Computer Methods of Analysis of Offshore Structures
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Module - 01
Lecture - 05
Beam Element - 2 (Part - 1)

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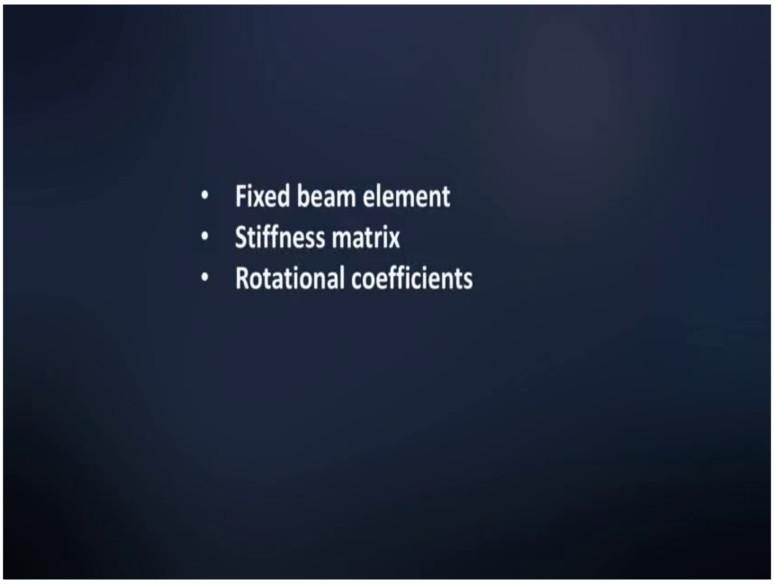

Friends, let us continue with the fifth lecture in module 1. Where we will discuss more features about beam element as a second part of the lecture.

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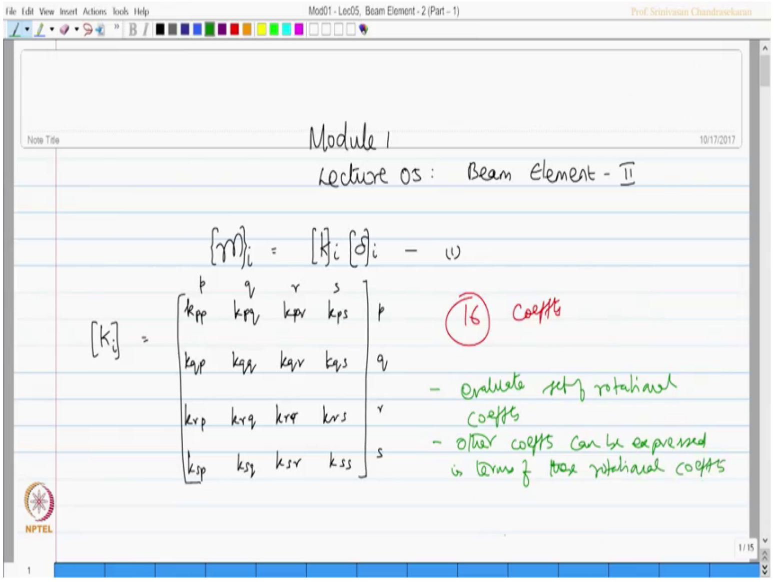

In the earlier lecture we said that m i is k i of delta i; m being a vector, k being a matrix and delta being a vector is a valid equation. If you look at the typical stiffness matrix of the i-th element which we derived in the last lecture we said let us have p q r s. Let us also indicate p q r s and stiffness coefficients are k pp k pq k pr k ps; row first and column next say k qp k qq k qr k qs; k rp k rq k rr k rs; and k sp k sq k sr k ss.

$$[k]_i = \begin{bmatrix} k_{pp} & k_{pq} & k_{pr} & k_{ps} \\ k_{qp} & k_{qq} & k_{qr} & k_{qs} \\ k_{rp} & k_{rq} & k_{rr} & k_{rs} \\ k_{sp} & k_{sq} & k_{sr} & k_{ss} \end{bmatrix}$$

So, there are rather 16 coefficients as you see in this equation, out of which one good thing is we need to only evaluate set of rotational coefficients, others can be expressed in terms of these rotational coefficients.

So, what are these rotational coefficients?

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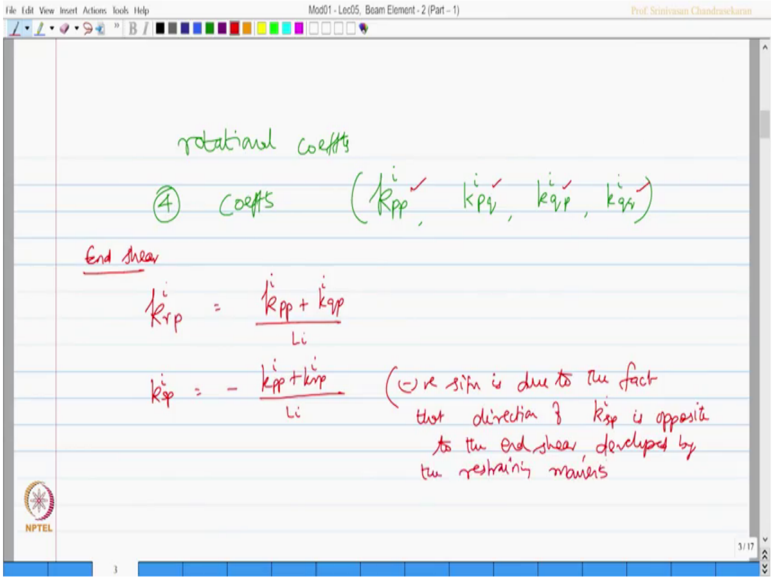

The rotational coefficients which we are interested is k pp of the i-th element, k pq of the i-th element, k qp of the i-th element, and k qq of the i-th element; these are the four coefficients. These are the four coefficients which are important. The remaining can be expressed in terms of this.

For example: we want to evaluate the end shear we can say k rp is actually k pp plus k qp of the i-th member divided by length of the member and k sp of the i-th member is minus of k pp plus k rp by L i. Carefully look at this figures: k pq k qq k pp and k qp, these are the four coefficients k pp k pq qp and qq these are the four coefficients which we are interested in fact we are interested in these four: if i am able to evaluate these four remaining 12 can be expressed in terms of this, ok.

So, now the negative sign in this case is due to the fact that direction of k sp is opposite to the end shear developed by the restraining moments. Let us try to understand how this happens.

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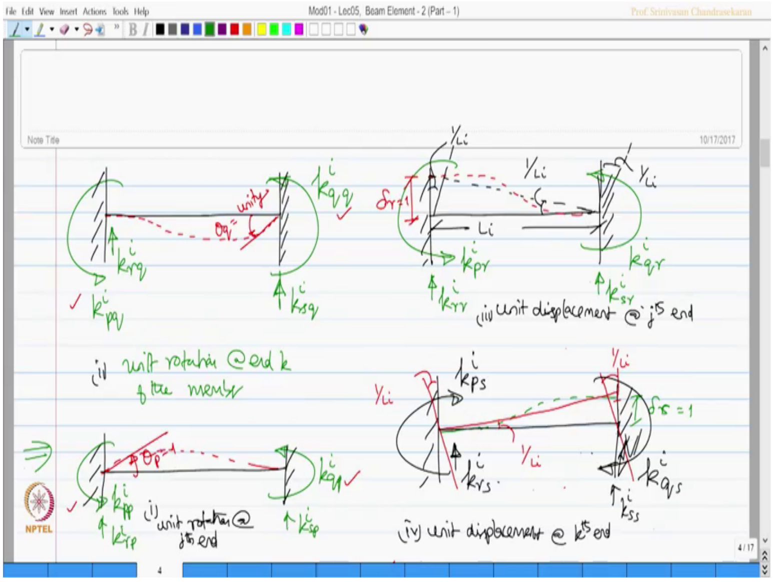

Let us look at this figure, that is this figure; look at this figure and I try to enlarge this figure again.

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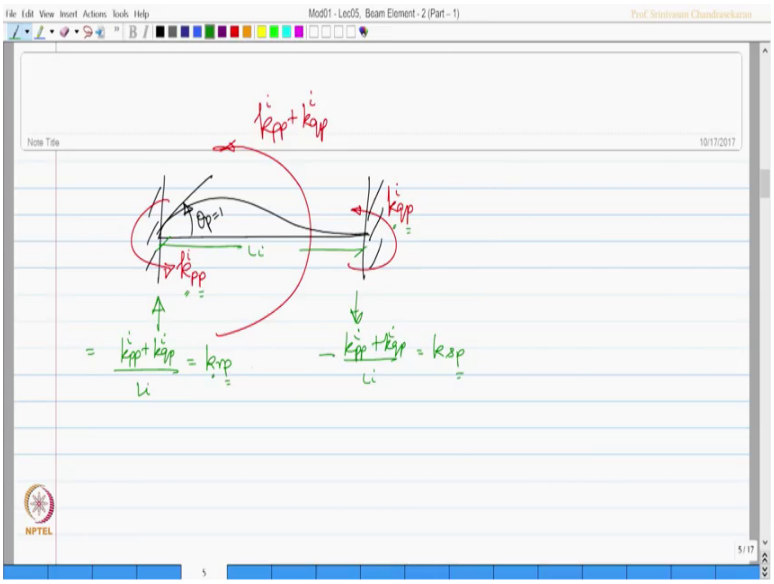

I have an element with both ends fixed; we have given unit rotation at the end p. So, now to control this, this will induce a moment which will be equal to k pp. This will also cause another moment which is k qp of the i-th element.

So, now this will have an anticlockwise moment which actually will be equal to k pp plus k qp is or no, this is got to be counter acted by the shear which will be creating a clockwise couple which will be actually equal to k pp plus k qp by L i; where L i is the length of the member. So, this will be also k pp plus k qp by L i. If you look at the original degrees of freedom we already said upward end shear is positive. So, this will be negative that is what we writing here k rp which is this value and this will be k sp.

Please recollect the second subscript indicates we have given unit displacement in the p-th degree and the first subscript indicates the respective forces in the degrees of freedom. So, this will be k rp it will be k sp which actually equal to negative. So, this is what we got.

So, the end shear can be expressed, once I know the rotational coefficients these four. So, by this manner the remaining coefficients can also be expressed in terms of these rotational coefficients if you know that. So, out of 16 coefficients we have we need to only evaluate these 4 coefficients; only these four are important remaining can be derived if we know these four coefficients.

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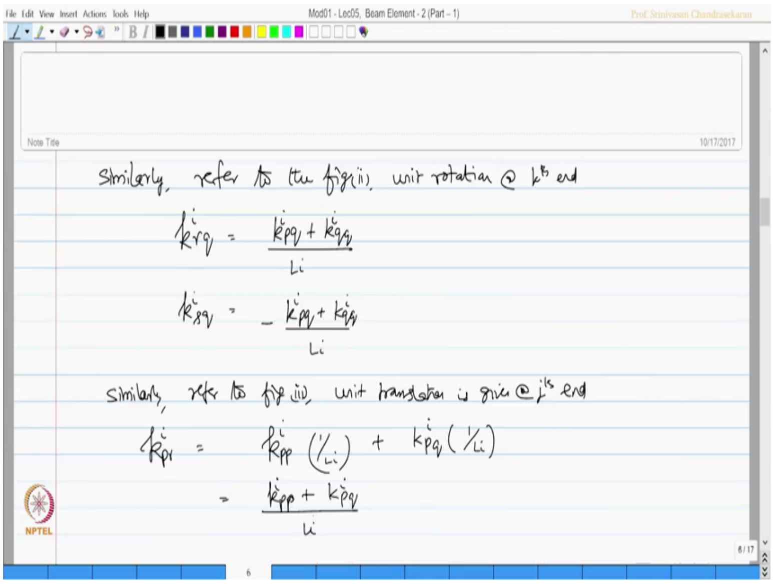

So, now the task is to evaluate these rotational coefficients. So, we can extend the similar logic further like. Similarly, refer to the figure I should say the number, I should say figure two this figure; referring to figure two which causes unit rotation at k-th end you can see here unit rotation at k-th end. K rq and k sq can be expressed as sum of the rotations which is k pq and k qq. So, k pq plus k qq of the i-th member by L i. And this will be same k pq plus k qq of the i-th member by L i, but with a negative sign.

Now, similarly refer to figure three where unit translation is given at j-th end. You can see the figure three: figure three unit translations at the j-th end. The rotational coefficients are k pr and k qr. So now, I can easily find this k pr as the rotation cost by this change in slow. So, k pr can be expressed as k pp of the i-th member with this rotation plus k pq of the i-th member with this rotation which will be k pp plus k pq divided by L i.

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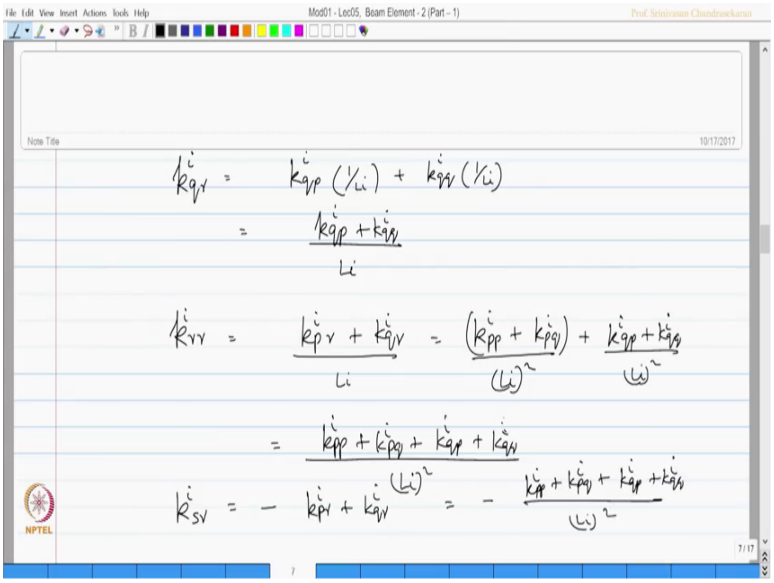

Similarly, k qr will be also equal to k qp of 1 by L i plus k qq of 1 by L i of the i-th member which will be written as k qp plus k qq of the i-th member by L i.

Now, I can say k rr is k pr plus k qr by L i, which is nothing but because k pr is given by this expression which is k pp plus k pq by L i. So, which should be k pp plus k pq by L i square is already L i here plus similarly k qr is k qp plus k qq. So, k qp plus k qq by L i square which means k pp plus k pq plus k qp plus k qq by L i square.

Similarly, k sr will be negative of k pr plus k qr which will amount to minus of k pp plus k pq plus k qp plus k qq of L i square.

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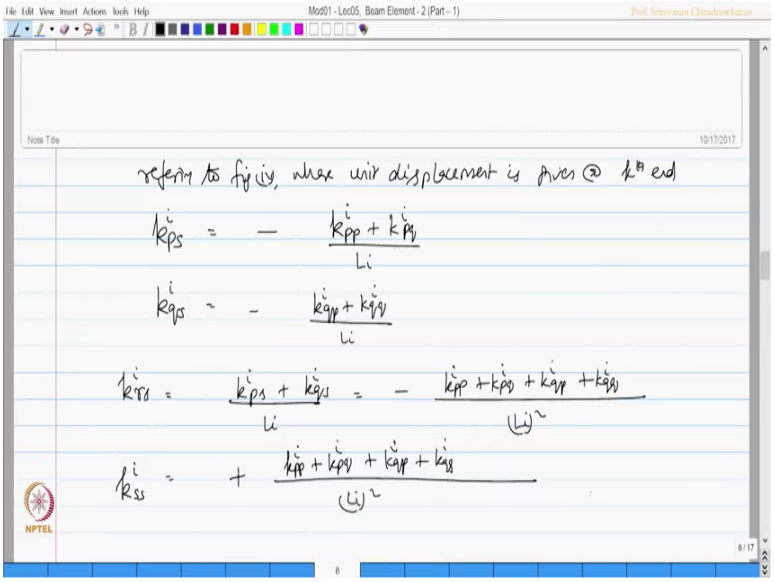

Now, referring to figure four where unit displacement is given at k-th end and see this figure; unit displacement at k-th end and we have a slope which is negative this is this way. The moment is this way clockwise where as the slope is on the other side. So, k ps that is this coefficient and this rotational coefficients k ps is minus of k pp plus k pq by L i k qs is again minus of k qp plus k qq of L i. Therefore, k rs will be k ps plus k qs by L i which will tell me minus of k pp k pq k qp plus qq by L i the whole square. Whereas, k ss will be positive of k pp plus pq qp and qq of L i square.

So, friends look at these equations which are end shear they are expressed in terms of the rotational coefficients, the end shear is again expressed in terms of rotational coefficients. So, the interest is the end shears can be expressed in terms of rotational coefficients.

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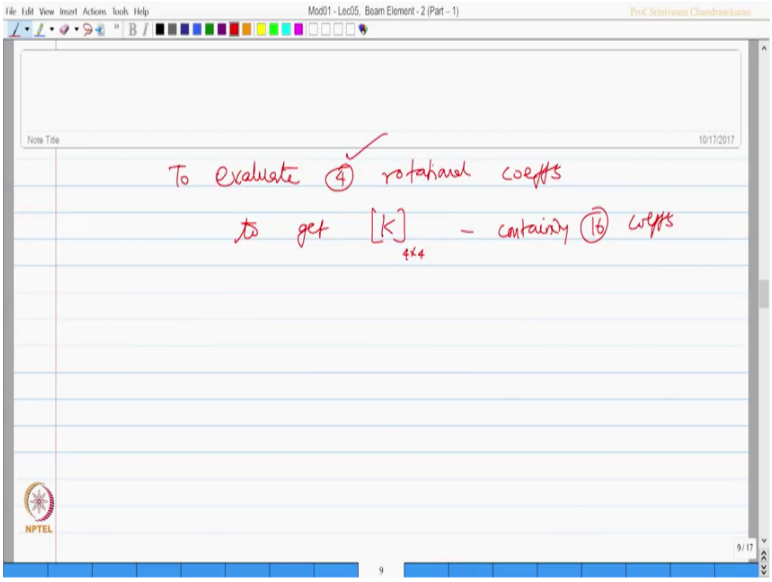

So, we are now interested only to evaluate the 4 rotational coefficients to get the full stiffness matrix of 4 by 4 containing 16 coefficients. So it is very simple, we do not have derive all the 16 we derive only the 4 we can get the remaining 12 automatically from these set of equations what we derive just now.

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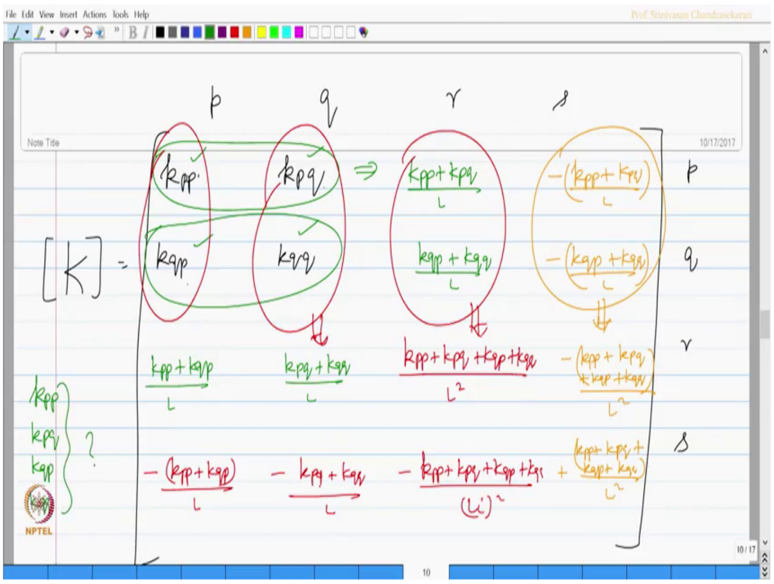

So, I can now make a matrix saying that the k matrix which will be p q r and s; p q r and s. This is k pp, this is k pq, and this is k qp, this is k qq; row first and column next. I evaluate only this four, remaining are derivable very simply this is k pp plus k pq by L that is sum of these two. And this is k qp plus k qq by L that is sum of these two by L. This value will be k pp plus k qp by L which will be sum of these two.

And this value will be minus of k pp plus k qp by L which will be as same as this with the negative sign. Similarly this value will be k pq plus k qq by L which is sum of these two. And this will be minus of k pq plus k qq by L. Now let us come to this argument, this will be k pp plus pq plus qp plus qq by L square which will be sum of these two by L and this term will be negative of k pp plus k pq plus k qp plus k qq the same with the negative sign.

The forth will be the same value with the negative sign that is minus of k pp plus k qp by L minus of k qp plus k qq by L. Then, this value will be the sum of these two by L, therefore minus k pp plus k qp plus k qp this is k pq, this is k pq k pq q p plus k qq by L square this will be plus of this value which is pp pq qp qq by L square.

So friends, if I am able to evaluate these four values: 1 2 3 and 4 that is this value this value and this and this all the four remaining all can be calculated with the help of this particular table. So, the job is now to evaluate these four rotation coefficients k pp, k pq, k qp, and k qq. So, we need to evaluate this that is all.