Computer Methods of Analysis of Offshore Structures
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Module - 02
Lecture - 17
Damping Estimate - 2 (Part – 1)

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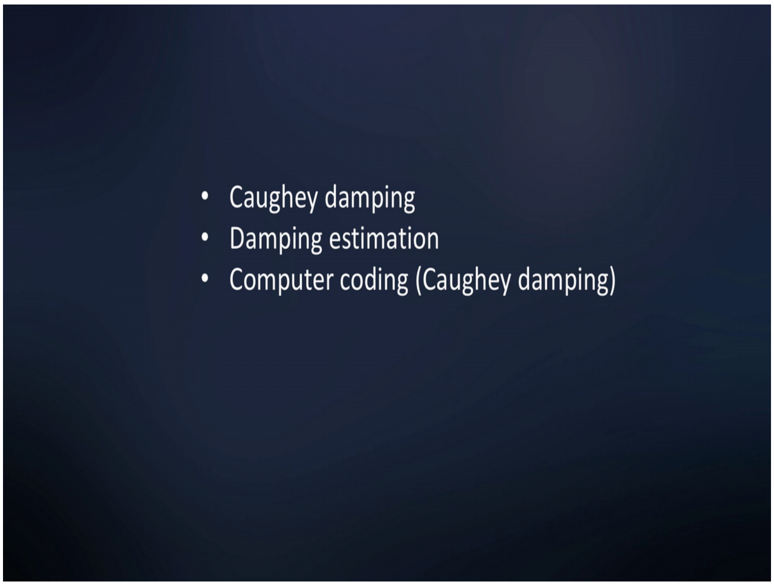

Friends, welcome to the 17th lecture, we will continue with the damping estimate. In the last lecture, we discussed about damping estimate using Rayleigh method, we derived the equation, we solve the problem by hand, we also gave you the computer code and we validated the results by the computer code with that of the solution what we had in hand where exactly the same answers more or less comparable were Rayleigh damping. Now the second issue which is also related to damping in offshore structures is.

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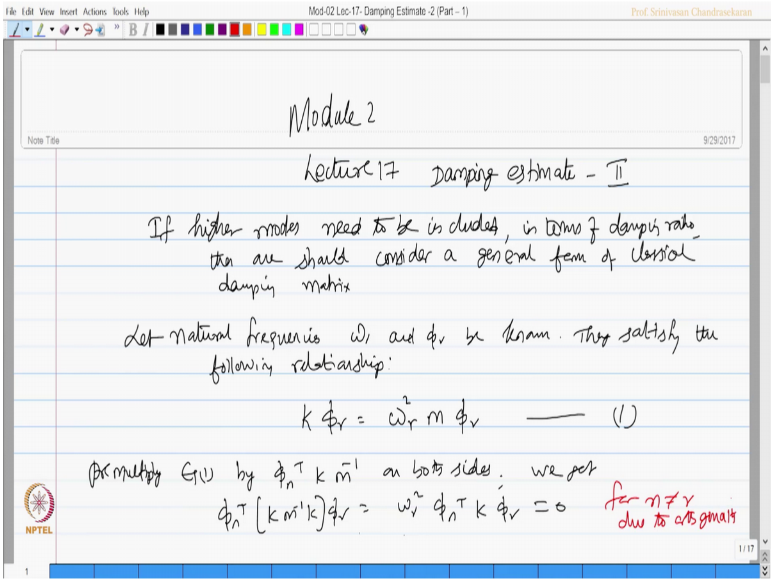

If you really want to include higher modes in terms of damping ratios, then one should take a general form of classical damping matrix. Remember classical damping matrix is assuming damping ratio more or less proportional to we say in all the modes entire space of the structure.

So, let natural frequencies omega r and phi r be known, they satisfy the following relationship; k phi r is omega square m phi r, pre multiply equation 1 by phi n transpose k m inverse on both sides; we get phi n transpose k m inverse k, I am just grouping, it phi r is omega r square phi n transpose k phi r, this is equal to 0 for n not equal to r because the modes are orthogonal.

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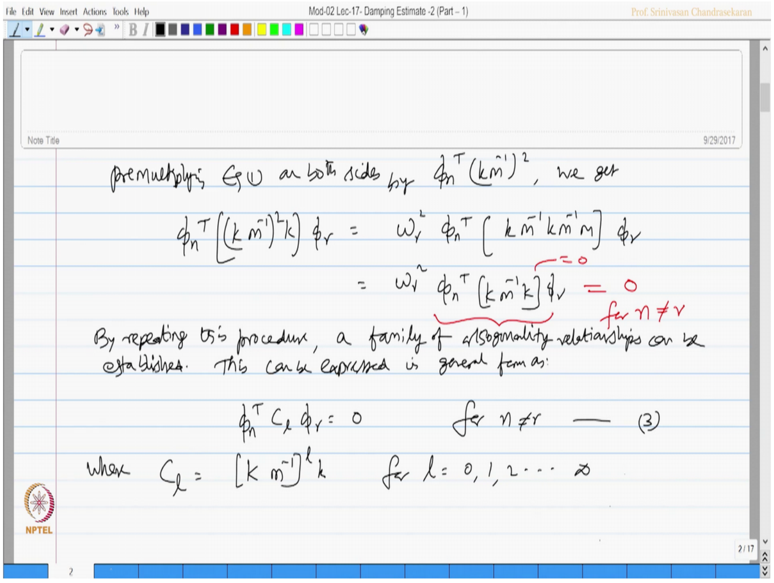

Pre multiplying equation 1 on both sides by phi n transpose, k m inverse square, we get phi n transpose k m inverse square k of phi r is omega r square, phi n transpose k m inverse k m inverse m of phi r.

I have just expanded k m inverse square like this, which is equal to omega r square phi n transpose k m inverse k of phi r. Please look at this equation omega r square phi n transpose k phi r omega r square phi n transpose k m inverse k, k phi r this term which is expressed here phi n transpose k m inverse k phi r phi n transpose k m inverse k phi r is already 0 [FL] I should say this is now 0 for n not equal to r. So, we repeating this by repeating this application or this procedure, we can establish a family of orthogonality relationships. Now this can be expressed in general form as phi n transpose C l phi r is 0 for n not equals r, where C l is k m inverse to the power l of k for l equal 0 1 to infinity.

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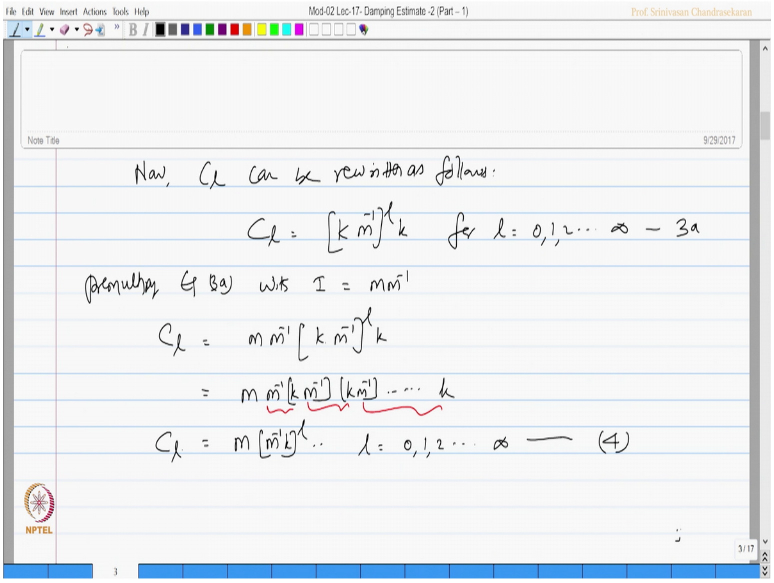

Now, C l can be rewritten as follows C l is equal to k m inverse to the power l of k, for l equals 0, 1, 2 infinity call equation 3 a. Pre multiply equation three a with I which is m m inverse. So, C l is equal to m m inverse of k m inverse l of k, which can be said as m m inverse k m inverse that is a k m inverse of l times of k, which can be m, m inverse k of l times of because I can group each one of them and so on this is true for l equal 0, 1, 2 up to infinity I call this equation number 4.

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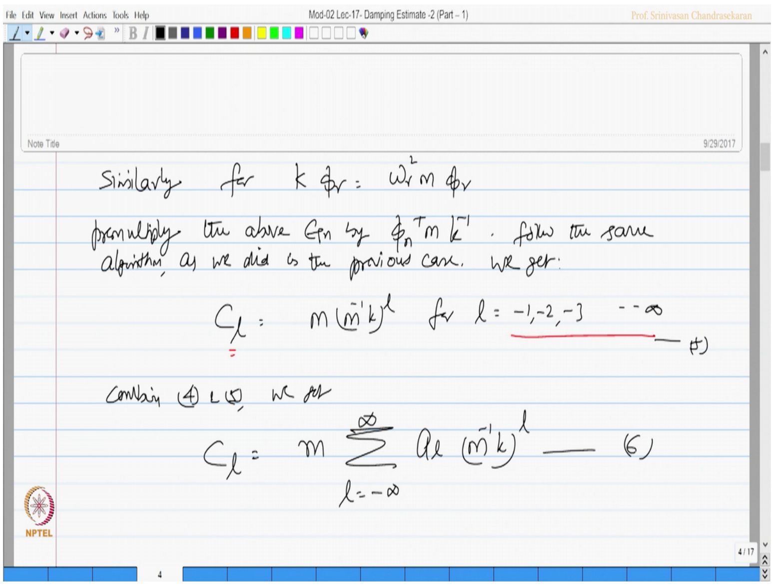

So, C l is equal to this equation, similarly for k phi r equals omega r square m phi r, pre multiply the above equation by phi n transpose m k inverse, follow the same algorithm as we did in the previous case. So, we get C l is equal to m, m inverse k to the power l for now l equals minus 1, minus 2, minus 3 to infinity equation 5. So, now, combine 4 and 5 equation 4 and 5, we get C l is m summation of l equals minus infinity to plus infinity because in this case, C l is valid in the minus range, in the previous case C l is valid in the plus range. So, combining these 2 I can say C l minus to plus infinity a l, m inverse k to the power l.

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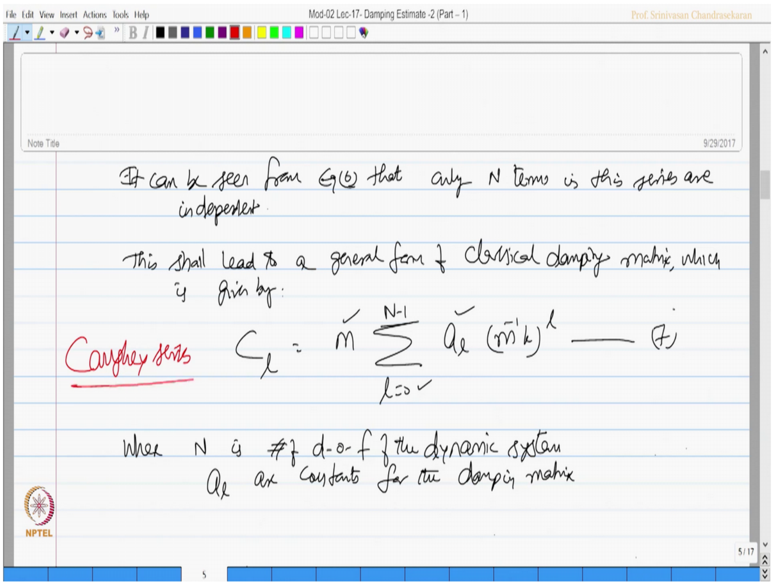

It can be seen in equation 6 that only n terms in this series are independent.

This shall lead to a general form of classical damping matrix, which is given by C l equals m l equal 0 to n minus 1 a l m inverse k to the power l, I call equation number 7, where n is number of degrees of freedom of the dynamic system and a l are constants for the damping matrix, let us consider first three terms of this series in 7.

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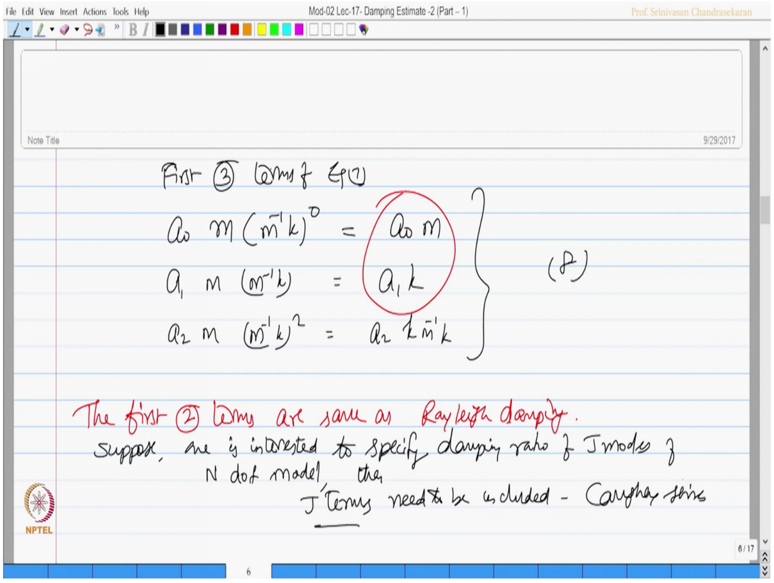

So, let us say a 0 the equation is I am considering first 3. So, let us I am putting this as 1 0 1 and 2 we are starting from 0. So, c 0 I am what interested. So, I let find out a l or a 0 a 0 into m see a 0 into m inverse k m into m inverse k right. So, m into m inverse k to the power 0, you can see here m a 0 m inverse to the power 0 which will give me a 0 m similarly i substitute this as one.

So, a 1 m m inverse k 1. So, a 1 m m inverse k will be equal to a 1 k, similarly a 2 m m inverse k square will be equal to a 2 k m inverse k I call this as equation number 8. One can very easily see here the first 2 terms are same as Rayleigh damping is it not? These 2 are same as Rayleigh damping. Suppose one is interested to specify damping ratio of J modes of n degree freedom system model, then J terms need to be included and this series is what we call as Caughey series. So, we include this in Caughey series.

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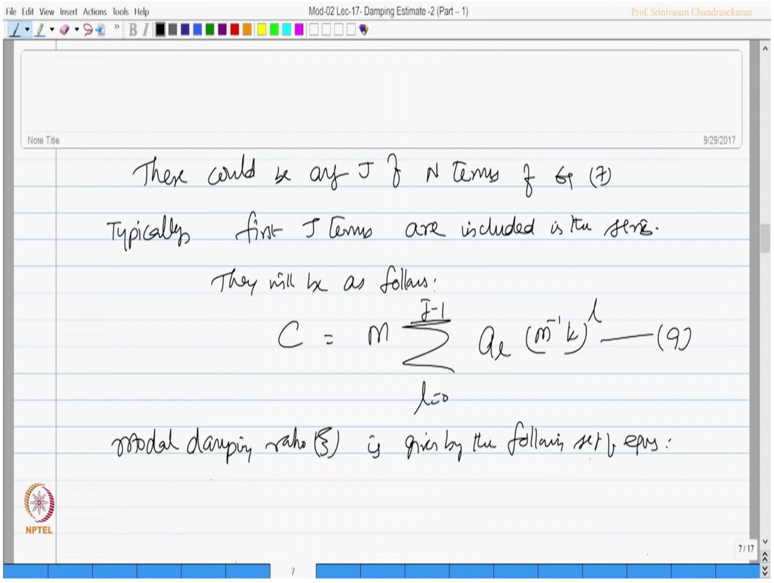

So, there could be any J of N terms of equation 7 equation 7 is this. There could be any J term J of N terms of equation 7 typically first J terms are included in the series and they will be as follows, C is going to be m summation l equal 0 J minus 1 a l, m inverse k of l equation 9. Then the modal damping ratio zeta is given by the following set of equations.

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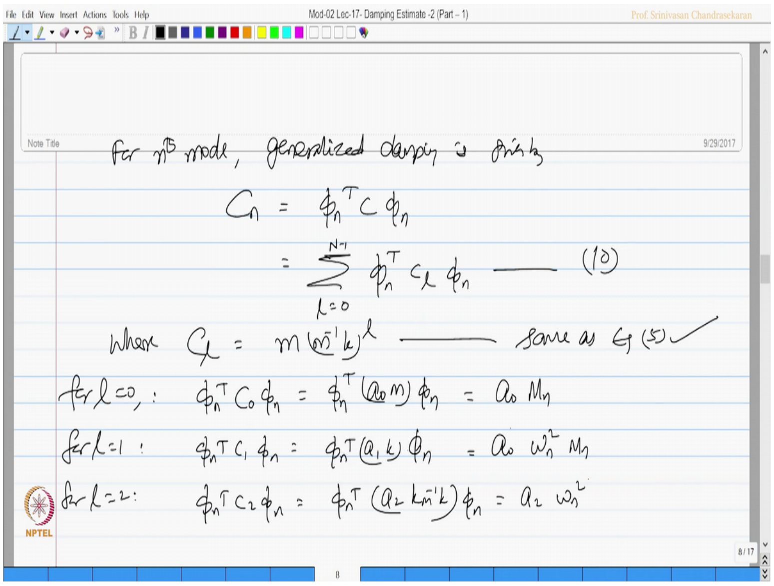

Let us say for n th mode we know the generalized damping is given by C n which is phi n transpose C phi n, which can be now said as summation of l equal 0 to N minus 1 phi n transpose C l phi n I call this equation number 10, where C l is m m inverse k to the power l which is same as equation 5, which we already had equation 5 m inverse k l same as that equation m inverse k l ok.

So, for l equal 0 because it just vary from 0 to n minus 1 l equals 0 phi n transpose C 0 phi n, will be phi n a 0 m of phi n because you know for l equal 0 we already have this term is a 0 m. So, a 0 m is added here similarly, for l equals one phi n transpose c one phi n will be phi n transpose a 1 k phi n, which is actually equal to a 0 M n this can be a 0 omega n square M n and for l equals 2 phi n transpose c 2 phi n can be phi n transpose a 2 k m inverse k of phi n which can be a 2 omega n square.