

Structural Health Monitoring (SHM)
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Lecture - 30

Damage identification using lumped mass and Element modal stiffness - Part 2

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(4) Damage identification using Modal Strain Energy

This is a Two stage process:

1st stage: to locate the damage using change in modal strain energy of the element

2nd stage: Extent of damage is determined by iterative scheme

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Further, damage identification can also be done using modal strain energy. This is actually a 2 stage process: first stage to locate the damage using change in modal strain energy of the element.

The second stage is to determine the extent of damage, by iterative scheme.

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Modal strain energy of j^{th} element of both undamaged & damaged states is given by:

$$MSE_{ij} = \Phi_i^T k_j \Phi_i \quad \text{--- (1)}$$

$$MSE_{ij}^d = \Phi_{di}^T k_{dj} \Phi_{di} \quad \text{--- (2)}$$

where d subscript identifies damaged state

where k_j element stiffness matrix of j^{th} element
 Φ_i is the i^{th} mode shape

Modal strain energy of j -th element of both undamaged and damaged case is given by modal strain energy $i j$ is $\phi_i^T k_j \phi_i$, modal strain energy damaged case is given by $\phi_{di}^T k_{dj} \phi_{di}$; where d stands for damage d subscript identifies damaged state, k_j is the element stiffness matrix of j -th element ϕ_i is the i -th mode shape.

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Change in Modal strain Energy ratio is indicated as $MSECR$

$$MSECR_j^i = \frac{|MSE_{ij}^d - MSE_{ij}|}{MSE_{ij}} \quad \text{--- (3)}$$

ϵ_j^i is a meaningful indicator of damaged elements

Damaged elements will have a significant change in the stiffness.
 - Stiffness will be degraded

Change in stiffness for the damaged element can be expressed as a fractional change of element stiffness matrix:

$$K^d = K + \sum_{j=1}^L \Delta k_j \quad \text{--- (4)}$$

Change in modal strain energy ratio is indicated as modal strain energy change ratio ok. So, the modal strain energy change ratio from j to i is given by modal strain energy $i j$ in

the damaged state, minus modal strain energy $i j$ the mod value by modal strain energy $i j$, that is a equation 3. Equation 3 is a meaningful indicator of damaged elements, damaged elements will have a significant change in the stiffness, because stiffness will be degraded. Therefore, change in stiffness for the damaged element can be expressed as a fraction of or a fractional change of elemental stiffness matrix this is given by the damaged k is given by k plus j equals 1 to 1 delta $k j$, which is equal to k plus delta to 1 alpha $j k j$, this is valid for minus 1 less than alpha j less than equal to 0 .

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$$= k + \sum_{j=1}^L \alpha_j k_j \quad \text{--- (6)}$$

(valid for $-1 < \alpha_j \leq 0$)

Now, change in Modal strain Energy is expressed as below:

$$MSE_{ij} = 2A \bar{\phi}_i^T k_j \bar{\phi}_i + \alpha_j \bar{\phi}_i^T k_j \bar{\phi}_i \quad \text{--- (6)}$$

In the above eqn, α_j is unknown.
To start with, this is assumed to be zero & iteration is set in.

Thus,

$$MSE_{ij} = 2A \bar{\phi}_i^T k_j \bar{\phi}_i \quad \text{--- (7)}$$

Modal strain Energy change can be determined for both damaged & undamaged state. By using the appropriate $(k_j, \bar{\phi}_i)$ - for damaged case, one can use $k_j, \bar{\phi}_i$.

Now, change in modal strain energy is expressed as below: modal strain energy $i j$ is 2 delta ϕ_i transpose $k_j \phi_i$ plus alpha j ϕ_i transpose $k_j \phi_i$. In the above equation alpha j is unknown remaining all are known actually ok. To start with this is assumed to be 0 and iteration in set it thus modal strain energy correction $i j$ is 2 delta ϕ_i $k_j \phi_i$.

Now, the modal strain energy correction or the modal strain energy change can be determined for both damaged and undamaged state, by using the appropriate stiffness and mode shape is it not. For damage we will use for damaged case one can use k damage and ϕ_i damage is it not.

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In the undamped case, following Eqn holds good:

$$\left[(k + \Delta k) - (\omega_i + \Delta \omega_i)^2 M \right] \left[\bar{\phi}_i + \Delta \bar{\phi}_i \right] = 0 \quad (8)$$

Now, $\Delta \bar{\phi}_i$ is expressed as a linear combination of mode shape of undamaged system $\bar{\phi}_k$ as given by:

$$\Delta \bar{\phi}_i = \sum_{k=1}^n \alpha_k \bar{\phi}_k \quad (9)$$

Sub (9) in (8) & neglecting higher order terms, we get

$$\alpha_k = - \frac{\bar{\phi}_r^T \Delta k \bar{\phi}_i}{(\omega_r - \omega_i)} \quad \text{for } r \neq i \quad (10)$$

So, in the undamped case following equation holds good k plus Δk minus $(\omega_i + \Delta \omega_i)^2 M$ times $(\bar{\phi}_i + \Delta \bar{\phi}_i)$ is 0.

Now, $\Delta \bar{\phi}_i$ is expressed as a linear combination of mode shape of undamaged system and is given by, now substituting equation 9, in equation 8, and neglecting higher order terms, we get α_k is minus $\bar{\phi}_r^T \Delta k \bar{\phi}_i$ by $(\omega_r - \omega_i)$ for $r \neq i$. Modal strain energy change is then given by modal strain energy change is given by $2 \bar{\phi}^T k_j \sum_{r=1}^n \alpha_r \bar{\phi}_r^T \Delta k \bar{\phi}_i$ of $\bar{\phi}_i$.

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MSEC is then given by:

$$MSEC_{ij} = 2 \Phi^T k_j \left(\sum_{r=1}^n - \frac{\Phi_r^T \Delta k \Phi_i}{(\omega_r - \omega_i)} \Phi_r \right) \quad \text{for } r \neq i \quad (11)$$

The earlier Eqn (4-5) of MSEC can be simplified as below:

$$MSEC_{ij} = \sum_{p=1}^L 2 \alpha_p \Phi_i^T k_j \left(\sum_{r=1}^n - \frac{\Phi_r^T k_p \Phi_i}{\omega_r - \omega_i} \Phi_r \right) \quad \text{for } r \neq i \quad (12)$$

once the damage is located, the damage sensitivity can be determined as below

Now, the earlier equations that is 4 to 5 of modal strain energy correction can be simplified as below, modal strain energy correction i j summation of 1 to L alpha P phi transpose k j r equals one to n phi r transpose k p phi i omega r minus omega i of phi r. For r not equals i this also for r not equals i equation 12. Now once the damage is located then damage sensitivity can be determine as below.

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$$\begin{Bmatrix} MSEC_{i1} \\ MSEC_{i2} \\ \vdots \\ MSEC_{ij} \\ \vdots \\ MSEC_{iP} \end{Bmatrix} = \begin{bmatrix} \beta_{i1} & \beta_{i2} & \dots & \beta_{iP} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{ij} & & & \beta_{iP} \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_j \\ \vdots \\ \alpha_P \end{Bmatrix} \quad (13)$$

where p is a suspected damaged site (location)
j is a element considered to compute MSEC

Modal strain energy correction in i 1, modal strain energy correction i 2, modal strain energy correction i j, is given by beta 1 1, beta 2 1, beta j 1, beta 1 2, beta 1 P, beta j P

multiplied by alpha 1, alpha 2, alpha p, equation 13 where p is the number of suspected damaged sites that is damaged locations j is the number of elements consider to compute the modal strain energy correction.

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$$\beta_{st} = -2 \sum_{r=1}^n \phi_i^T k_s \phi_r^T k_i \phi_i / ((\omega_r - \omega_i) \phi_r)$$
 for $r \neq i$

Where ϕ_i is the element sensitivity coefficient of MSEC

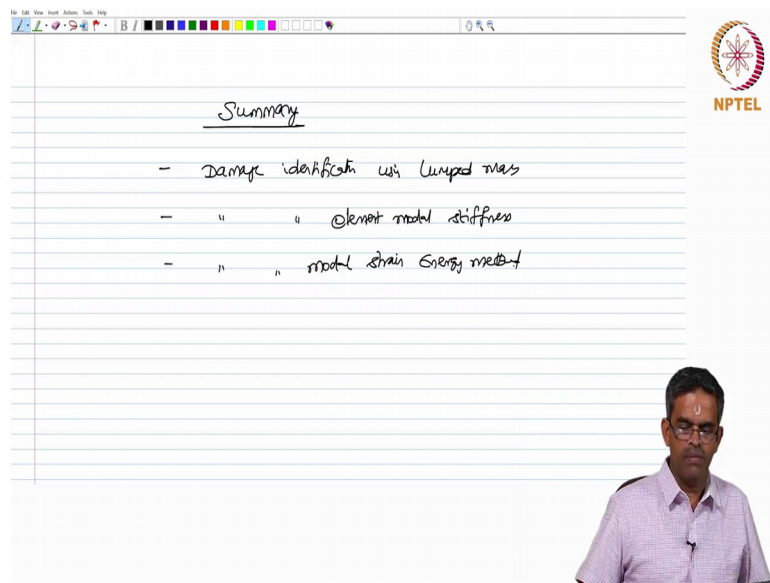
MSEC are experimentally measured and then substituted in Eq. (13). To obtain the fractional change in stiffness of the damaged system (elements)

Once, initial value of α_p (as referred in Eq. (12)) is obtained, values of MSEC can be updated for each iteration, until convergence is reached

Beta s t is given by minus twice r equals 1 to n phi i transpose k s phi r transpose k k i phi i omega r minus omega i of phi r for r not equals i. Where, beta s t is the element sensitivity coefficient of modal strain energy correction. Now modal strain energy correction are experimentally measured and then substituted in equation 13 to obtain the fractional change, in stiffness of the damage system or let us say the damaged elements.

Once initial value of alpha p as referred in equation 12 is obtained values of modal strain energy correction can be updated for each iteration until, convergence is reached.

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The image shows a digital whiteboard interface. At the top, there is a menu bar with options like 'File', 'Edit', 'View', 'Insert', 'Action', 'Tools', and 'Help'. Below the menu bar is a toolbar with various drawing tools. The main area of the whiteboard contains the following text:

Summary

- damage identification using lumped mass
- " " element modal stiffness
- " " modal strain energy method

In the bottom right corner, there is a small video feed of a man wearing glasses and a light-colored shirt. The NPTEL logo is visible in the top right corner of the whiteboard area.

In this lecture we learnt how to do damage identification using lumped mass. We also learnt how to do damage identification using element modal stiffness. We have also learnt how to do damage identification using modal strain energy method. We will discuss the further methods in the next lecture.

Thank you very much.