

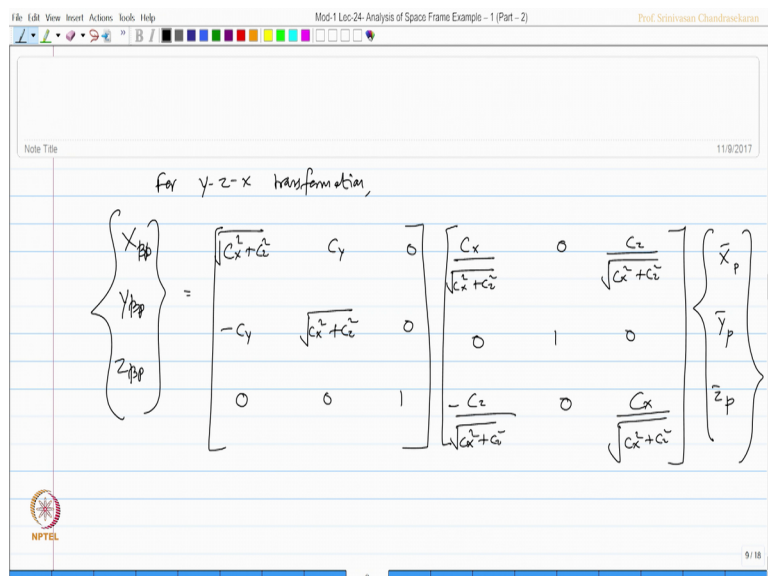
Computer Methods of Analysis of Offshore Structures
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Module - 01
Lecture - 24
Analysis of Space Frame Example - 1 (Part - 2)

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So, for y-z-x transformation, we already derived yesterday that $x_{\beta P}$, $y_{\beta P}$ and $z_{\beta P}$; P is transfer the point P and β is the transformation is actually given by C_x square plus C_z square root C_y 0 minus C_y C_x square plus C_z square root 0 0 0 1 multiplied by C_x by root of C_x square z square 0 C_z by root of C_x square plus C_z square 0 1 0 minus C_z by root of C_x square C_z square 0 C_x by root of C_x square C_z square.

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$$\begin{Bmatrix} x_{\beta P} \\ y_{\beta P} \\ z_{\beta P} \end{Bmatrix} = \begin{bmatrix} C_x & C_y & C_z \\ \frac{-C_x C_y}{\sqrt{C_x^2 + C_z^2}} & \sqrt{C_x^2 + C_z^2} & \frac{-C_y C_z}{\sqrt{C_x^2 + C_z^2}} \\ \frac{-C_z}{\sqrt{C_x^2 + C_z^2}} & 0 & \frac{C_x}{\sqrt{C_x^2 + C_z^2}} \end{bmatrix} \begin{Bmatrix} x_p \\ y_p \\ z_p \end{Bmatrix}$$

$$x_{\beta P} = C_x x_p + C_y y_p + C_z z_p$$

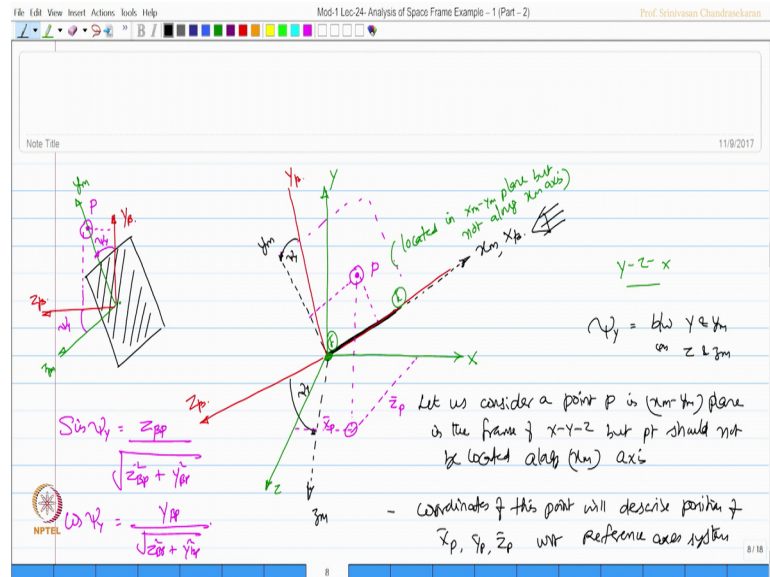
$$y_{\beta P} = \dots$$

$$z_{\beta P} = \dots$$

We derive this matrix yesterday multiplied by $x_{\beta P}$, $y_{\beta P}$, $z_{\beta P}$ when you do this multiplication, I will find a relationship between $x_{\beta P}$, $y_{\beta P}$, $z_{\beta P}$ as C_x , C_y , C_z minus C_x , C_y by root of C_x square plus C_z square root of C_x square plus C_z square minus C_y , C_z by root of C_x square plus C_z square minus C_z by root of C_x square C_z square 0 C_x y root of C_x square plus C_z square multiplied by $x_{\beta P}$, $y_{\beta P}$, $z_{\beta P}$.

So, if we expand this, I can straight away write $x_{\beta P} = C_x x_p + C_y y_p + C_z z_p$ and so on $y_{\beta P}$ and $z_{\beta P}$ can be written; simply I can write this.

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Now, very interestingly looking at this figure coordinates of the point P on $x_m y_m$ plane is now given by $y_{\beta P}$ and $z_{\beta P}$ is it not; how can I say that? Let us say I view this from here and draw a section; let us say my y_m is vertical and my z_m is to the left of that and my y_{β} is to the right of y_m , so this y_{β} and z_{β} . Obviously, will be the left of z_m is it not z_{β} .

And if you take this point P somewhere on y_m this is y_m ; is it not; this is y_m ; let us say I rub this arrow, this is y_m , if you take this point P and I call this is point P ; now this distance and this distance will be simply known to us and this value is what we call as ψ_y is also equal to this. So, now, the coordinates of this point will be given.

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Coordinates of point P (x_m - y_m) plane is now given by $(y_{\beta P}, z_{\beta P})$

Coordinates of point P, wrt reference axes system

$P(4, 4, 0)$

Coordinates of point P wrt its end frame member B $P(0, 0, -3)$

Thus position vector is given by \hat{P} end wrt x - y - z system $(4, 4, 0)$

\hat{P} end wrt x - y - z system $(0, 0, 0)$

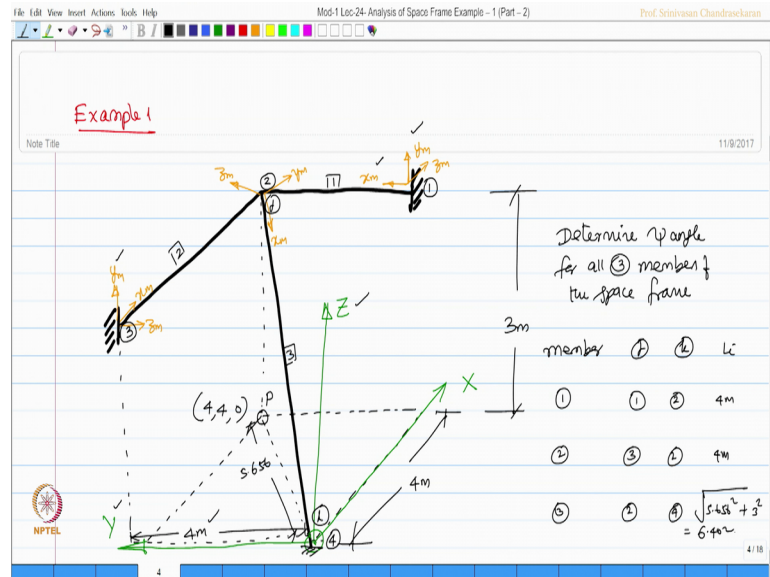
$$\begin{Bmatrix} \hat{x}_P \\ \hat{y}_P \\ \hat{z}_P \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ -3 \end{Bmatrix}$$

$L_1 = L_2 = 6.402m$

Coordinates of point P on the x_m , y_m plane is now given by $y_{\beta P}$ and $z_{\beta P}$; is it not; look at this figure.

$y_{\beta P}$ and $z_{\beta P}$ correct. So, I can now find $\sin \psi$ as simply $z_{\beta P}$ by root of $z_{\beta P}^2 + y_{\beta P}^2$. Similarly $\cos \psi$ can be simply $y_{\beta P}$ by root of $z_{\beta P}^2 + y_{\beta P}^2$, I will use this relationship now. So, the coordinates of point P with reference to the reference axes system; let us see; what is that coordinates of point P with reference to reference system should be of the point P; let us say this figure of the point P.

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Now, I call this as my point P, I am choosing this point on x y m plane, I call this as my point P for the problem. So, the coordinates of this point with the reference axes system will be 4 comma 4 comma 0 with reference to j-th end, it will be 0 comma 0 comma minus 3, correct; this end will be I write down that here coordinates of point P will be 4 comma 4 comma 0.

Similarly, the coordinates of point P with reference to the j-th end of the member 3 that is measured from the j-th end will be 0 comma 0 comma minus 3, you can see here with reference to j-th end, this is j-th end of this member, this is the j-th end of this member. So, 0 comma 0 comma minus 3 because z is positive; so, thus the position vectors is given by x p, y p and z p, simply 0 0 and minus 3.

Now, what are the coordinates of the j-th end with respect to x-y-z axes system; we can see here coordinates of the j-th end with respect to this will be 4 comma 4 comma 3; 4 comma 4 comma 3. Similarly coordinates of the j-th end with respect to x-y-z system, you can see here; this will be the origin. So, it is 0; is it not; this is the j-th end. So, it is 0, now the length of the member which is 3 is also known to us which is 6.402 meters; we already computed that we can see here we already computed that.

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The screenshot shows a presentation slide with a white background and blue horizontal lines. At the top, there is a menu bar with 'File', 'Edit', 'View', 'Insert', 'Actions', 'Tools', and 'Help'. Below the menu bar is a toolbar with various icons. The main content area contains three handwritten equations for direction cosines:

$$C_x = \frac{x_i - x_j}{L_i} = \frac{0 - 4}{6.402} = -0.625$$
$$C_y = \frac{y_i - y_j}{L_i} = \frac{0 - 4}{6.402} = -0.625$$
$$C_z = \frac{z_i - z_j}{L_i} = \frac{0 - 3}{6.402} = -0.469$$

At the bottom left, there is a logo for NPTEL. At the bottom right, there is a page number '12' and a date '11/9/2017'.

Now, let us compute C_x , C_y and C_z ; the direction cosines which will be simply x_k minus x_j by L_i , $y_k - y_j$ by L_i , $z_k - z_j$ by L_i ; let us do that $0 - 4$ by 6.402 , $0 - 4$ by 6.402 , $0 - 3$ by 6.402 which will give me minus 0.625 , minus 0.625 , minus 0.469 ; these are my direction cosines; once I get this, I can use this relationship $X B P$ will be equal to this C_x , C_y , C_z by this $Y B P$ will be equal to this row by column and $Z B P$ will be this row multiplied by the column.

So, let us do this relationship. So, I say x beta P which is given by $C_x x$ bar P plus $C_y y$ bar P plus $C_z z$ bar p .

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$$X_{pp} = C_x \bar{x}_p + C_y \bar{y}_p + C_z \bar{z}_p$$

$$0 + 0 + (0.469 \times 3) = 1.407$$

$$Y_{pp} = \frac{-C_x C_y}{\sqrt{C_x^2 + C_z^2}} \bar{y}_p + \frac{C_x C_z}{\sqrt{C_x^2 + C_z^2}} \bar{y}_p - \frac{C_y C_z}{\sqrt{C_x^2 + C_z^2}} \bar{z}_p$$

$$= - \frac{(-0.625)(-0.469)(-3)}{\sqrt{0.625^2 + (0.469)^2}} = +1.125$$

$$Z_{pp} = \frac{-C_z}{\sqrt{C_x^2 + C_z^2}} \bar{y}_p + \frac{C_x}{\sqrt{C_x^2 + C_z^2}} \bar{z}_p = \frac{(-0.625)(-3)}{\sqrt{(0.625)^2 + (0.469)^2}} = +2.40$$

So, C_x and C_y are anyway 0 because the x_P and y_P are 0 plus C_z is 0.469 into 3. So, that gives me this values as 1.407 $y_{beta P}$; if you look at the equation, this will be minus C_x , C_y by root of C_x square plus C_z square of $x_{bar P}$ plus root of C_x square plus C_z square of $y_{bar P}$ minus $C_y C_z$ root of C_x square C_z square of $z_{bar p}$.

We know these values are 0. Therefore, this term will not be there let us substitute directly for the last term which will be minus of minus 0.625 minus 0.469 minus 3 divided by root of 0.625 square plus 0.469 square which will be plus 1.125 $z_{beta P}$ is given by minus C_z of C_x square plus C_z square of $x_{bar P}$ plus C_x by root of square of C_x and C_z of $z_{power of P}$.

We know that this value is further 0; let us substitute only for this value which will be minus 0.625 into minus 3 divided by root of 0.625 square plus 0.469 square which gives me this value as plus 2.4.

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$$\sin \psi = \frac{Z_{\beta P}}{\sqrt{Z_{\beta P}^2 + Y_{\beta P}^2}} = \frac{2.40}{\sqrt{(2.4)^2 + (1.125)^2}} = 0.905$$
$$\cos \psi = \frac{Y_{\beta P}}{\sqrt{Z_{\beta P}^2 + Y_{\beta P}^2}} = \frac{1.125}{\sqrt{(2.4)^2 + (1.125)^2}} = 0.424$$
$$\psi = \tan^{-1} \left\{ \frac{0.905}{0.424} \right\} = \underline{\underline{64.897^\circ}}$$

Let us now compute sin psi y which is given by Z beta P by root of z beta P square plus Y beta P square we already derive this expression.

Let us substitute that now 2.40 by square root of 2.40 square plus 1.125 square which becomes 0.905 cos psi y which has been also derived as y beta P by z beta P square plus y beta P square which will be 1.125 by 2.4 square plus 1.125 square which gives me 0.424.

So, now psi y can be said as tan inverse of 0.905 by 0.424, because sin by cos will give you tan and the angle is tan inverse of that which gives me 64.897 degrees, but let us carefully mark psi y depending upon the figure. Let us take the member.