

Structural Health Monitoring (SHM)
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Lecture – 22

Part- 2: Estimation of Structural Health using Static SHM

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Damage severity index (α)

$$\alpha = \frac{E I_{und} - E I_{dam}}{E I_{und}}, \quad 0 < \alpha < 1.$$

Now, the problem is reduced to minimize the function

$$f(\alpha, \delta, a) = \sum_{j=1}^k \left| \frac{\Delta \epsilon_j^t - \Delta \epsilon_j^m}{\Delta \epsilon_j^m} \right| \quad \text{subjected to the condition}$$

$$0 \leq a + \delta \leq 1$$

$$0 \leq \alpha \leq 1$$

$$\delta \leq L$$

One can establish damage severity index alpha. So, alpha is given by E I undamaged minus E I damaged by E I undamaged, where alpha varies from 0 to 1.

Therefore, now the problem actually is reduced to minimize the function which is function of alpha, delta and a; which can be given by summation of j equals 1 to k mod value of delta epsilon j t minus delta epsilon j m by delta epsilon j m.

Subjected to the condition that 0 less than a plus delta less than 1, 0 less than alpha less than 1 and delta is far compared to 1 lesser.

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beam is divided into small lengths (Δ) - discretization

$$l = \left(\frac{L}{n}\right)$$

$$\Rightarrow \Delta = l = \left(\frac{L}{n}\right)$$

$$\Rightarrow a_i = a_i = \Delta(i-1) \text{ for } i: 1, 2, \dots, n$$

Hence minimize

$$f(\alpha, a_i) = \sum_{j=1}^k \left| \frac{\Delta \epsilon_j^k - \Delta \epsilon_j^n}{\Delta \epsilon_j^n} \right| \text{ subjected to}$$

$a_i = 1, 2, \dots, n$
for $0 \leq \alpha < 1$

So, to execute this beam is divided into small lengths l this is discretization such that l is L by n and therefore, Δ is L by n and therefore, a_i equals Δ times $(i-1)$ for i equals 1 to n .

Hence minimize f of α and a_i which is summation of j equals 1 to k , $\Delta \epsilon_j^k - \Delta \epsilon_j^n$ by $\Delta \epsilon_j^n$ mod value subjected to a_i is $1, 2, n$ for 0 less than α less than 1 .

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(i) Vibration-based damage detection

Hypothesis: Structural damage can be characterized by local modifications of stiffness

- modification is defined, in turn affects the modal parameters.

Procedure:

Member is subjected to an external excitation

- It can be forced vibration (for model is Gpr)
- (or) ambient vibration under natural loading cases.

Let us now talk about the second method which is vibration based damage deduction. The hypothesis began this method is that structural damage can be characterized by local modification of stiffness. The change in stiffness or modification in stiffness in turn affects the modal parameter. Let us see how the procedure works. Member will be subjected to an external load excitation. This excitation can be a forced vibration which is possible for a model in experiments.

Because it is not possible to create a forced vibration for a prototype system and the service is automatically created by the external loads, for of course, model scale one can create a forced vibration or alternatively it can be an ambient vibration under the natural loading cases.

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- Modal parameters are estimated from the vibration data
 - These parameters are used as input for damage identification (damage detection)

Let us consider

- changes in modal parameter as ΔV
- stiffness reduction factor (SRF) as $\{\alpha\}$
- weightage of each term in the stiffness matrix as \bar{W}
- Analytical data be "A"
- Experimental data be "E"

So, for this model parameters are established and estimated from the vibration data which is a standard procedure. Now, these parameters are used as input for damage identification or we can also call this as damaged detection

Let us consider changes in model parameter has delta V. Let us consider stiffness reduction factor that is SRF as alpha vector, let us consider the weightage of each term in the stiffness matrix of the member as W bar and let analytical data be represented by capital A and experimental data be represented by capital E, is not in smallest, is experimental data.

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Damage identification can be done as follows:

$$J = \left\| \bar{W} \left\{ \Delta V^{Analy}(\alpha) - \Delta V^{Exp} \right\} \right\|^2$$

The problem here is to minimize the above function, J, subject to the condition that

$$1 \leq \alpha \leq 0 \text{ is valid}$$

↑
stiffness reduction factor

Expanding $J = \left\{ \Delta V^{Analy}(\alpha) - \Delta V^{Exp} \right\}^T (\bar{W})^T \left\{ \Delta V^{Analy}(\alpha) - \Delta V^{Exp} \right\}$

Therefore, damage identification can be done as follows; J will be equal to some function which will be the weighted value of change in model parameter from the analysis of that of alpha minus change in model parameter of that of the experiments and square. Now, the problem here is to minimize the above function J subject to the condition that 1 less than alpha less than 0 is valid, this is what we call as stiffness reduction factor.

Therefore, expanding J will become delta V analytical of alpha minus delta V experimental the transpose W bar square of delta V analytical of alpha minus delta V experiment.

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Damage detection and quantification can be assessed from three objective functions, as given below:

$$J = \sum_{i=1}^{nm} w_i^2 \left[\left(\frac{\lambda_i^{Analy}}{\lambda_i} \right) - \left(\frac{\lambda_i^{Damaged} - \lambda_i^{Undamaged}}{\lambda_i^{Undamaged}} \right) \right]^2$$

where $n m$ = measured modes in analysis
 λ_i = i -th eigenvalue

Damage detection and quantification can be assessed from three objective functions as given below. J is equal to algebraic sum of i equals 1 to $n m$, w square λ_i of α minus λ_i by λ_i analytical minus λ_i damaged minus λ_i undamaged by λ_i undamaged which is obtained experimental the whole square where number of measured modes in the analysis is indicated as $n m$, α is the i -th Eigen value.

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(b) Mode shape changes

Change in mode shape characteristics can be done using the following relationship:

$$J = \sum_{i=1}^{nm} w_i^2 \sum_{j=1}^{np} \left(\left[\phi_{ij}(t) - \phi_{ij} \right]^{Analy} - \left[\phi_{ij}^{Damaged} - \phi_{ij} \right]^{Exp} \right)^2$$

where $n p$ = measured pairs
 ϕ_{ij} - j -th component of i -th mass is the normalized mode shape

When I want to trace the mode shape changes this can be done using the following relationship. In that case the minimization function will be equal to sum of i equals 1 to nm W bar ϕ_i square summation of j equals 1 to np ϕ_{ij} of α minus ϕ_{ij} which is analytical minus ϕ_{ij} of damaged state minus ϕ_{ij} of undamaged state, which is obtained by experimental the whole square, where number of measured points is np and ϕ_{ij} is the j -th component of i -th mass in the normalized that is important.

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(ii) Frequency changes, combined with mode shape

Following function is valid:

$$J = \sum_{i=1}^{nm} \bar{W}_{\lambda_i}^2 \left(\left[\frac{\lambda_i(\alpha)}{\lambda_i^2} - \lambda_i^0 \right]^{Anst} - \left[\frac{\lambda_i^{Damped}}{\lambda_i^{Undam}} - \lambda_i^0 \right]^{Bst} \right)^2 + \sum_{i=1}^{nm} \bar{W}_{\phi_i}^2 \sum_{j=1}^{np} \left([\phi_{ij}(\alpha) - \phi_{ij}^0]^{Anst} - [\phi_{ij}^{Damp} - \phi_{ij}^{Undam}]^{Bst} \right)^2$$

I can also do this by looking at the frequency changes. Frequency changes combined with mode shape can also be done. Following function is valid. J equals algebraic sum of i equals 1 to nm W bar λ_i square of λ_i α minus λ_i λ_i^0 analytical λ_i damaged minus λ_i undamaged by λ_i undamaged experimental square plus this is arising from one part of the problem. The second is from the change of mode shapes this is the frequency change.

Now, the second part of the equation will give you the change in the mode shape i equals 1 to nm W bar ϕ_i square summation j equals 1 to np ϕ_{ij} α minus ϕ_{ij} which is analytical minus ϕ_{ij} damaged minus ϕ_{ij} undamaged experimental the whole square.

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The image shows a digital whiteboard interface with a toolbar at the top. The word "Summary" is written at the top center. Below it, there are two main bullet points:

- static method of SHM
 - governing Eqn to minimize
- vibration-based SHM
 - frequency change
 - change in mode shape
 - combination of the above

To the right of the second bullet point, there is a vertical line and the text "damaged & undamaged condition". In the bottom right corner, there is a video feed of a man in a blue shirt. The NPTEL logo is visible in the top right corner of the whiteboard area.

So, friends, in this lecture we learnt about introduction to static method of structural health monitoring, we also learnt about the governing equation to minimize and we also saw how it becomes a minimization problem.

We also learnt about vibration based structural health monitoring, how the frequency change, change in mode shape and combination of these two between the damaged and undamaged case is helpful in estimating the vibration based characteristics or deviations of vibration characteristics in the structure caused because of damage.

Thank you very much.