

**Structural Health Monitoring (SHM)**  
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**Lecture – 33**  
**Vibration based health monitoring scheme – Part 1**

Friends, welcome to the 9th lecture in module 2.

In this lecture, we are going to compare various Vibration-based methods of Structural Health Monitoring, which are essentially used for Damage identification. We will call it lecture number one simply comparison ok.

Let us see the benefits merits and demerits of different methods which are used essentially for damage identification, understanding the crux of which method can be used for what kind of problem ok. We look into that angle very strictly.

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Fundamental idea of vibration-based monitoring

- to detect damage based on the fact that

damage-induced vibration changes structural properties like mass, stiffness, damping.

detection through change is comparison to an undamaged model is useful to detect damage

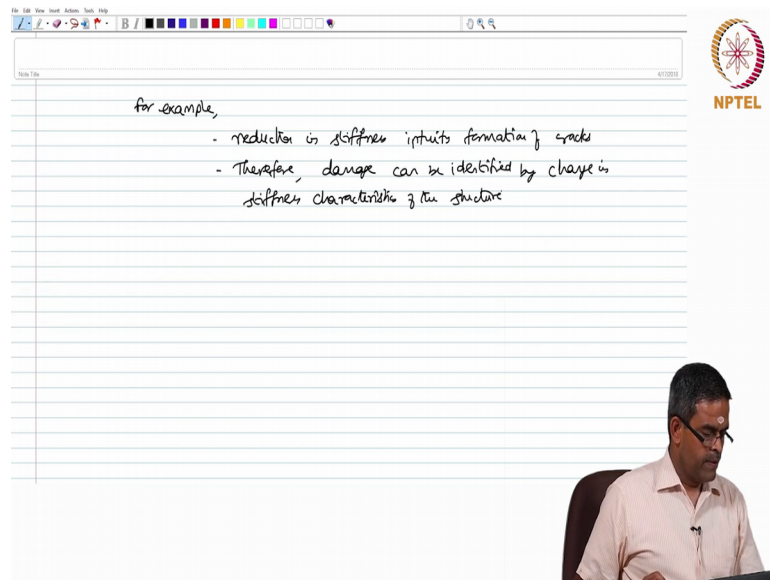
- what are those parameters, which are compared
- frequency
- modal shape
- modal damping

Before answering this question and comparison, let us ask what is the fundamental idea?

What is the fundamental idea of vibration-based monitoring? Let us ask this question because all methods circumscribe on this objective. The fundamental idea is to detect damage, based on the fact that damage induced vibration changes structural properties like mass, stiffness, damping.

These properties are significantly changed; detecting these changes in comparison to an undamaged model is useful to detect damage. So, by that logic what are those parameters which are compared? The essential parameters compared or frequency mode shape and modal damping.

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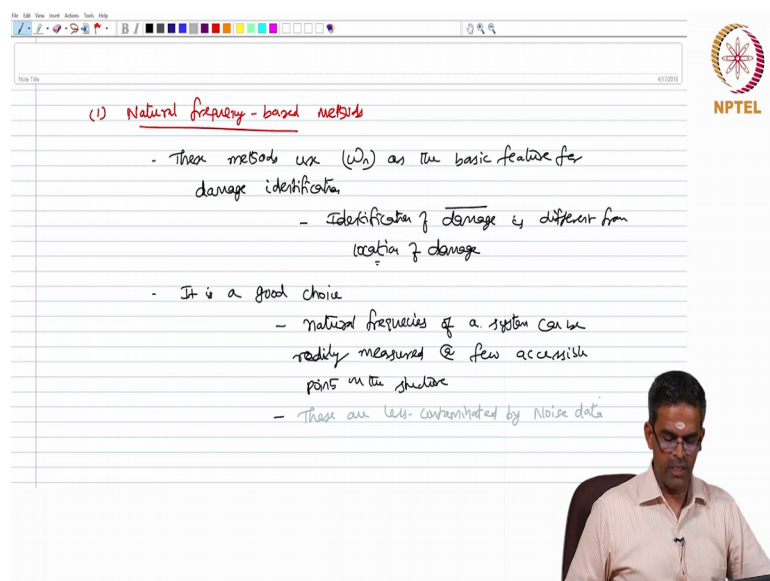
for example,

- reduction in stiffness intuit formation of cracks
- Therefore, damage can be identified by change in stiffness characteristics of the structure

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Let us say for example, reduction in stiffness intuit formation of cracks. Therefore, damage can be identified by change in stiffness characteristics of the structure. Let us see different methods and now, start comparing and contrasting the method within itself.

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The slide shows a digital whiteboard interface with a toolbar at the top. The handwritten text on the board reads:

(1) Natural frequency-based methods

- These methods use  $(\omega_n)$  as the basic feature for damage identification
  - Identification of damage is different from location of damage
- It is a good choice
  - Natural frequencies of a system can be readily measured @ few accessible points on the structure
  - These are less contaminated by noise data

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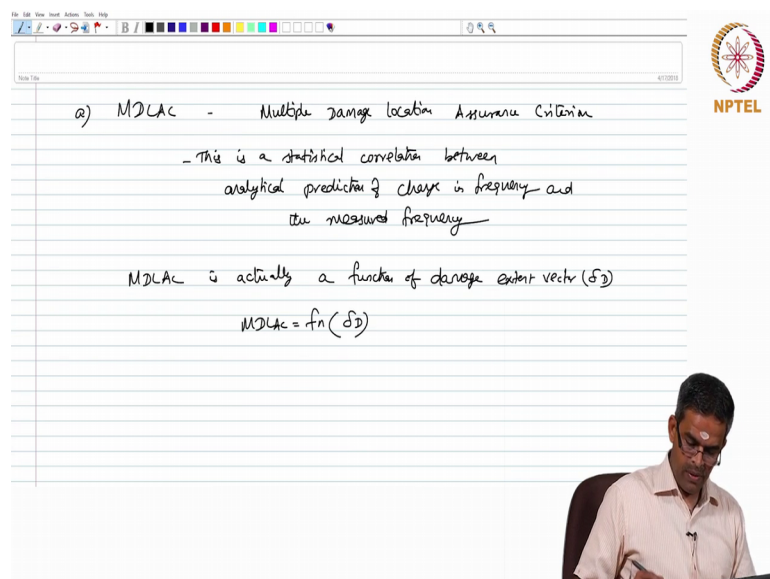
Let us say the first method is Natural frequency-based methods. We will pick up this set of methods which work essentially on natural frequency measurements of the system to detect damages.

In general, these methods use natural frequency as the basic feature for damage identification please note that identification of damage is different from location of damage. We first identify that there is a damage in the system; then, we have to locate, where the damaged system has occurred? What are the extension of the damage? What would be the influence of the damage and structural health?

All this will follow the first format is to identify that there is a damage in the system. So, we are talking about that methods which are useful in detecting or identifying the damage and not going to an extent of saying the location of the damages identified ok. In general, frequency based methods which are used to detect damage based on the natural frequency is a good choice.

For a simple reason natural frequencies in a system of a structural system can be readily measured at few accessible points on the structure. They are easily measurable. Number one, most importantly natural frequencies are generally less contaminated by other noise data. So, this makes these kind of methods for damage detection more powerful.

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The image shows a presentation slide with handwritten text. The text is as follows:

a) MDLAC - Multiple damage location Assurance Criterion

- This is a statistical correlation between analytical prediction of change in frequency and the measured frequency

MDLAC is actually a function of damage extent vector ( $\delta_D$ )

$$MDLAC = f_n(\delta_D)$$

The slide also features the NPTEL logo in the top right corner. In the bottom right corner, there is a video inset showing a man in a light-colored shirt speaking.

Let us see the first method in this which is MDLAC which abbreviates for Multiple Damage Location Assurance Criteria. Essentially, this is a statistical correlation between analytical prediction of change in frequency and the measured frequency. MDLAC is actually a function of damaged extent vector that is delta D.

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The slide content is as follows:

$$MDLAC(\delta D) = \frac{|\{\Delta f\}^T \delta f(\{\delta D\})|^2}{(\{\Delta f\}^T \{A\}) \cdot (\{\delta f(\{\delta D\})\}^T \{\delta f(\{\delta D\})\})} \quad \text{--- (1)}$$

where  $\delta f$  - analytical prediction of frequency change  
 $\Delta f$  - measured frequency change

MDLAC provides a good prediction of both

- location of damage
- size of damage (extent of damage)

@ one or more sites

So, MDLAC is a function of delta D. MDLAC can be given by delta f transpose del f del D mod value square divided by delta f transpose delta f multiplied by del f, which is a function of delta D and transpose of the value multiplied by the left which is a function of del D; where, I call equation number 1; where, del f is the analytical prediction of frequency change and del delta f is the measured frequency change.

Both are change in frequency. That is why the term del and delta are associated. Both are frequency changes; this is one is analytical, one is experimental. Now interestingly MDLAC provides a good prediction of both location of damage and size of damage, what we can say otherwise extent of damage at one or even.

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(b) SDI (Single Damage Indicator)

- It is useful to locate and quantify the damage in flexural members.
- It is good to locate & quantify cracks in beams.
- This method uses change in  $\omega_n$  to detect the damage.
- Fractional change in Modal Energy is related to the fractional change in frequency, which has occurred due to damage.

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The next available method is b which is SDI which is called Single Damage Indicator. This method is useful to locate and quantify the damage in flexural members. Essentially, this method is good to locate and quantify cracks in beams to be very specific.

This method anyway uses change in natural frequency to detect the damage. Fractional change in modal energy is related to the fractional change in frequency which has occurred due to damage.

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SDI is used to indicate damage location.

$$SDI_j = \left[ \sum_{i=1}^{NM} @_{ij} \right]^{-1/2} \quad (2)$$

where  $@_{ij}$  = Error index  
= used to represent localization error for  $i^{\text{th}}$  mode in  $j^{\text{th}}$  location

$$@_{ij} = \frac{\sum_{k=1}^{NM} Z_k}{Z_i} - \frac{F_{ij}}{\sum_{k=1}^{NM} F_{kj}} \quad (3)$$

where  $Z_i$  is the fractional change in  $i^{\text{th}}$  eigenvalue due to damage

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SDI that is single damage indicator is used to indicate damage location that is why it is called indicator. SDI  $j$  is given by algebraic sum of  $i$  equals 1 to number of modes is square  $i j$  to the power minus half.

We discussed this in the previous lecture as well. This is for our elaborate understanding we are re discussing this again; where,  $e_{ij}$  is termed as error index. This is used to represent localization error for  $i$ th mode, in  $j$ th location; that is why it is  $e_{ij}$ .

$e_{ij}$  is given by  $Z_i$  by summation  $k$  equals 1 to number of modes  $Z_k$  minus  $F_{ij}$  by summation  $k$  equals 1 to number of modes  $F_{kj}$ . Where,  $Z_i$  is the fractional change in  $i$ th eigen value due to damage.

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This is given by

$$Z_i = \frac{\Delta \omega_i^2}{\omega_i^2} \quad \rightarrow$$

Further, sensitivity of the  $i^{\text{th}}$  modal stiffness of  $j^{\text{th}}$  element is given by

$$F_{ij} = \frac{K_{ij}}{K_i} \quad \rightarrow$$

This is given by  $Z_i$  is equal to  $\Delta \omega_i^2$  by  $\omega_i^2$ . Further, sensitivity of the  $i$ th modal stiffness of  $j$ th element is given by  $F_{ij}$  which is  $K_{ij}$  by  $K_i$ .