Computer Methods of Analysis of Offshore Structures
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Module - 01
Lecture - 04
Beam Element - 1 (Part - 2)

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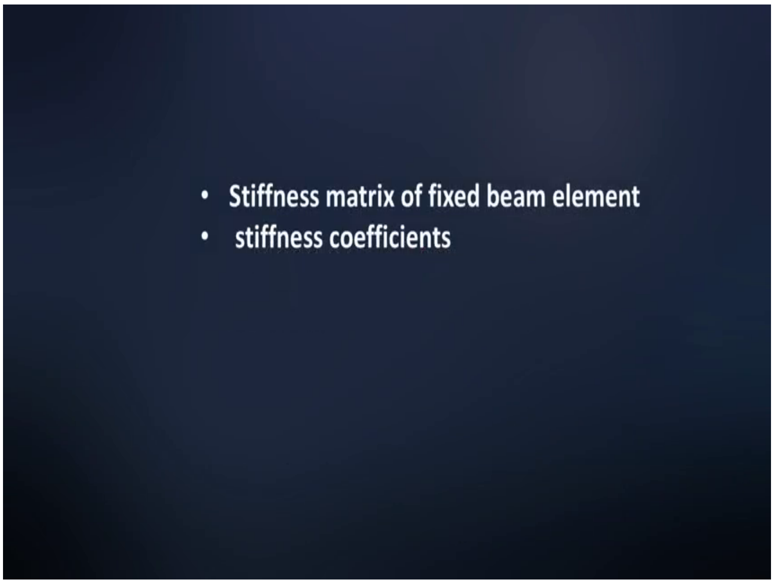

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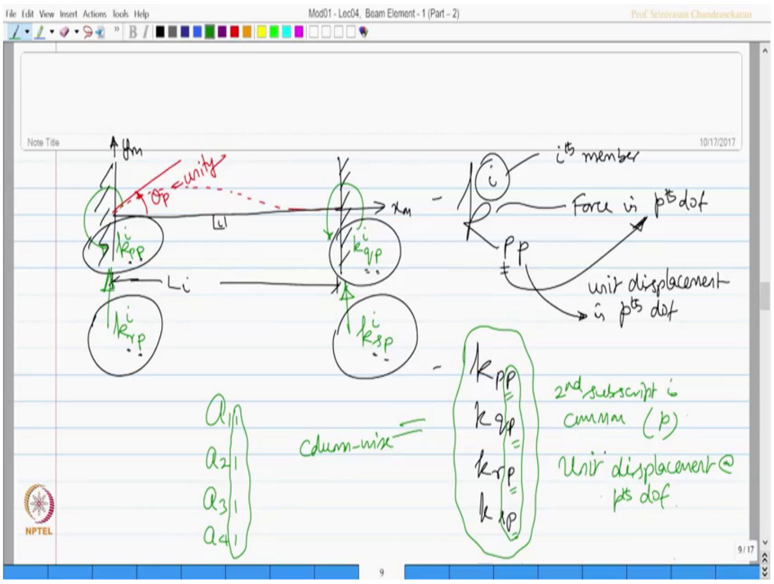

This is my beam with fixed at both the ends xm and ym axis on an i-th member, the length of the member is Li. I give unit rotation at p. At p alone keeping all other degrees of freedom restrained. So, I give unit rotation I draw a tangent, I measured this as theta p which is unit here. Now this will invoke members with end forces, I call this as k pp of the i-th member, I call this as k qp of the i-th member, this value will be k rp of the i-th member, and this value will be k sp of the i-th member.

Let us try to understand how did we get this notation. Take for example, this notation kipp what does it mean. This is actually force in p-th degree of freedom. So, the first p represents this. By giving unit displacement in p-th degree of freedom, the second one represents this. And this notation represents this is meant for i-th member.

Similarly, anyone can read this force in q-th degree of freedom by giving unit displacement in the p-th degree. And similarly this will be force in the r-th degree of freedom by giving unit displacement in the p-th degree. And this will be force in the s-th degree of freedom by giving unit displacement in the p-th degree.

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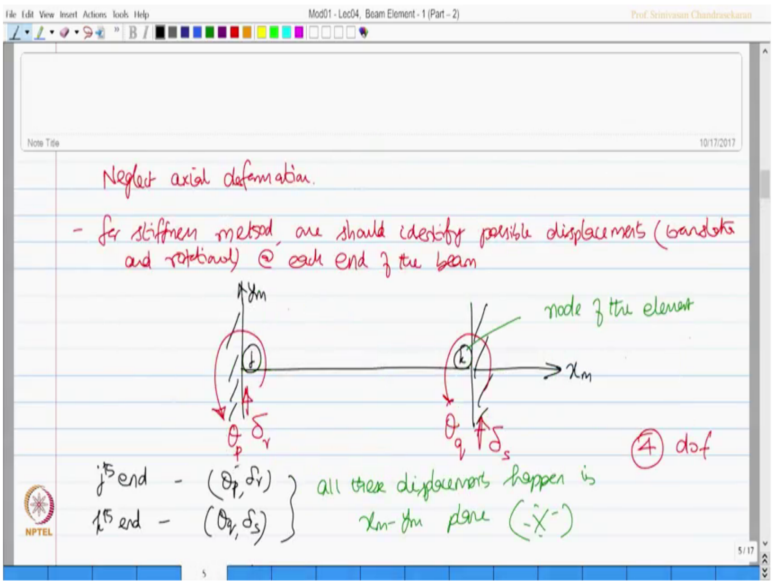

How did we get this p q r s? Please look at this convention p q r s are what we have already taken, the rotation is called as p and q and the displacements, translational are called as r and s. So, I am using the same thing here this is p and p, q and p, r and p, s and p. So, there is one more important thing I want to you to observe, you can see the second subscript in all the notations in k pp, k qp, k rp, k sp. The second subscript in all the notations is p is common; the second subscript is common which is p here, which indicates that we have given unit displacement at p-th degree ok

On the other hand you will see that the stiffness coefficients will be generated column wise. This is the first column now, because all the second notations of this are same. It is similar to a 11 a 21 a 31 a 41 which indicates it is the first column and different rows. So, stiffness matrix will be generated and the coefficients are prepared and derived column wise. So, that is very important.

Now, let find out these values. I have given you unit rotation at the end j and I have got k pp, k qp, k rp, and k sp ok.

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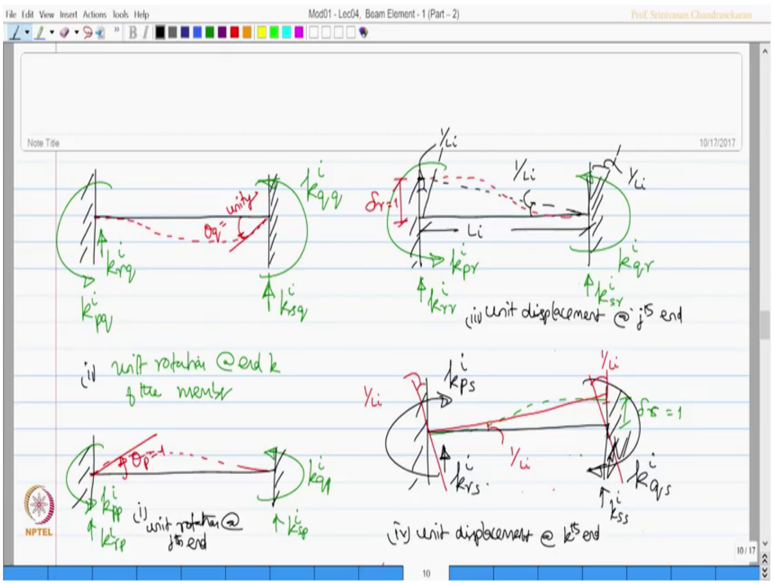

Similarly, I can also draw for unit rotation at theta q. So, I will try to get the moment directions as k qq i, k pq i, k rq i, and k sq i. You can note down the second subscript in all coefficients or q which implies that we have given unit rotation at q-th degree of freedom.

So, this will be imposing unit rotation at the end k of the member, so that is this figure. Similarly, I can also cause unit displacement and to the j-th end and unit displacement at the k-th end. So, let us say I want to create a unit displacement at this end, so this is going to be unity. So, this will impose moment which will be k pr i which will be k qr i and this will be k rr i and this will be k sr i.

Similarly, on the other hand let us try to have unit displacement here. Say this is my delta s which is unity. So now I am going to mark the degrees of freedom and the moments which is k qs i, which is k ps i, which is k rs i, and k ss i. Interestingly, I can also draw a tangent between the initial and the final line of the beam and I can say that the beam has undergone indicates a rotation which is actually 1 by L i where L i is the length of the member which is also now equal to 1 by L i.

You will also have the same fashion here which is 1 by L i. Similarly we can also do this by joining the initial and the final line of this member and saying that this rotation is 1 by L i which will cause rotation here and here which will be again 1 by L i and 1 by L i, ok.

So, for completion sake let us draw this figure also back here. So, this is unit rotation or unit displacement at j-th n, this is unit displacement at k-th end. And let us draw this figure for completion sake in the same sketch and give unit rotation here. So, this is going to be k pp i, this is going to be k qp i, this is going to be k rp i, and this is k sp i which is same as this figure which I have just reproduced here and I am saying the title is unit rotation at j-th end of the member.

So four figures this is actually figure one, this is figure two, this is figure three, and this is figure four. I am sorry for the order but this what, already we have explained in the last slide. Now, we have four figures our objective is to find out these stiffness coefficients for unit displacement and each degrees of freedom, ok.

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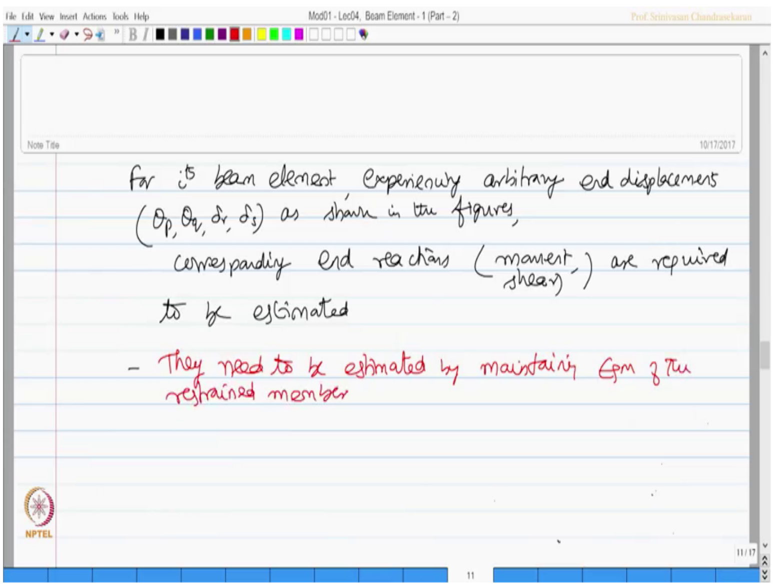

Let us write down the compatibility equation for i-th beam element, experiencing arbitrary end displacements namely: theta p, theta q, delta r, and delta s. I have shown in the figures. Corresponding end reactions, what could be the end reactions; it could be the moment, it could be shear are required to be estimated that is how i jump. They need to be estimated by maintaining equilibrium of the restrained member. So, we need a governing a equation they are the as follows.

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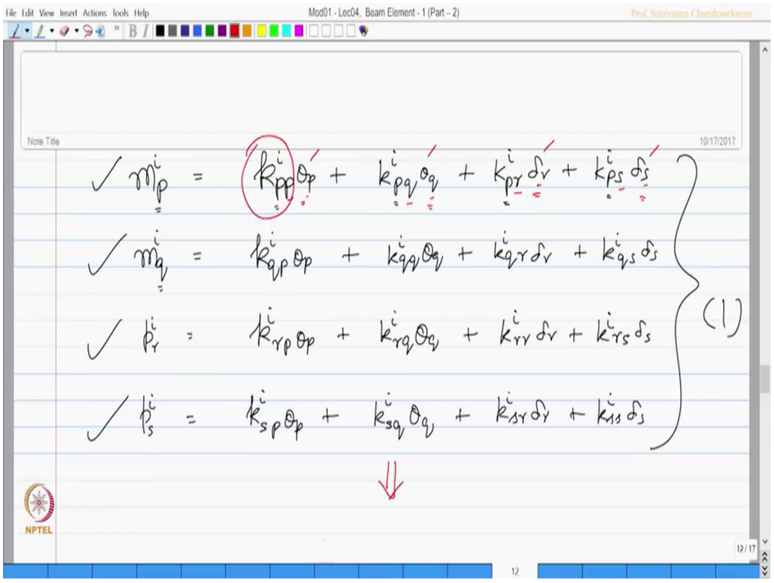

M p that is the end moment of the i-th member in the p-th degree of freedom will be k pp theta p plus k pq theta q plus k pr delta r plus k ps delta. So, one can very easily see here. If you talk about the end moment at p-th degree of freedom all will be related to p and contributions from each degree like p q r s have been taken. The moment you say the second subscript is p you get theta p, q theta q, r delta r s delta s. We also know p and q are rotations and r and s are displacements. So, I am using theta for rotations and delta for displacements.

In the same order now we can write m i q, I expect you to write this it is very simple. So, since it is q all first subscript will remain as q; sp k q. So, it is a p theta p i-th number plus k qq, therefore theta q plus k qr, and therefore delta r of the i-th member plus k qs delta s very good. So, that is what we have to write. It is very simple to remember if we understand the notation very carefully.

Similarly I want to find the force that the shear at the r-th end. So obviously, this is going to be the first subscript is going to be r caused because of p, second subscript is p and theta p. So, r q theta q plus rr delta r plus rs delta s. I do not think any confusion in this. Similarly, the fourth one p s i; so k sp theta p k sq theta q plus k sr delta r plus k ss delta s; I think that is what it is done.

So, I call this equation as a equation of set 1. So, what are these equations giving me? These equations are giving me the end moments and end shear for arbitrary displacements theta p theta q delta r and delta s which are unity at respective degrees of freedom and we are trying to find out the forces.

The moment I say these displacements are unity in magnitude; these reactions will actually become nothing but the stiffness coefficients which are evaluated. And they will all be the forces or the end reactions, and each one of these value will be the stiffness coefficient.

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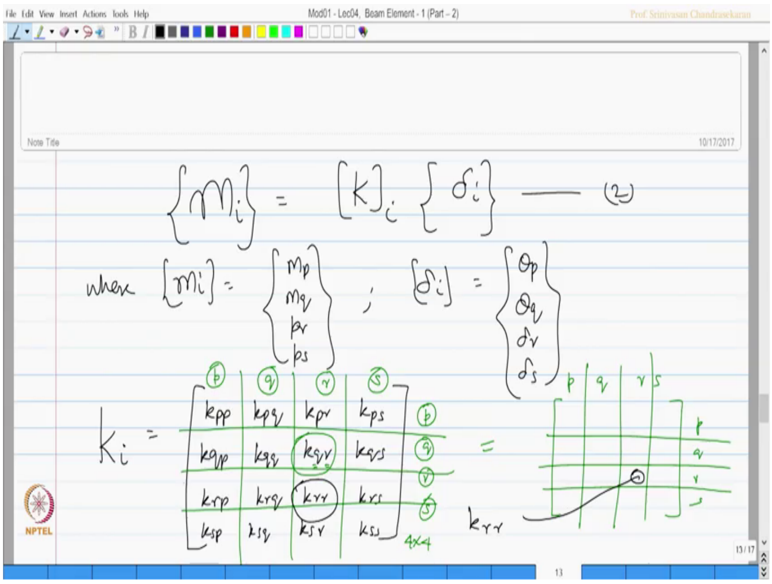

So, now I can generalize this equation by a statement saying m i will be k i into delta i. The generalized equation where; m i vector is actually m p, m q, p r, and p s. And delta vector is actually equal to theta p theta q delta r delta s. And k i will be the stiffness matrix which will be k pp k pq k pr k ps, k qp k qq k qr k qs, k rp rq rr rs, sp sq sr ss.

To remember this let us use some shortcut method to really reproduce this in a simple term. Please remember this it is very easy, I call this as p q r s columns, these are all columns and I call these rows also as p q r s these are rows. It is a 4 by 4 matrix. So, if you want to remember this it is very easy row first column next.

So, qr row q column r, so you can easily reproduce this matrix. So, I can rewrite this matrix in a very simple form simply understanding saying p q r s p; q r s. If you want to really write down this element, this element, this element will be actually k row first and column next which is similar to exactly this. So now, this equation what we derived is a fundamental expression for a beam element. Our job is to now found out what would be these stiffness coefficients by giving unit displacement and the respective degrees of freedom.

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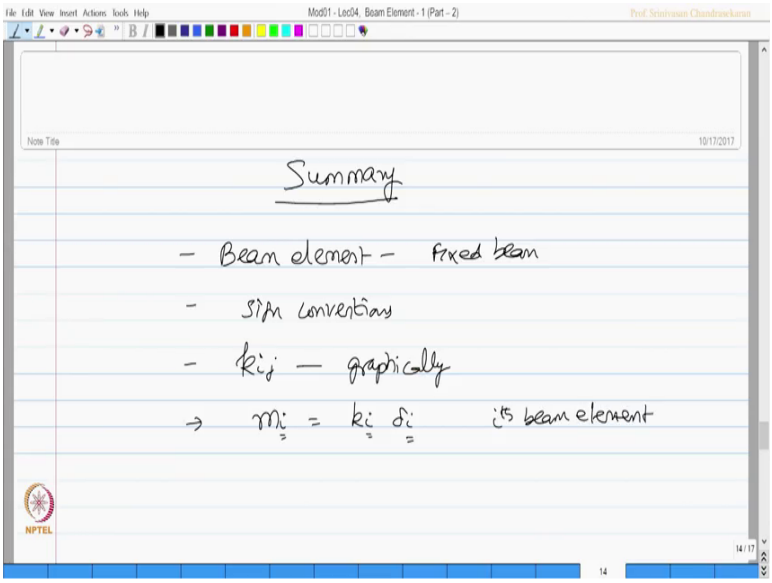

So, friends let us look at the summary of this lecture. In this lecture we introduce a conventional beam element which is actually a fixed beam. We also recommended certain sign conventions which are required to follow the derivation. We expressed in simple terms how to derive k ij graphically. And then we landed up in the expression saying m i is k i of delta i, where i represents about the whole event for i-th beam. So, there can be n number of such elements in a given frame.

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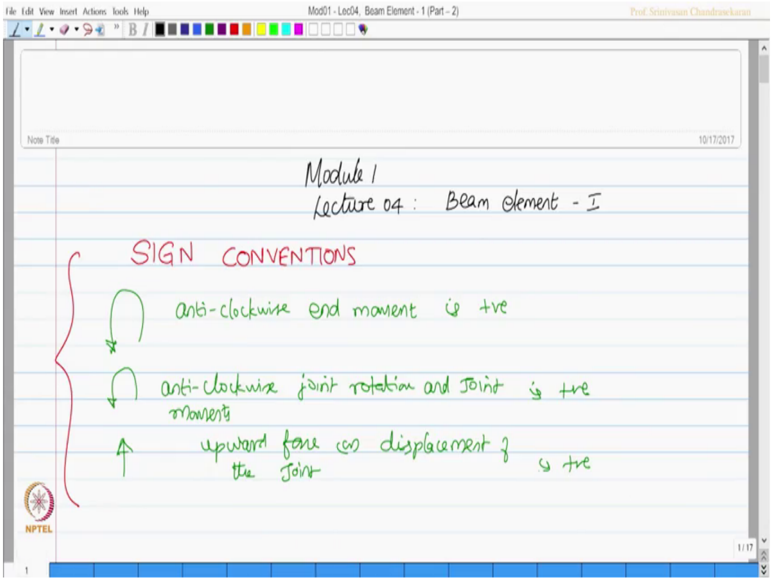

I hope you have understood this; I want you to browse through this very quickly, and understand once again how we have followed this. And I hope we will continue to discuss more in detail in the next lecture.

Thank you.