Computer Methods of Analysis of Offshore Structures
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Module - 01
Lecture - 04
Beam Element - 1 (Part - 1)

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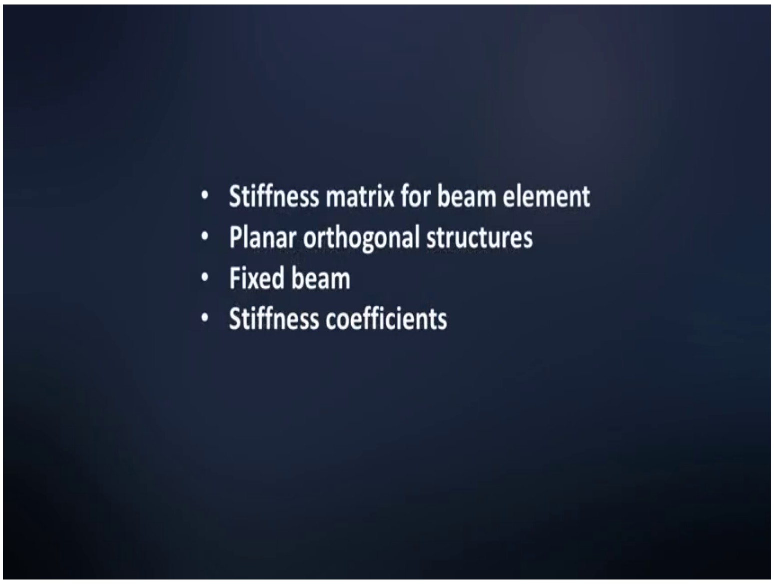

Friends, let us continue with the lectures on module 1 of the course on Computer Methods of Structural Analysis. In this lecture 4 we are going to derive the standard stiffness matrix for a beam element which will be one of the basic element to be used in structural analysis of planar orthogonal frames. There are some sign conventions which we need to follow before we derive the stiffness matrix for the beam element.

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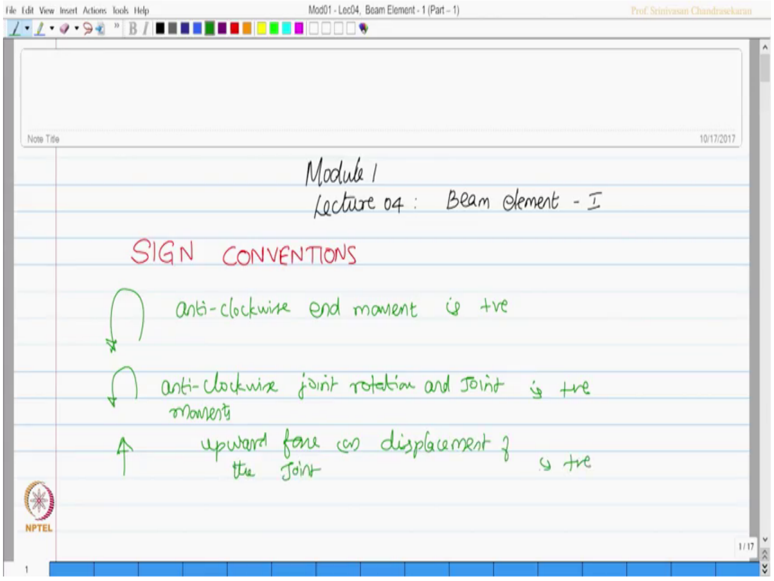

Let us see the sign conventions: anticlockwise moment is positive. Anticlockwise joint rotation and joint moments is positive. Upward force or displacement of the joint is positive.

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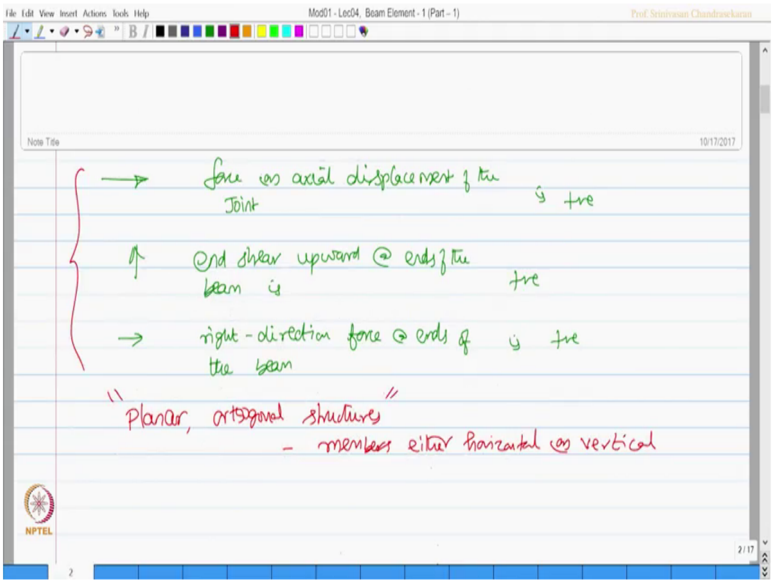

Force towards the right or axial displacement towards the right of the joint is positive. End shear upward at the ends of the beam is positive. Right direction force at the ends of the beam is positive.

So, we are now going to restrict the derivation to planar, orthogonal units or structures which will have members either horizontal or vertical. Let us have this sign conventions very clearly understood and we are going to limit our discussion now only to planar orthogonal structures.

Let us take a beam element.

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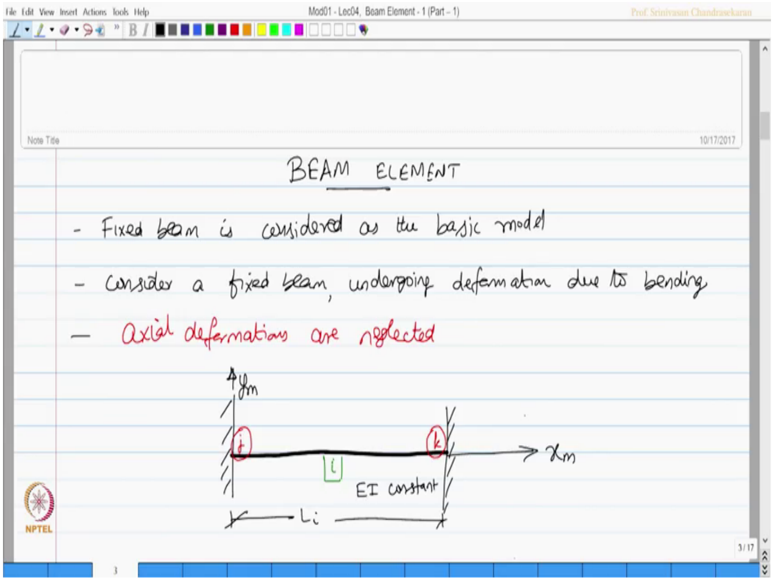

Fixed beam is considered as a basic model. So, let us mark or consider a fixed beam undergoing deformation due to bending, we are neglecting the axial deformation. So, if that is the case, the standard fixed beam looks like this. It has both end fixed, this is beam element. The beam element has i as its index, it has got two joints j and k, this is my x axis of the element x m and this is my y axis of the element y m, and the length of the member is L i, and the beam has constant EI for the standard element.

Let us explain this in terms of some mathematical conditions.

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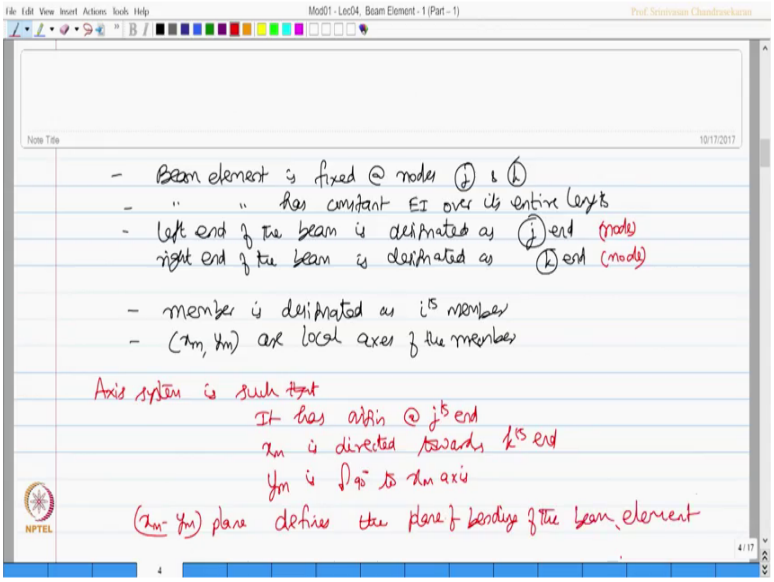

The considered beam element is fixed at nodes j and k, you can see here they are fixed at nodes j and k; has constant EI over its entire length. Left end of the beam is designated as j-th node. Right end of the beam is designated as k-th node; let us say j-th node and k-th node. Member is designated as i-th member xm ym are local axes of the member.

It is very important to note the axis system. Axis system is such that it has origin at j-th end, x m is directed towards k-th end; that is very important please see this figure here. Origin is at j-th end and x m is directed towards the k-th end. Y-m is counter clock wise 90 degrees to x-m axis.

Therefore, xm ym plane defines the plane of bending of the beam element. So, this is a conventional way of explaining the standard beam element, these are the sign conventions, member is designated as I, j and k ends, Li, EI constant, xm ym orientation, and we have explained this all in words as well as graphics.

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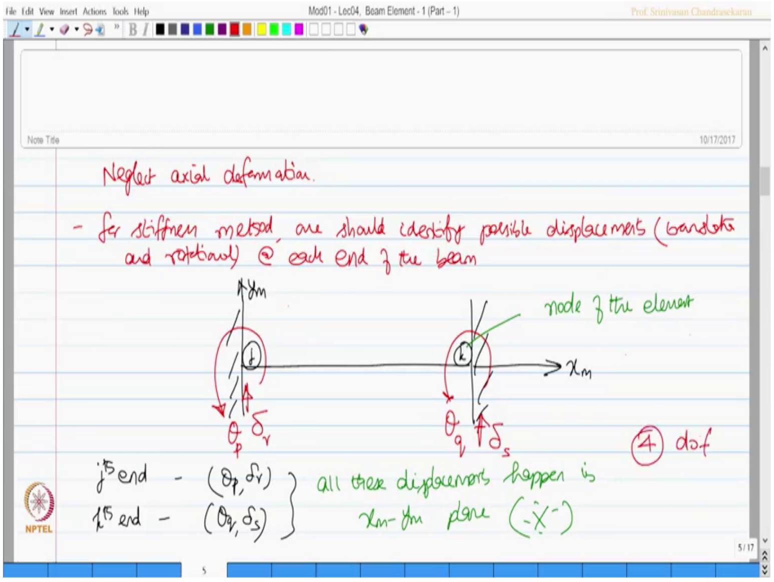

Now, we will neglect the axial deformation; we will neglect the axial deformation. Now for stiffness method we already said one should identify possible displacements both translational and rotational at each end of the beam. So, let us see what are the possible translations and rotations at each end. So this is my member, these are the supports, this is my j-th end, this is my k-th end, this is my x m and y m. The possible rotations at end it can be theta, again on the right can be theta, the vertical displacement can be delta and can the delta.

Now there is a confusion that thetas and deltas are available at both the ends. To make a very clear distinct difference between these notations I have to use a subscript. Some textbooks use subscript related to the joint j; let us say this can be called as theta j delta j theta k delta k, but I do a different notation here. I am going to say this is the theta p theta q delta r delta s; please note down the style of notation. I first marked the rotations theta p and theta q, then I marked the translations delta r and delta s as p q r s. So, there is an order; please understand this order, because this order is very important to make the computer program more comfortable and more concession compact.

So, let us now write down what are the displacements at j-th end; j-th end we have theta p delta r, and at the k-th end we have theta q delta s. And please note all these displacements happen in x m y m plane. So, no out of plane bending in this case, its happening in x one y m plane; this is very very important.

I hope you have no doubt in the sign convention and in the standard notation what we have followed. I have explained why we are using this p q r s and I also explained you what is the order of selecting this notation theta p theta q delta r delta s. So, as you re-correct from the lecture stiffness method demands you to identify the displacements rotational and translational which are available at each end of the beam.

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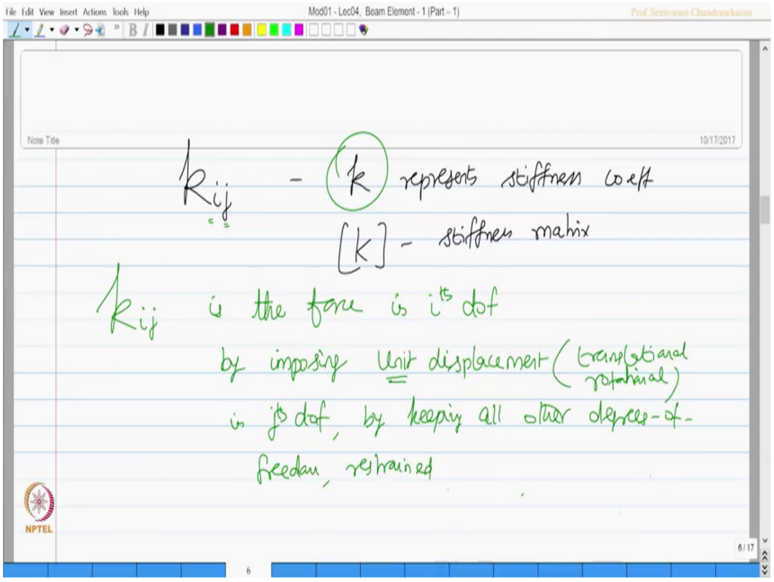

Now, we are interested in deriving k ij. K actually represents the stiffness coefficient, while capital K represents the stiffness matrix. Please do not get confused with this k and this k. This k is actually the node of the element. This k what I am talking about is actually the stiffness coefficient which I am going to derive. A stiffness matrix capital K and small k is stiffness coefficient. So, there are subscripts here i and j.

So now, by definition k ij is the force in i-th degree of freedom by imposing unit displacement. This displacement can be translational, rotational; does not matter but unit value unit displacement in j-th degree of freedom by keeping all other degrees of freedom restrained.

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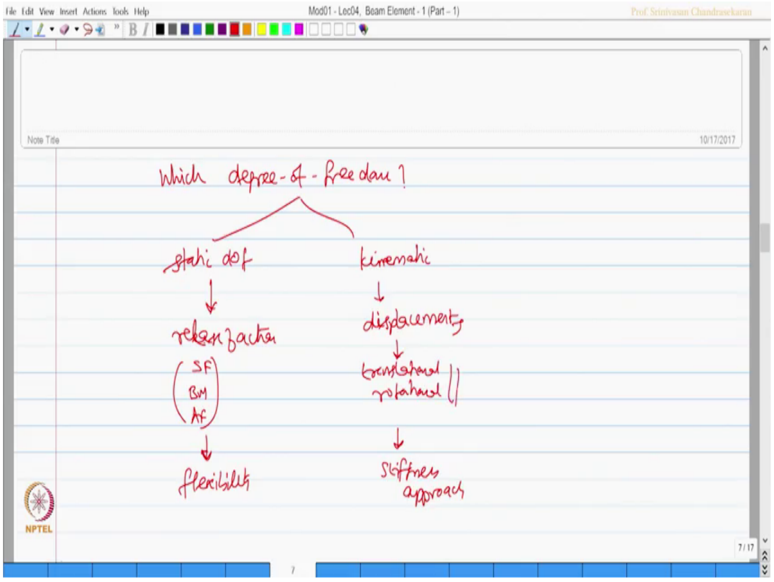

Now the moment I say degree of freedom there is a confusion here, which degree of freedom you are talking about. There are two degrees of freedom: one is we called static degree of freedom, other is kinematic degree of freedom. We know static degree is a freedom are related to release of actions, like shear force, bending moment, axial force etcetera. Kinematic degrees of freedom are related to displacements. And we already said static degree of freedom will be associated to flexibility approach, and kinematic of freedom will be associated to stiffness approach.

So, the moment I say here degrees of freedom I am only talking about the displacements which can be translational or rotational does not matter, both are allowed right. So, that is degree of freedom what we are talking about. So, k ij by classical definition is this. So, what I am going to do is. There are now four degrees of freedom in this problem: two rotations and two translations at each end j and k. So, total four degrees of freedom. So, what I should do? I should give unit displacement in each one of them and find the forces in all degrees of freedom by keeping the remaining degrees of freedom restrained that is what I am going to do, ok.

So, let us also try to make another statement saying that.

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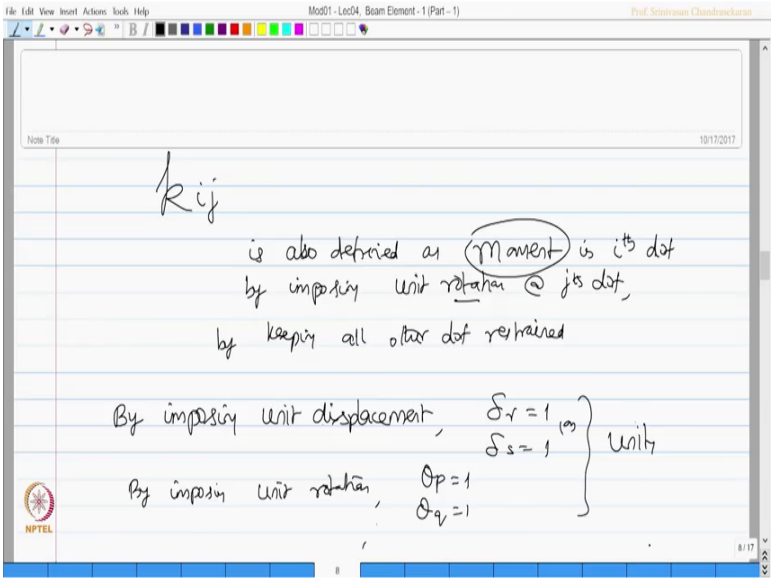

K ij is also called or also defined as moment in i-th degree of freedom by imposing unit rotation at j-th degree of freedom by keeping all other degrees of freedom restrained. So, it is actually an extension of the same logic except that I said instead of unit displacement I specifically said unit rotation. And when the moment I give unit rotation I will get not the force but the moment, ok.

Therefore, by imposing unit displacement we can say either delta r is unity or delta s is unity. By imposing unit rotation we mean that either theta p is unity or theta q is unity; these are all unity.

Having said this one is now interested to find either the forces or the moments which are caused in all degrees of freedom by imposing either unit displacement or unit rotation in respected degrees of freedom. So, let us do this one-by-one.