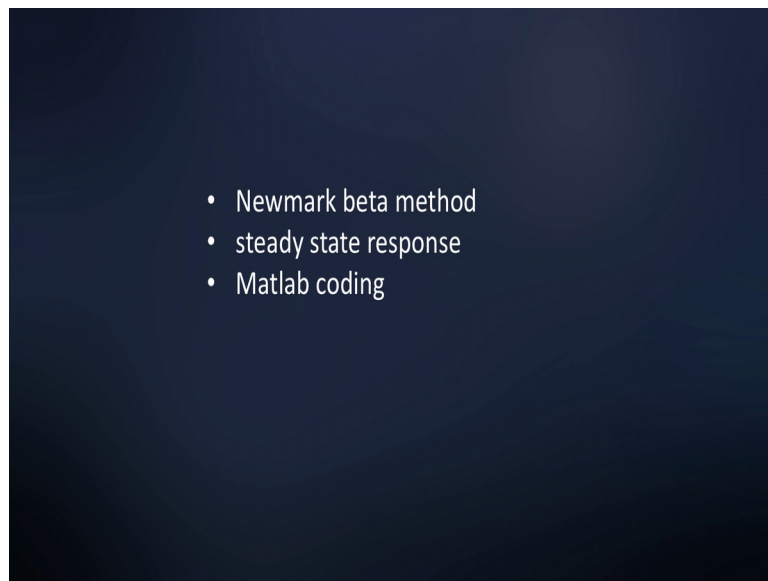


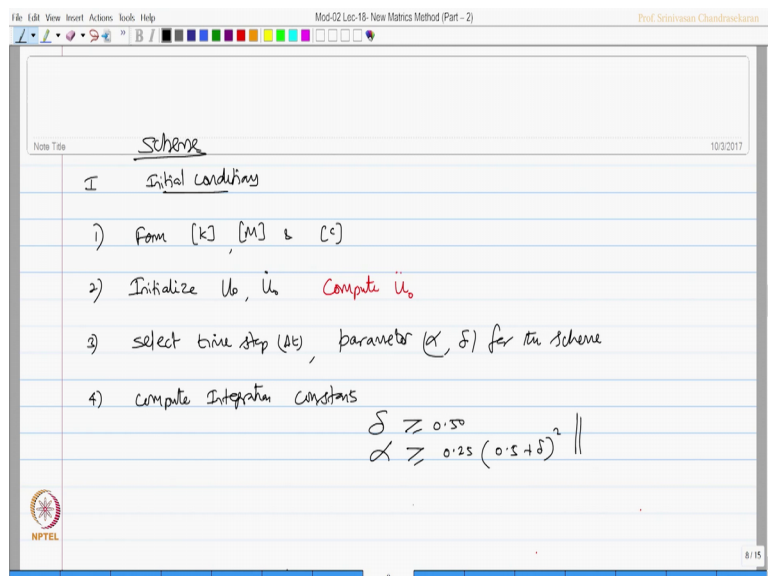
Computer Methods of Analysis of Offshore Structures
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Department of Ocean Engineering
Indian Institute of Technology, Madras

Module - 02
Lecture - 18
Newmark's method

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Suggested by Newmark's beta scheme as the following steps, the first step is related to initial conditions. So, it says form the k matrix, m matrix and c matrix, which we already know. The second step initialized U_0 and \dot{U}_0 and compute \ddot{u}_0 , third select time step Δt and then the parameters α and β for the scheme.

Fourth step compute integration constants; there are many integration constants which are to be computed for the system, β value should be greater than or equal to 0.5 and α should be greater than or equal to $0.25 + 0.5\beta$.

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Integration Constants

$$a_0 = \frac{1}{\alpha \Delta t^2} \quad a_3 = \frac{1}{2\alpha} - 1 \quad a_6 = \Delta t (1 - \beta)$$

$$a_1 = \frac{\delta}{\alpha \Delta t} \quad a_4 = \frac{\delta}{\alpha} - 1 \quad a_7 = \beta \Delta t$$

$$a_2 = \frac{1}{\alpha \Delta t} \quad a_5 = \frac{\Delta t}{2} \left(\frac{\delta}{\alpha} - 1 \right)$$

So, a_0 these are integration constants 1 by $\alpha \Delta t^2$, a_1 $\alpha \Delta t$, a_2 1 by $\alpha \Delta t$, a_3 1 by 2α minus 1 , a_4 δ by α minus 1 , a_5 Δt by 2 δ by α minus 2 , a_6 Δt 1 minus β and a_7 $\beta \Delta t$.

So, these are integration constants, once I do this I go to the fifth step.

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(5) form effective stiffness matrix

$$\hat{k} = K + a_0 M + a_1 C$$

(6) Triangularize $\hat{k} = L D L^T$

for each time step,

1) compute effective load @ time $(t+\Delta t)$

$$\hat{F}_{t+\Delta t} = F_{t+\Delta t} + M (a_0 u_t + a_2 \dot{u}_t + a_3 \ddot{u}_t) + C (a_4 u_t + a_5 \dot{u}_t + a_6 \ddot{u}_t)$$

This is form effective stiffness matrix which I called as K hat which is given by original K plus a 0 M plus a 1 c; then triangularize k hat that is k hat will be equal to L D L transpose. For each time step now compute the effective load at new time step T plus delta t that is F hat T plus delta t will be equal to F T plus delta t plus m times of a 0 u T plus a 2 u dot T plus a 3 u double dot 3 plus c times of a 1, u T plus a 4 u dot t, plus a 5 u double dot t.

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(2) solve for displacement @ time $(t+\Delta t)$

$$L D L^T u_{t+\Delta t} = \hat{F}_{t+\Delta t}$$

(or)

$$\hat{k} u_{t+\Delta t} = \hat{F}_{t+\Delta t}$$

$$u_{t+\Delta t} = (\hat{k})^{-1} \hat{F}_{t+\Delta t}$$

(3) compute acc & vel @ new time step $(t+\Delta t)$

$$\ddot{u}_{t+\Delta t} = a_0 (u_{t+\Delta t} - u_t) - a_2 \dot{u}_t - a_3 \ddot{u}_t$$

$$\dot{u}_{t+\Delta t} = \dot{u}_t + a_6 \dot{u}_t + a_7 \ddot{u}_{t+\Delta t}$$

The next step is solve for displacement, at time T plus delta t that is a new time step. So, $L D L^T$ transposes U^T plus delta t is F^T plus delta t, because LDL transpose is actually equal to K^T . So, stiffness into displacement will give you the force on the other hand I can always say $K^T u$ plus delta t is actually equal to F^T plus delta t.

So, I want U^T plus delta t that is a displacement at the new time step is given by K^T inverse F^T plus delta t in the third step compute acceleration and velocity at new time step, T plus delta t which can be given by u''^T plus delta t will equal to $a_0 U^T$ plus delta t minus $a_2 U^T$ minus $a_3 u''^T$ and the velocity at the new time step will be equal to u'^T plus $a_6 u''^T$ plus $a_7 u''^T$ plus delta t. So, that is a scheme available to us, let us try to take an example problem and see how are you going to solve this.

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Example $M = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ $K = \begin{bmatrix} 6 & -2 \\ -2 & 4 \end{bmatrix}$ $F = \begin{bmatrix} 0 \\ 10 \end{bmatrix}$

$u_0 = u'_0 = 0$

$M \ddot{u} + K u = F(t)$ — undamped system

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} + \begin{bmatrix} 6 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \end{bmatrix}$$

$\begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \end{bmatrix}$ — check this answer

The example problem what I going to take is the 2 degree freedom system simple problem, where the mass matrix is 2 0 0 1 and the stiffness matrix is 6 minus 2 minus 2 and 4 and the force vector is given by 0 and ten. So, let us say $u_0 = u'_0 = 0$. So, you know $m u'' + k u = f$ of T where I am taking un damped system.

So, one can say 2 0 0 1 of u_1 and u_2 plus 6 minus 2 minus 2 4 of u_1 u_2 is 0 and 10. So, by this logic I can always find u_1'' and u_2'' by taking inverse of this matrix and multiplication and I will get this value as 0 and 10. Please check this value

please check this answer by a simple mathematics as we can simply multiply these 2 because this value is actually 0 is it not this value 0.

So, I will take an inverse of this matrix and try to multiply with this vector get this value check this answer. Once I get this I want to perform important step the important step is.

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The image shows a digital whiteboard with the following handwritten content:

$$M = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \quad k = \begin{bmatrix} 6 & -2 \\ -2 & 4 \end{bmatrix} \rightarrow \omega_1^2 = 2, \quad \phi_1 = \begin{Bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{Bmatrix}$$

$$\omega_2^2 = 5, \quad \phi_2 = \begin{Bmatrix} 0.5\sqrt{5} \\ -\sqrt{5} \end{Bmatrix}$$

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{\sqrt{2}} = 4.443s$$

$$T_2 = \frac{2\pi}{\sqrt{5}} = 2.81s$$

$$\Delta t = \frac{T_2}{10} = 0.281s \text{ (6\% example)}$$

$\Delta t < \Delta t_{cr} = \frac{T_{min}}{\pi} = \frac{2.81}{\pi} = 0.9$
 $\therefore \Delta t < \Delta t_{cr}$, the solution will be stable

You know mass matrix is actually 2 0 0 1 and k matrix is 6 minus 2 minus 2 1 4, one can easily use the existing computer programs which I gave you to find out the frequencies. So, let us do that I will get omega 1 square as 2 and the corresponding 5 1 as 1 by root 3 and 1 by root 3.

Similarly, omega 2 square is 5 and the corresponding 5 2 I will get this as 0.5 root 2 by 3 and minus root 2 by 3. Therefore, I can get T 1 has 2 pi by omega 1 which is 2 pi by root 2 which is 4.443 seconds and T 2 is 2 pi by root 5 which is 2.81 seconds. So, delta t should be actually equal to the ratio between these 2 values, which I take it as maybe delta t will be considered as one tenth of the lowest period I take it as 0.28 seconds in this example.

We should also check that this delta t should be less than or equal to delta t c r which should be T minimum by pi that is 2.81 by pi which becomes 0.9. Since, delta t is less than delta tcr the solution will be stable. So, we have ensured this once I do this then let

me calculate let us calculate the integration constants for alpha equals 0.25 for del equals 0.5 and delta t equals 0.28.

(Refer Slide Time: 11:35)

The image shows a slide with handwritten calculations for integration constants α_0 through α_7 . The parameters are $\alpha = 0.25$, $\delta = 0.5$, and $\Delta t = 0.28$.

$$\alpha = 0.25 \quad \delta = 0.5 \quad \Delta t = 0.28$$

$$\alpha_0 = \frac{1}{\alpha \Delta t^2} = \frac{1}{(0.25)(0.28)^2} = 51.02 \quad \alpha_4 = \frac{\delta}{\alpha} - 1 = \frac{0.5}{0.25} - 1 = 1$$

$$\alpha_1 = \frac{\delta}{\alpha \Delta t} = \frac{0.5}{0.25(0.28)} = 7.14 \quad \alpha_5 = \frac{\Delta t}{2} \left(\frac{\delta}{\alpha} - 2 \right)$$

$$= \frac{0.28}{2} \left(\frac{0.5}{0.25} - 2 \right) = 0$$

$$\alpha_2 = \frac{1}{\alpha \Delta t} = \frac{1}{0.25(0.28)} = 14.286$$

$$\alpha_6 = \Delta t (1 - \delta) = 0.28(1 - 0.5) = 0.14$$

$$\alpha_3 = \frac{1}{2\alpha} - 1 = \frac{1}{(2 \times 0.25)} - 1 = 1 \quad \alpha_7 = \delta \Delta t = 0.5(0.28) = 0.14$$

So, a 0 is 1 by alpha delta t square which is 1 by 0.25 0.28 square which gives me 51.02 a 1 is del by alpha delta t which is 0.5 by 0.25 of 0.28 which gives me 7.14, a 2 is 1 by alpha delta t which is 1 by 0.25 and 0.28 which is 14.286 and a 3 is 1 by 2 alpha minus 1 2.25 minus 1, which is 1 and a 4 is minus 1 which is 0.5 by 0.25 minus 1 which is 1 again a 5 delta t by 2 del by alpha minus 2 which is 0.28 by 2 0.5 by 0.25 minus 2 which become 0.

A 6 is delta t 1 minus del which is 0.28 1 minus 0.5 which is 0.14 and a 7 is del delta t which is 0.5 into 0.28 which is 0.14. So, I have got all the integration constants required form a numerical scheme.

(Refer Slide Time: 13:44)

$$\hat{K} = K + a_0 M + a_1 C$$

$$= \begin{bmatrix} 6 & -2 \\ -2 & 4 \end{bmatrix} + 51.02 \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 108.04 & -2 \\ -2 & 55.02 \end{bmatrix}$$

$$(\hat{K})^{-1} = \begin{bmatrix} 0.0093 & 0.00034 \\ 0.00034 & 0.0182 \end{bmatrix} \text{ check this value}$$

So, the next step could be compute \hat{K} which is K plus $a_0 m$ plus $a_1 c$. So, this is 0 in my case. So, I should say is going to be 6 minus 2 minus 2 4 plus a 0 is 51.02, 2 0 0 1 which gives me \hat{k} has.

108.04 minus 2 minus 2 55.02, I can use the standard subroutine to compute the inverse. So, I get \hat{k} inverse as you can check this value 1 0 0 9 3, 0.0 0 0 3 4, 0.0 0 0 3 4 and 0.0 1 8 2 I request you to please check this value before you proceed it is a standard solution you can obtain this.

(Refer Slide Time: 14:55)

for each time step, we need to evaluate the following.

$$a) \hat{F}_{t+dt} = \begin{bmatrix} 0 \\ 12 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} (51.02 \dot{u}_t + 14.286 \ddot{u}_t + 1.0 \ddot{u}_t)$$

$$b) \hat{K} u_{t+dt} = \hat{F}_{t+dt}$$

$$c) \ddot{u}_{t+dt} = 51.02 (u_{t+dt} - u_t) - 14.286 (\dot{u}_t) - 1.0 \ddot{u}_t$$

$$d) \dot{u}_{t+dt} = \dot{u}_t + 0.14 \ddot{u}_t + 0.14 \ddot{u}_{t+dt}$$

So, for each time step we need to evaluate the following let us see what are they we need to calculate F at $t + \Delta t$ which is the original F plus m times of $2 \dot{u}$ plus $1.0 \ddot{u}$ plus $14.286 u$ plus $1.0 \ddot{u}$, then we need to find K at $t + \Delta t$ is equal to F at $t + \Delta t$ then one need to find out \ddot{u} at $t + \Delta t$ as $51.02 u$ plus Δt minus u minus 14.286 of \dot{u} minus $1.0 \ddot{u}$.

Then I need to also find out \dot{u} at $t + \Delta t$ as \dot{u} plus $0.14 \ddot{u}$ plus $0.14 \ddot{u}$ plus Δt we need to know this let us quickly see how we can write the computer program for this and solve the problem using computer code.

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Mod 02, Lec-18 - New Matrix Method (Part - 2) Prof. Srinivasan Chandrasekaran


EXAMPLE PROBLEM

2

► Calculate the displacement response of the system for which the governing equilibrium equation is

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{U}_1 \\ \ddot{U}_2 \end{Bmatrix} + \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{Bmatrix} + \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 2 \end{Bmatrix}$$

Using Newmark's method. Use $\alpha=0.25$, $\delta = 0.5$ and $\Delta t=0.28$.

 NPTEL

So, the same example is being considered now you know this my mass matrix this my mass matrix this my stiffness matrix, I want to consider the system with the mass value the damping value and the stiffness value and that is my initial force vector, I am using alpha and del and delta t as you see from here.

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Mod-02 Lec-18 New Matrix Method (Part - 2) Prof. Srinivasan Chandrasekaran

NEWMARK METHOD

3

MATLAB code:

```
% Enter dof
dof=2;
% Enter Mass matrix
M=[1 0 0 1]; % mass in kg
% Enter Stiffness Matrix
K=[2 -1 -1 1]; % Stiffness in N/m
% Enter damping matrix
C=[2 -1 -1 1]; % Stiffness in Ns/m
% Enter external force vector
F=[0;2];
fprintf('Mass Matrix \n')
disp(M);
fprintf('Stiffness Matrix \n')
disp(K);
fprintf('Damping Matrix \n')
disp(C);
```

INPUT

- ❖ The Inputs given for calculating the frequency and mode shape of MDOF system are
 - ❖ Degrees of Freedom
 - ❖ Mass Matrix
 - ❖ Stiffness Matrix
 - ❖ Damping matrix
 - ❖ Force vector
- ❖ Check the size of the matrices while giving the input (Here, it is a 3x3 matrix).
- ❖ The Mass values are given in 'kg', stiffness values in 'N/m' and damping values in Ns/m.

So, now let us enter the degree of freedom which is 2, the mass matrix is entered and the stiffness matrix is entered and the damping matrix of course, computer in entered in this example and the force vector is entered.

Then let us verify this by printing the mass matrix, the stiffness matrix and the damping matrix. So, we need to give the inputs as degrees of freedom mass matrix stiffness damping and force what we did here. So, please check the units of mass and stiffness matrices and damping values before you enter be very careful about the units.

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NEWMARK METHOD - contd

4

Initial Calculations

```
% Initialize displacement, velocity and acceleration
dis=[0;0]; %Initial displacement vector
vel=[0;0]; %Initial velocity vector
% Selection of parameters
al=0.25; %alpha
del=0.5; %delta
% step size
dt=0.28; %step size should be less than Tn/pi
% Total time
t=20; %Time in seconds
%% Calculation of acceleration vector
acc=(inv(M))*(F-(K*dis)-(C*vel));
```

INPUT

- ❖ For the initial calculations, the following inputs are given:
 - ❖ Initial displacement
 - ❖ Initial velocity
 - ❖ Alpha and beta values corresponding to Newmark's method.
 - ❖ Step size.
 - ❖ Total time for calculation of response.
- ❖ This initial acceleration is calculated from the equation of motion.

Once I do this, then I do initial calculation the following inputs are required I need to give initial displacement initial velocity initial displacement velocity are set to be 0 and I have to compute the initial acceleration which I will compute later, then I need to give the alpha and beta values which are entered here the del value is referred as beta in the literature that is why this method is call Newmarks beta method ok.

You also decide the step size that the delta t should be lesser than the delta t critical, then you estimate what is the total time you require for calculate in the response in this case we have taken as 20, then compute the initial acceleration vector which is required to solve the problem. So, this is computed from the original equation of motion.

(Refer Slide Time: 18:48)

```

Mod-02 Lec-18 Newmark Method Part- 2
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5
NEWMARK METHOD - contd
Integration constants
%% Calculation of Integration constants
a0=1/(al*dt*dt);
a1=del/(al*dt);
a2=1/(al*dt);
a3=(1/(2*al))-1;
a4=(del/al)-1;
a5=(dt/2)*(del/al)-2);
a6=dt*(1-del);
a7=del*dt;
%% Formation of Effective stiffness matrix
Keff=K+(a0.*M)+(a1.*C); %Effective stiffness matrix
tt=t/dt; %Number of time steps
FinalDis=zeros(dof,round(tt));
FinalVel=zeros(dof,round(tt));
FinalAcc=zeros(dof,round(tt));
FinalDis(:,1)=dis;
FinalVel(:,1)=vel;
FinalAcc(:,1)=acc;

```

Once I do this I now estimate the integration constants a 0, a 1, a 2 and so on once I do this then I compute the effective stiffness matrix which is k hat using the equation then I find the final displacement velocity in acceleration and keep on iterating it.

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NEWMARK METHOD - contd

6

Calculation of displacement response

```
for i=2:round(t/dt)
    Feff=F+(M*(a0.*dis)+(a2.*vel)+(a3.*acc))+(C*(a1.*dis)+(a4.*vel)+(a5.*acc));
    disdt=(inv(Keff))*Feff;
    accdt=(a0.*(disdt-dis)-(a2.*vel)-(a3.*acc));
    veldt=vel+(a6.*acc)+(a7.*accdt);
    dis=disdt;
    vel=veldt;
    acc=accdt;
    FinalDis(:,i)=disdt;
    FinalVel(:,i)=veldt;
    FinalAcc(:,i)=accdt;
end
```

NPTEL

❖ For each time step, the following parameters are calculated:

- ❖ Displacement.
- ❖ Velocity.
- ❖ Acceleration.

So, calculate the displacement responses, then plot the values and try to understand how they vary.

(Refer Slide Time: 19:16)

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NEWMARK METHOD - contd

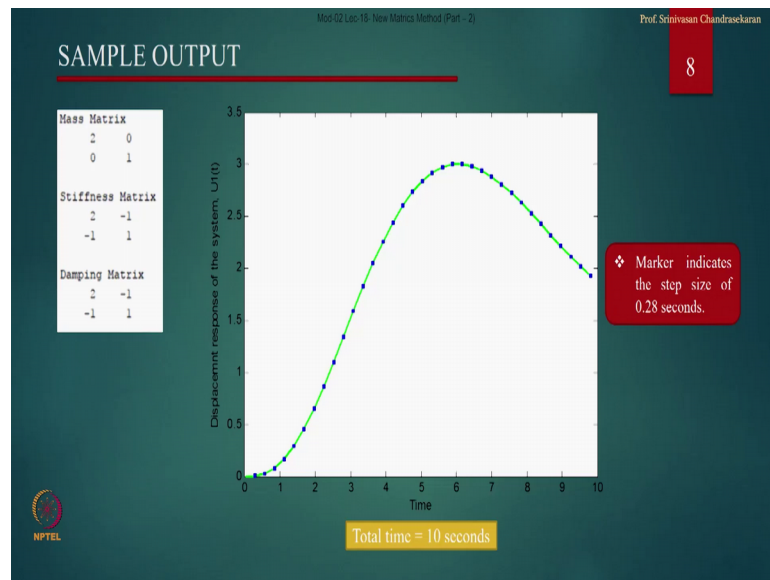
7

MATLAB code for plot

```
%% plot
ttp=zeros(1,length(FinalDis));
ttp(1)=0;
for i=2:length(FinalDis)
    ttp(i)=ttp(i-1)+dt;
end
plot(ttp,FinalDis(1,:),'-ks','LineWidth',2,'MarkerSize',3,'MarkerEdgeColor','b');
xlabel('Time');
ylabel('Displacemnt response of the system, U1(t)');
```

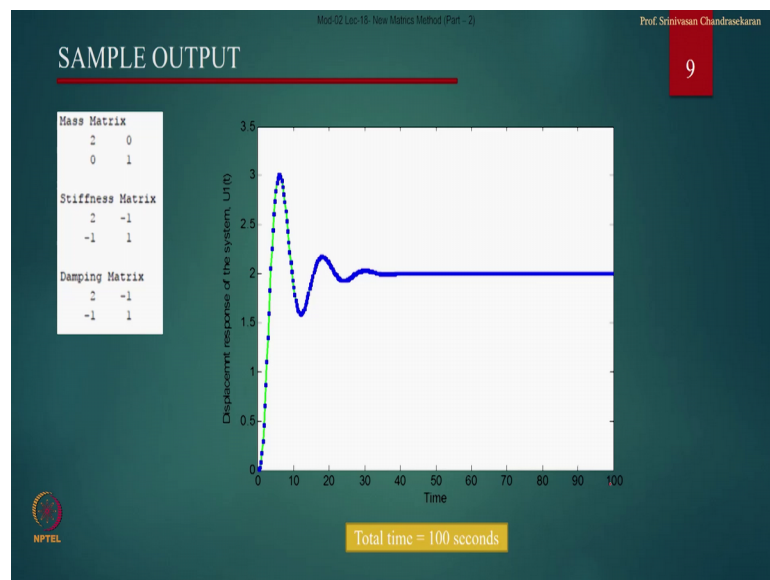
NPTEL

(Refer Slide Time: 19:21)



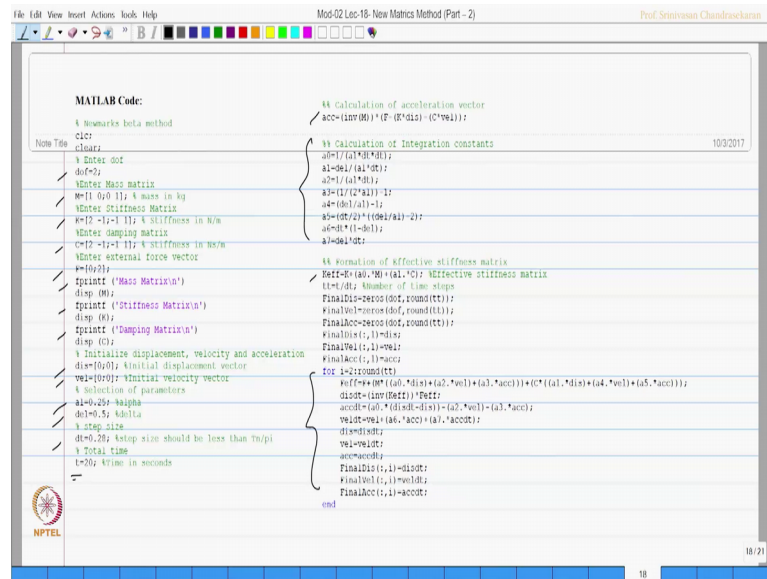
If I try to plot them the mass matrix the stiffness matrix and damping matrix are given as a sample output, this is the output at every 0.28 you get the plot value the marker and the total time taken is 10 seconds.

(Refer Slide Time: 19:41)



If I try to do this for hundred seconds, you will see that initially for 10 seconds the values are changing, then it becomes constant then after a specific time it becomes completely unchangeable it is a steady stable output is available from the scheme.

(Refer Slide Time: 20:02)



```
MATLAB Code:
% Remarks beta method
clear;
% Enter dof
dof=2;
%Enter Mass Matrix
M=[1 0;0 1]; % mass in kg
%Enter Stiffness Matrix
k=[2 -1;-1 1]; % stiffness in N/m
%Enter damping matrix
c=[2 -1;-1 1]; % stiffness in N*s/m
%Enter external force vector
%F=[0;0];
fprintf('Mass Matrix\n');
disp(M);
fprintf('Stiffness Matrix\n');
disp(k);
fprintf('Damping Matrix\n');
disp(c);
% Initialize displacement, velocity and acceleration
dis=[0;0]; %Initial displacement vector
vel=[0;0]; %Initial velocity vector
% Selection of parameters
a1=0.25; %alpha
a2=0.5; %beta
% step size
dt=0.25; %step size should be less than Tn/pi
% Total time
t=20; %time in seconds

% Calculation of acceleration vector
acc=(inv(M))*(F-(k*dis)-(c*vel));

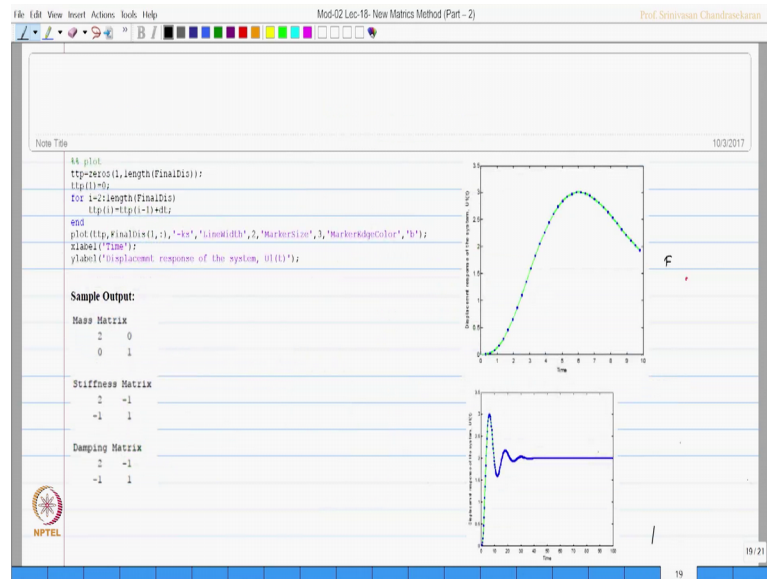
% Calculation of integration constants
a0=1/(a1*a2);
a1=a1/(a1*dt);
a2=1/(a1*a2);
a3=(1/(2*a1))-1;
a4=(a2/a1)-1;
a5=(dt/2)*((a2/a1)-2);
a6=a1*(1-a2);
a7=a2*dt;

% Formation of Effective stiffness matrix
kEff=k+(a3.*c)+(a4.*c); %Effective stiffness matrix
l=length(dof); %Number of time steps
FinalDis=zeros(dof,round(t/dt));
FinalVel=zeros(dof,round(t/dt));
FinalAcc=zeros(dof,round(t/dt));
FinalDis(:,1)=dis;
FinalVel(:,1)=vel;
FinalAcc(:,1)=acc;
for i=2:round(t/dt)
    %F=[0;0];
    %F=(a0.*(dis+(a2.*vel)+(a3.*acc)))+(c.*(a1.*dis)+(a4.*vel)+(a5.*acc));
    disdt=(inv(kEff))*F;
    accdt=(a0.*(disdt-dis))-(a2.*vel)-(a3.*acc);
    veldt=vel+(a6.*acc)+(a7.*accdt);
    dis=disdt;
    vel=veldt;
    acc=accdt;
    FinalDis(i,1)=disdt;
    FinalVel(i,1)=veldt;
    FinalAcc(i,1)=accdt;
end
```

So, let us see the computer coding of this. So, this is my computer coding which we just now explained. So, we have entered the degrees of freedom the mass matrix the stiffness matrix and also the damping matrix we have entered the initial force vector we have computed and we have printed the mass matrix the stiffness and damping for verification.

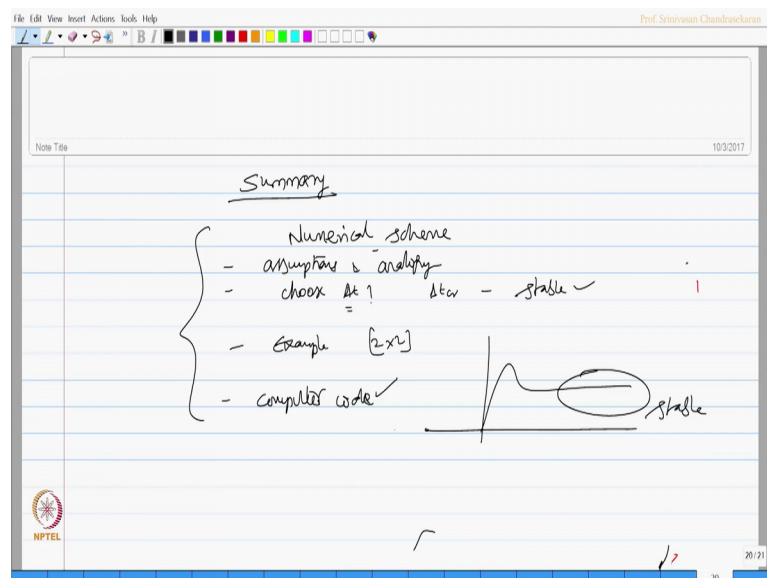
We gave the initial displacement in velocity as 0 we computed the initial acceleration we also fixed the alpha and beta value for the scheme, we also taken the time step we want to do it for total 20 seconds. We calculated integration constants then we computed the k effective k hat matrix, and then found out for a scheme of iteration the velocity displacement and acceleration as you see in the scheme.

(Refer Slide Time: 20:58)



If you look at the answers we have plotted at every Δt value the steps at every 0.28 when we extend this for 100 seconds you will see there is a steady stable solution available in the scheme.

(Refer Slide Time: 21:18)



So, friends in this lecture we understood how to solve an equation of motion using numerical scheme. What are the basic assumptions and analogy followed in this scheme, how to select or choose the time step for numerical integration, how to compute the critical time step. So, that the time step chosen is lesser than a critical. So, to get a stable

unconditional solution we solved an example problem of 2 by 2 matrix and we found out that after along iteration the solution becomes still.

So, the computer coding is available to solve this problem to solve this problem. So, one can try to repeat the solution by some other example and see how this can be comfortably followed by you.

Thank you very much.