Computer Methods of Analysis of Offshore Structures
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Module - 02
Lecture - 18
Newmark's method

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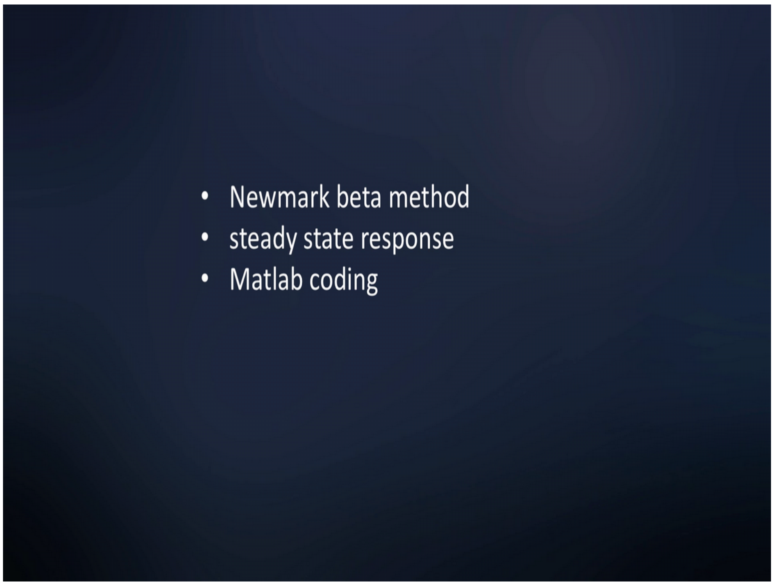

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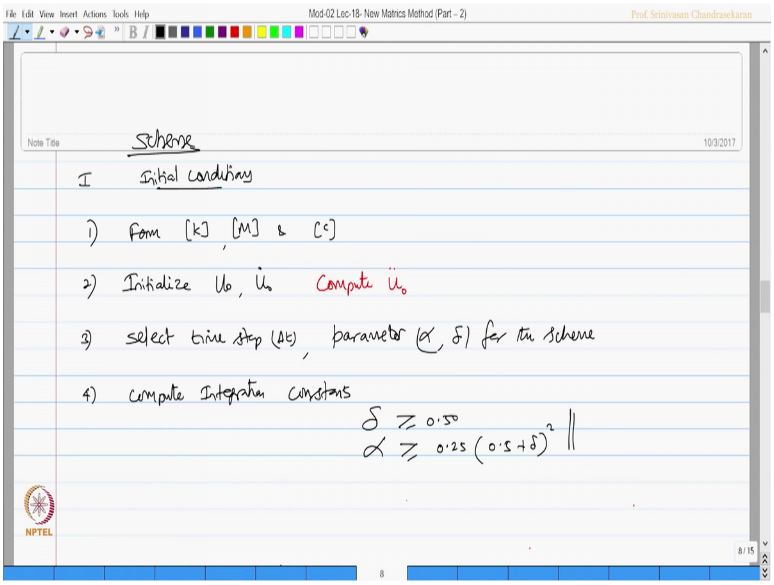

Suggested by Newmark's beta scheme as the following steps, the first step is related to initial conditions. So, it says form the k matrix, m matrix and c matrix, which we already know. The second step initialized U 0 and U dot 0 and compute u double dot 0, third select time step delta t and then the parameters alpha and del for the scheme.

Fourth step compute integration constants; there are many integration constants which are to be computed for the system, del value should be greater than or equal to 0.5 and alpha should be greater than or equal to 0.25 of 0.5 plus del the whole square.

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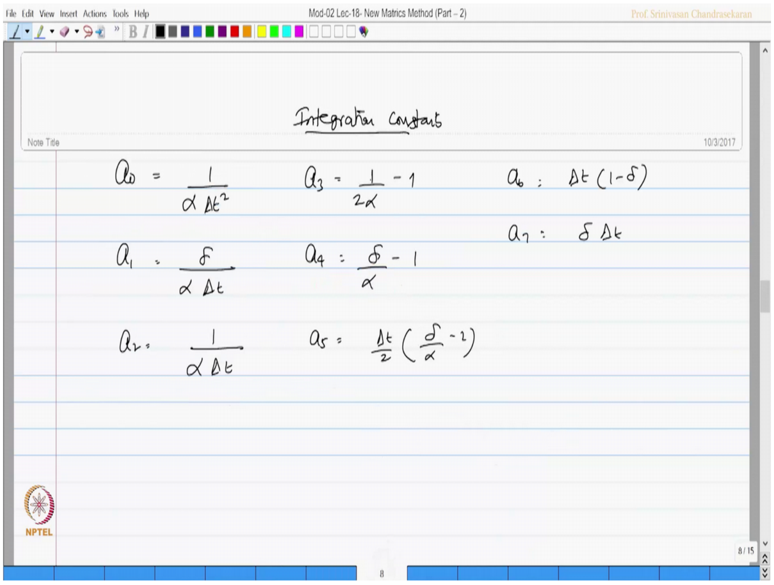

So, a 0 these are integration constants 1 by alpha delta t square, a 1 alpha delta t, a 2 1 by alpha delta t, a 3 1 by 2 alpha minus 1, a 4 del by alpha minus 1, a 5 delta t by 2 del by alpha minus 2, a 6 delta t 1 minus del and a 7del delta t.

So, these are integration constants, once I do this I go to the fifth step.

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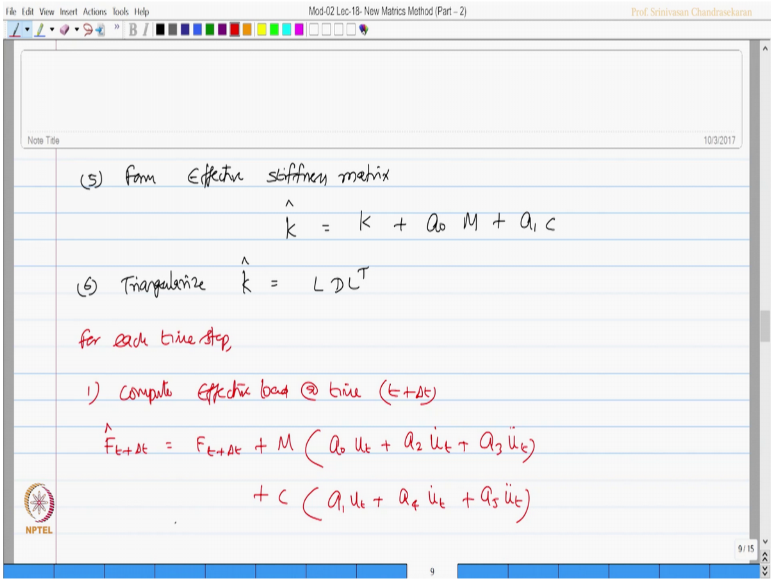

This is form effective stiffness matrix which I called as K hat which is given by original K plus a 0 M plus a 1 c; then triangularize k hat that is k hat will be equal to L D L transpose. For each time step now compute the effective load at new time step T plus delta t that is F hat T plus delta t will be equal to F T plus delta t plus m times of a 0 u T plus a 2 u dot T plus a 3 u double dot 3 plus c times of a 1, u T plus a 4 u dot t, plus a 5 u double dot t.

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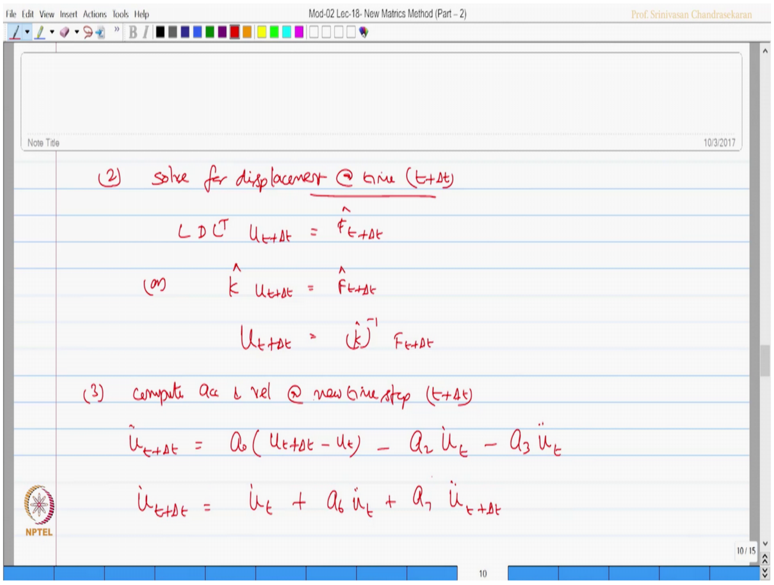

The next step is solve for displacement, at time T plus delta t that is a new time step. So, L D L transposes U T plus delta t is F hat T plus delta t, because LDL transpose is actually equal to k hat. So, stiffness into displacement will give you the force on the other hand I can always say k hat u plus delta t is actually equal to F T plus delta t.

So, I want U T plus delta t that is a displacement at the new time step is given by k hat inverse Ft plus delta t in the third step compute acceleration and velocity at new time step, T plus delta t which can be given by u double dot T plus delta t will equal to a 0 u T plus delta t minus u T minus a 2 U dot T minus a 3 u double dot T and the velocity at the new time step will be equal to u dot T plus a 6 u double dot T plus a 7 u double dot T plus delta t. So, that is a scheme available to us, let us try to take an example problem and see how are you going to solve this.

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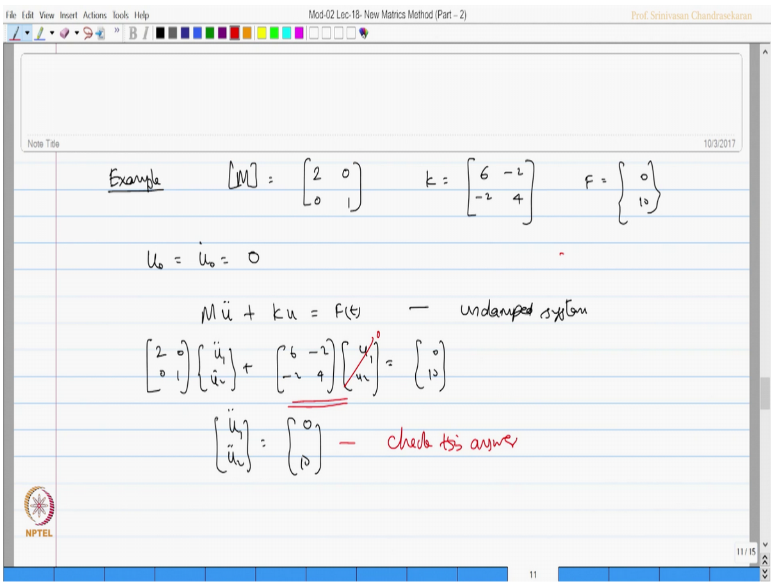

The example problem what I going to take is the 2 degree freedom system simple problem, where the mass matrix is 2 0 0 1 and the stiffness matrix is 6 minus 2 minus 2 and 4 and the force vector is given by 0 and ten. So, let us say u 0 u dot 0 is 0. So, you know m u double dot plus k u is f of T where I am taking un damped system.

So, one can say 2 0 0 1 of u 1 and u 2 plus 6 minus 2 minus 2 4 of u 1 u 2 is 0 and 10. So, by this logic I can always find u 1 double dot and u 2 double dot by taking inverse of this matrix and multiplication and I will get this value as 0 and 10. Please check this value please check this answer by a simple mathematics as we can simply multiply these 2 because this value is actually 0 is it not this value 0.

So, I will take an inverse of this matrix and try to multiply with this vector get this value check this answer. Once I get this I want to perform important step the important step is.

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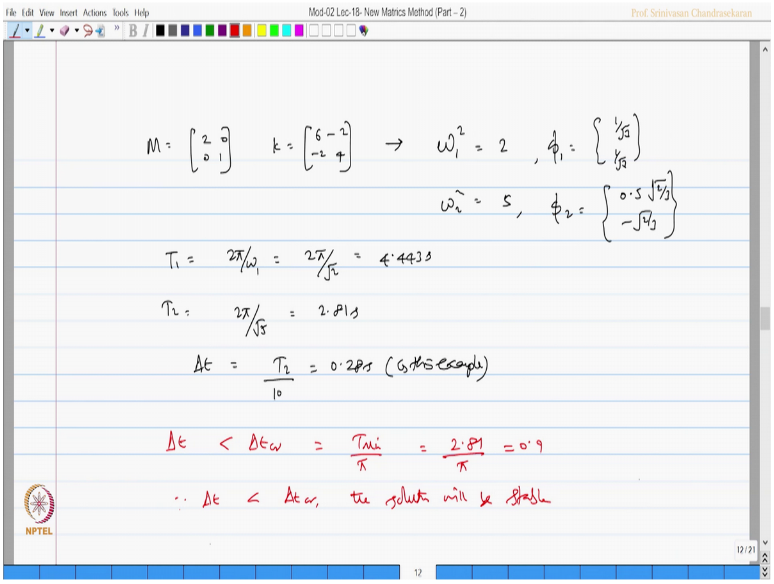

You know mass matrix is actually 2 0 0 1 and k matrix is 6 minus 2 minus 2 1 4, one can easily use the existing computer programs which I gave you to find out the frequencies. So, let us do that I will get omega 1 square as 2 and the corresponding 5 1 as 1 by root 3 and 1 by root 3.

Similarly, omega 2 square is 5 and the corresponding 5 2 I will get this as 0.5 root 2 by 3 and minus root 2 by 3. Therefore, I can get T 1 has 2 pi by omega 1 which is 2 pi by root 2 which is 4.443 seconds and T 2 is 2 pi by root 5 which is 2.81 seconds. So, delta t should be actually equal to the ratio between these 2 values, which I take it as maybe delta t will be considered as one tenth of the lowest period I take it as 0.28 seconds in this example.

We should also check that this delta t should be less than or equal to delta t c r which should be T minimum by pi that is 2.81 by pi which becomes 0.9. Since, delta t is less than delta tcr the solution will be stable. So, we have ensured this once I do this then let me calculate let us calculate the integration constants for alpha equals 0.25 for del equals 0.5 and delta t equals 0.28.

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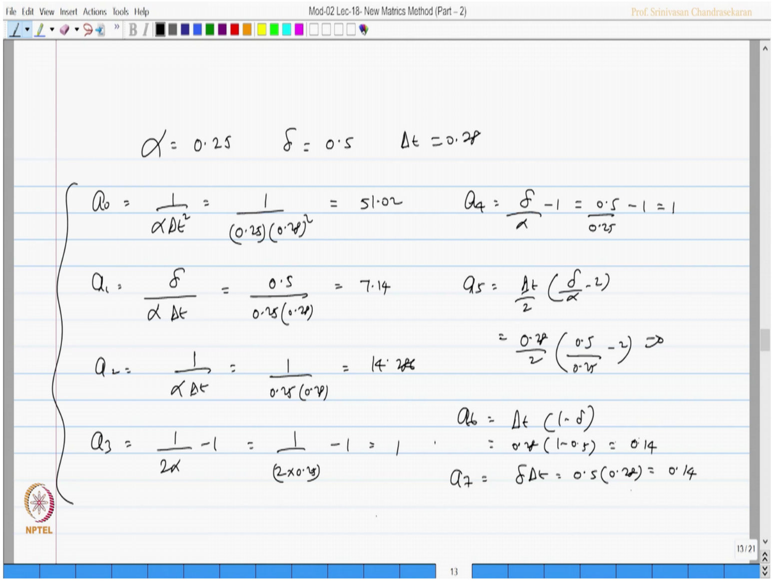

So, a 0 is 1 by alpha delta t square which is 1 by 0.25 0.28 square which gives me 51.02 a 1 is del by alpha delta t which is 0.5 by 0.25 of 0.28 which gives me 7.14, a 2 is 1 by alpha delta t which is 1 by 0.25 and 0.28 which is 14.286 and a 3 is 1 by 2 alpha minus 1 2.25 minus 1, which is 1 and a 4 is minus 1 which is 0.5 by 0.25 minus 1 which is 1 again a 5 delta t by 2 del by alpha minus 2 which is 0.28 by 2 0.5 by 0.25 minus 2 which become 0.

A 6 is delta t 1 minus del which is 0.28 1 minus 0.5 which is 0.14 and a 7 is del delta t which is 0.5 into 0.28 which is 0.14. So, I have got all the integration constants required form a numerical scheme.

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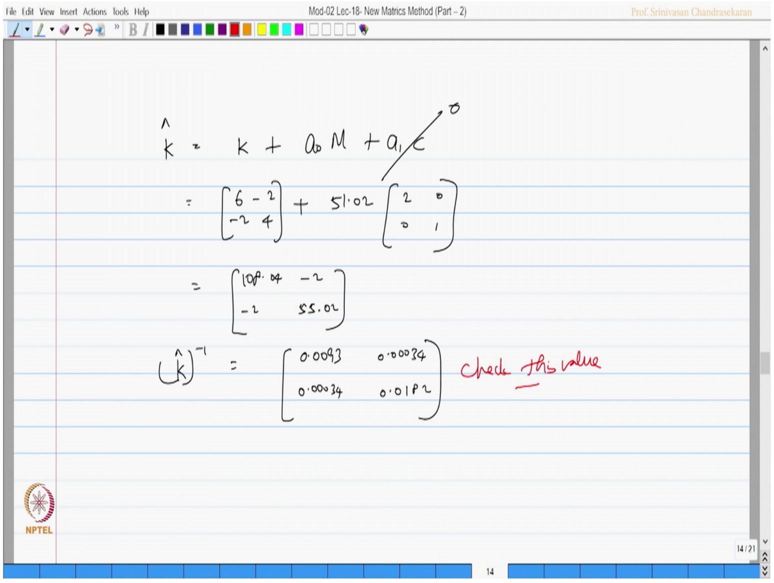

So, the next step could be compute K hat which is K plus a 0 m plus a 1 c. So, this is 0 in my case. So, I should say is going to be 6 minus 2 minus 2 4 plus a 0 is 51.02, 2 0 0 1 which gives me k hat has.

108.04 minus 2 minus 2 55.02, I can use the standard subroutine to compute the inverse. So, I get k hat inverse as you can check this value 1 0 0 9 3, 0.0 0 0 3 4, 0.0 0 0 3 4 and 0.0 1 8 2 I request you to please check this value before you proceed it is a standard solution you can obtain this.

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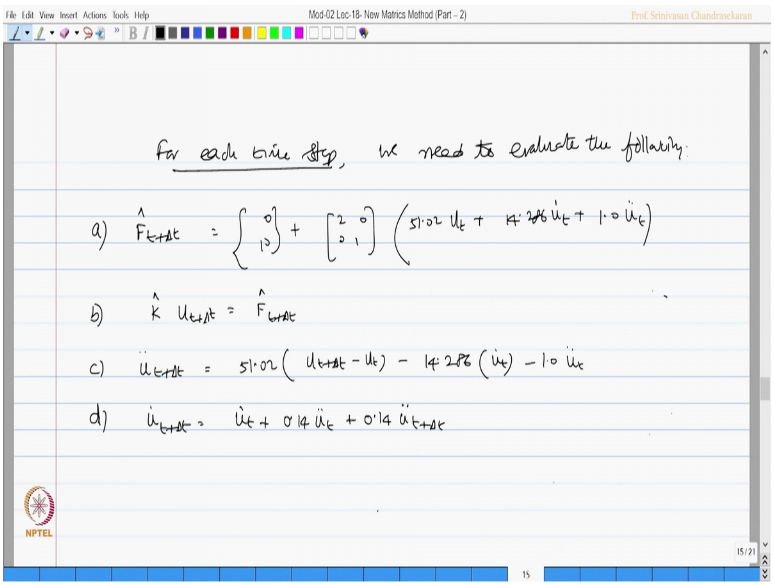

So, for each time step we need to evaluate the following let us see what are they we need to calculate F hat t plus delta t which is the original F plus m times of 2 0 0 1 of 51.02 u T plus 14.286 u dot T plus 1.0 u double dot t, then we need to find K hat u t plus delta t is equal to F hat t plus delta t then one need to find out u double dot t plus delta t as 51.02 u t plus delta t minus u t minus 14.286 of u dot t minus 1.0 u double dot t.

Then 1 need to also find out u dot t plus delta t as u dot t plus 0.14 u double dot t plus 0.14 u double dot t plus delta t we need t o know this let us quickly see how we can write the computer program for this and solve the problem using computer code.

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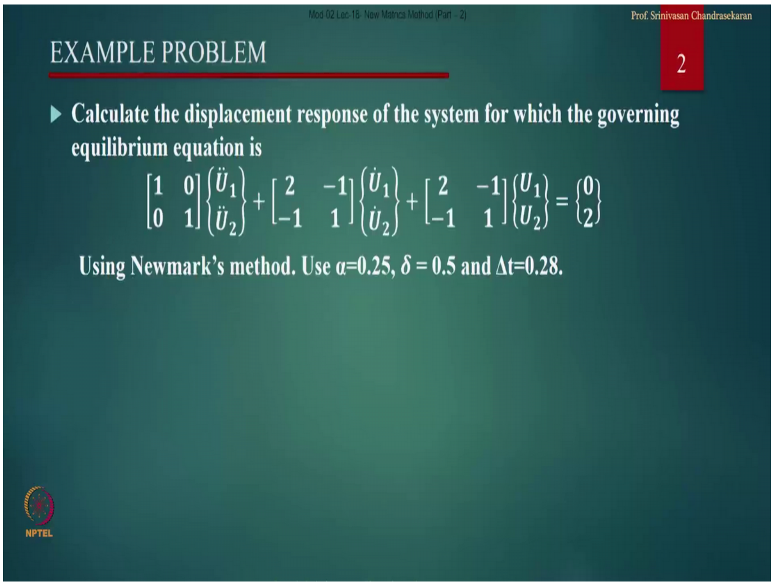

So, the same example is being considered now you know this my mass matrix this my mass matrix this my stiffness matrix, I want to consider the system with the mass value the damping value and the stiffness value and that is my initial force vector, I am using alpha and del and delta t as you see from here.

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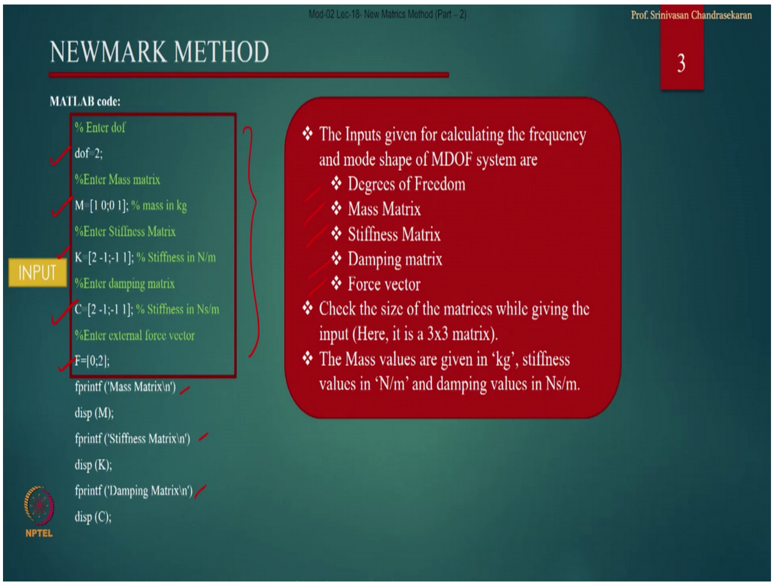

So, now let us enter the degree of freedom which is 2, the mass matrix is entered and the stiffness matrix is entered and the damping matrix of course, computer in entered in this example and the force vector is entered.

Then let us verify this by printing the mass matrix, the stiffness matrix and the damping matrix. So, we need to give the inputs as degrees of freedom mass matrix stiffness damping and force what we did here. So, please check the units of mass and stiffness matrices and damping values before you enter be very careful about the units.

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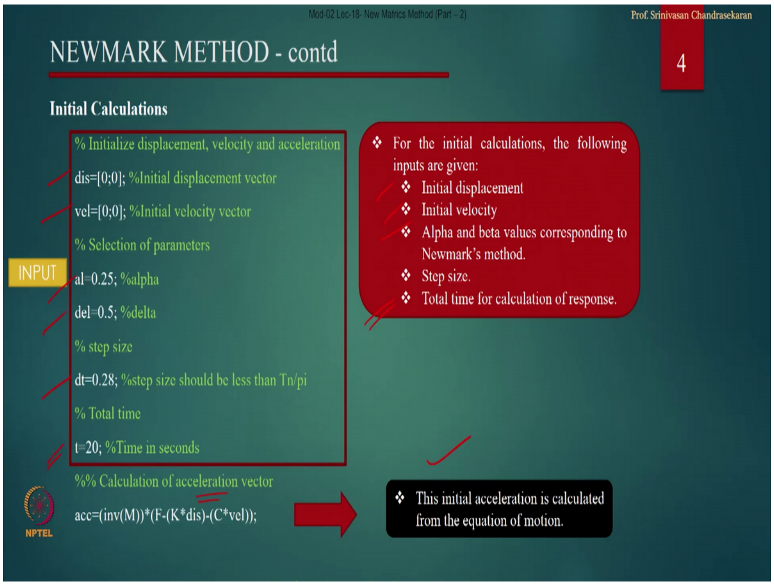

Once I do this, then I do initial calculation the following inputs are required I need to give initial displacement initial velocity initial displacement velocity are set to be 0 and I have to compute the initial acceleration which I will compute later, then I need to give the alpha and beta values which are entered here the del value is referred as beta in the literature that is why this method is call Newmarks beta method ok.

You also decide the step size that the delta t should be lesser than the delta t critical, then you estimate what is the total time you require for calculate in the response in this case we have taken as 20, then compute the initial acceleration vector which is required to solve the problem. So, this is computed from the original equation of motion.

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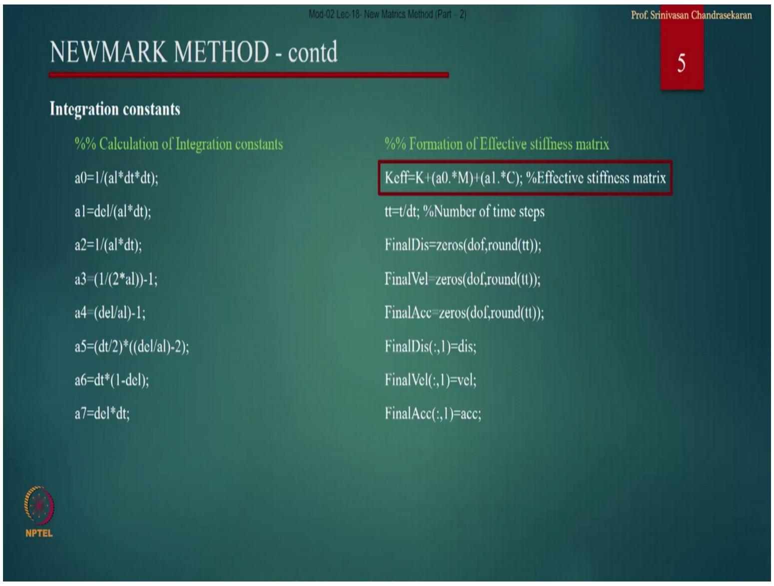

Once I do this I now estimate the integration constants a 0, a 1, a 2 and so on once I do this then I compute the effective stiffness matrix which is k hat using the equation then I find the final displacement velocity in acceleration and keep on iterating it.

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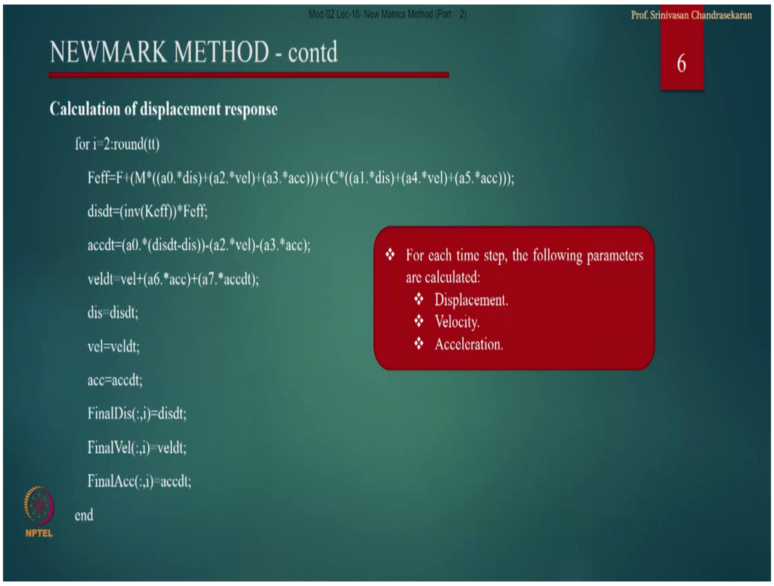

So, calculate the displacement responses, then plot the values and try to understand how they vary.

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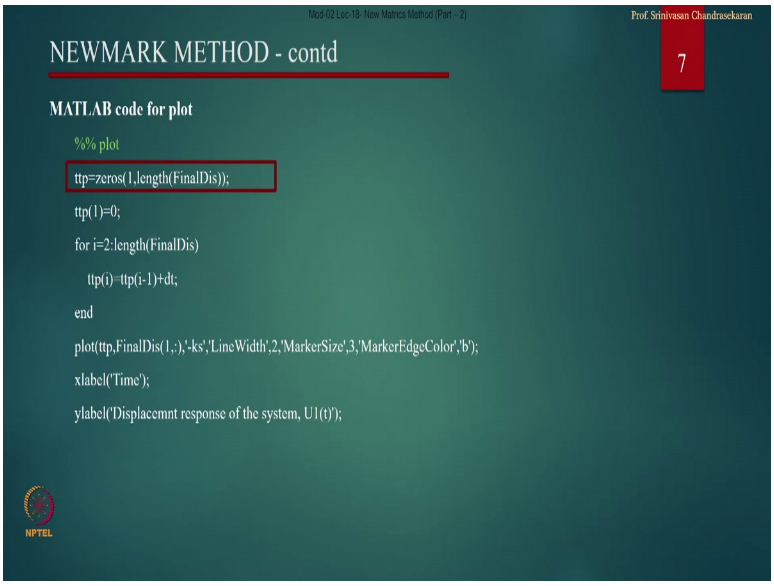

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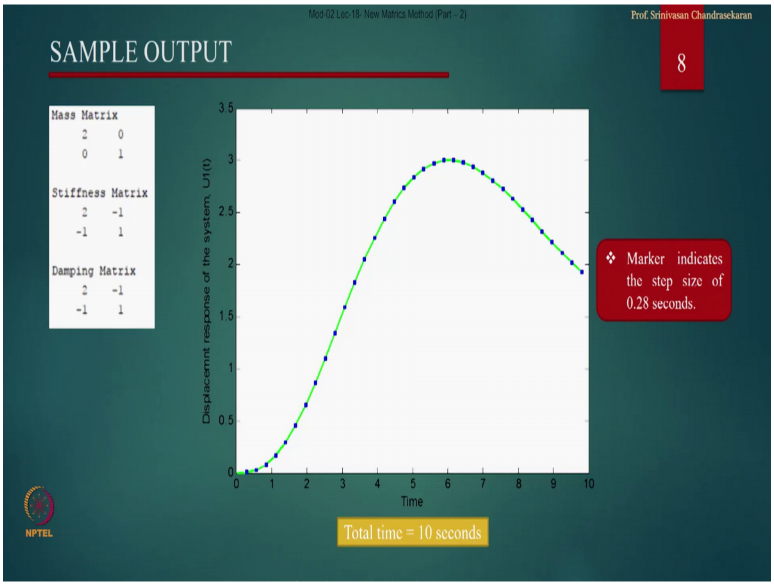

If I try to plot them the mass matrix the stiffness matrix and damping matrix are given as a sample output, this is the output at every 0.28 you get the plot value the marker and the total time taken is 10 seconds.

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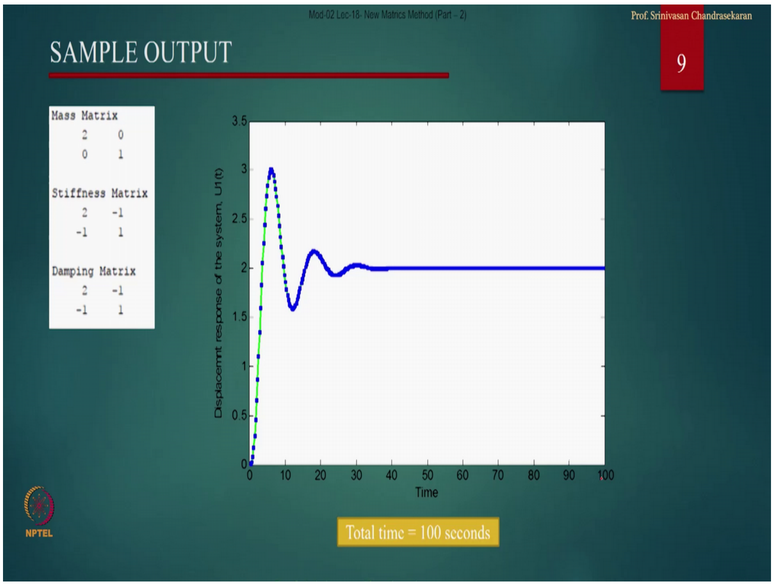

If I try to do this for hundred seconds, you will see that initially for 10 seconds .the values are changing, then it becomes constant then after a specific time it becomes completely unchangeable it is a steady stable output is available from the scheme.

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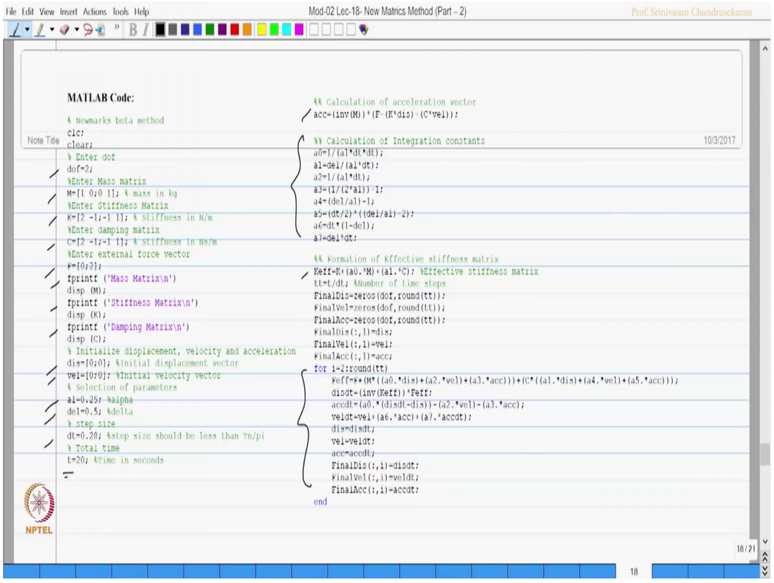

So, let us see the computer coding of this. So, this is my computer coding which we just now explained. So, we have entered the degrees of freedom the mass matrix the stiffness matrix and also the damping matrix we have entered the initial force vector we have computed and we have printed the mass matrix the stiffness and damping for verification.

We gave the initial displacement in velocity as 0 we computed the initial acceleration we also fixed the alpha and beta value for the scheme, we also taken the time step we want to do it for total 20 seconds. We calculated integration constants then we computed the k effective k hat matrix, and then found out for a scheme of iteration the velocity displacement and acceleration as you see in the scheme.

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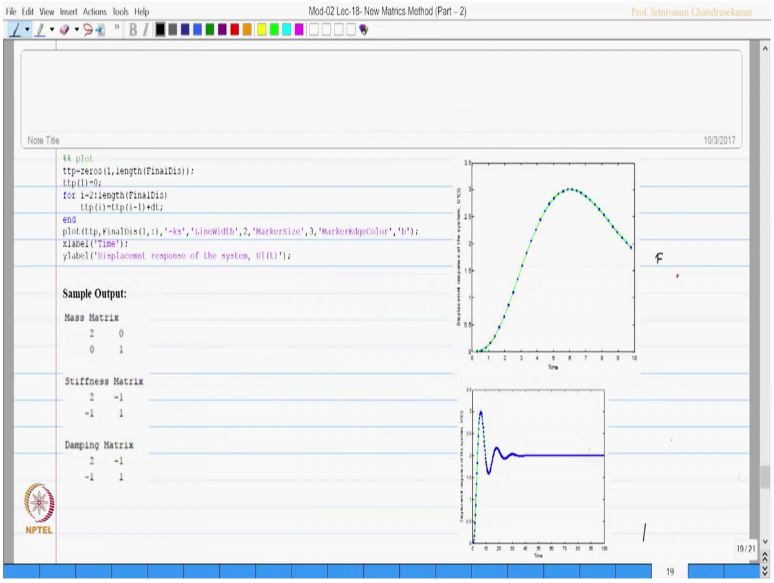

If you look at the answers we have plotted at every delta t value the steps at every 0.28 when we extend this for 100 seconds you will see there is a steady stable solution available in the scheme.

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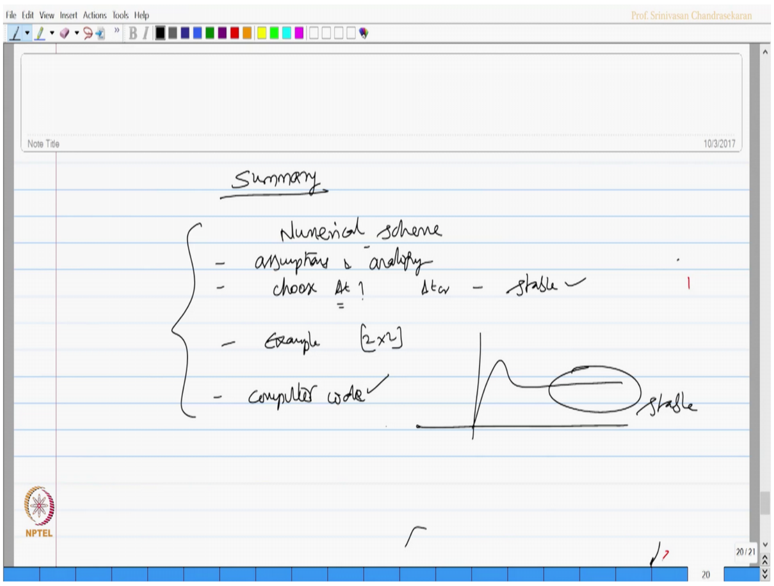

So, friends in this lecture we understood how to solve an equation of motion using numerical scheme. What are the basic assumptions and analogy followed in this scheme, how to select or choose the time step for numerical integration, how to compute the critical time step. So, that the time step chosen is lesser than a critical. So, to get a stable unconditional solution we solved an example problem of 2 by 2 matrix and we found out that after along iteration the solution becomes still.

So, the computer coding is available to solve this problem to solve this problem. So, one can try to repeat the solution by some other example and see how this can be comfortably followed by you.

Thank you very much.