


**Structural Health Monitoring (SHM)**  
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**Lecture – 25**  
**Vibration based health monitoring scheme – Part 1**

(Refer Slide Time: 00:19)



Module 2  
Lecture 5: Vibration-based SHM scheme.

Civil Engg structures - with the recent advancement of sensors actuators and computational capabilities have become smart structures

- Intelligent enough to undergo a self-diagnosis so as to develop early warnings in case of any critical health state.

SHM methods - should address certain basic themes/purposes

- ✓ - To deal with reliable functional components to avoid malfunctioning of the scheme and thus ensure public safety
- SHM methods should be effective and efficient such that functional losses to the structural system can be avoided

Friends, welcome to the module – 2, lecture – 5 where we are going to discuss vibration based structural health monitoring scheme we all agree that civil engineering structures with the recent advancements of sensors actuators and computational capabilities have become smart structures. They are intelligent enough to undergo a self diagnosis so as to develop early warnings in case of any critical health state.

So, under this objective let us see the structural health monitoring methods. Should address certain basic themes or let us say a purposes should fulfill certain basic purposes. It should deal with reliable, functional components to avoid malfunctioning of the system or the scheme and thus ensure public safety.

So, reliability of the system is the first objective. The second could be the SHM methods should be effective and efficient such that functional losses to the structural system can be avoided when we ask a question how do we get functional loss to a structural system.

(Refer Slide Time: 04:26)

- structural system, if not functioning properly will face a downtime for repair — can lead to economic loss

- SHM methods should enable to revisit the design principles towards light-weight structures

- maintenance & assessment are more effective in light weight structures.

Considering the factors such as

- causes for the damage
- material and functional degradation
- load path shifting etc

a damaged structure can be expressed in terms of

If the structural system is not functioning properly we will face a downtime for repair and that downtime can lead to economic loss. SHM methods should enable to revisit the design principles towards lightweight structures because maintenance and assessment are more effective in lightweight structures.

Now, considering the factors such as; causes for the damage, material and functional degradation, load path shifting etcetera.

(Refer Slide Time: 07:09)

general, non-linear, time-varying, spatially discrete and a coupled system as below:

$$M(\theta_d, \theta_e, x, t)\ddot{x} + g(x, \dot{x}, \theta_d, \theta_e, t) = f_{app}(t) + f_{exp}(t) \quad \text{--- (1)}$$

where  $[M]$  is the mass matrix

$g$  is the force vector, which is a function of Elastic damping force depends on vel, displacement & time.

$\theta_d$  damage parameter (for example, crack length, loss of stiffness, loss of mass etc)

$\theta_e$  indicates influence of environmental forces and operation conditions on the structure's health

A damaged structure can be expressed in terms of general non-linear, time varying, spatially discrete and a coupled system as below;  $\theta_d$ ,  $\theta_e$ ,  $x$ ,  $t$  the  $x$  double dot plus  $g$  which is a function of displacement velocity  $\theta_d$ ,  $\theta_e$  and time will be equal to  $f_{\text{operational}}$  plus  $f_{\text{experiment}}$ , where  $m$  is the mass matrix  $g$  is the force vector which is a function of elastic damping force depends on velocity displacement and time.

$\theta_d$  is damaged parameter, it can be any issues for example, crack length, loss of stiffness, loss of mass can be a damage parameter.  $\theta_e$  indicates influence of environmental forces and operational conditions on the structure as health.

(Refer Slide Time: 10:12)

for example, temperature, humidity, change of mass distribution etc.

$f_{op}$  - operational loads  
 $f_{ex}$  - experimental loads (scaled magnitude of operational loads)

- Damage function ( $\theta_d$ ) is non-linear and can be expressed as below:

$$\theta_d = \Gamma(\theta_d, \theta_e, x, \dots) \quad (2)$$

while doing such analysis,  
 evaluation of damage (damage identification)  
 and dynamic response of the system under damaged  
 condition takes place in 2 different time scale.

For example, it can be temperature, humidity, change of mass distribution etcetera  $f_{op}$  refers to operational low enough experimental refers to experimental loads which are essentially scaled magnitude of operational loads.

Further the damage function  $\theta_d$  is again non-linear and can be expressed as below  $\theta_d$  is a function of  $\theta_d$ ,  $\theta_e$ ,  $x$ ,  $\dot{x}$  and time. While doing such analysis evaluation of damage that is damage identification and dynamic response of the system under damage condition takes place in two different time scale.

(Refer Slide Time: 12:44)

one may be slowly time-varying  
other may be rapidly varying with time

It is necessary that  
we shall account  
for such variations  
carefully

- Damage identification in Linear Systems

Dynamic response of 'n' dof system can be expressed as:

$$M\ddot{x} + c\dot{x} + kx = f(t) \quad \text{--- (3)}$$

If the system is undamped (or practically, lightly damped), then the characteristic features of the structural system (like  $\omega_n$ ,  $\phi_n$ ) can be determined using classical Eigen solver theory.

One may be slowly time varying, the other may be rapidly varying with time. Therefore, it is important, that is necessary that theta d shall account for such variations carefully, that is a word of caution, ok. Let us extend this logic to study damage identification in linear systems dynamic response of n degree of freedom system can be expressed as  $M \ddot{x} + c \dot{x} + kx = f(t)$ .

If the system is undamped or practically to say so, lightly damped then the characteristic features of the system like natural frequency more shape can be determined using classical Eigen solver theory.

(Refer Slide Time: 15:20)

$$(K - \omega_n^2 M) \phi_n = 0 \quad (4)$$

Alternatively,

Correction parameters to represent modal changes in the element level of the structural system can be expressed as below:

$$\Delta k = \sum_j k_j \Delta a_j$$

$$\Delta c = \sum_j c_j \Delta a_j$$

$$\Delta m = \sum_j M_j \Delta a_j$$

This says  $K$  minus  $\omega$  square  $m$   $\phi_n$  is 0. Alternatively, one can use correction parameters to represent the model changes in the element level of the structural system. This can be expressed as below  $\Delta k$ ,  $\Delta c$  and  $\Delta m$ .

(Refer Slide Time: 16:56)

Where general damage parameter ( $\theta d$ ) is now replaced with linear matrix correction  $\Delta a$ . (Correction parameter)

The correction parameter which localizes and quantifies the damage, can be determined by solving the 'Inverse problem' which is

Minimize the weighted sum of components of data error,  $\epsilon$

Minimize, the following function with  $\epsilon$

$$J = \epsilon^T W_\epsilon \epsilon + \Delta a W_a \Delta a \quad (5)$$

$$\epsilon = S \Delta a - r$$

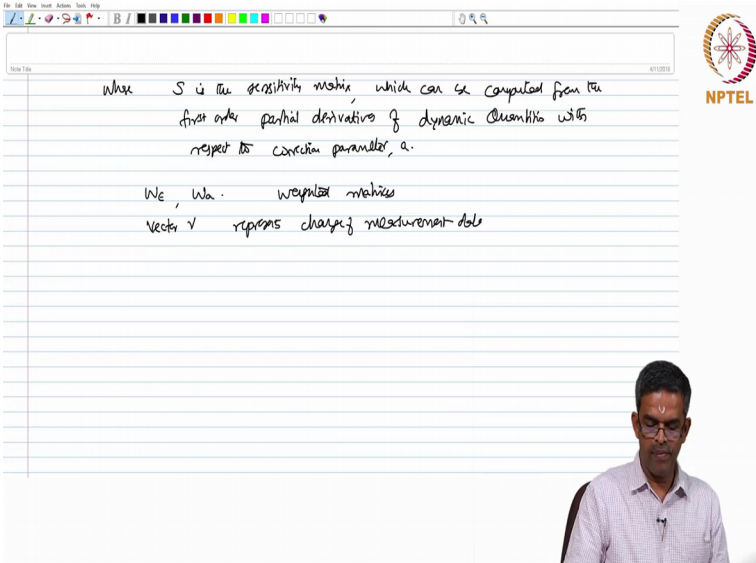
Where, general damage parameter  $x$  plus  $\theta d$  is now replaced with a linear matrix correction  $\Delta a$ . So,  $\Delta a$  is called the correction parameter.

Now, the correction parameter which localizes and quantifies the damage because you can see from equation  $\Delta a_j$  is the parameter which quantifies damage on mass,

damping and stiffness, ok. So, this essentially localizes and quantifies the damage can be determined by solving the inverse problem which is minimizing the weighted sum of components of the data error epsilon.

So, now, the problem is now reduced to a minimization function, minimizing the following function with epsilon. The function is epsilon transpose weighted function plus delta a weightage of a plus delta a and epsilon is S delta a minus r.

(Refer Slide Time: 19:56)



The image shows a digital whiteboard interface with a toolbar at the top. The whiteboard contains the following handwritten text:

Where  $S$  is the sensitivity matrix, which can be computed from the first order partial derivatives of dynamic quantities with respect to connection parameter,  $a$ .

$W_\epsilon$ ,  $W_a$  - weighted matrices

Vector  $r$  represents change of measurement data

In the bottom right corner, there is a video feed of a man with glasses and a light-colored shirt, looking down. To the right of the whiteboard is the NPTEL logo.

Where,  $S$  is the sensitivity matrix which can be computed from the first order partial derivative of the dynamic quantities with respect to the connection parameter  $a$ .  $W_\epsilon$  and  $W_a$  are called weighted matrices. The vector  $r$  represents change of measurement data.