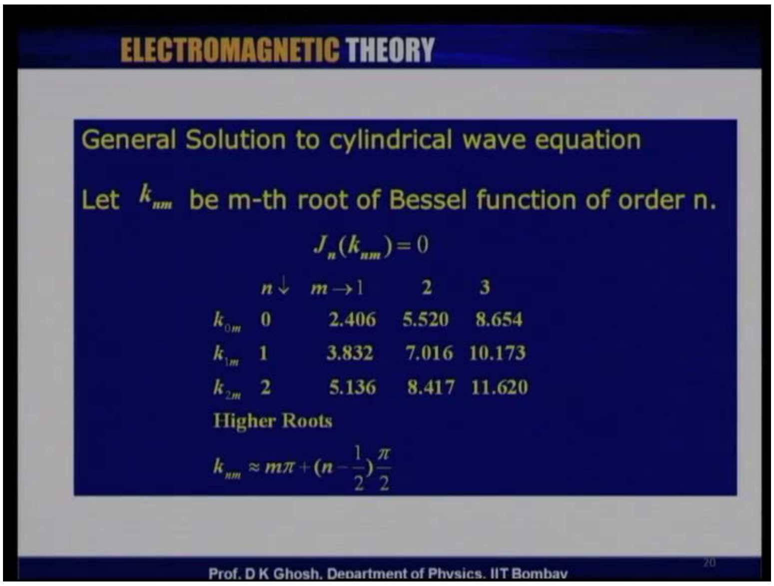
Now, the second function, which is also the second solution of the Bessel function is what is known as n of nu, it is given a name Neumann function. It is also called Bessel function of the same second kind. Now, it turns out that these functions, at the origin they are divergent, but at long distances, again they are sinusoidal functions.

(Refer Slide Time: 30:57)



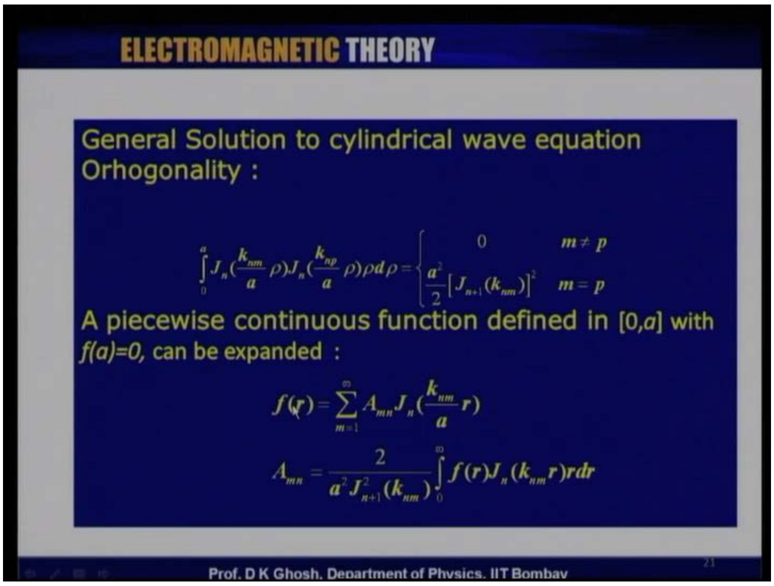
One sometimes expresses the solutions in terms of linear combinations of Bessel functions of first kind and second kind or that is Bessel function and Neumann function and these are then called the Henkel functions of first kind and second kind.

(Refer Slide Time: 31:17)



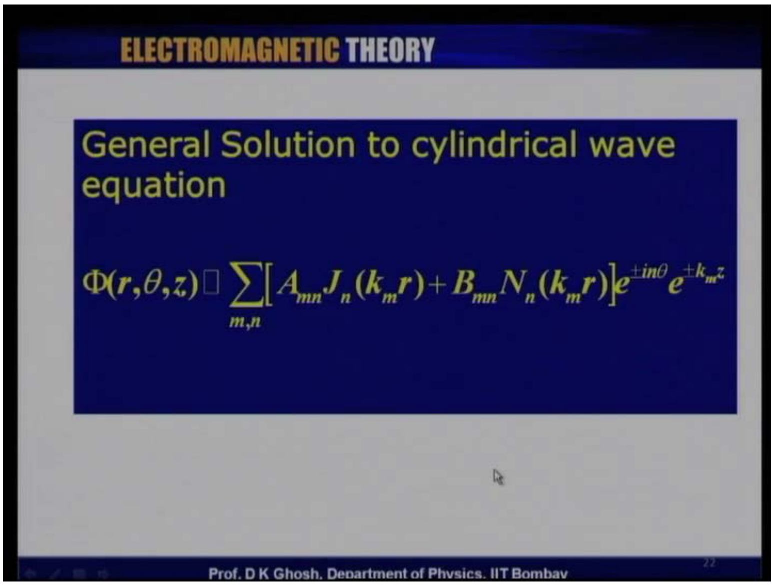
Other than knowing the nature of the solution, I am not really going to be talking about any detailed properties of the solution. Now, let us look at what is the general solution of this cylindrical wave equation. Now, I had shown you that the Bessel functions oscillate. In order words, Bessel functions have several 0's. Just look at this picture. So, you notice that this is, take any degree for example, this is the Bessel function of degree 1. You notice it becomes 0 here, then 0 here and 0 there etcetera. Now, these are 0's of Bessel functions, which are very well documented. You can see that, if the n is equal to 0 or mu equal to 0, the Bessel functions, roots of the Bessel functions are at 2.406, 5.520 to 8.654 etcetera and this table gives you the locations of the 0's of the Bessel functions. For larger values, it sort of goes like this.

(Refer Slide Time: 32:21)



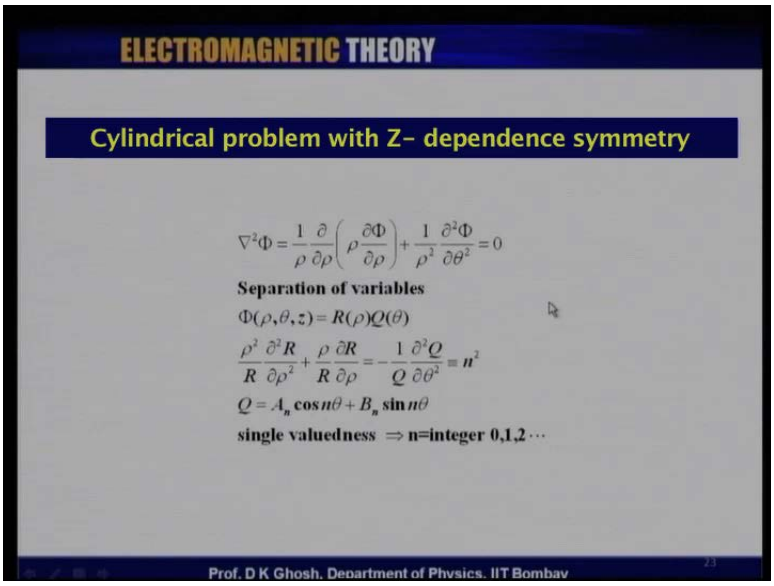
Bessel functions also satisfy some orthogonality property. That is, if you take the product of J m with J n and integrate over rho d rho, then you get 0, unless the degree of this Bessel functions are the same. When they are equal, then we get this expression here. Now, any piece wise continuous function, which is defined in the range 0 to a, such that f of a equal to 0, can be expanded in terms of Bessel functions. Like the way we did with the Legendre polynomials, I can use this orthogonality property and this form of the function f of R to determine what the a mn's are in terms of an integral over an f R and the product of a Bessel function.

(Refer Slide Time: 33:27)



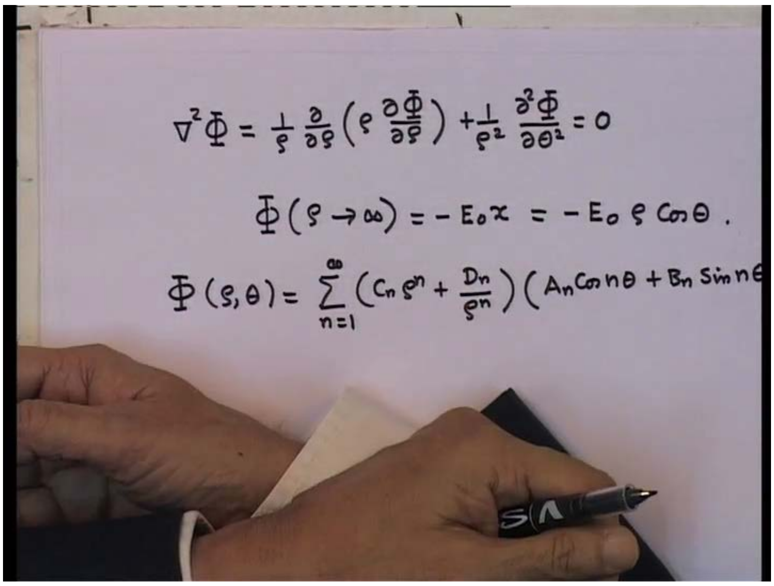
As a result, the general solution to the cylindrical wave equation has other than E to the power plus minus i n theta, and that is the solution for q and E to the power plus minus k m z, which is the solution for the z part. The other part, the radial part or the radial part is given by linear combination of Bessel function of the first kind and the second kind.

(Refer Slide Time: 33:59)



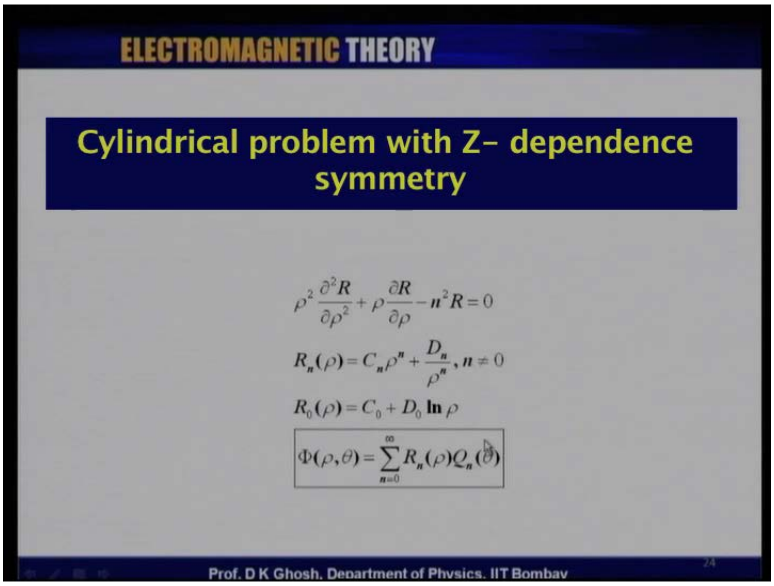
Once again, in doing applications, I will take a rather simple case. I will assume that my problem has symmetry and its solution has no z dependency. If that happens, my del square phi becomes like this.

(Refer Slide Time: 34:23)



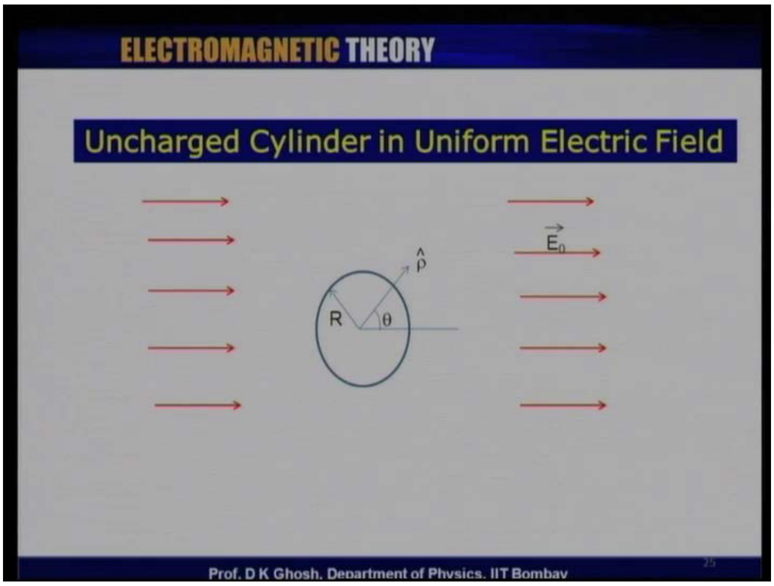
So, let me rewrite this del square phi. Since, there is no more z dependence, this is equal to 1 over rho d by d rho of rho d phi by d rho plus 1 over rho square d square phi over d theta square equal to 0. This is rather simple to solve. By doing a separation of variable, you find out that this is equal to that and as we talked about earlier, the solution for q has to be sin cosine function, with n being an integer and you substitute it here.

(Refer Slide Time: 35:06)



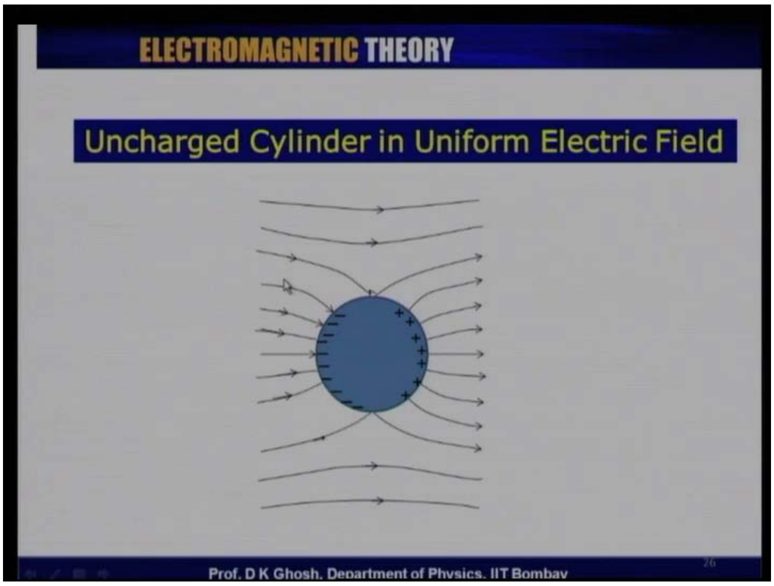
This is the equation that you get. Remember that when z was there, I had obtained a Bessel equation. But this equation is rather simple because, this equation has a power series solution. You can check that. If I take a combination of rho to the power n and 1 over rho to the power n, this equation is satisfied. The only exception that exist if the value of n is equal to 0, then I get rho square d square R by d rho square plus rho d R by d rho equal to 0, which is a first order differential equation in d R by d rho. It can be very trivially solved. The solution in that case is R 0 of rho is c 0 plus a logarithm of rho. So therefore, the expression for phi is R n of rho, which in general is a power series in rho and 1 over rho times q n of theta, which is a linear combination of cos theta and cosine theta and sin n theta.

(Refer Slide Time: 36:18)



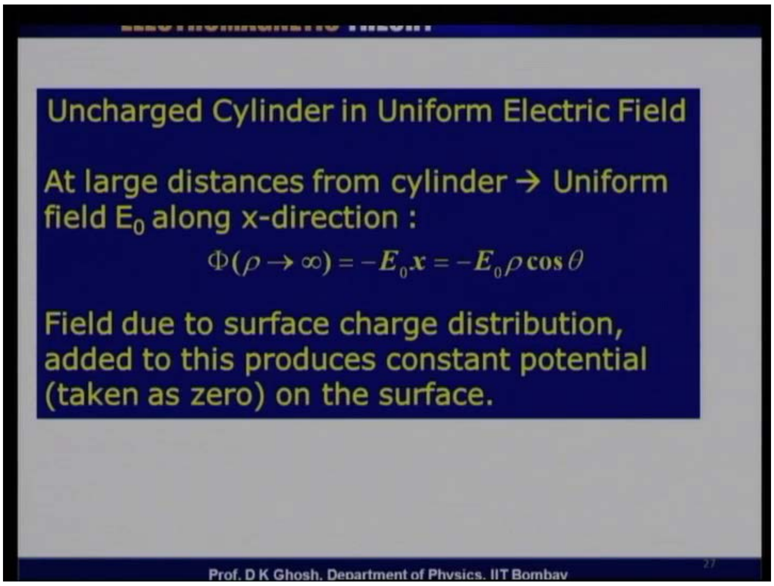
Now, to look at a specific example, let me take a problem very similar to what we did for the spherical problem. There, I had put in a sphere in an electric field, which is otherwise uniform. That is, uniform electric field at large distance. Now, this time, what I am going to do is to put a cylinder, so cylinder is here, perpendicular to the plane of the screen and this is the electric field that I have got.

(Refer Slide Time: 37:01)



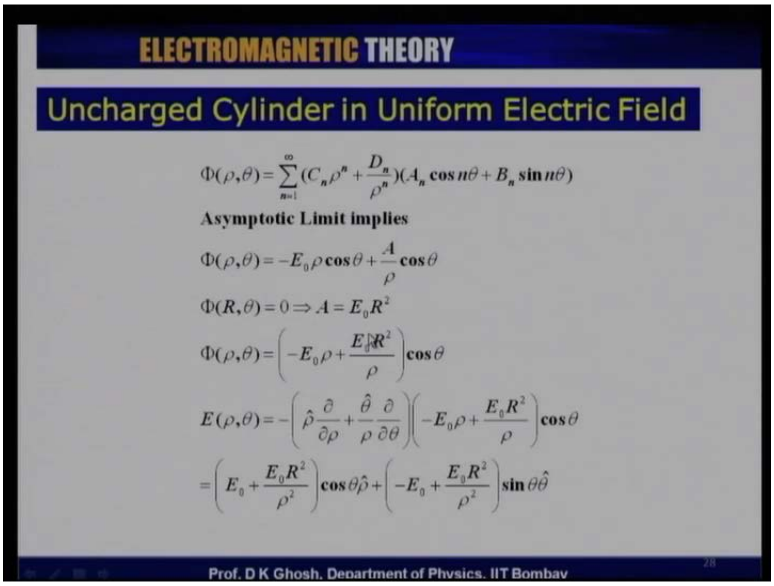
So, I put in a cylinder there and let us look at the modifications that I have. Now, physical picture is very much the same, because I have a conductor, the lines of forces must arrive at or leave from the conductor surface normally. At large distances, I must have the potential corresponding to an electric field.

(Refer Slide Time: 37:26)



So, let us, the principle is very similar. So, once I say, now this time I have taken the electric field at large distances along the x direction. Which means, at large distances, my potential rho going to infinity is minus E 0 times x, which is nothing but minus E 0 times rho times cosine of theta. Now, once again, I must cancel this potential on the surface of the sphere. I can take the potential on the surface of the cylinder to be a constant, which I have taken as 0. So, a charge distribution must now arise on the surface of the cylinder, which will cancel the potential, in which I have put in this charge distribution.

(Refer Slide Time: 38:33)



So, let us look at how does it go. So, I have the expression for phi function of rho theta and we have said this is n is equal to 1 to infinity of c n rho to the power n plus d n by rho to the power n times a n cos n theta plus B n sin n theta. Now remember that, only if I had a line charge, I would have expected a potential with a logarithmic term. Since, there is no line charge here, I have not taken n is equal to 0 term, which we had shown that gives me a logarithmic solution. So, that is why this sum goes from 1 to infinity.

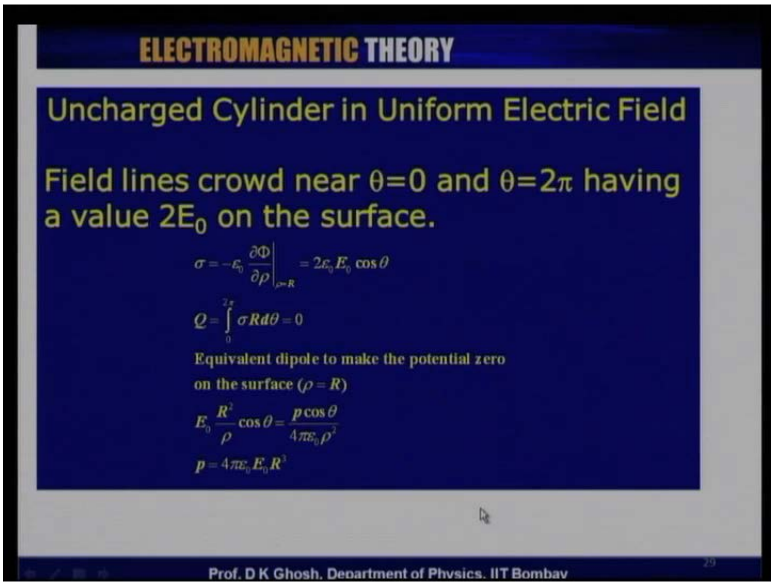
Now, what is my asymptotic limit? My asymptotic limit is that the potential goes as minus E 0 rho cos theta. Now, if the potential goes as minus E 0 rho cos theta, it tells me that all the B n's must be 0, because I do not have a sin theta anywhere. So, in principle, I can have a n cos n theta. But however, I do not have any cos 2 theta 3 theta terms. So, as a result, the only term that I can have is a 1 cos theta. Now, this tells me that since this is just the cos theta term, so, I take n is equal to 1. So, I get c rho a 1 cos theta and for convenience, I have said, c 1 times a 1 must be is equal to E 0.

So, this is what I get there. Similarly, I have said c times, a times d must be equal to another constant a. So therefore, the potential form is like this. Now, I now need to determine the constant a, which is the only unknown quantity there. Now, this is done by realizing that the potential on the surface of the cylinder is equal to 0. So, if I take small R rho is equal to capital R, then I get phi R theta is equal to 0, which tells me that a is equal to E 0 R square. So, phi of rho theta is equal to minus E 0 rho plus this constant E

0 R square by rho times cos theta. So, I got an expression for the potential and now, I need to find out the potential on the electric field that is there.

So, let us look at the electric field. The electric field is negative gradient of this potential. The negative gradient, gradient is rho d by d rho plus 1 over rho d by d theta along the unit vector theta. This a fairly straight forward third differentiation to be done. Now, when you do this differentiation, you find a term which is proportional to cos theta, which is E 0 plus E 0 R square by rho square cos theta times rho and a term, which is obtained by the differentiation of this cos theta, which gives me a sin theta term, which is minus E 0 E 0 R square by rho square.

(Refer Slide Time: 42:18)



Now, notice what is happening. Once I know the electric field, I can find out what is the normal component of the electric field and that will give me the charge distribution. So, notice that the field lines crowd near theta equal to 0 and theta equal to 2 phi. If you take theta is equal to 0, this term of course goes and you will be left with cos theta, which is equal to 1. If you put rho is equal to R, that is on the surface of the sphere, you notice that the field strength becomes twice E 0. So, that is what we get on the surface of the cylinder. The charge density on the surface of the cylinder is given by minus E 0 normal component, namely d phi by d rho at rho is equal to R. It gives me 2 times epsilon 0 E 0 cos theta. Now, once again, I can find out what is the total amount of charge induced on the surface. q is the theta integral from 0 to 2 phi of sigma R d theta and the sigma is proportional to cos theta. So therefore, this is equal to 0.

Now, notice one interesting thing. Like the case of the sphere, I have a potential expression, where I have minus E 0 rho cos theta which was already there and this is modified by a term, which is proportional to 1 over R. So, what we do is this. We are now asking the question, what is the equivalent dipole moment that can be put at the origin, so that, the potential will become 0 on the surface. Now, this is simply done by equating that extra term that we had, with the expression for the potential due to an electric dipole on the surface of the cylinder.

So, put rho is equal to R and that gives you the equivalent dipole moment is 4 phi epsilon 0 E 0 R cube. So, this is the solutions of Laplace equation in the cylindrical coordinates. There are more formal ways of solving Laplace’s equations. Some of them we will be talking as we go along. But what I would do next time is to tell you that, instead of just taking up mathematical ways of solving as we have been doing, there are some beautifully inutility ways of solving these problems. One of them is what is known as a method of images, where you replace the original problem by an equivalent problem which you intuitively get. Then, using the fact that, if I have obtained a solution of the Laplace equation corresponding to a given boundary condition, then the solution that I obtain must be unique. This is because of the uniqueness of the solution.