Computer Methods of Analysis of Offshore Structures
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Module - 01
Lecture - 03
Matrices (Part - 1)

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Friends, let us continue with the discussion on module 1. In this lecture we will discuss about some special properties of matrices which are useful for computer methods of structural analysis.

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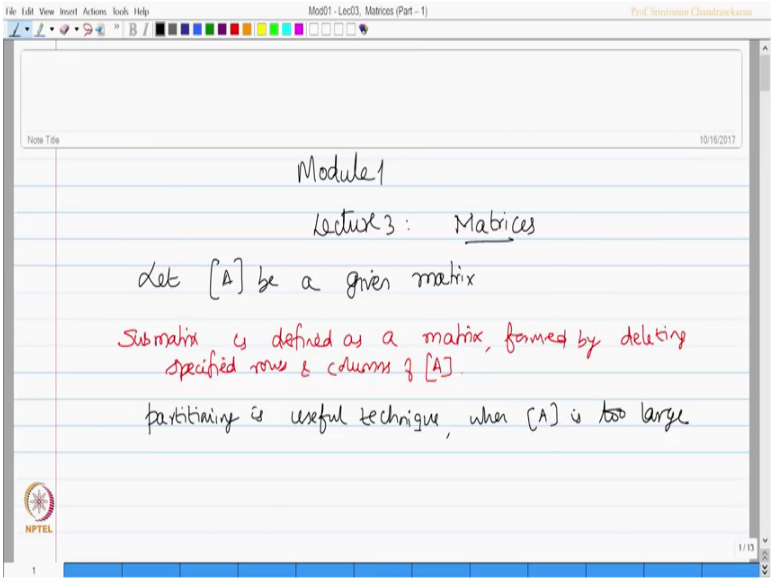

Let A be given matrix then as sub matrix is defined as a matrix form by deleting specified rows and columns of the matrix A. Instead of doing the deletion there is an alternative for this: you can also partition the matrix. We will see the advantages of partitioning quite a while from now. This is an useful technique when the matrix size is too large.

Let us explain this by a set of algebraic equations.

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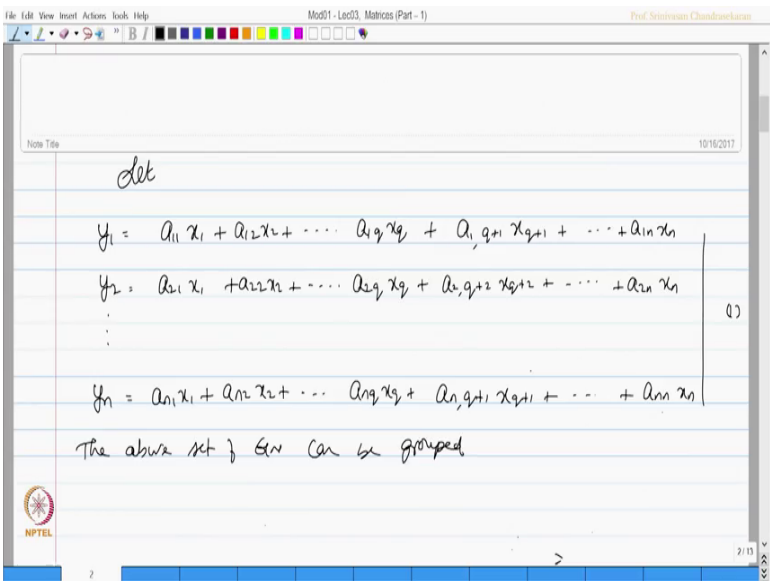

then the above set of eqations can be also grouped.

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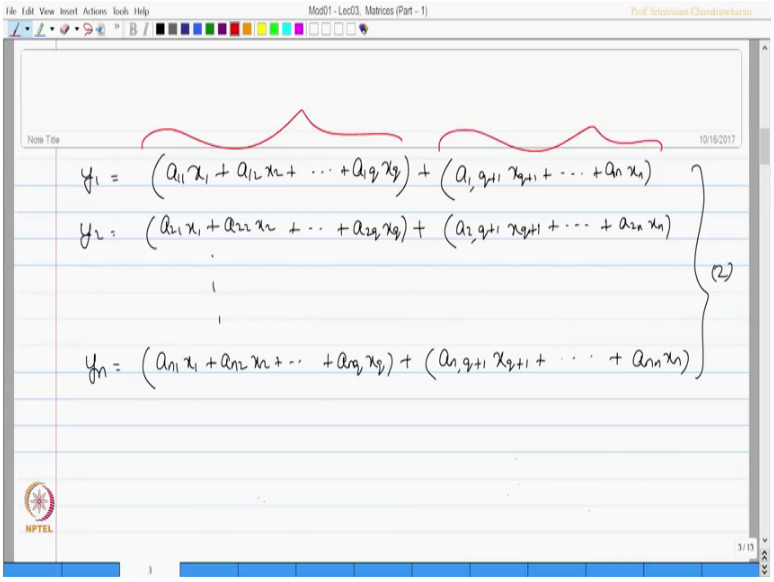

Let us see how I can say this as y 1 is equal to a 11 x 1 plus a 12 x 2 plus a 1q x q plus a 1 q plus 1 x q plus 1 plus a 1n x n. Similarly, y 2 is also grouped as: a 21 x 1 plus a 22 x 2 plus a 2q x q and then plus a 2 q plus 1 x q plus 1 plus a 2n x n. By this logic y n can be expressed as: a n1 x 1 plus a n2 x 2 plus a n q x q plus a n q plus 1 x q plus 1 plus a nn x n, I call this as set of eqations 2 .

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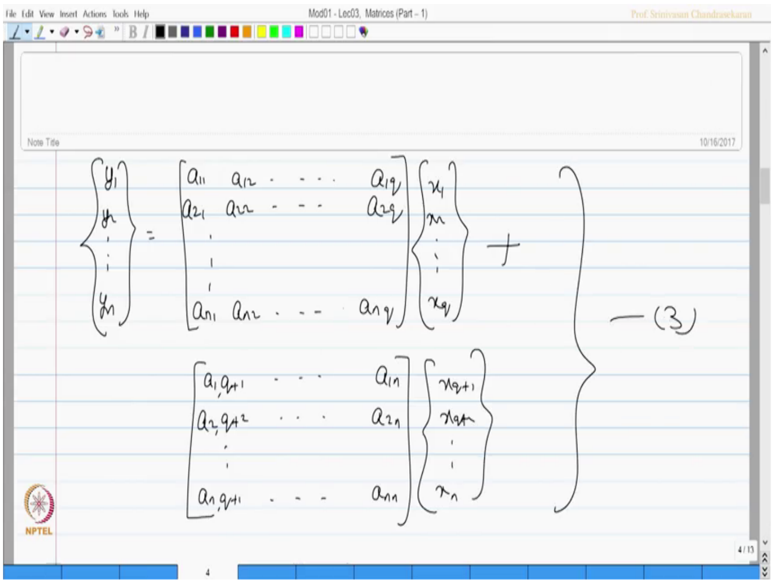

Now, let me express both the set of equations in a matrix form: y 1 y 2 y n can be expressed as a 11 a 12 a 1q, a 21 a 22 a 2q, a n1 a n2 a nq of x 1 x 2 x q plus a 1 q plus 1 a 1n, a 2 q plus 2 a 2n, a n q plus 1 a nn multiplied by x q plus 1 x q plus 2 x. I call this as equation 3. Equation 3 is a matrix representation of equation 1 and 2. In fact, 1 is a general equation, whereas 2 is a grouped equation. So, this is one group this is another group; till q is one group then q plus 1 is another group. I express equation 2 in two groups: first group and second group. till q the first group and q plus 1 onwards the second group second group this.

Let us see what is the advantage of doing this.

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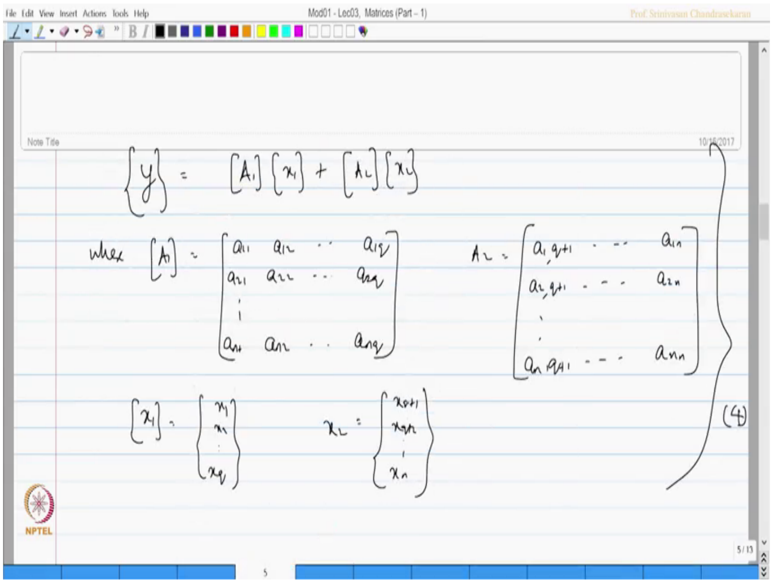

Now, I can write the vector y as A 1 and vector x 1 plus A 2 and the vector x 2, where A 1 is actually a 11 a 12 a 1q, a 21 a 22 a 2q, a n1 a n2 a n q. And A 2 is actually A 1 comma q plus 1 which goes till a 1n, a 2 comma q plus 1 which goes till 2 n, a n comma q plus 1 which goes a nn. And x 1 is x 2 x 2 of x q and x 2 is x of q plus 1 q plus 2 till x n.

I call this equation as equation 4, ok. Now let us see the size.

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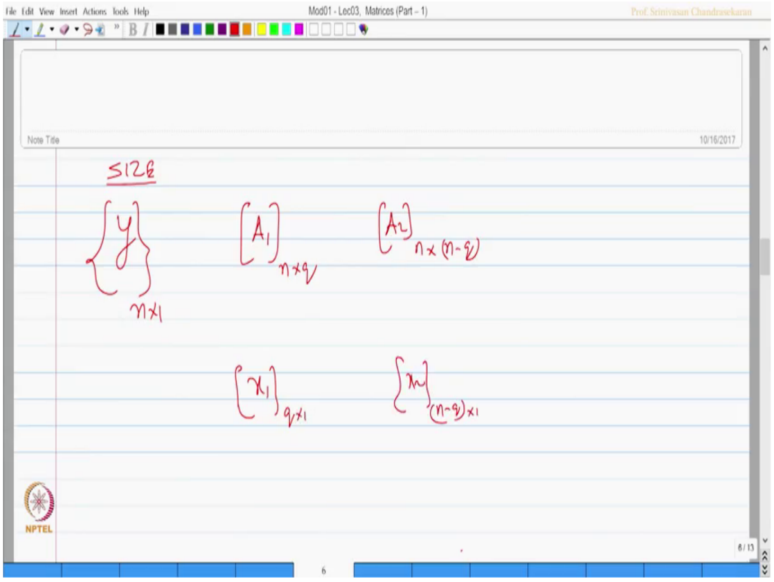

Let us write down y, we can see the size y is n into 1 A 1 is n by q, A 2 is n by n minus q, x 1 is q into 1, and x 2 is n minus q into 1.

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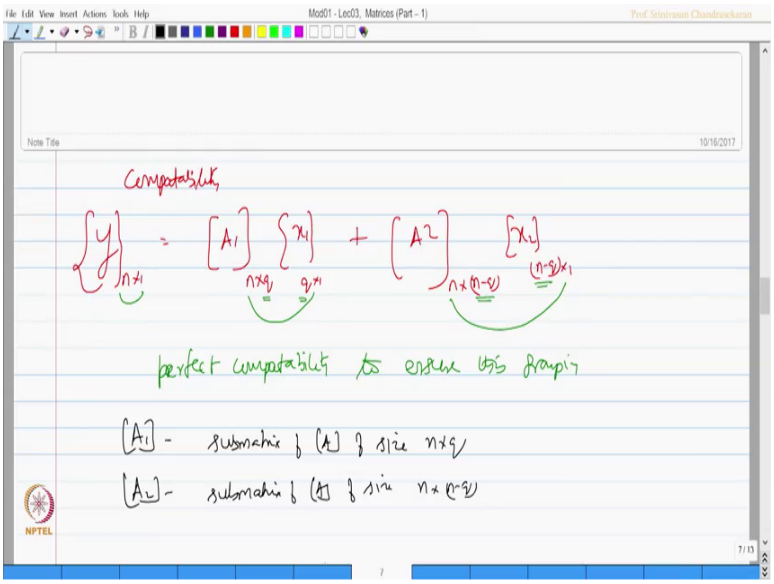

So, there should be a perfect compatibility between the respective multiplying mtrices; that is y will have n plus n into 1 which will have A 1 of n q and vector x 1 of q 1: n q n q 1 plus A 2 which will be n of n minus q A 2 which will have a vector x 2 which is n minus q of 1. So, you can see the compatibility the number of columns and number of rows of the adjacent multipliers should be same; number of columns and number of rows should be same. So, ultimately this will result in a matrix of n isto 1, this will also n cross 1 I get n plus 1 .

So, there should be a perfect compatibility to ensure this grouping. Now, I can say that A 1 is a sub matrix of A of size n by q. And A 2 is a sub matrix of a of size n cross n minus q.

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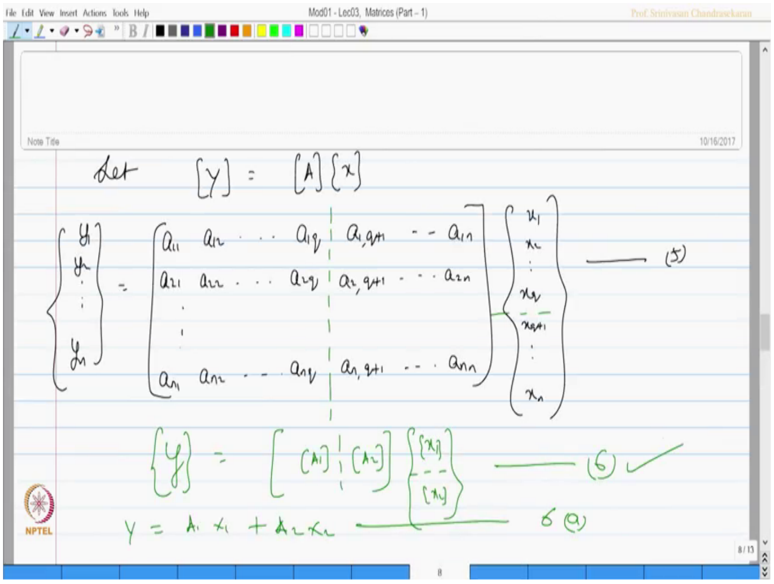

Having said this, let y be A x. That is, y 1 y 2 y n should be a 11 a 12 a 1q then a 1q plus 1 till a 1n. Similarly a 21 a 22 a 2q a 2 q plus 1 a 2n. Going till a n1 a n2 a nq a n comma q plus 1 going till a nn, multiplied by x 1 x 2 going till xq then x q plus 1 going till x n. that is the whole equation. I call this as equation 5, which is same as the original equation but please understand I am going to now group them. So, what I am going to do is, I am gong to now put partition lines; these are the two partition lines.

So, now I am writing y as two matrices A 1 partitioned A 2 which are sub matrices multiply by the vector x 1 partition x 2, ok. So, now I can say y is A 1 x 1 plus A 2 x 2.

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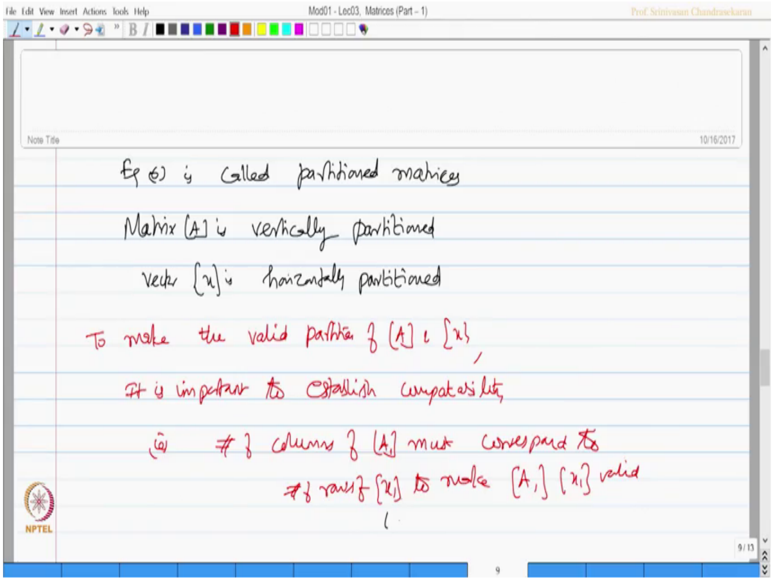

Now the equation 6 is called partitioned matrix. To be very precise matrix A is vertically partitioned, you can see here matrix A is vertically partitioned and vector x is horizontally partitioned. Now very important, to make valid partition of A and the vector x it is important to establish compatibility that is number of columns of A let us say A 1 must correspond to number of rows of x 1 to make A 1 x 1 valid. That is very important, ok.

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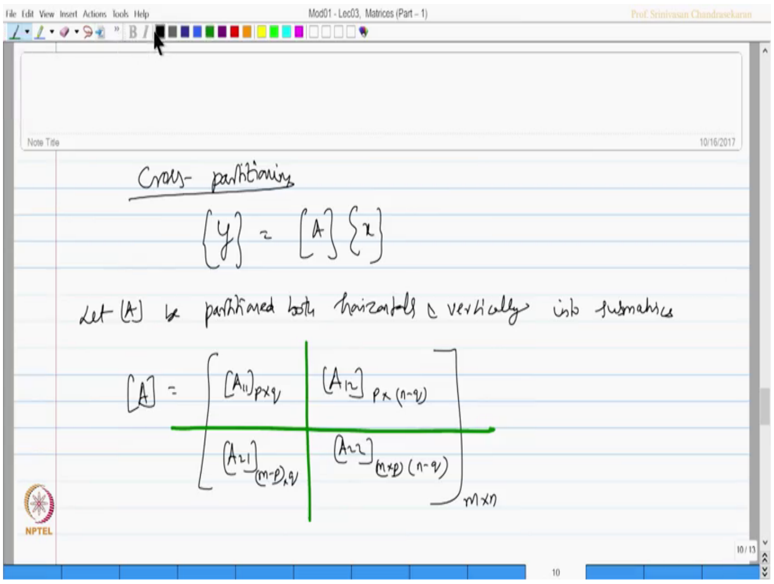

Let us call something about cross partitioning. So far we have seen vertical and horizontal partitioning of the matrix, let us consider the same equation again y is actually A of x. Let A be partitioned both horizontally and vertically into sub matrices. Let us say how to do that. A now will be expressed as A 11 which will be of size p by q, A 12 which will be of size p into n minus q, A 21 which will be of size m minus p into q, and A 22 will be of size m minus p into n minus q. And the whole matrix is of size m by n . Now, I draw a partition vertical and horizontal.

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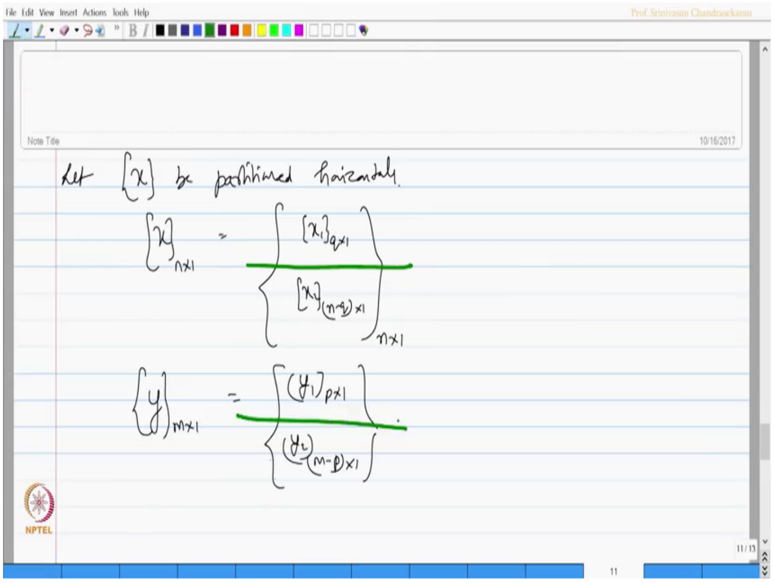

Similarly, let us talk about x vector. Let x also be portioned horizontally. So, x vector which is n by 2 will be actually equal to x 1 of q into 1 and x 2 of n minus q into 1. Now I have a partition which will be horizontal, which will give me a size as n by 1. And therefore, the resulting matrix y which will be m into 1 will also be a partition value which is y 1 of p into 1 and y 2 of m minus p into 1 and I will have a partition of this matrix which is going to be horizontal, therefore.