Computer Methods of Analysis of Offshore Structures
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Module - 02
Lecture - 16
Damping Estimate (Part – 2)

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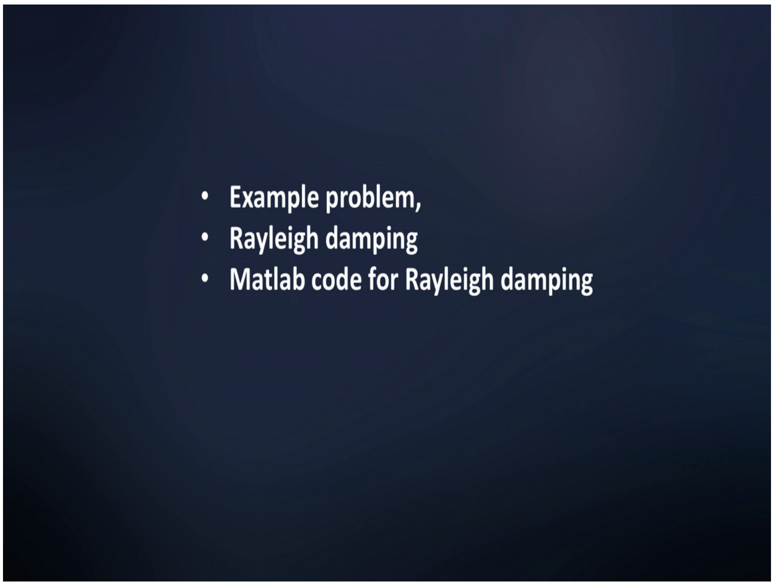

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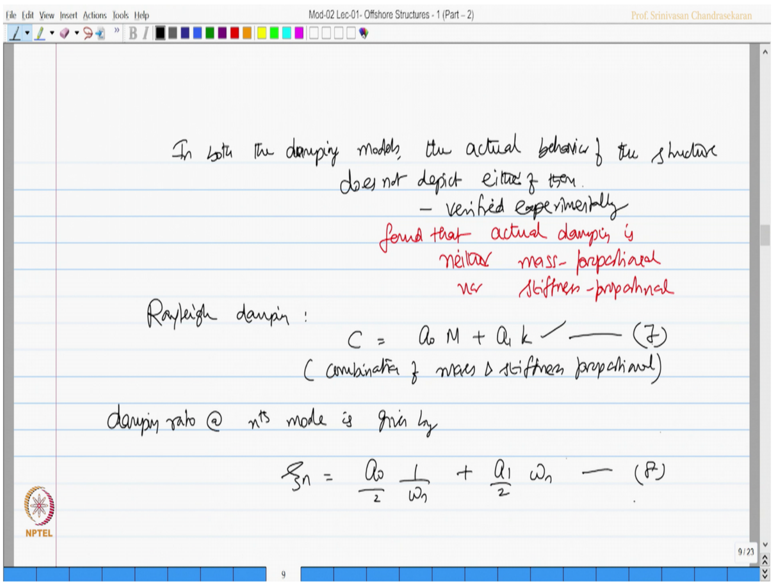

Rayleigh proposed damping which includes both C is going to be a 0 m plus a 1 k, therefore it is a combination of mass and stiffness proportional. In that case damping ratio at n-th mode is given by you know the damping ratio for stiffness proportional is equation 6.

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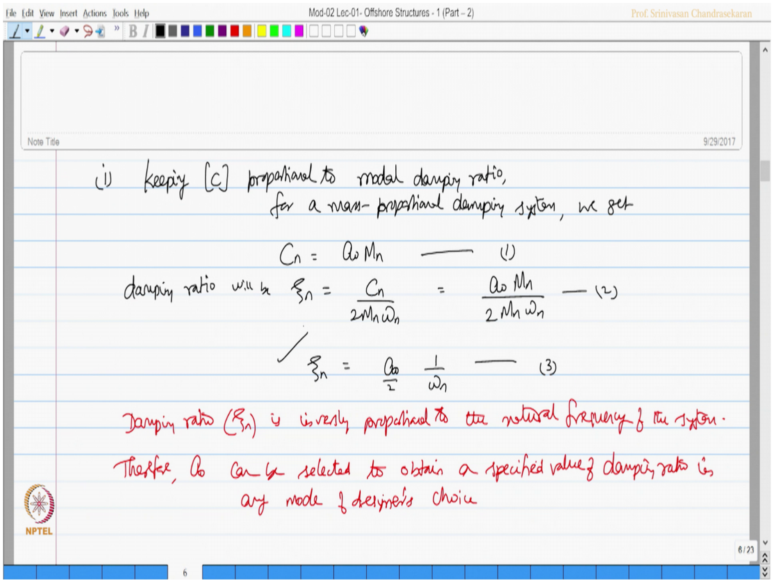

Damping ratio for mass proportional is equation 3, therefore damping proportional for a combination of this could be simply the addition of this that is a 0 by 2 1 by omega n because for mass proportional damping is inversely proportional to omega plus a 1 by 2 omega n because for stiffness proportional damping ratio is directly proportional to omega n. I call this equation number 8. Of course, this is my equation number 7 given by Rayleigh.

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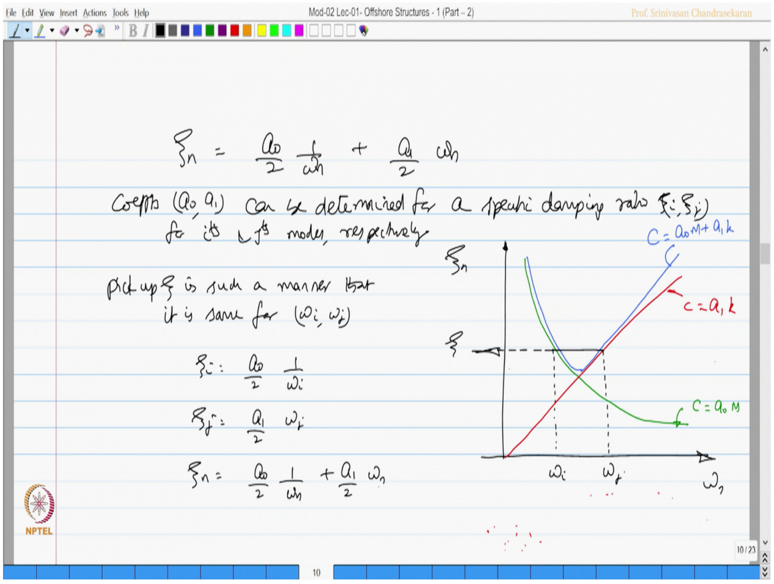

So, therefore, once we said zeta n as actually a 0 by 2 1 by omega n plus a 1 by 2 omega n. The coefficients a 0 and a 1 can be determined for a specific damping ratio zeta i and zeta j respectively for i-th and j-th modes. So, now, the damping will look like the plot as shown now the original damping which is mass proportional is omega n and this is the zeta n. The mass proportional damping looks inversely proportional to omega increase this is I should say C is equal to a 0 m whereas, the stiffness proportional damping is directly proportional to omega n. So, I should say in this case C is a 1 k Rayleigh gave a combination of these 2 it says that let it be a combination of mass proportional and the stiffness proportional, this is Rayleigh damping.

So, for any specific value of omega i and omega j, this is omega i this omega j I should be able to get the same zeta, so pickup zeta in such a manner that it is same for omega i and omega j as shown in the figure. So, zeta i is going to be a 0 by 2 1 by omega i and zeta j is a 1 by 2 omega j hence zeta n is a 0 by 2 1 by omega n plus a 1 by 2 omega n.

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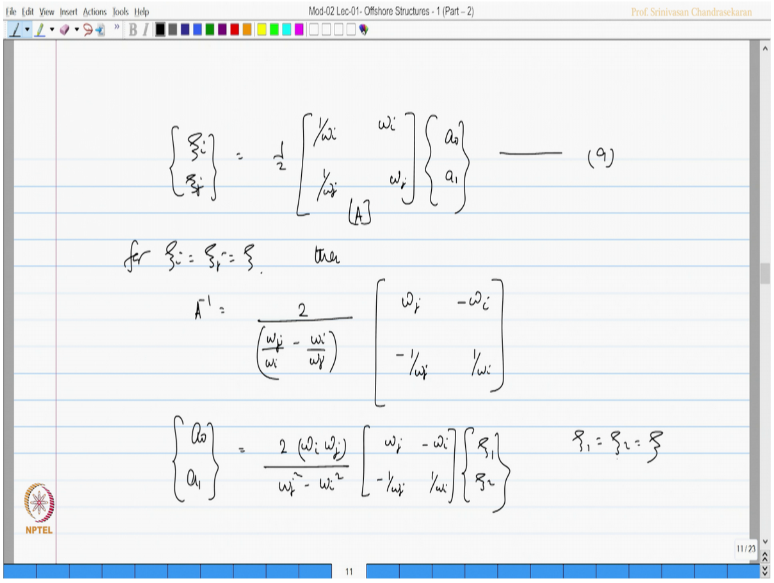

So, I can now say zeta i zeta j is actually equal to half of I am expressing this in a matrix form, I am expressing this relationship in a matrix form 1 by omega i omega i, 1 by omega j omega j of a 0 a 1. One can read this equation and similarly understand zeta i is equal to a 0 by 2 into 1 by omega i plus a 1 by 2 omega i which is as same as this.

So, now, for zeta i zeta j is same as zeta. Then I call this matrix as a matrix let us work out a inverse is very simple to find out that it is going to be twice of omega j by omega i minus omega i by omega j of omega j minus omega i minus 1 by omega j 1 by omega i. I can easily find a 0 a 1 which are the coefficients of the damping matrix as twice of omega i omega j by omega j square minus omega i square of omega j minus omega i minus 1 by omega j 1 by omega i of zeta 1 and zeta 2.

We now say zeta 1 zeta 2 are same zeta that is a condition, you see here omega 1 zeta 1 omega 2 zeta 2 they should be same zeta for this condition to be imply.

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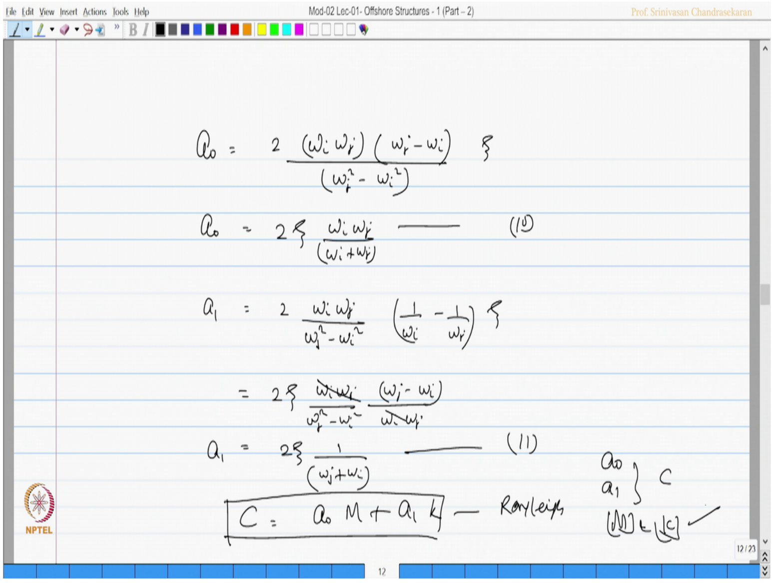

I can now say a 0 is going to be twice of omega i omega j into omega j minus omega i of zeta divided by omega j square minus omega i square. I am just finding out this value a 0 is twice of this zeta 1 this zeta 2, there is no zeta 1 zeta 2 there are zeta divided by this that is what I am writing omega i omega j omega j omega i zeta minus omega i zeta. I can see that is what I am writing and so on, which can be simplified as 2 zeta of omega i omega j by omega i plus omega j where this can be denominator expanded as a square minus b square I get the product I can cancel this I will get this value that is my a 0 equation number 10.

Similarly, reading from the second row of this equation a 1 will be equal to twice of omega i omega j by omega j square minus omega i square of 1 by omega i minus 1 by omega j because this omega j is minus is negative j of zeta. This says 2 zeta omega i omega j into omega j minus omega i by omega i omega j of omega j square minus omega i square. So, this goes away I can now say it is going to be 2 zeta 1 by omega j plus omega i, I got a 1 equation 11.

Now, C is actually equal to a 0 m plus a 1 k which is given by Rayleigh. So, for the known values of a 0 and a 1 one can find C matrix because m and k are already known.

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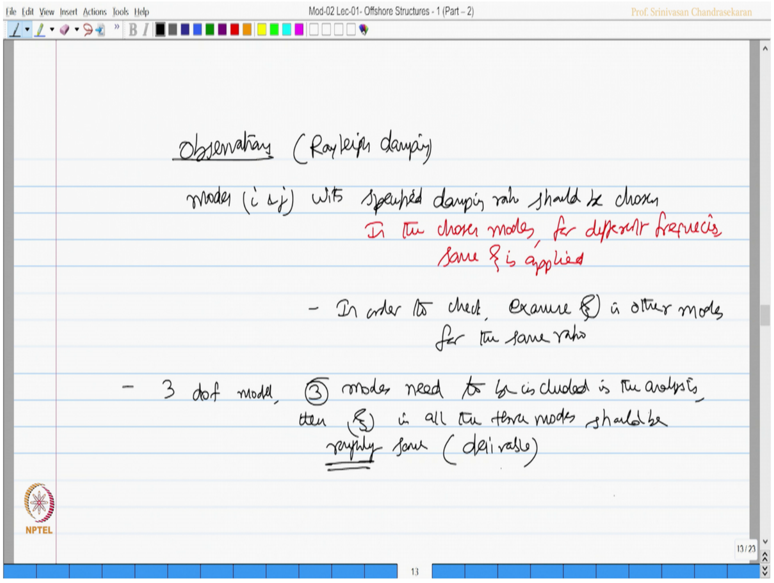

There are some observations on this condition some observations are - in applying this procedure of Rayleigh damping the modes i and j with specify damping ratio should be chosen. The condition is in the chosen modes for different frequencies same damping ratio is applied.

So, you should be able to choose 2 different frequencies which has the same damping ratio. Further in order to check examine damping in other modes for the same ratio. For example, if it is a 3 degree freedom system model 3 modes need to be included in the analysis then the damping ratio in all the 3 modes should be roughly same, I should say the word roughly same because you cannot get exactly the same ratio which is desirable to apply this method.

Let us take an example problem which we already solved.

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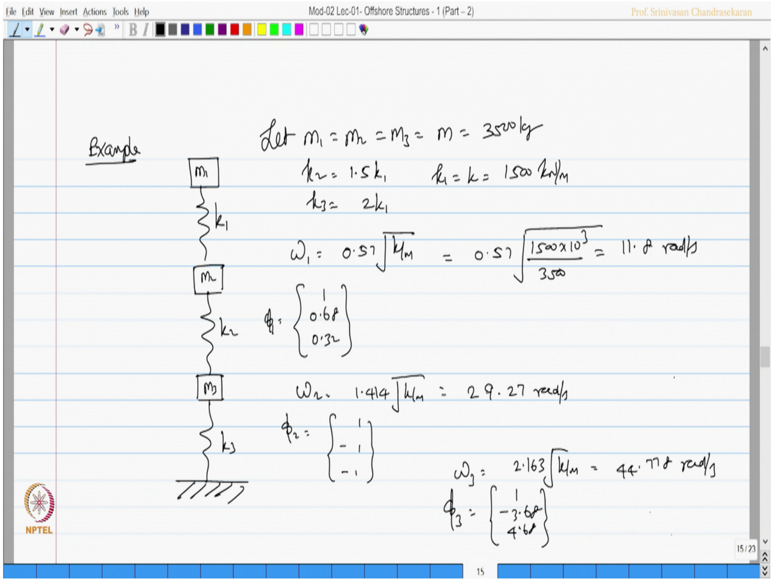

The example problem is let us say this is m 1, m 2 and m 3; this is k 1, k 2 and k 3. Let m 1 and m 2 and m 3 be a constant value which is 3500 kg, k 1 k 2 and k 3 be a constant value not constant let us take it like this k 2 is 1.5 k 1, and k 3 is 2 k 1, and k 1 is k which is 1500 kilonewton per meter let us say this data is given.

So, now, the mass matrix and k matrix are known to us which is required I can apply any standard procedure what we discussed in the last lectures and find out omegas. Let us say I determined omega by influence coefficient method and the values are omega 1 is 0.57 root k by m which is 0.57 root of 1500 kilonewton divided by 3500, let us say 11.8 radians per second. And the corresponding phi 1 is 1.68 and 0.32. Similarly omega 2 is 1.414 root k by m. So, we substituting we get 29.27 radians per second and the corresponding phi 2 is 1 minus 1 and minus 1.

Similarly, omega 3 is 2.163 root k by m which equals to 44.778 radians per second and the corresponding phi 3 is 1 minus 3.68 and 4.68.

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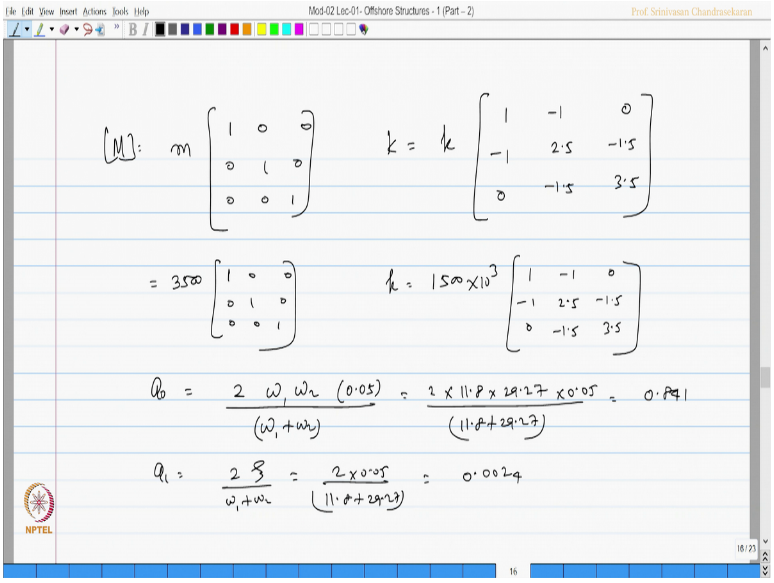

So, the mass matrix is now known to me which is m times of 1 0 0, 0 1 0, 0 0 1. From the equation of motion by force Newton's force method I can also find the k matrix I leave this is an homework to you, k will be actually equal to 1 minus 1 0, minus 1 2.5 minus 1.5 and 0 minus 1.5 and 3.5 that is my k matrix now. So, m matrix can also be said as 3500, so much 1 0 0, 0 1 0, 0 0 1 in kg and k matrix can be now said as 1500 into 10 power 3 of 1 minus 1 0, minus 1 2.5 minus 1.5, 0 minus 1.5 3.5.

So, now let us quickly compute a 0 which is 2 omega 1 omega 2. I am taking first 2 modes and I am assuming zeta to be 5 percent 0.05 divided by omega 1 plus omega 2 that is the equation for, you can see here for a 0 which is 2 zeta omega 1 omega 2 and sum of this and a 1 is 2 zeta by this sum. Let us do that which is going to be 2 into 11.8 into 29.27 into 0.05 divided by 11.8 plus 29.27 which gives me as 0.841.

Similarly, a 1 is 2 zeta by omega 1 plus omega 2, 2 0.05; 11.8 plus 29.27 which is 0.0024. So, now, I can find the C matrix as a 0 m plus a 1 k.

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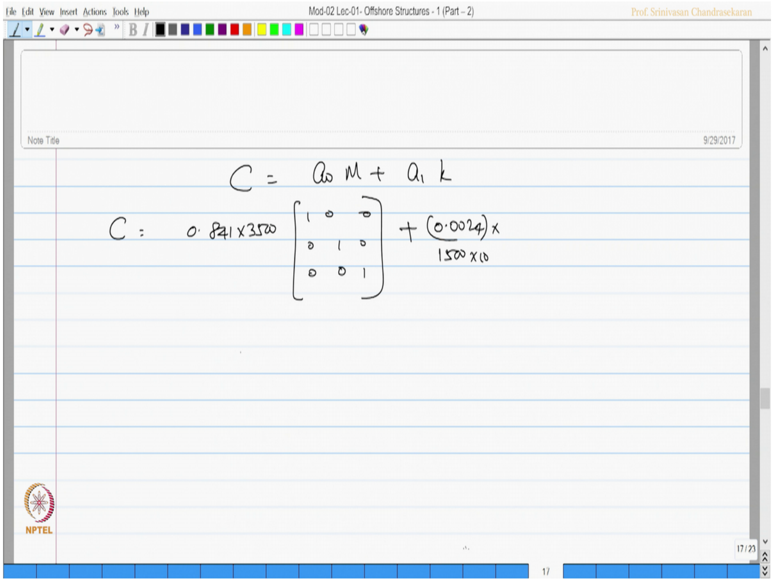

So, that is C is going to be 0.841 into 3500 of 1 0 0, 0 1 0, 0 0 1 that is my mass matrix plus 0.0024 that is my a 1 into 1500 into 10 power 3.

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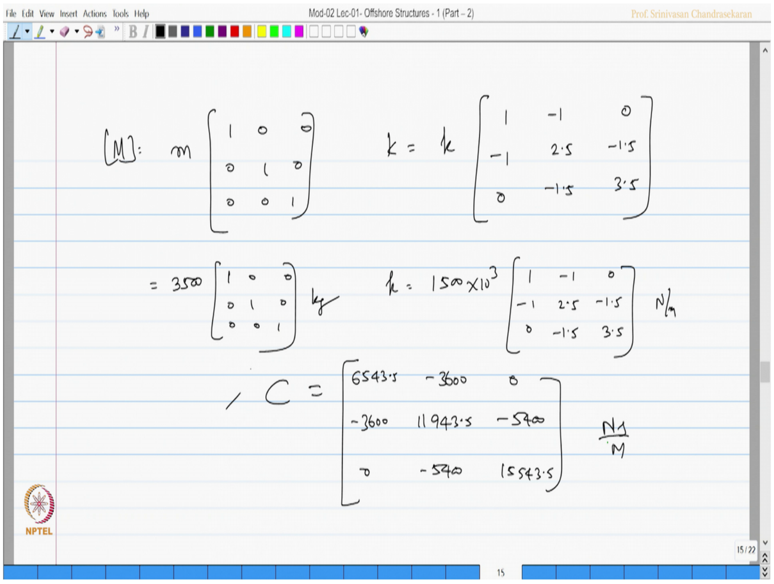

Summing them all we get C matrix as 6543.5 minus 3600 0, minus 3600, 0 11943.5 minus 540, minus 5400 15543.5 in so many Newton second per meter because k matrix is in Newton per meter mass matrices is in kg, C matrix is this.

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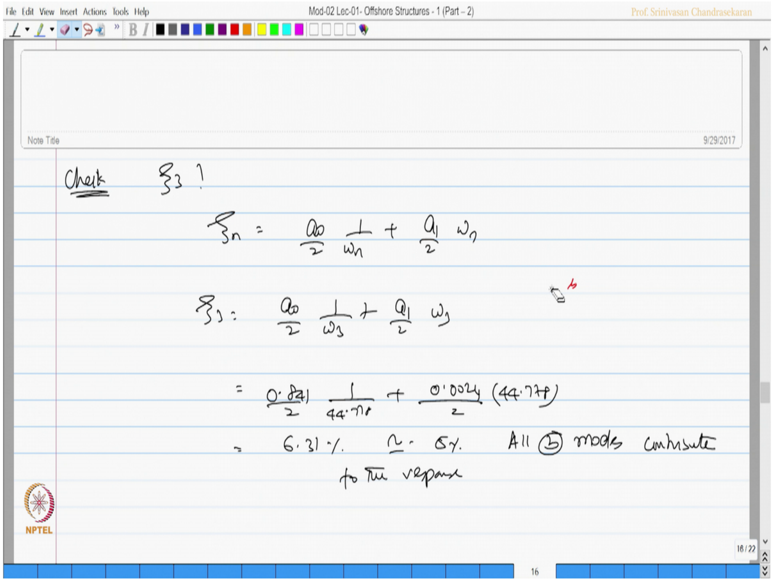

Now, I want to check what is my zeta 3, that is checking for the third mode zeta 3 I want to check. So, we know zeta n is a 0 by 2, 1 by omega n plus a 1 by 2 omega n. So, zeta 3 is a 0 by 2 omega 3 plus a 1 by 2 omega 3; a 0 we know is 841 by 2 by 44.778 plus a 1 is 0.0024 and omega 3 is 44.778 when you estimated this I get this as 6.31 percent which is very close to 5 percent hence we can say all 3 modes contribute to the response.

So, Rayleigh damping is very easy to obtain a problem we will take the numerical example of this through a computer code.

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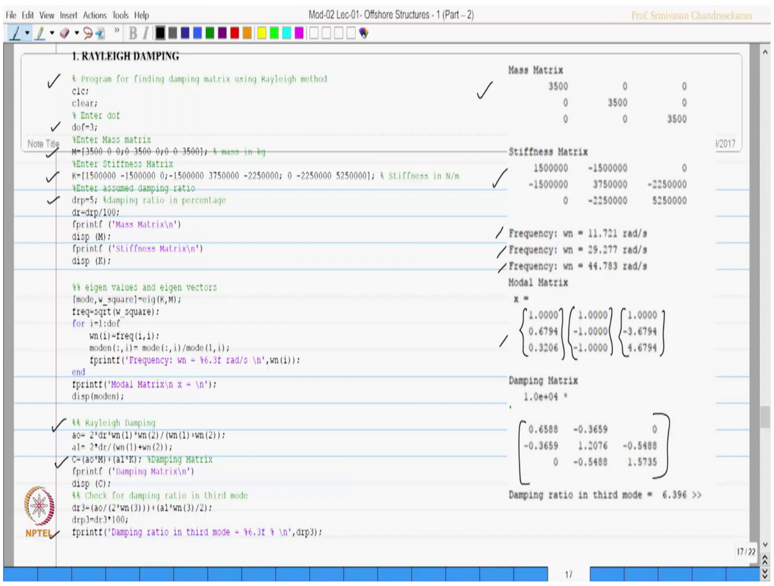

The computer code in this program now shows how the code is written for Rayleigh damping this is for a Rayleigh damping. You have to enter the degrees of freedom, you have to enter the mass matrix in our case is the mass matrix. Similarly one can enter the stiffness matrix, we have entered the stiffness matrix then enter the damping ratio we have taken 5 percent. Once you do that then the Rayleigh constants are evaluated and then the damping matrix is evaluated and it is printed and check for the third one. So, the sample output looks like this, this is my mass matrix, this is my stiffness matrix and these are the input frequencies which we already got 11.721, 29.277 and 44.783 by computer program.

The modal matrix what I get is what I gave you this is my first mode; this is my second mode this is my third mode. And the damping matrix is what I get here 3 by 3 which is exactly same let us say 6588 I by hand I got 6543. So, that is what, it is going to be same. And check for the third mode damping ratio I get 6.396 whereas, we got third mode as 6.31. So, the computer program exactly solves the problem in the same style as we did by hand a sample output is shown to you. And let us see the summary now.

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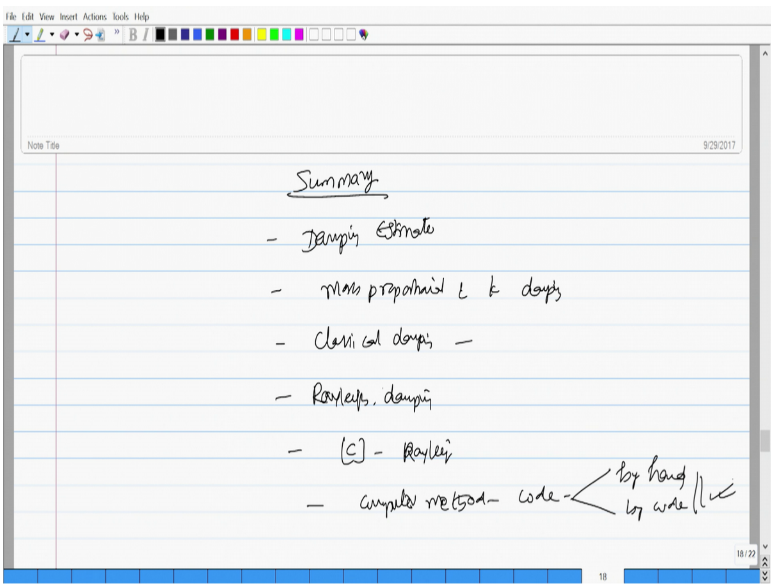

So, in this lecture friends we learned how damping estimate is important in offshore structure, what is mass proportional and stiffness proportional damping, what is classical damping, what is the problem with the classical damping and why Rayleigh damping is appropriate to offshore structures and how to solve see from Rayleigh damping. We also learned the computer method, we also have the access to the computer code, we solve the problem both by hand and by computer code and we found the answers are agreement closely with them.

I hope you will be able to program this in MATLAB and do couple of problems and realize how easy and convenient is Rayleigh damping for damping estimates in offshore structures.

Thank you very much.