Computer Methods of Analysis of Offshore Structures

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Module - 02
Lecture - 12
Dynamic Analysis - 1 (Part - 2)

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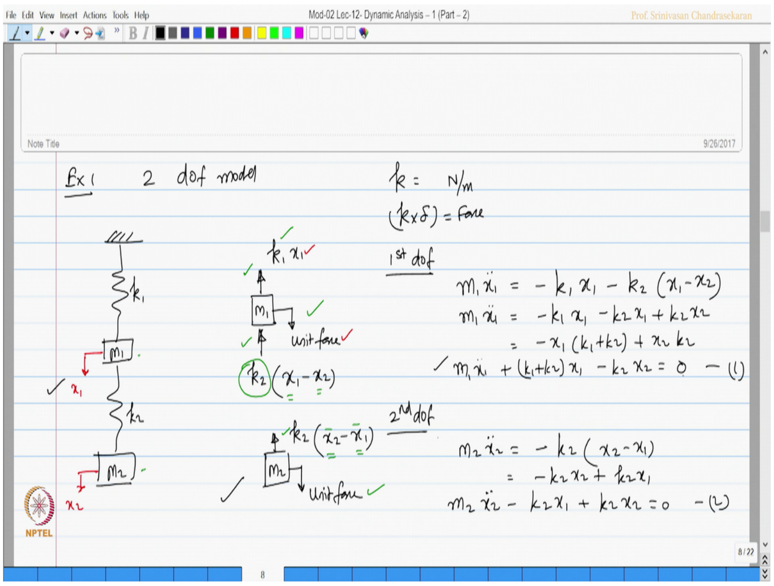

The example 1 which we now take to start with is a 2 degree freedom system model a spring mass system. Will take up this as an example let us say my example problem is 2 degree freedom system model which has mass m 1 and m 2, the degrees of freedom are marked here as x 1 and x 2, the model does not have damping, but it has a restoring force k 1 and k 2.

So, I want to first to draw the free body diagram to write the equations of motion. So, listen carefully how do we write them, draw the free body diagram and then derive the equations of motion using Newton's method. There are 5 methods based on which you can derive equations of motion simple harmonic method, Newton's method, energy method, Rayleigh method and using D Alembert's principle. So, the details of these methods individually can be examined and studied and understood from the reference textbooks and NPTEL courses which I cited in the previous slides. We will not talk about those methods in detail in this particular course we will simply take one simple easy method which is Newton's force method and try to derive the equations of motion.

So, now, this is my mass m 1 when the mass is allowed to put downwards by this x 1 let us say the force the mass will be restored by the spring. So, I give in unit force and it will be restored by stiffness and displacement because you know stiffness is actually force by displacement. If I multiply stiffness with displacement I will get the force. So, when the mass m 1 start to move by newly force and the first degree of freedom x 1 k 1 spring will restore the mass.

At the same time the k 2 spring will be compressed and will apply a force in the opposite direction because k 2 will be compress now will oppose this motion and that will be equal to the stiffness of that spring multiplied by the point where you are measuring the displacement and the point where the spring is connected. It is very clear please understand here. The x 1 is the point where you are measuring the displacement of the force given and x 2s to the point where the spring k 2 is connected k 2s connected between x 1 and x 2, so I say x 1 x 2. The direction of k 2 is simple, m 1 is trying to pull down k 2 will oppose m 1, so apposition, m 1 tries to move down k 1 is trying to pull it up. So, the direction the force value the direction and the force value are very clear in this particular figure.

Let us do this similarly for mass m 2. So, same logic mass m 2 I apply unit force along x 2 along x 2 when I apply that the spring k 2 will try to oppose that and that is going to be equal to k 2 multiplied by the point where the unit forces apply minus the point where the spring is connected. It is very simple, k 2 is connected between x 2 and x 1 you can see that x 2 and x 1. And first coordinate will be where you are measuring the force second is where you are connecting this force the direction is very simple mass m 2 is pulled down therefore, k 2 will try to full it down. I do not think any confusion is there in marking the arrow directions of the stiffness of the springs marking the arrow direction of the unit force and therefore, now this is me free body diagram. Now, we are ready to write the equation of motion using Newton's law.

So, I want write equation of motion. So, let us say first degree I am writing a force balance equation we know that force equal to mass and acceleration. So, m 1 x 1 double dot will be acting down that is the acceleration force given by the x 1 double dot which is actually a derivative of displacement x 1 which will be actually equal to minus k 1 x 1 negative sign because m 1 x 1 double dot is acting in the downward direction whereas, k 1 x 1 is a restoring force in the opposite direction therefore, minus sign; similarly again a minus sign for k 2 which is x 1 minus x 2.

So, let us say m 1 x 1 double dot is minus k 1 x 1 minus k 2 x 1 plus k 2 x 2 which is minus x 1 of k 1 plus k 2 plus x 2 of k 2. Rearranging m 1 x 1 double dot plus k 1 plus k 2 of x 1 minus k 2 of x 2 is 0 that is my first equation of motion. How can you say this first equation of motion? Because the first equation of motion the coordinates are x 1 and x 2 they are connecting because I applied unit force which is having acceleration in the first degree of freedom.

Now, let us do for the second degree of freedom. So, look at this figure now. So, m 2 x 2 double dot is equal to minus of k 2 of x 2 minus x 1 which is minus k 2 x 2 plus k 2 x 1. So, rearranging m 2 x 2 double dot minus k 2 x 1 plus k 2 x 2 is 0 second equation. So, I have written the equations of motion I can write also them in a matrix form because computer methods require more simple method of representing these equations let us say equation 1 and 2 will be represented in matrix form.

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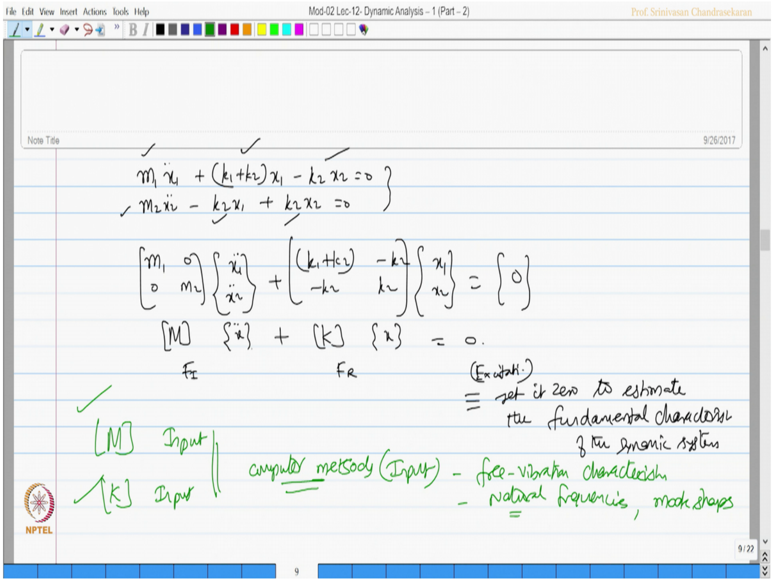

Let me write the equations first m 1 x 1 double dot plus k 1 plus k 2 of x 1 minus k 2 of x 2 is 0 there is a first equation the second equation is m 2 x 2 double dot minus k 2 x 1 plus k 2 x 2 is 0. I want to represent this in matrix form. So, I can say now m 1 0 0 m 2 of x 1 double dot x 2 double dot plus k 1 plus k 2 minus k 2 minus k 2 k 2 this is stiffness matrix, x 1 x 2 is 0. One can read this first row m 1 x 1 double dot plus that is this term, k 1 plus k 2 of x 1 this term, minus k 2 of is 0. So, I get the equation.

Similarly, the second row m 2 x 2 double dot minus k 2 x 1 plus k 2 x 2 is 0. So, the equations are now converted to a matrix form this matrix is called mass matrix this is your displacement vector this is stiffness matrix this is displacement vector acceleration vector. So, I say this is going to be a. So, inertia force, restoring force, excitation force of course, this is 0 in this case we have to set it to 0, we have to set it to 0 to estimate the fundamental characteristics of the dynamic system. That is why there other ways called free vibration characteristics. They are also called natural frequencies because under no excitation force these frequencies are demonstrated by the system, that is why they call natural frequencies and mode shapes.

So, writing equation of motion gives me 2 inputs – one, the mass matrix is an input for me analysis the stiffness matrix is also an input from analysis. From a computer method, I should be able to derive the mass and stiffness matrix and give them as an input. Once I have this input I want to estimate the frequency and mode shape.

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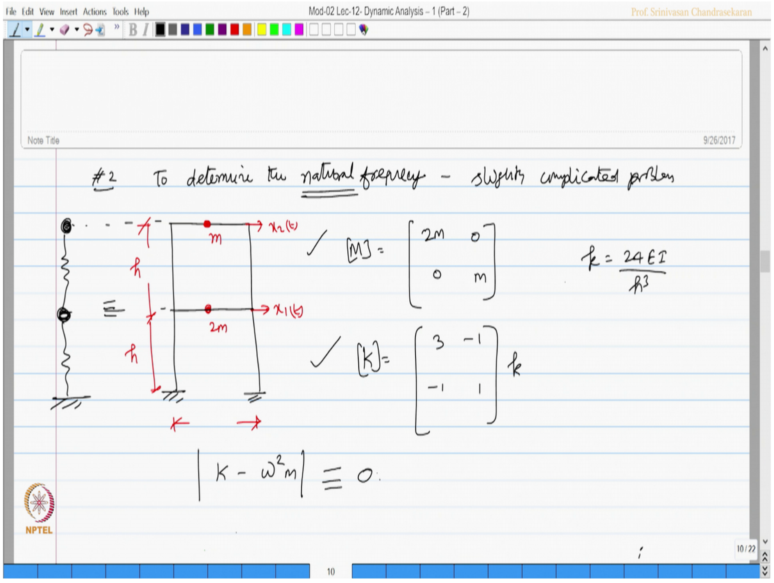

So, step number 2 to determine the natural frequency as I said we are doing this problem by hand. So, we will do this solution the classical eigen solver method then will introduce you to the computer methods and solve the same problem very easily using a computer program. So, equation of motion can be easily written for this problem.

So, now to estimate the natural frequency we will take slightly a complicated problem. So, we a taking a frame of single bay 2 storey, the mass is lumped at both the locations I call this as m and this as 2m and this is my degree of freedom x 1 of t and x 2 of t and the frame has dimensions as given in the screen now the height of the frame is h and h.

So, let us know that the mass matrix in simple terms very quickly can be identified from this which is going to be 2m 0 0 m. I am not deriving the equation of motion I believe that you understood from the previous slide how equation motion can be easily derived by using force balance method suggested by Newton by drawing free body diagram. I believe that you know that. So, this is an equivalent system now, the equivalent system is mass spring mass spring and so on.

So, the stiffness matrix when you do the equation of motion will be 3 minus 1 minus 1 1 with the multiplayer of k where k is 24 EI by h cube because we are looking for a bending stiffness. So, I have the mass matrix now, I have this stiffness matrix norm which is also derived from the first principles by writing equation of motion for a simple problem. For this problem similarly I play the same technique is a small homework given to you, you will be able to array the masses stiffness matrix as shown on the screen now.

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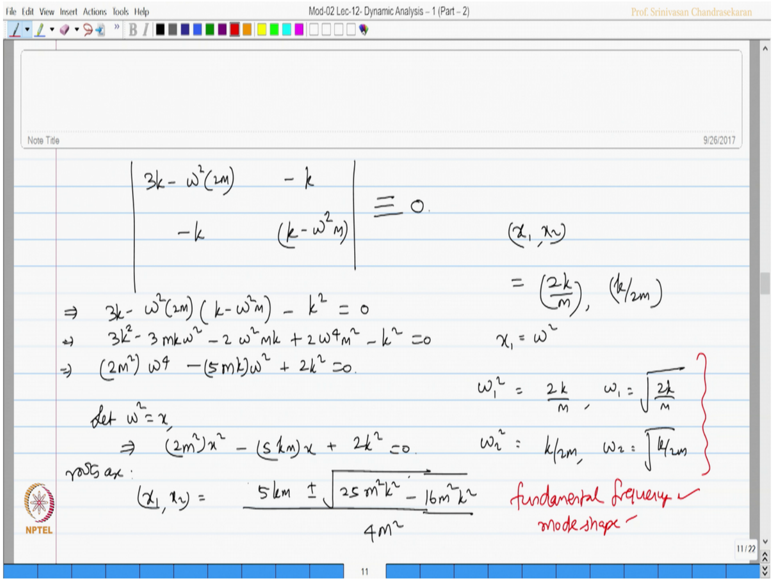

So, let us sin the natural frequency for this system. So, it is very simple find the determinant of k minus omega square m and set it to 0. So, let us do that determinant of do you know k matrix is 3 k minus k minus k k and m matrix 2m 0 0 m. So, I am writing it here 3 k minus omega square 2m there is k minus omega square m minus k minus k k minus omega square y I said the determinant to 0. So, doing that expanding I get 3 k minus omega square 2m into k minus omega square m minus k square is 0.

Again expanding 3 k minus 3 k square 3 m k omega square minus 2 omega square y m k plus 2 omega 4 m square minus k square is 0. So, rewriting 2m square of omega 4 minus 3 plus 2 that is 5 m k of omega square plus 2 k square is 0. Let omega square v x. So, the equation of becomes 2m square x square minus 5 km x plus 2 k square is 0 the simple quadratic of second order I can find the roots of this quadratic which is x 1 and x 2 which will be minus b plus or minus b square minus 4 ac whole by 2 a let us do that. So, 5 km plus are minus root of b square, so 25 m square k square minus 4 ac, so 4 to 8 to 16. So, minus sixteen m square k square a whole divided by 2 a. So, that gives me 4 m square. So, which gives me x 1 x 2 after simplification as 2 k by m and k by 2m there are 2 roots now.

We also know x 1 is omega square. So, omega 1 square can be said as 2 k by m therefore, 1 can find omega 1 has root of 2 k by m and k value is known m value if known one can find omega in radians per second. Similarly omega 2 square is k by 2m therefore, omega 2 is going to be root of k by 2m. So, knowing the values of k and m 1 can estimate omega 1 and omega 2. As we have seen, we have found out the natural frequencies of the system. But we did not know which is the lowest frequency or the fundamental frequency how to identify the fundamental frequency, the fundamental frequency can be identified only by estimating the mode shapes and then from the mode shape can qualify and find out what is the fundamental frequency.

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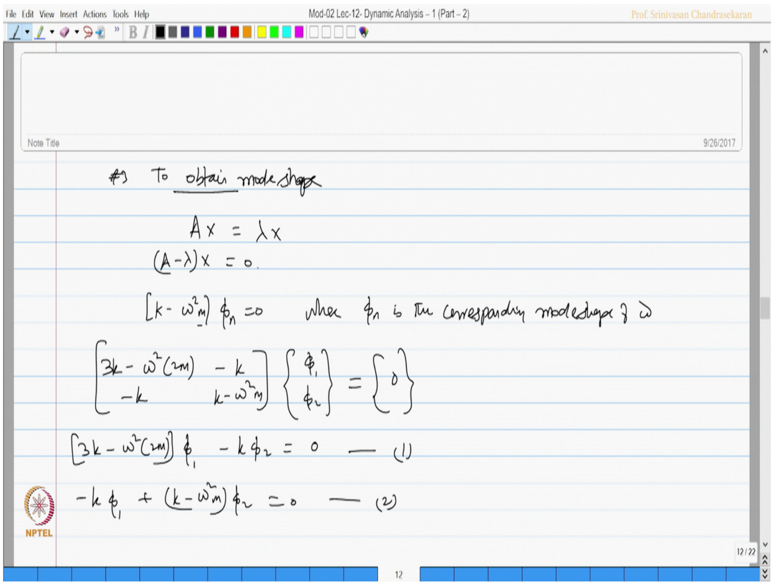

So, the next step is the estimate the mode shape, the estimate the mode shape the governing equation is simply a x is lambda x that is a minus lambda x is set to 0 that is k minus omega square 1 of phi n is 0 where phi n is the corresponding mode shape of omega let us do that.

So, we have 3 k minus omega square 2m minus k minus k k minus omega square m multiplied by let us say phi 1 phi 2 set it to 0, that is 3 k minus omega square 2m of phi 1 minus k phi 2 is 0 that is the first equation we get. Minus k phi 1 plus k minus omega square m 5 to is 0 is a second equation here. I want to find the first mode shape. So, what I do is in these 2 equations substitute omega 1 square as 2 k by m which we already got can see here omega 1 square is 2 k by m.

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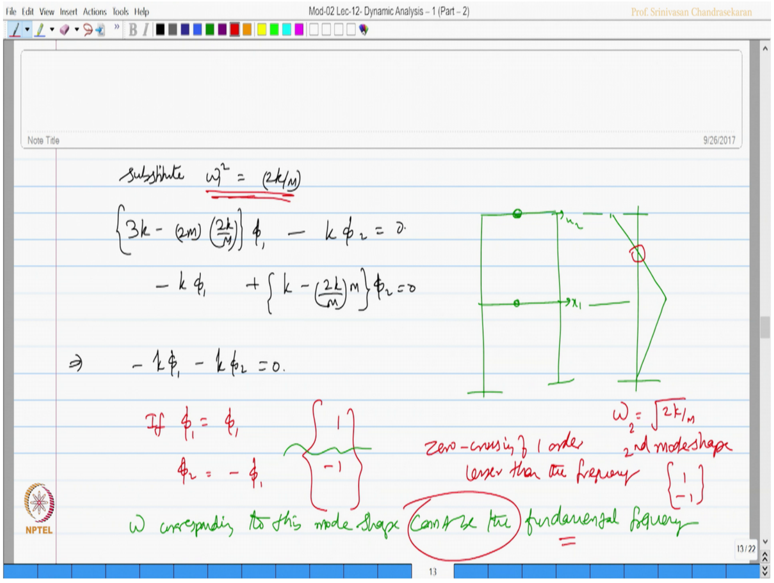

So, substitute that to get the first one. So, we say 3 k minus 2m of omega square that is 2 k by m off phi 1 and just substituting the value the previous equation minus k phi 2 is 0 minus k phi 1 plus k minus 2 k by m off m of phi 2 is 0 simplifying I get minus k phi 1 minus k phi 2 is 0. If phi 1 is phi 1 phi 2 is minus 5, so the mode shapes corresponding are 1 and minus 1. Since there is a zero crossing between them the frequency corresponding to this mode shape cannot be the fundamental frequency because the fundamental frequency will have no zero crossing what did mean the zero crossing let us try to plot this.

I have a system of this type, you can see here from this figure this is x 1 and x 2. So, x 1 and x 2 let us try to plot this. So, this is positive and this is negative because this is 1 this 1. So, when a joined them I get 1 0 closely. Fundamental frequency cannot have any 0 cross it means the omega value corresponding to root of 2 k by m is not the first frequency, but the second frequency and this is the second mode shape. So, that is why I said just by saying omega 1 it does not mean that the first frequency fundamental frequency should be qualified based upon the characteristic of the mode shape. So, we found the mode shape, we got the characteristic there is no zero crossing then we can say first frequency if there is 1 crossing then second frequency it means zero crossing will be of 1 order lesser than the frequency. So, if it is second frequency one zero crossing third frequency two zero crossing first frequency no zero crossing. So, we should workout the first frequency. So, as we substituted omega 1 square as this value let us do the same logic and substitute.

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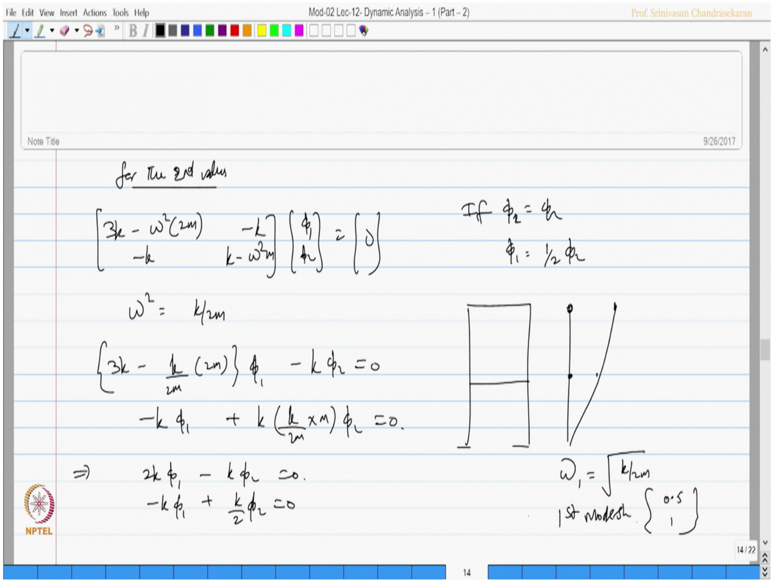

For the second, for the second value that is substitute 3 k minus omega square 2m minus k minus k k minus omega square m off again phi 1 and phi 2 there is a next mode shape set to 0, but in this case substitute omega square as k by 2 m. So, 3 k minus k by 2m of 2m of phi 1 minus k phi 2 is 0 which gives me and the next equation is minus k phi 1 plus k of k by 2m of m of phi 2 is 0.

So, simplifying I get 2 k phi 1 minus k phi 2 is 0 or minus k phi 1 plus k by 2 phi 2 is 0. So, if phi 1 phi 2 is phi 2 then phi 1 is half of phi 2 from this equation what does it mean is in this system these are the mass points phi 1, phi 2 is 1 and this is half. So, I get the mode shape like this, so there is no zero crossing. The corresponding omega now is square root of k by 2m and this is the first mode shape which is 0.5 and 1. So, the earlier mode shape was correspondingly 1 and minus 1.

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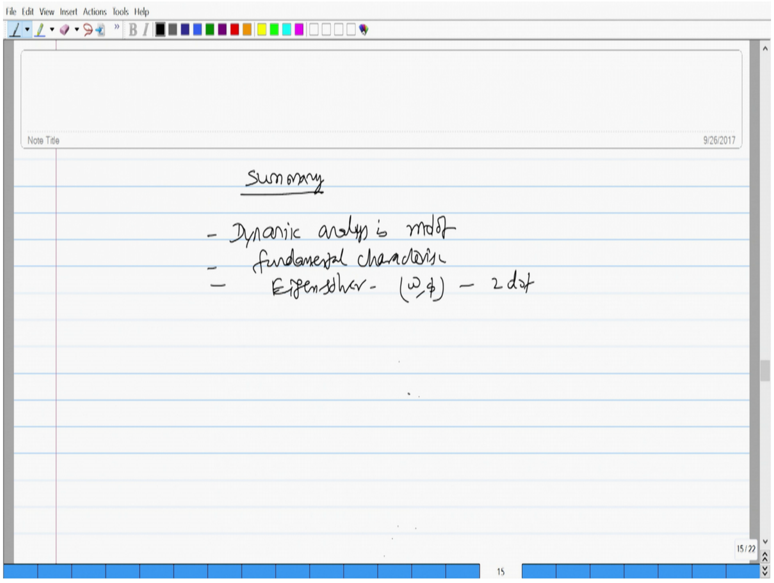

So, friends in this lecture we have started understanding the dynamic analysis of multi degree freedom systems we are solving the problem by hand to estimate fundamental characteristics of the system. So, in this example we have used eigen solver method to find omega and phi of a 2 degree freedom system model. We will continue the discussion in the next lecture.

Thank you very much.