

**Computer Methods of Analysis of Offshore Structures**  
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**Module – 02**  
**Lecture – 07**  
**Environmental Loads - 1 (Part – 2)**

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- Macro and micro scale variation
- Wave loads
- Airys theory

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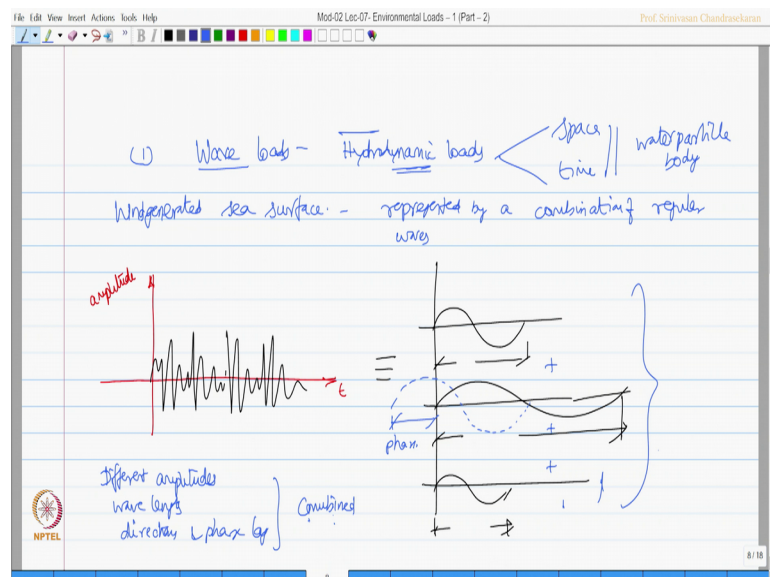
The image shows a digital whiteboard with handwritten notes comparing macro and micro scale variations. The notes are organized into two columns. The left column is titled 'Macro-scale variation' and lists four points: 1) average wind velocity over a period of time (10min), 2) tidal current, 3)  $H_s$  and  $T_z$  (peak periods) of the wave spectrum, and 4) Peak Ground Acc (PGA) of the motion. The right column is titled 'micro-scale variation' and lists two points: - It causes sufficient impact on the response, and - gives rise to dynamic effects. The whiteboard interface includes a toolbar at the top with various drawing tools and a status bar at the bottom showing the NPTEL logo and the slide number 7/18.

Macro-scale variation	micro-scale variation
1) average wind velocity over a period of time (10min)	- It causes sufficient impact on the response
2) tidal current	- gives rise to dynamic effects
3) $H_s$ and $T_z$ (peak periods) of the wave spectrum	
4) Peak Ground Acc (PGA) of the motion	

Let us take an example of a macro scale variation. One example can be an average wind velocity over a period of time; usually this period is taken as 10 minutes, second variation of a tidal current, they will not cause any major effect in the response. Three, significant wave height and zero crossing or let us say peak periods of the wave spectrum; fourth can be peak ground acceleration PGA of the earthquake motion. These are some examples of macro scale variation. Coming to micro scale variation in this case the variation is so, minimum it causes sufficient impact on the response.

So, interestingly friends micro scale variation actually gives rise to dynamic effects.

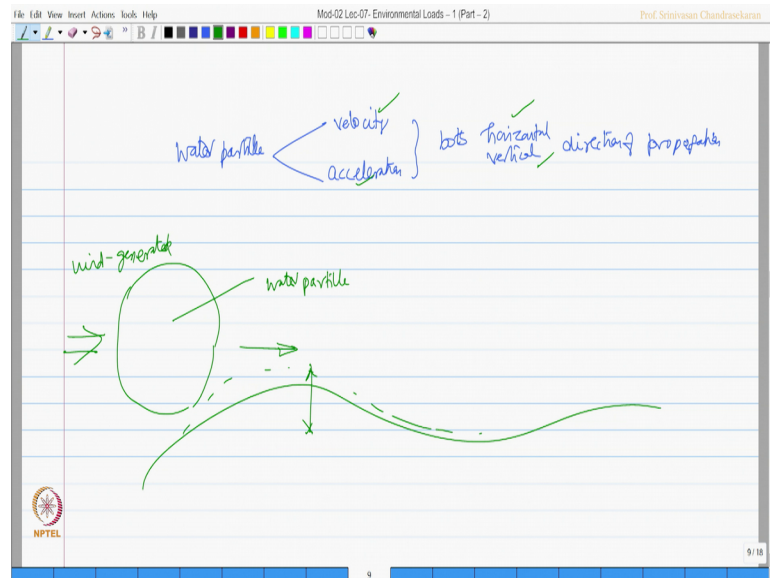
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Having said this let us talk about one important variety today which is wave loads; also called as hydrodynamic loads the reason is they vary both in space and in time that is why it is dynamic, it varies because of variations caused by water particle body therefore, they are called hydro. So, hydrodynamic loads generally friends waves are generated by wind. So, wind generated sea surface, need to be represented it is generally represented by combination of regular waves. For example, if I have a typical sea surface elevation which looks this is time and this is let us say amplitude variation, if it varies typically like this this can be equal to series of regular waves with different period and even with different phase. So, sea surface elevation generated by wind can be represent by combination of these kinds of regular waves.

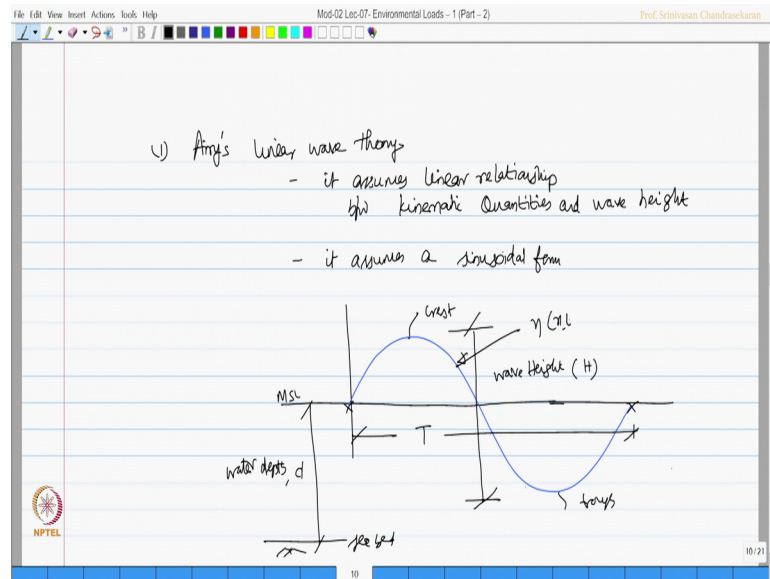
So, regular waves of different amplitudes, different wave lengths, different directions and phase lag or combined to form the input load.

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How are they expressed in theory they are expressed using water particle movements, water particle velocity and acceleration in both horizontal and vertical directions of propagation. On the other hand imagine a ball, throw it on a surface of the sea the ball will roll it means the ball moves ahead horizontally, the ball moves also vertical it has an velocity and acceleration this ball is nothing, but the water particle, how does this ball move? This ball is moved because of wind generated waves, actually wind pushes, blows this ball or this water particle at different directions. So, they are expressed by wave elevation and amplitude.

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There are many theories available in the literature; the basic theory which confirms to estimation of water particle kinematics is Airy's wave theory. This is commonly used because it assumes linear relationship between kinematic quantities and wave height usually it assumes a sinusoidal form. So, if you draw a specific wave, the difference between the crest and the trough is called wave height, the average of this is called mean sea level and if this is my sea bed this is called my water depth indicated as small  $d$  and from 1 0 to next 0 is called as a wave period and so on.

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$$\eta(x,t) = \frac{H}{2} \cos(kx - \omega t) \quad (1)$$

where  $\eta(x,t)$  - sea-surface elevation  
 $H$  - wave height  
 $\omega$  = wave frequency =  $\frac{2\pi}{T}$ ,  $T$  - wave period (s)  
 $k$  = wave number =  $\frac{2\pi}{\lambda}$   
 $\lambda$  = wave length  $\approx 1.56 T^2$   
 $C_p$  - phase speed =  $\frac{\omega}{k} = \frac{\lambda}{T}$

The sea surface profile is expressed by  $\eta(x, t)$ , where  $\eta(x, t)$  is called sea surface elevation which is given by  $\frac{H}{2} \cos(kx - \omega t)$ , where  $\eta(x, t)$  is called sea surface elevation,  $H$  is the wave height,  $\omega$  is a wave frequency which is  $\frac{2\pi}{T}$  where  $T$  is a wave period usually expressed in seconds,  $k$  is called wave number which is given by  $\frac{2\pi}{\lambda}$ , where  $\lambda$  is called wave length which is approximately equal to  $1.56 T^2$ .  $C_p$  is called phase speed which is  $\frac{\omega}{k}$ .

Which I will transform to  $\lambda$  by  $T$ , because  $\omega$  is  $\frac{2\pi}{T}$  and  $k$  is  $\frac{2\pi}{\lambda}$  which gives me this.

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The image shows a digital whiteboard with handwritten notes on water particle kinematics. The notes include the following equations:

- Horizontal velocity: 
$$\dot{u}(x, y, t) = \frac{\omega H}{2} \frac{\cosh(ky)}{\sinh(kd)} \cos(kx - \omega t) \quad (2)$$
- Vertical velocity: 
$$\dot{v}(x, y, t) = \frac{\omega H}{2} \frac{\sinh(ky)}{\sinh(kd)} \sin(kx - \omega t) \quad (3)$$
- Horizontal acceleration: 
$$\ddot{u}(x, y, t) = -\frac{\omega^2 H}{2} \frac{\cosh(ky)}{\sinh(kd)} \sin(kx - \omega t) \quad (4)$$
- Vertical acceleration: 
$$\ddot{v}(x, y, t) = \frac{\omega^2 H}{2} \frac{\sinh(ky)}{\sinh(kd)} \cos(kx - \omega t) \quad (5)$$

Next to the equations is a diagram of a wave profile. The vertical axis is labeled  $y$  and the horizontal axis is labeled  $x$ . The mean sea level (MSL) is indicated by a horizontal line. The water depth is labeled  $d$ . The wave profile is shown as a sinusoidal wave. A note next to the diagram says "Shreechakra modifications (Wheeler, 1970; Chakrabarti, 1971; Hogben)".

So, once the sea surface elevation from equation one is known then I can find the water particle kinematics which are nothing, but velocity and acceleration in horizontal and vertical direction. Horizontal means the direction of wave propagation, vertical means the direction of water depth. So, horizontal water particle velocity  $U \cdot t$  is given by  $\frac{\omega H}{2} \frac{\cosh(ky)}{\sinh(kd)} \cos(kx - \omega t)$  equation 2. The vertical water particle velocity is given by  $\frac{\omega H}{2} \frac{\sinh(ky)}{\sinh(kd)} \sin(kx - \omega t)$  equation 3. Friends in this equation there are variables of  $x, t$  and  $y$  if you really see  $\omega, h, k, d$  or know  $x$  and  $t$  are variables then what is  $y$ .

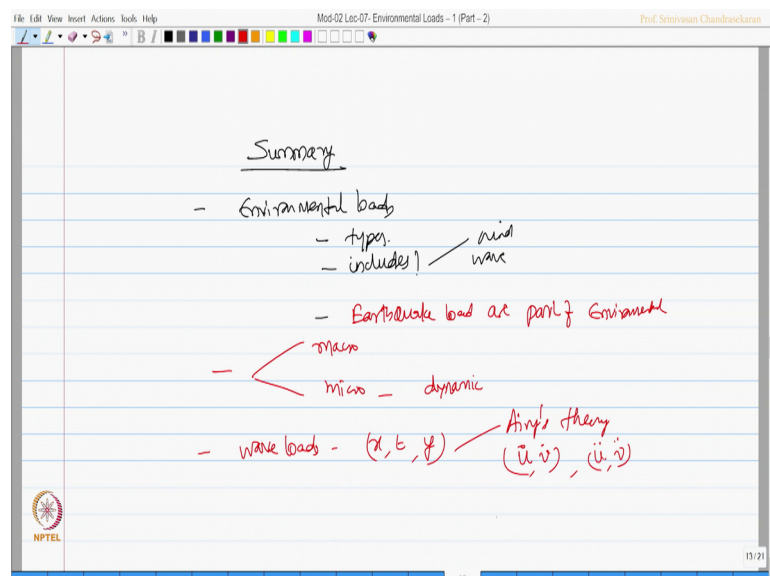
If you have a typical sea surface elevation if this is my mean sea level, as I said it will vary in space. So,  $y$  is measured from here and this is my sea bed, this is my water depth

d which is indicated here. So, friends water particle velocity in horizontal and vertical directions vary along time, along direction of propagation and along depth as well you see all three variables are there, if I differentiate once I get the acceleration.

So, I say  $u$  double dot  $\times t$  which nothing, but the derivative of  $u \times \text{dot}$  equation 2, I will get  $\omega^2 H$  by 2,  $\cos$  hyperbolic  $k y$  by  $\sin$  hyperbolic  $k d$ ,  $\sin k x$  minus  $\omega t$ . Similarly derivative of the vertical will be minus  $\omega^2 H$  by 2  $\sin$  hyperbolic  $k y$  by  $\sin$  hyperbolic  $k d$  of  $\cos k x$  minus  $\omega t$ , I call this is as equation number 4 and equation number 5.

So, friends Airy's theory describes the sea surface elevation and enables us to compute the water bodily kinematics in the direction of propagation, in time variation and along depth as well. Now only limitation with this theory is, this theory is valid only till the mean sea level. If you want to stretch and include the variations above and below the mean sea level, then I should apply different stretching modifications suggested by various researchers like Wheeler in 1970, Chakravarthy in 1971 and Hogbor. Details of these extensions can be seen in the literature recommended for studies in the website of NPTEL of this course. So, let us quickly see the summary of today's lecture.

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The summary says this lecture started with discussions on environmental loads, we have understood what are the types of environmental loads, what include environmental loads what are included like wind load wave load etcetera we have also learnt that the

earthquake loads are part of environmental loads, environmental loads can cause macro scale variation and micro scale variation, they can influence dynamic response of the system.

We have also said that wave loads start propagating the direction variation of  $x$   $t$  and along the depth, Airy's theory is one of the simplest theory which gives me the water particle velocity in horizontal and vertical direction and of course, the water particle acceleration in horizontal vertical direction whose equation are presented in this lecture. So, we should try to understand how these loads can be quantified for analysis purpose we will try to do this in the next lecture.

Thank you very much.