Advanced Design of Steel Structures
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Lecture - 40
Shear center - 3

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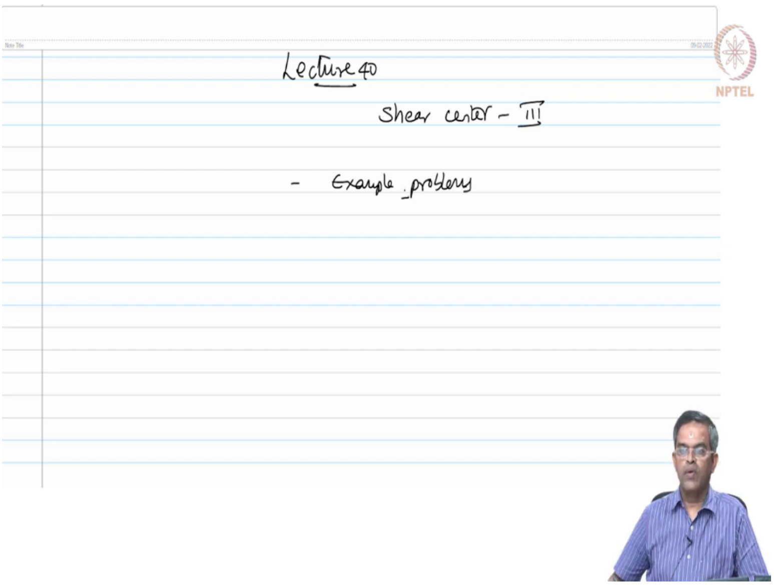

Friends welcome to lecture-40 of the course Advanced Steel Design. We are working on estimating more examples on Shear center. I will call Shear center lecture-3. In this lecture we will learn more example problems of locating shear center.

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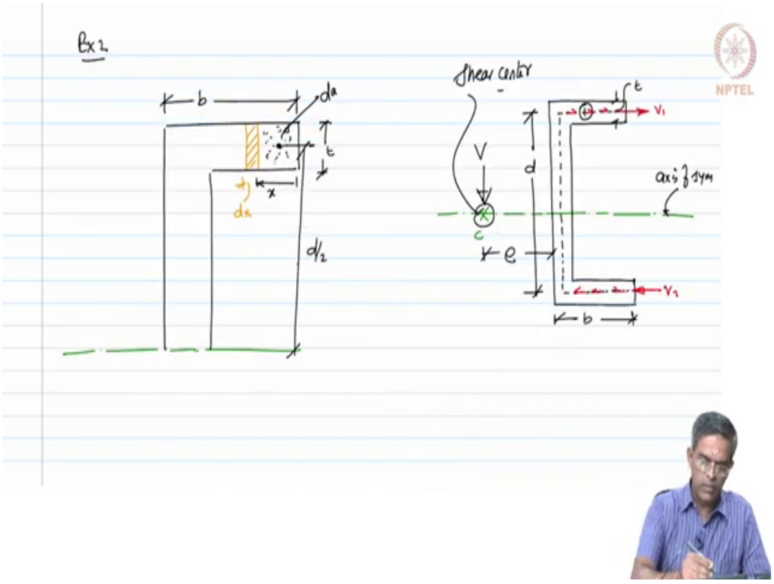

So, example 2. We will take up a channel section. We know the shear flow this is going to be. So, this is I calls V_1, I call this as V_2 and we also know that this section has an axis of symmetry.

So, the shear center will lie on this axis let us say somewhere here. Let us assume that the load is applied at the shear center to avoid twisting of the cross section. We call the eccentricity or offset of the shear center from the phase of the flange or e. Let us say the depth of the web is d and the total breadth of the flange is b. Let us say the section has a uniform thickness t throughout.

So, a thinned section and we already know the green line represents axis of symmetry. Shear center will have to lie this is the shear center. We will have to lie along this line of symmetry. Let us take up element 1 this is my element 1, and draw an enlarge view of this.

Let us cut a section at a distance x measured from the tip and this is my da. Let us call this thickness of the strip as dx and the c g of this da from the shear center is $\frac{d}{2}$ from the figure, $\frac{d}{2}$ from the figure. And we know that this is b breadth of the flange. Let us copy this figure and move to the next page.

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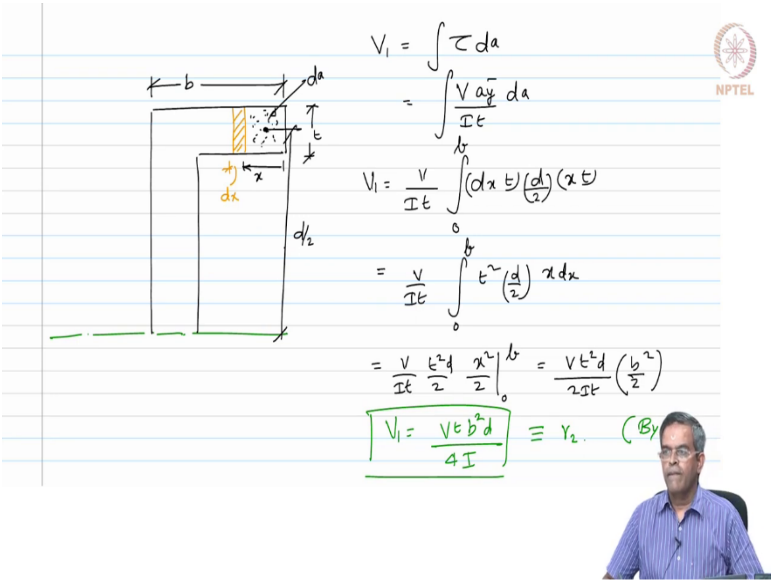

So, now let us write V_1 will be

$$V_1 = \int \tau \, da = \int \frac{Va\overline{y}}{It} \, da$$

$$a = tx$$

$$\overline{y} = \frac{d}{2}$$

$$da = (dx)t$$

$$V_1 = \int_x^b \frac{V}{It}(xt)\frac{d}{2}dx(t)$$

$$= \int_x^b \frac{V}{It}(xt)^2 \frac{d}{2}dx$$

$$= \frac{Vt^2 d}{2It}\int_x^b x \, dx$$

$$V_1 = \frac{V\,dt^2}{2It}\left(\frac{b^2}{2} - \frac{x^2}{2}\right)$$

So, I can now write V_1 as

$$V_1 = \frac{Vdt^2}{2It}\left(\frac{b^2}{2}\right)$$

Now we can also say by symmetry is equal to V_2, is not it from the figure? Further look at this figure I will copy this figure again.

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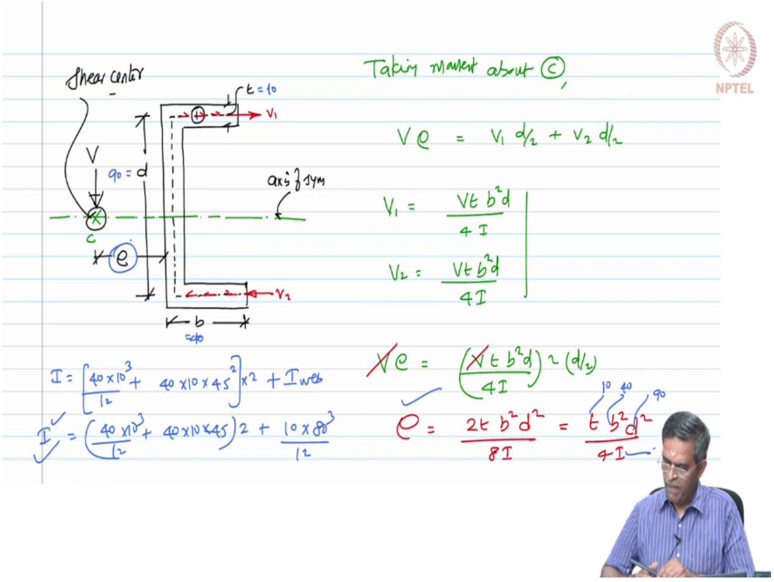

We can now say taking moment about c

$$Ve = V_1\frac{d}{2} + V_2\frac{d}{2}$$

$$V_1 = \frac{Vtb^2d}{4I}$$

$$V_2 = \frac{Vtb^2d}{4I}$$

$$Ve = \left(\frac{Vtb^2d}{4I}\right)^2\frac{d}{2}$$

$$e = \frac{2tb^2d^2}{8I} = \frac{tb^2d^2}{4I}$$

So, for a given section if I know these values let us say for example, b is 40 and d is 100 and t is 10.

So, I can find quickly I for this which will be

$$I = \left[\frac{40\times10^3}{12} + 40\times10\times45^2\right]\times2 + I_{web}$$

$$I_{web} = \frac{10\times80^3}{12}$$

So, I get I now. So, now, in this equation I know t, t is 10 mm. I know b, b is 40 mm. I know d, d is 90 and I know I which is here. So, I can find e. And e will be the offset of the shear center from the cg. So, friends we have a very easy example these examples were easy because they had at least one axis of symmetry. So, the shear center was located in that axis of symmetry.

So, it is very simple that if you have a problem where the section has got a symmetric axis, we can easily locate the shear center. Let us take up one more example which is again simple, but slightly tricky in terms of its equation. Let us try to do that. We will take another example.

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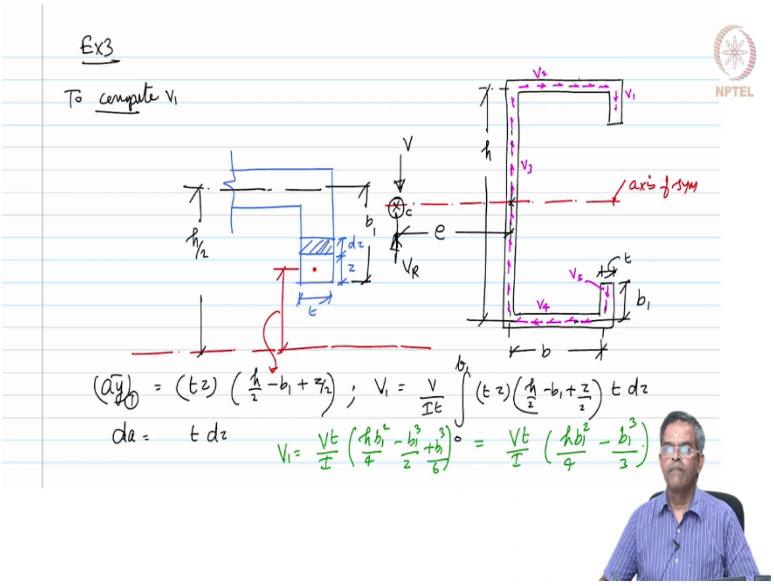

We will call this example 3. We say it is a very common section being used in marine structures a thin section having uniform thickness. So, let us say the dimensions are marked as shown in the figure this is b.

Let us say this dimension is b_1, the overall depth is h all centroid center, uniform thickness t throughout, this section has an axis of symmetry. So, from this point let us mark the shear center somewhere here and we call this offset as e. So, let us say the V_R and V e are acting here to avoid twisting of the cross section.

So, now let us mark the shear flow. We call these forces. So, this is V_1 and this is V_2 and this is V_3 this is V_4 and this is marked as V_5. Let us now start computing these internal shears. So, I want to compute V_1. So, I am drawing an enlarged view of this. Let me cut a section of thickness dz at a distance z from here and I am marking the centroidal axis or the axis of symmetry from here. And we know this distance is h by 2 from here, and this was b_1.

So, I have expression for V_1 which will be

$$V_1 = \int_0^{b_1} \frac{Va\bar{y}}{It} da$$

$$a\bar{y} = Q = (tz)\left(\frac{h}{2} - b_1 + \frac{z}{2}\right)$$

$$V_1 = \int_0^{b_1} \frac{V}{It}(tz)\left(\frac{h}{2} - b_1 + \frac{z}{2}\right) dz(t) = \frac{Vtb_1^2}{I}\left\{\frac{h}{4} - \frac{b_1}{3}\right\}$$

Let us try to find V_5. So, let me write this.

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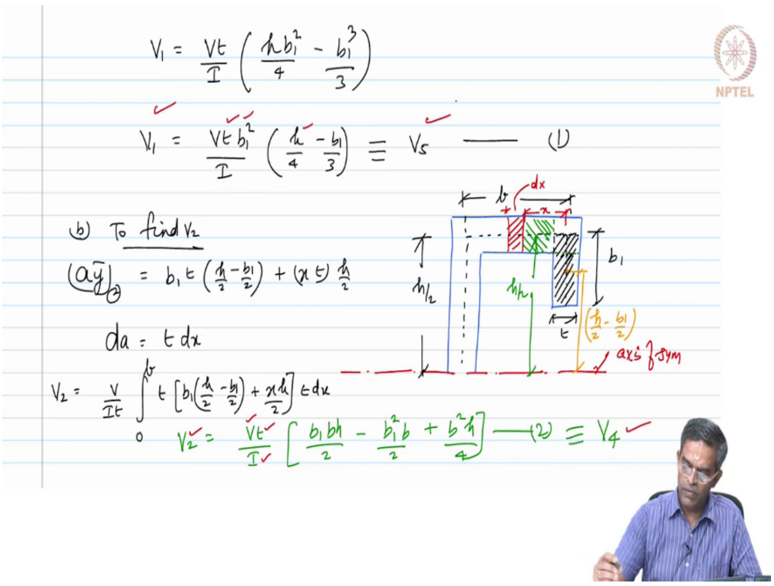

Which is V_1 which is identically same as V_5 because equation number 1.

Now, let us do it for V_2 we will draw a separate figure. This is my axis of symmetry. Let us mark these dimensions. So, thin section. So, I am marking all the dimension centroid. This is b_1, uniform thickness t, this is b and we know this distance is h by 2. Let me cut a strip at a distance x of thickness dx from here. We will also divide this into two parts.

So, now I want to find V_2. Let us find out what is a y bar of piece 2.

$$V_2 = \int \frac{Va\overline{y}}{It} da$$

$$da = tdx$$

$$a\overline{y} = Q$$

$$= (xt)\frac{h}{2}$$

$$V_2 = \int_0^b \frac{V}{It}(xt)\frac{h}{2} t\, dx$$

$$V_2 = \frac{Vt}{I}\frac{b^2 h}{4}$$

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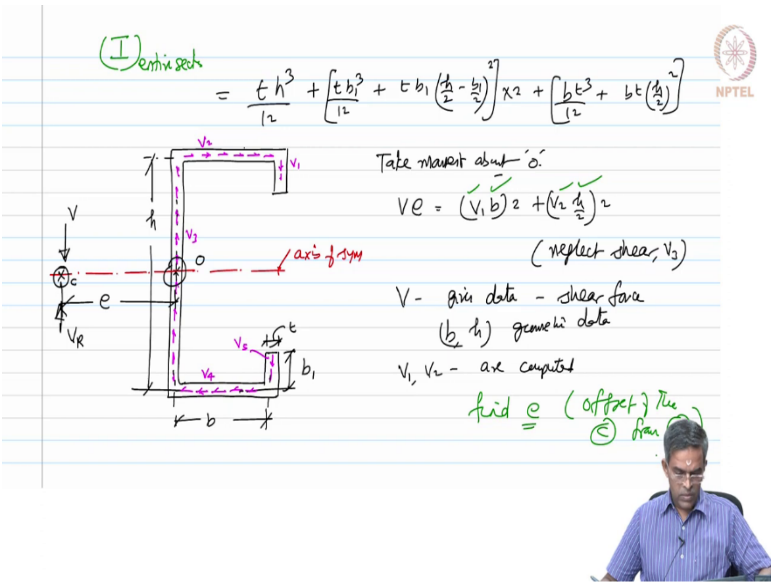

$$I_{entire\ section} = \frac{th^3}{12} + \left[\frac{tb_1^3}{12} + tb_1\left(\frac{h}{2} - \frac{b_1}{2}\right)^2\right]\times 2 + \left[\frac{bt^3}{12} + bt\left(\frac{h}{2}\right)^2\right]$$

So, we have I here now let us take moment about we call this point as O. So,

$$Ve = \left(V_1 b\right)2 + \left(V_2 \frac{h}{2}\right)2$$

We neglect shear V_3.

So, now V is a given data this is a shear acting force acting on the section. b h or geometric data which is known look at this equation. If I know V if I know V if I know t and if I can compute I. I can compute V_2 and V_4. Similarly, if I know t if I know b 1 and h I can compute V_1 and V_5. So, in this equation e_1 V_1 V_2 are computed.

So, I know V_1 V_2 also. So, I can easily find it which is the offset of the shear center from the point O. So, it is very simple. So, we can take a quick example.

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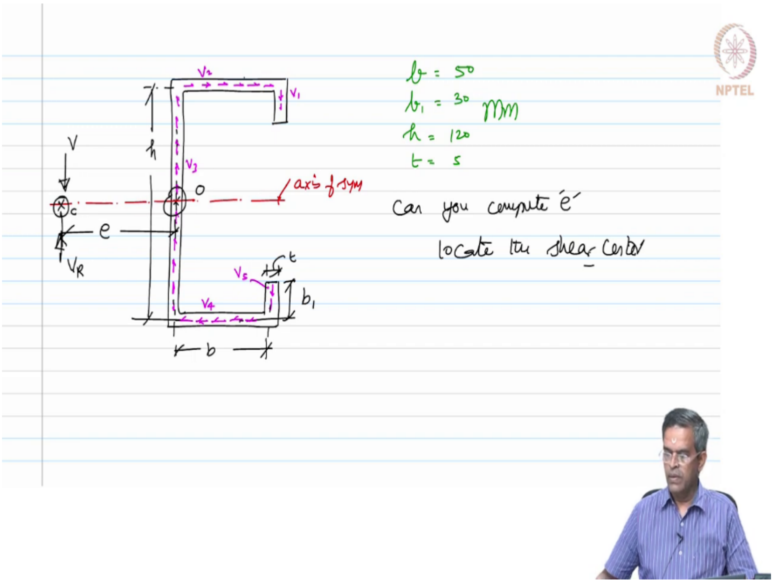

Let us say let b be 50, b 1 be 20 or 30, h be 120 and t be 5 mm. All dimensions are given.

So, I know all dimensions can you compute e? Therefore, locate the shear center. It is a very straightforward problem there is no issue in this. Let us do one more problem.

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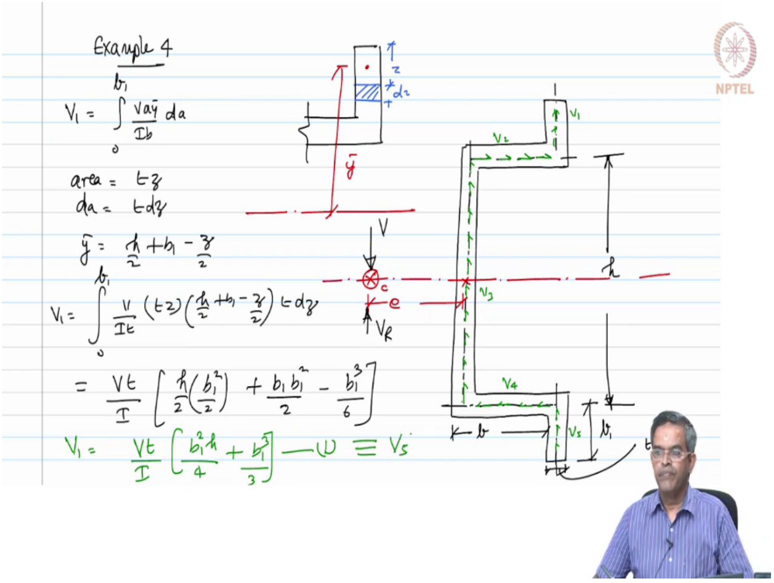

Which is example 2, 3 we will do example 4. So, the cross section is drawn like this. Let me mark. Let us say this dimension is b which is centroid center.

Let us take this dimension as b 1 and this dimension as h. Let us say it is having uniform thickness t throughout. So, we know that this section has one axis of symmetry, this intersection is marked as O and we mark this as my shear center, we mark this x and t as e from here. Let us assume that the shear and the reaction forces are acting at this point to avoid twisting of the cross section.

Let us mark the shear flow now as shown the figure. Say this is V_1, this is V_2, this is V_3 and this is V_4 and this is V_5. Now to locate V_1. We know it is going to vary say this is V_1. So,

$$V_1 = \int_0^{b_1} \frac{Va\bar{y}}{It} da$$

$$area = tz$$

$$da = tz$$

So, let us say area we will consider a strip marked here I will draw this figure separately. So, we will consider a strip at a distance z from here for a thickness dz.

And I want to know the cg of this from here.

$$\overline{y} = \frac{h}{2} + b_1 - \frac{z}{2}$$

$$V_1 = \int_0^{b_1} \frac{V}{It}(tz)\left(\frac{h}{2} + b_1 - \frac{z}{2}\right)tdz$$

$$V_1 = \frac{V}{It}\left[\frac{h}{2}\left(\frac{b_1^2}{2}\right) + \frac{b_1 b_1^2}{2} - \frac{b_1^3}{6}\right]$$

$$V_1 = \frac{V}{It}\left[\frac{b_1^2 h}{4} + \frac{b_1^3}{3}\right] \equiv V_5$$

We will take equation number 1. Identically friends this will also be equal to V_5. Now let us take the next part that is V_2, I will draw that separately.

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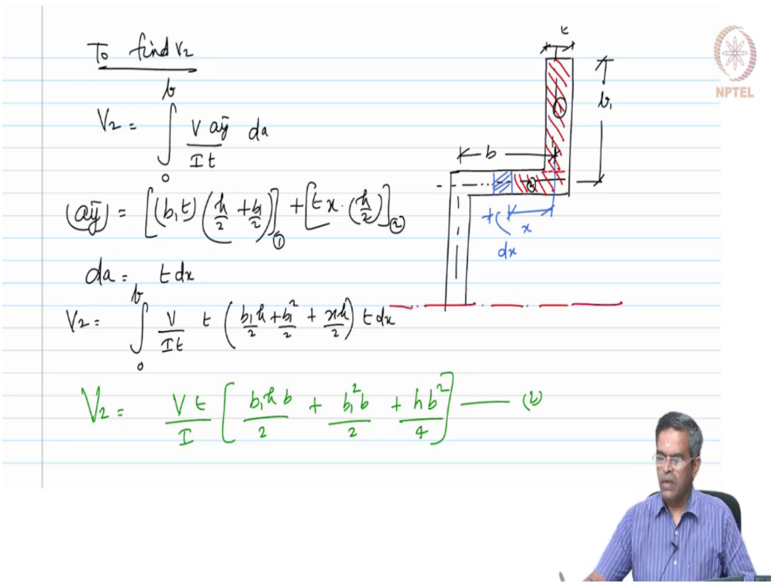

This may line of symmetry. We know this is b_1. I am marking the center line. We know this is b_1 and the thickness is t uniform. I will cut the strip here at a distance x from here and let the thickness be dx.

Let us hatch this portion and I want to find the cg of this. That is what we want to find. So, let us divide this into two parts and for our simplicity let us find V_2. So, we know V_2 is going to integrate you know this dimension is b.

$$V_2 = \int_0^b \frac{V a\bar{y}}{It} da$$

$$(a\bar{y}) = \left[(b_1 t)\left(\frac{h}{2} + \frac{b_1}{2}\right)\right]_1 + \left[tx\left(\frac{h}{2}\right)\right]_2$$

$$da = t\, dx$$

$$V_2 = \int_0^b \frac{V}{It} t\left(\frac{b_1 h}{2} + \frac{b_1^2}{2} + \frac{xh}{2}\right) t\, dx$$

$$V_2 = \frac{V}{It}\left[\frac{b_1 hb}{2} + \frac{b_1^2 b}{2} + \frac{hb^2}{4}\right]$$

Now I want to find the moment of inertia of the whole section let us copy this figure.

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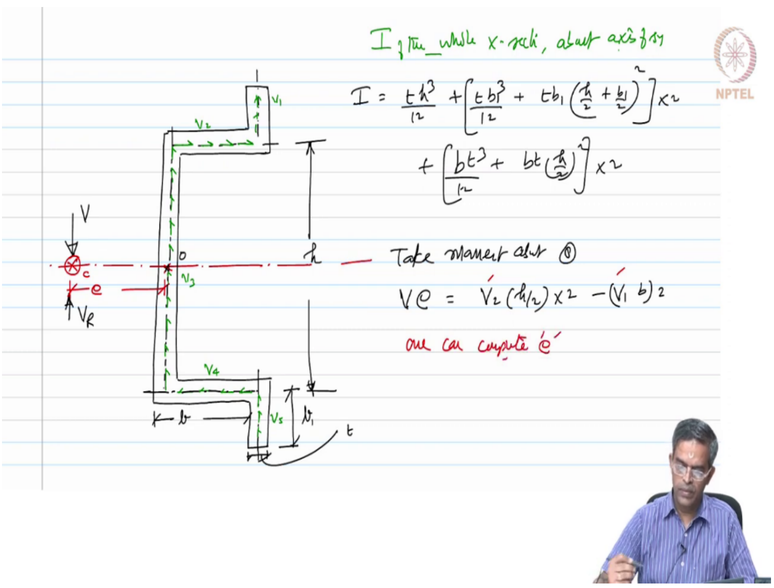

So, we want to find the moment of inertia of the whole cross section about axis of symmetry. So, let us do that.

$$I = \frac{th^3}{12} + \left[\frac{tb_1^3}{12} + tb_1\left(\frac{h}{2} + \frac{b_1}{2}\right)\right]\times 2 + \left[\frac{bt^3}{12} + bt\left(\frac{h}{2}\right)^2\right]\times 2$$

So, I get I. Now let us take moment about O.

$$Ve = V_2\left(\frac{h}{2}\right)\times 2 - \left(V_1 b\right)\times 2$$

So, friends V_2 is a function of I the geometric property. V_1 is a function of I geometric property. So, in this equation V_2 V_1 will have V. So, V goes away and for all the dimensions known one can compute e.

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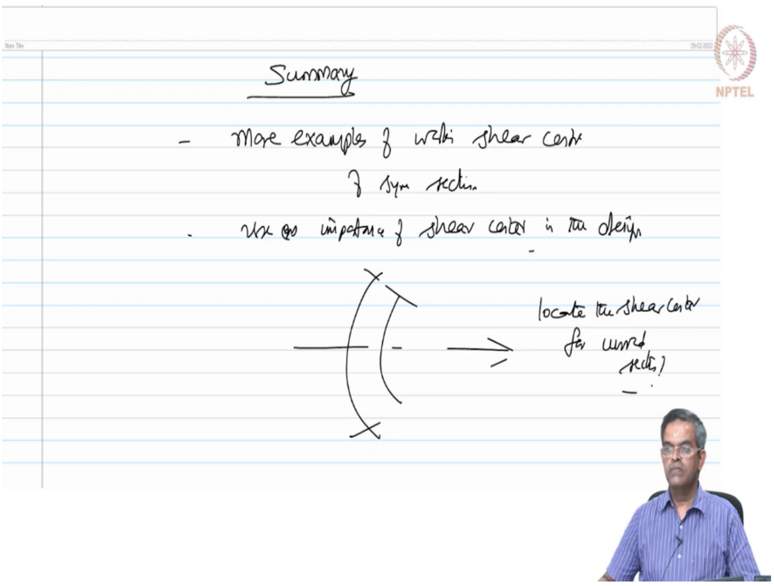

So, friends we have learnt more examples of working shear center of symmetric sections. We have understood the use or the importance of shear center in the design. So, in the next lecture we will start talking about shear center of curved beams. Suppose if we have a section which is curved how do we locate the shear center? That is what we will focus on in the next lecture.

Thank you very much friends have a good day bye.