

Structural Health Monitoring (SHM)
Prof. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Lecture – 28
Structural health monitoring methods: 1 – Part 2

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II To estimate mass & stiffness of a shear model building from the model test

Let undamped shear building, expressed by the eqn of motion as below:

$$M \ddot{x} + kx = F \quad (2)$$

Characteristic eqn to determine Eigenvalue mode shape is given by:

$$(k - \omega_i^2 M) \phi_i = 0.$$

where ω_i - Eigen value
 ϕ_i - corresponds mode shape

So, the second argument is to estimate mass and stiffness of a shear model building from the model test. Let undamped shear building expressed by the equation of motion as below, that is $M \ddot{x} + kx = F$. The characteristic equation to determine Eigen value and mode shape is given by $(k - \omega_i^2 M) \phi_i = 0$ where ω_i is the Eigen value and ϕ_i is a corresponding mode shape.

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Stiffness matrix of the shear building is given by:

$$[K] = \begin{bmatrix} k_1 & -k_2 & & & & \\ -k_2 & k_2+k_3 & -k_3 & & & \\ & & & & & \\ & & & & & \\ & & & & -k_{n-1} & k_{n-1}+k_n & -k_n \\ & & & & & -k_n & k_n \end{bmatrix}$$

Stiffness matrix of the shear building is given by k_1 plus k_2 minus k_2 minus k_2 k_2 plus k_3 minus k_3 and so on; let us say minus k_{n-1} minus k_{n-1} plus k_{n-1} k_{n-1} minus k_n and k_n .

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Mass matrix is given by

$$[M] = \begin{bmatrix} m_1 & & & & \\ 0 & m_2 & & & \\ & & & & \\ & & & & \\ & & & & m_n \end{bmatrix}$$

Mass matrix is given by M_1, M_2, M_n ; expanding the above equations.

minus 2 2 n minus 2 in which, the last two rows and columns of B matrix are eliminated;
 b dash is 2 n minus 2 by 1 vector in which last two members of b vector are eliminated.

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$\{b'\}$ is $(2n-2) \times 1$ vector in which last 2 members of $\{b\}$ are eliminated

Now, solving for the unknown mass & stiffness parameters (k_i, m_i) we get:

$$\{b'\} = ([B']^T [B']^{-1}) [0]'^T \begin{Bmatrix} 0 \\ \vdots \\ \omega_l \phi_{ln} \\ \omega_r \phi_{rn} \end{Bmatrix}$$

$$k_n = \frac{\omega_l \phi_{ln}}{\phi_{ln} - \phi_{rn}}$$

Now, solving for the unknown mass and stiffness parameters, that is k_i and M_i , we get b dash is equal to B prime transpose B dash inverse B dash transpose 0 omega l phi l n , omega r phi r n , which tells me k_n is equal to omega l phi l n or phi l n minus phi l n minus 1.

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Mass and stiffness parameter, obtained from the above set of equations are relative values of m_n ($\because m_n$ is considered to be Unity)

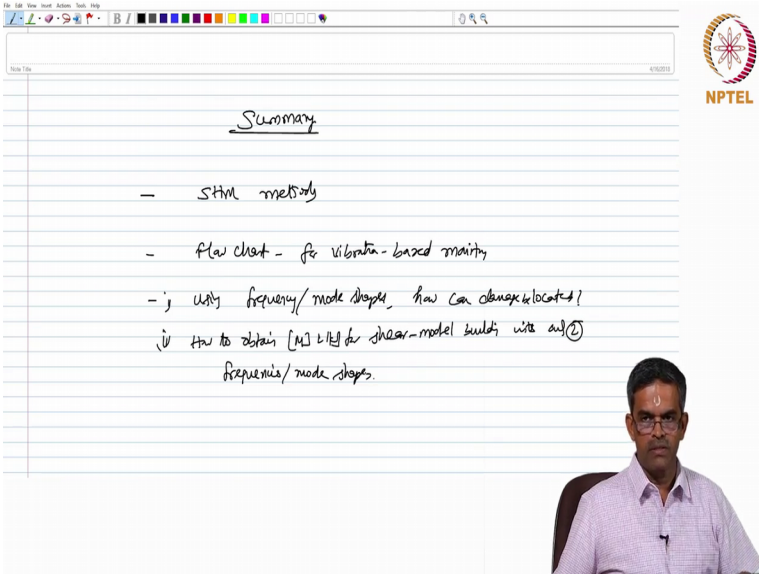
Advantages of this method

- (1) Only mode shape & frequency of 2 modes are required
- (2) This can be applicable only to linear modal analysis
- (3) This is valid only for undamped systems

Now, the mass and stiffness parameters obtained from the above equation or relative values of M_n , because you can see here M_n is taken to be unity. Therefore, the relative value γ because M_n is considered to be unity. This method has second advantage; one, only mode shape and frequency of two modes are required, see here l and n .

This method has a limitation, this can be applicable only to shear model buildings. Thirdly, this is valid only for undamped systems. So now, let us see the summary friends.

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Summary

- Stiff method
- Flow chart - for vibration-based monitoring
- using frequency/mode shape, how can damage be located?
- iv) How to obtain $[M]$ & $[K]$ for shear-model building with only 2 frequencies/mode shapes.

In this lecture, we are attempting to learn Structural Health Monitoring methods. We also saw the flowchart for vibration based monitoring.

We learnt two methods; one, using frequencies and mode shapes; how can damage located damage be located; second, how to obtain mass and k for shear model building with only two frequencies and mode shapes. We will continue to see other methods in the coming lecture.

Thank you.