

Computer Methods of Analysis of Offshore Structures
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Module - 02
Lecture – 22
New Generation Offshore Structures (Part – 02)

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$[K] = \begin{bmatrix} k_{11} & 0 & 0 & 0 & k_{15} & 0 & 0 & k_{18} & 0 \\ 0 & k_{22} & 0 & 0 & 0 & 0 & 0 & k_{27} & 0 \\ 0 & 0 & k_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ k_{35} & k_{36} & k_{37} & k_{38} & k_{39} & 0 & 0 & 0 & 0 \\ 0 & k_{42} & 0 & k_{44} & 0 & 0 & 0 & k_{47} & 0 \\ k_{51} & 0 & 0 & 0 & k_{55} & 0 & 0 & k_{58} & 0 \\ 0 & 0 & 0 & 0 & 0 & k_{66} & 0 & 0 & k_{69} \\ 0 & k_{72} & 0 & k_{74} & 0 & 0 & k_{77} & 0 & 0 \\ k_{81} & 0 & 0 & 0 & k_{85} & 0 & 0 & k_{88} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{99} \end{bmatrix}$

9x9

- i) $[K]$ - square
- ii) unsymmetric
- iii) off-diagonal terms represent coupling effects
- iv) force in these dof is present for unit displacement is all dof
- v) these is strongly coupled with other dof
- vi) stiffness offered by the ball joint is not considered

Free rotation

So, that kind of stiffness which is offered or resistance in fact, offered by the ball joints is not considered in the derivation.

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Change in tension in tethers, due to unit displacement is given by:

$$\Delta T_i \quad (i=1,2,\dots,9) = \frac{AE}{L} \left[\sqrt{x_i^2 + l^2} - l \right] \quad (1)$$

where unit displacement is given in i^{th} dof

$$k_{11} = \frac{3(T_0 + \Delta T)}{\sqrt{x_i^2 + l^2}} \quad (2)$$

$k_{21} = 0$ (due to unit displacement, offered only in surge dof)
 $k_{41} = 0$ ($\because k_{21} = 0$)

$$k_{31} = \frac{3(T_0 + \Delta T) \cos \theta_1 - 3T_0}{[\sqrt{x_i^2 + l^2} - l] \cos \theta_1} \quad (3)$$

The diagram shows a triceratops-like structure with three legs. The legs are represented by lines connecting the top of the structure to the bottom. The tension in the legs is labeled as $(T_0 + \Delta T_i)$. The angle between the legs and the vertical is labeled as θ_1 . A blue arrow indicates a surge displacement of the sea level.

having said this let us now derive the stiffness coefficients; change in tension in tethers due to unit displacements is given by I say ΔT_i and i is equal to 1,2 to 9 can be equal to axial stiffness change in length x_i square minus l square the whole root minus l , where unit displacement is given in i^{th} degree of freedom.

Now, let say the deck is supported by the ball joints which in turn connects to the buoyant leg, which in turn is connected or anchored to the sea bed and this becomes my mean sea level undergoes a surge displacement which is indicated here, there is going to be a change in tension which is T_0 plus ΔT_i .

So, now k_{11} will be T_0 plus ΔT_i or T_1 by the new length x_i square plus l square root, there will be 3 such legs where triceratops has 3 legs equation number 2. k_{21} will be 0 due to unit displacement offered only in surge degree of freedom. Since k_{21} is 0 k_{41} will also be 0 since k_{21} is 0.

k_{31} will be given by T_0 plus ΔT_1 of $\cos \theta_1$. So, this is angle θ_1 , $\cos \theta_1$ 1 minus $3 T_0$ and 3 divided by root of x_1 square plus l square minus l of $\cos \theta_1$.

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$k_{51} = -k_{11} \bar{h}$ (restoring moment is offered in opposite direction to that of surge displacement)

$k_{61} = 0$ (yaw motion is not active)

$k_{71} = k_{91}$ (roll in the deck, due to surge is buoyant leg = 0. Yaw in the deck, due to " " " ")

$k_{81} :$

$k_{81} \theta_{81} = \left\{ k_{11} (H_{cg} - \bar{h}) x_1 \right\} - \left\{ k_{31} \left(\sqrt{x_1^2 + l^2} - l \right) x_1 \right\} \quad \text{--- (4)}$

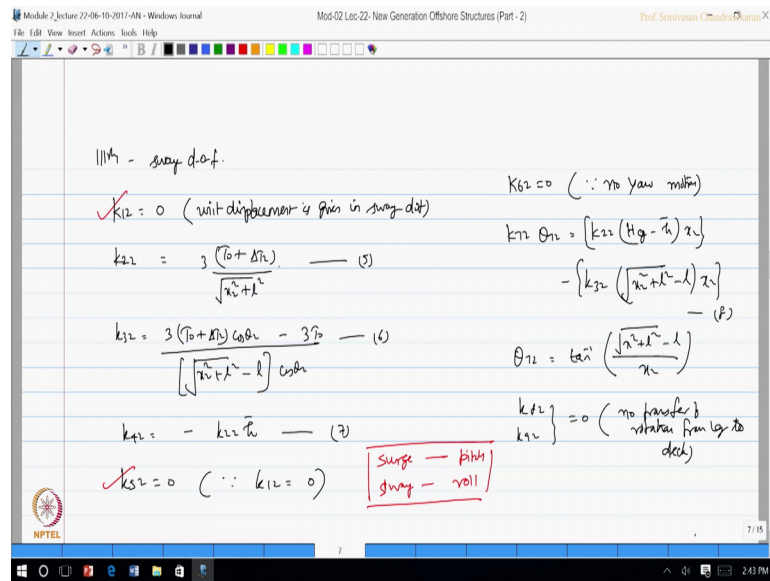
$\theta_{81} = \tan^{-1} \left(\frac{\sqrt{x_1^2 + l^2} - l}{x_1} \right) \quad \text{--- 4.}$

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Let say equation 3; k_{51} will be k_{11} into \bar{h} a negative sign, negative sign indicates the restoring moment is offered in opposite direction to that of surge displacement. So, that the re centering moment, k_{61} will be 0 because yaw motion is not activated.

Now, interestingly k_{71} and k_{91} that is roll in the deck due to surge in buoyant leg or yaw in the deck due to surge in the buoyant leg will be 0. k_{81} will be present which can be computed as k_{81} into θ_{81} will be equal to k_{11} of H_{cg} minus \bar{h} into x_1 minus k_{31} of root of x_1 square plus l square minus l of x_1 , this equation is 4. So, now, we have the first column derived of the stiffness matrix.

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Similarly, we can also do for the sway degree of freedom, k_{12} will be 0 because unit displacement is given in sway degree of freedom, k_{22} similar to 1 1 will be T_0 plus ΔT_2 divided by root of x_2 square plus l square into 3 times say this is equation 5. k_{32} will be T_0 plus ΔT_2 of $\cos \theta_{12}$ 3 times minus $3 T_0$ by root of x_2 square plus l square minus l of $\cos \theta_{12}$.

k_{42} will be minus k_{22} into \bar{h} , k_{52} will be 0 because 1 2 is 0. Friends please note surge invokes roll sway invokes roll. So, this is when surge is absent pitch is also absent similarly, when sway is absent roll is also absent. So, k_{52} is 0, k_{62} is 0 since no yaw motion, k_{72} into θ_{12} will be given by $k_{22} H_{cg}$ minus \bar{h} of x_2 , minus k_{32} root of z_2 square plus l square minus l of x_2 equation number this is 7 this is 8.

Now, interestingly θ_{12} simply is \tan^{-1} of root of x_2 square plus l square minus l by x_2 , by that logic θ_{81} can also be said as \tan^{-1} of x_1 square plus l square root minus l by x_1 , 4 a k_{82} and k_{92} will be 0 because there is no transfer of rotation from the leg to the deck.

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stiffness coeffs in heave dof

$k_{13} = 0$ (unit displacement is offered in heave dof)
 $k_{23} = 0$ (unit displacement is offered in heave dof)

$k_{33} = 3 \left(\frac{AE}{L} \right) + \rho g A_{wp} \quad \text{--- (10)}$

$\left. \begin{matrix} k_{43} \\ k_{53} \\ k_{63} \\ k_{73} \\ k_{83} \\ k_{93} \end{matrix} \right\} = 0$ (no transfer of rotation due to ball joints)

Let us talk about stiffness coefficients in heave degree of freedom. K_{13} will be 0 because unit displacement is offered in heave degree by that logic k_{23} will also be 0, k_{33} will be actually equal to the axial stiffness of all the legs plus ρg of A_{wp} , which is now equation 10. K_{43} , k_{53} , k_{63} , k_{73} , k_{83} and k_{93} will be all 0 because no transfer of rotation due to ball joints.

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coeffs in roll dof

$k_{14} = 0$ (unit rotation is offered in roll dof)

$k_{24} = \frac{3(T_0 + \Delta T_4)}{\sqrt{S_{10}^2 + L^2}} \quad \text{--- (11)}$

$S_{10} = (H_{cg} - \bar{h}) \sin \theta_4 \quad \text{--- (12)}$

S_{10} in the deck is positive due to roll in the buoyant leg - ball joints only restrict transfer of rotation

S_{10} is transferred to deck/bcs vice-versa

$\Delta T_4 = \frac{AE}{L} \frac{P_4}{2} \cos \theta_4 (h_4) = \Delta T_4' \quad \text{--- (13)}$

Let us talk about coefficients in roll degree of freedom k_{14} will be 0, because unit rotation is offered in roll degree. K_{24} will be T_0 plus ΔT_4 divided by square root of

SwD square plus l square of 3 times we call this equation number 11, where SwD is Hcg minus h bar of sin theta 4 equation 12; interestingly please note sway in the deck is possible due to roll in the buoyant leg.

Please understand ball joints only restrict transfer of rotations, but sway is a displacement. So, sway is transferred to the deck and buoyant leg vice versa. So, delta T 4 which is change in tension because of roll degree of freedom is given by AE by l, Pl by 2 cos theta 4 of theta 4 which also same as T 4 theta. So, let us call this as equation number 13.

Where in a triceratops with a triangular deck these being 3 legs this being the wave direction this dimension is pb and this dimension is pl and this is my x axis. So, I am talking about rotation about this axis. So, Pl will be involved here and we also know that cg lies at pl by 2 and this distance is pb by 3.

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Eqn of forces in heave direction, for unit rotation is roll dof, we get

$$k_{34} = \frac{3(T_0 + \Delta T_4) \cos \theta_4 - 3T_0}{Z_b} \quad (14)$$

where $Z_b = \bar{h} - \left(\frac{e_4}{\tan \theta_4}\right) \quad (15)$

$$k_{44} \theta_4 = F_b e_4 + (T_0 + \Delta T_4)(s_1 - e_4) + (T_0 + \Delta T_4)(s_2 + e_4) - W_{deck} S_{wD}$$

$$\begin{aligned} e_4 &= \bar{h} \sin \theta_4 & S_{wD} &= (H_{cg} - \bar{h}) \sin \theta_4 \\ s_1 &= \frac{pl}{2} + e_4 & Z_b &= (H_{cg} - \bar{h}) - \left(\frac{S_{wD}}{\tan \theta_4}\right) \\ s_2 &= \frac{pl}{2} - e_4 \end{aligned} \quad (15)$$

So, now trying equilibrium of forces in heave direction for unit displacement or rotation in roll degree, we get k 3 4 as T0 plus delta t 4 of cos theta 4, minus 3 T0 by Zb where equation number 14 can be assigned to this, where Zb where equation number 14 can be assigned to this where Zb is h bar minus e 4 by tan theta 4, equation number 14 a.

Now, k 4 4 theta 4 can be said as buoyancy of e 4 plus T0 plus delta T 4 of s 1 minus e 4 plus T0 plus delta T 4 of s 2 plus e 4 minus w deck into SwD, e 4 is h bar sin theta, 4 s 1

is p_1 by 2 plus e_4 and s_2 is p_1 by 2 minus e_4 , S_wD is anyway we said it is H_{cg} minus \bar{h} bar of $\sin \theta_4$.

Let us also insert 1 more expression Z_t , which is H_{cg} minus \bar{h} bar minus S_wD by $\tan \theta_4$ call all these equations as 15.

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$\checkmark k_{54} = 0$ (unit rotation in roll dof - not affecting pitch BUS)
 $k_{64} = 0$ (due to no yaw motion)
 $\checkmark k_{74} \theta_{74} = \left[k_{24} S_wD (H_{cg} - \bar{h}) \right] - k_{34} \left(\frac{p_b}{3} \right) Z_t \quad (16)$
 (only due to transfer of translational responses by the buoy leg)
 $\checkmark \theta_{74} = \tan^{-1} \left(\frac{Z_t}{p_b} \right) \quad (17)$
 $\checkmark \left. \begin{matrix} k_{54} \\ k_{64} \end{matrix} \right| = 0$ due to no transfer of rotation to deck from buoyant leg
 $k_{74}, k_{64}, k_{54} = 0$ (Zero from rotation)

k_{54} will be 0, because unit rotation in roll is not affecting of buoyant leg; k_{64} is 0 due to non-sway sorry due to no yaw motion.

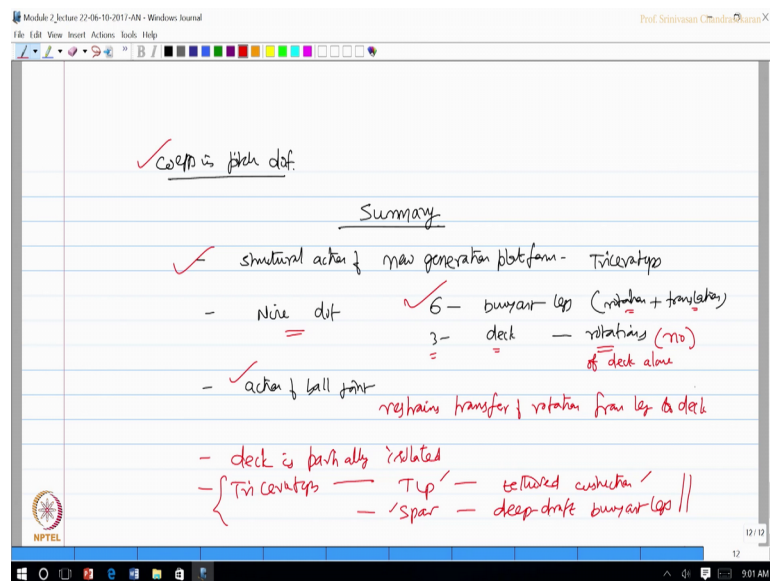
k_{74} into θ_{74} will be k_{24} into S_wD , H_{cg} minus \bar{h} bar minus k_{34} of p_b by 3 of Z_b call this equation number 16 this is only due to. So, friends k_{54} will be 0 this is because of the reason that unit rotation given in roll degree of freedom does not affect pitch of the buoyant leg, unit rotation given in roll degree of freedom does not affect pitch degree of freedom of the buoyant leg.

Similarly, $k_{64} = 0$ because there is no yaw motion in the buoyant leg, now 1 can estimate k_{74} which can be given by equation 16, which is k_{74} is given by the equation as you see here. It arises from 2 components one is because of k_{24} and other is because of k_{34} . Please note that these components are only due to transfer of translational responses; you know 2 and 3 are the translational responses 4 of course, is the rotation given to estimate the stiffness coefficient because of unit rotation in the roll degree in the buoyant leg.

So, by simplifying equation 16 we can get k_{74} , provided θ_{74} is given by this expression shown in equation 17. k_{84} and k_{94} will be 0 because there is no transfer of rotation to the deck from the buoyant leg; friends, please note that 8 and 9 are degrees of freedom of the deck now the buoyant legs are 6 degrees of freedom 1 to 6 and 7 8 9 are degrees of freedom of the deck.

So, 8 and 9 has no responses on the deck due to rotation given in the buoyant leg, because the ball joints do not transfer this. So, k_{74} , k_{84} and k_{94} which arise because of rotation from the buoyant leg is kept as 0, then I may ask me a question how do we get k_{74} when there is no transfer of rotation. Please note k_{24} and k_{34} are translational responses of the buoyant leg due to inert rotation given in the buoyant leg and translational responses are transferred therefore, k_{74} is active.

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So, friends, we will derive the coefficient of pitch degree of freedom and further in the next lecture let us look at the summary; we have learnt that the structural action of a new generation platform by name triceratops is highly innovative triceratops has 9 degrees of freedom, 6 degrees of the freedom for the buoyant leg 3 rotations and 3 translations about the respective axes x y and z.

And of course, the deck has an additional 3 degrees of freedom which are all the translations and no rotations are transferred to the deck. So, I should say no rotations are transferred therefore; they are ideally rotations of the deck alone. So, I should write here

rotations of the deck alone which will be independent of the rotation of the buoyant leg, we have learnt also that action of ball joints restrain transfer of rotations from the buoyant legs to the deck and therefore, deck is partially isolated.

So, triceratops derives advantages of tethered construction from that of a TLP and a deep draft construction of the buoyant legs from that of a spar. So, it is an hybrid new innovative, new generation platform for which we are trying to derive the stiffness coefficients to form the stiffness matrix from the first principles which can be aided by computer methods.

Thank you.