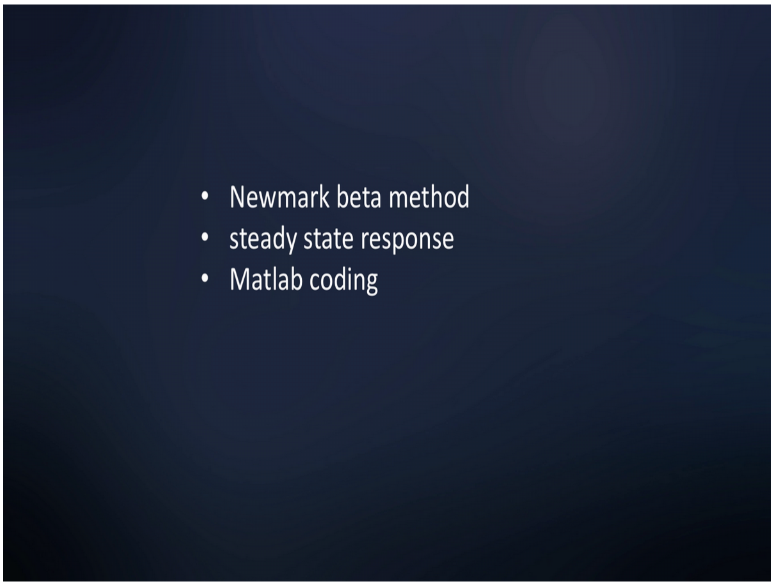
Computer Methods of Analysis of Offshore Structures
Prof. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

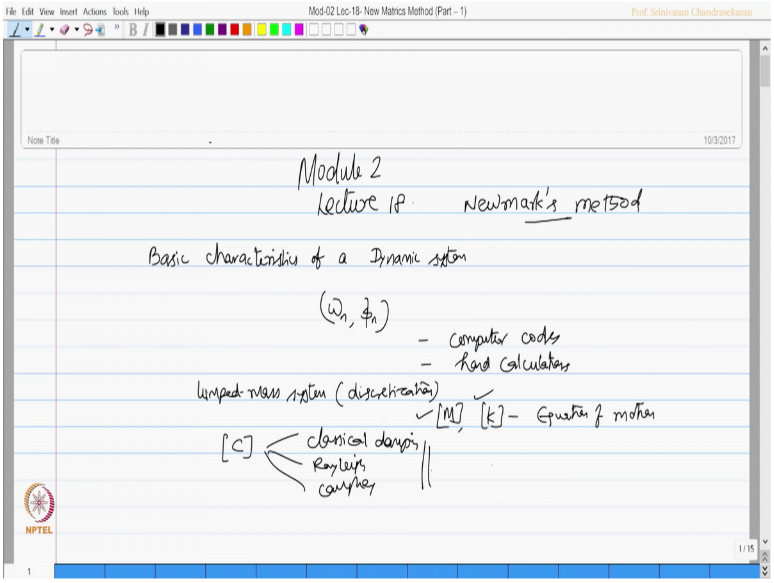
Module - 02
Lecture – 18
Newmark's method

(Refer Slide Time: 00:17)



Friends, let us continue with a discussion on numerical methods in computer analysis of offshore structures we are discussing lecture on module 2. In module 2, we have given exposure to computer course on dynamic analysis. Now we understand how to estimate the basic characteristics of dynamic system, which essentially are the natural frequencies and the mode shapes.

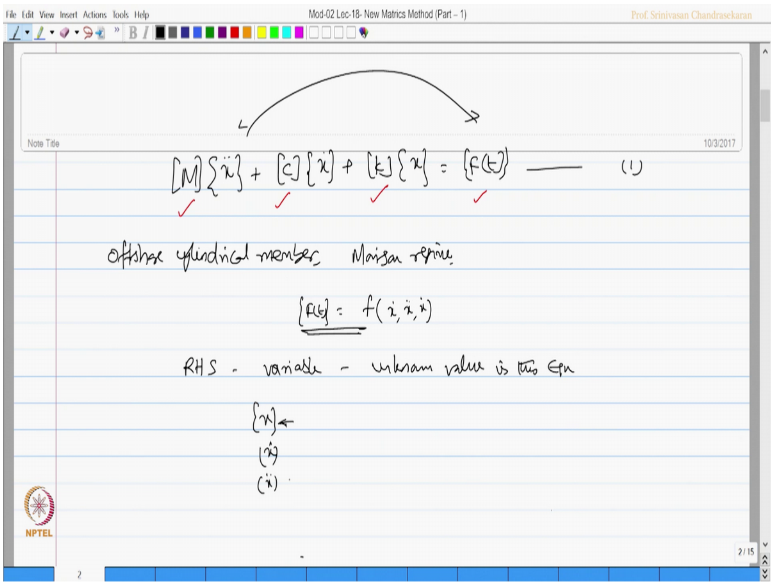
(Refer Slide Time: 00:52)



We already know that this can be estimated by different methods.

We have computer codes to estimate them, we also have hand calculations to verify them further for here lumped mass system or with discretization principle. We know how to obtain the mass matrix and from the equation of motion how to get the stiffness matrix. Once I know the mass and stiffness matrices I can always estimate the damping matrix by classical damping by Rayleigh damping and by Caughey damping.

(Refer Slide Time: 02:02)



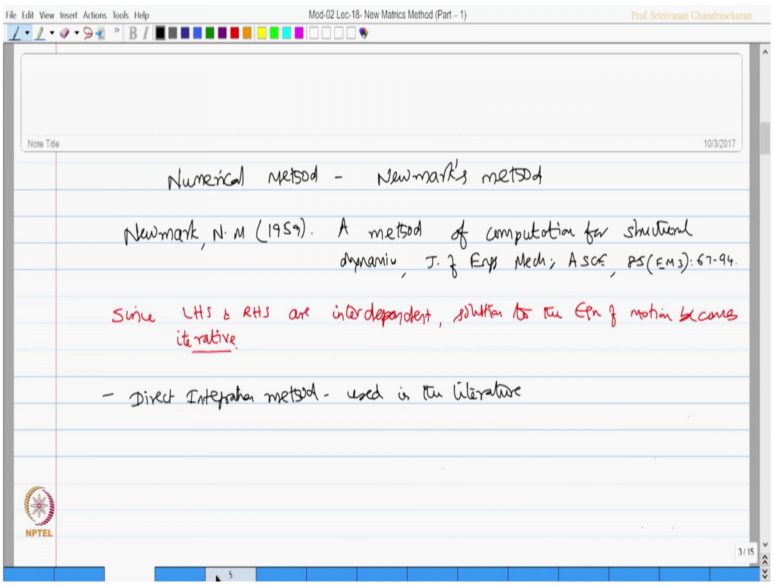
So, now for a given system I already have the variables of equation of motion as written here, this is my classical equation of motion for a multi degree freedom system model. So, we know the mass matrix, we know the k matrix for a given zeta value we know the damping matrix and from the different equations and coding available, I can always estimate the aerodynamic the wind loads, the wave loads, the current etcetera at any desired point at any special variation in a given system, may be take a for example, a cylinder.

Now, interestingly how to solve this problem. Let us take a very classical difficulty which is arising in this problem. If you consider an offshore cylinder cylindrical member which falls in the Morison Regime, we already know that force at any instant time is a function of the structural displacement and velocity and acceleration.

So, interestingly the equation of motion at c in equation number 1 are coupled, because the right hand side of this equation has a variable, which actually an unknown value in this equation. In fact, when you solve this equation you are trying to get the displacement then the velocity and then the acceleration. So, that is an unknown actually, if this is not known you will not be able to find the force vector.

So, now there is a strong coupling existing between the right hand side of this equation and left hand side of this equation, how to solve this.

(Refer Slide Time: 04:31)

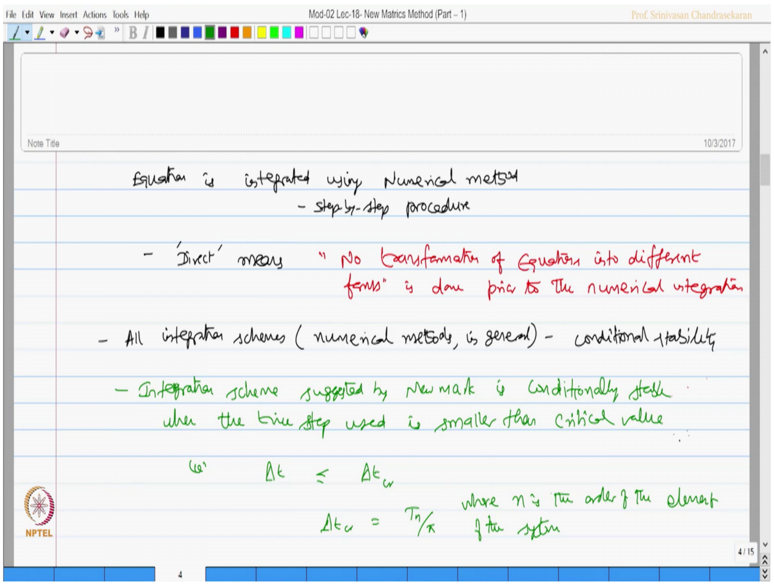


There are various methods available in the literature we will take one classical example of a numerical method and solve a simple problem by hand with this method now, let us explain a computer code then try to show the validated results between the computer code results and that of solved by hand.

A numerical method is popular to solve such equations of motion is Newmark's method, Newmark method was suggested by Newmark N.M 1959, a method of computation for structural dynamics journal of engineering mechanics ASCE, 85 EM 3, 67-94.

Interestingly since the left hand side and right hand side are interdependent, solution to the equation of motion becomes iterative, literature used direct integration method.

(Refer Slide Time: 07:04)

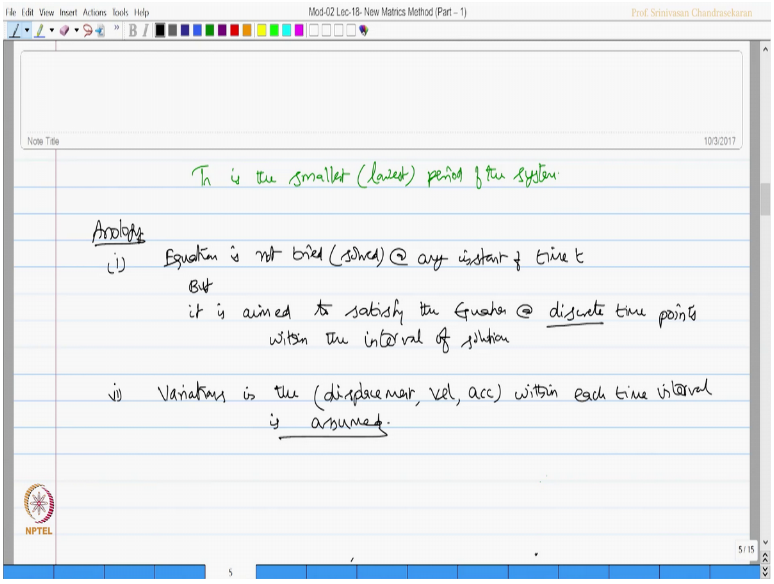


So, according to this method the equation is integrated using numerical method, which is a step by step procedure.

So, the term direct means no transformation of equations into different forms is done prior to the numerical integration that is why this method is called direct integration method. To make this integration scheme conditionally stable, because all integration schemes in particular are numerical schemes in general needs to be a certain for its conditional stability.

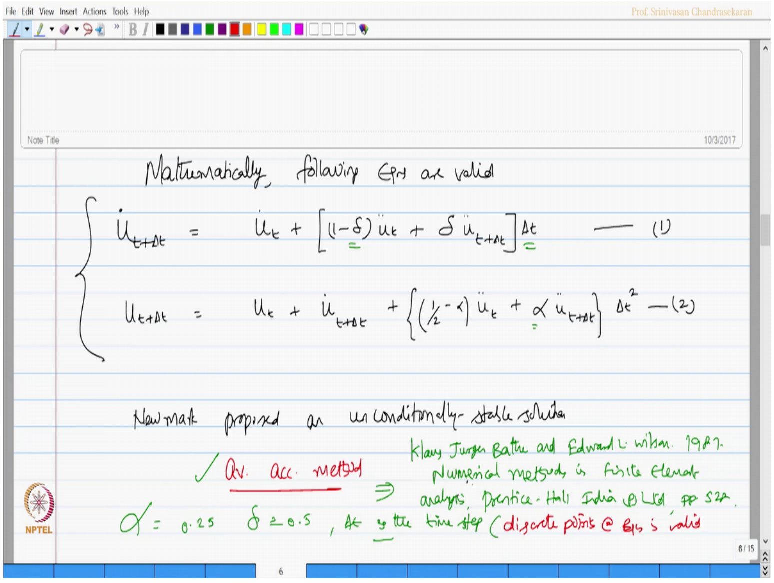
This method says that the integration scheme suggested by Newmark is conditionally stable, when the time step used is smaller than a critical value, that is the time step used delta t should be lesser than or equal to delta t cr which is actually equal to delta t cr is actually equal to tn by pi, where n is the order of element of the system.

(Refer Slide Time: 10:24)



And tn is a smallest period natural period on the system. So, the basic analogy in this method is equation is actually not try or it is not solved at any instant of time t, but it is aimed to satisfy the equation at discrete time points that is very very important. Discrete time points within the interval of solution that is a first analogy. The second assumption is that variations in the variables that in my case displacement velocity and acceleration within each time interval is assumed.

(Refer Slide Time: 12:39)



So, what does it mean is mathematically, following equations are valid u dot of t plus delta t is equal to u dot of t, plus 1 minus del of u double dot t plus del u double dot t plus delta t of delta t call equation number 1; u t plus delta t is equal to ut plus u dot t plus delta t plus half of half minus alpha u double dot t, plus alpha u double dot t plus delta t multiplied by delta t square. So, these equations are valid with respect to this analogy.

So, based on this Newmark proposed an unconditional stable solution Newmark proposed an unconditionally stable solution, this is called average acceleration method. More reference can be seen at Klaus Jurgen bathe and Edward Wilson 1987, numerical methods in finite element analysis, prentice hall India private limited, pp 528 it is another reference which is parallelly available in the literature, which helps you to understand the average acceleration method suggested by Newmark's beta.

So, in the above equation if you see the variables del and delta t and alpha. So, alpha is considered as 0.25 and del is considered as 0.5 and delta t is a time step for the solution. So, these are nothing, but the discrete points the discrete time intervals at which the equation is valid, that is what happening.