Computer Methods of Analysis of Offshore Structures
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Module - 01
Lecture - 23
Z-Y-X Transformation for 3d Analysis (Part - 1)

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Friends, let us continue to discuss about the transformation matrices or rotation matrix rather to be developed for 3 d analysis. This is lecture 23 and module 1, where we have going to talk about a new transformation process.

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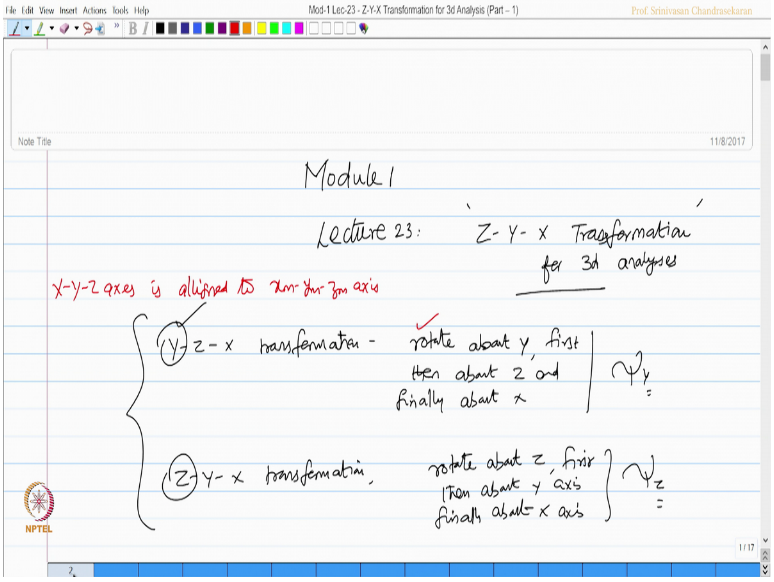

Which is Z-Y-X transformation in the last lecture, we discussed about Y-Z-X transformation, we said that rotate about Y first, then about z and finally, about x that is Y-Z-X to get the angle psi Y.

Now, we are going to do Z-Y-X transformation which in the same algorithms says that rotate about X axis first, then about y axis and finally, about X axis. So, friends we will be getting what is called phi Z here. So, this second subscript stands for the first axis about which you are rotating the whole process the question comes why this is done in simple terms X-Y-Z axes is aligned to x m y m z m axes that is we are doing it.

So, this Y is called rotation because we are rotating we are rotating about the 3 axes subsequently successively therefore, it is called rotation matrix.

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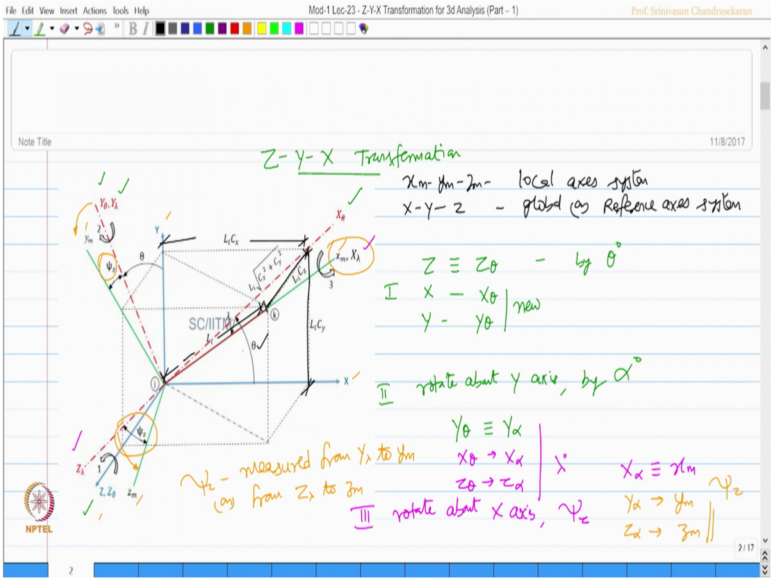

So, let us do this procedure; now let us look at this figure, there is a vector oriented arbitrarily which is marked in red colour the local axes are X m, Y m and Z m; the global axes of reference is X-Y-Z. So, X m, Y m, Z m are local axes and X-Y-Z are global or rather reference axes system looking in this figure we are now first going to rotate about Z axes when you do that Z will become Z zeta because I rotate by angle of theta degrees and X will become X theta and Y will become Y theta which will be knew, but Z and Z theta will be same.

So, you get X theta here and y theta here that is a first step in the second step you rotate about Y axes because it is Z-Y-X transformation by an angle alpha. So, when you do that Y theta will be as same as Y alpha you see here and X theta will become X alpha Y theta will become Z theta will become Z alpha. So, once you do this you know Z alpha and X alpha you rotate this by alpha degrees.

In the third and final stage you rotate about X axis; so, by doing that by an angle psi. So, I should say psi Z by doing that I get X alpha as same as X m that is what we get here Y alpha will now change to Y m and Z alpha will now align with Z m. So, the angle between these 2 that is Y alpha and Y m are Z alpha and Z m is called psi Z.

So, psi Z is measured from Y alpha to Y m or from Z alpha to Z m it is measured from here measured from here its measured from here. So, the idea is to align the reference axes X-Y-Z to that of X m Y m and Z m respectively. So, that is the stages involved. So, I must get the rotation matrix.

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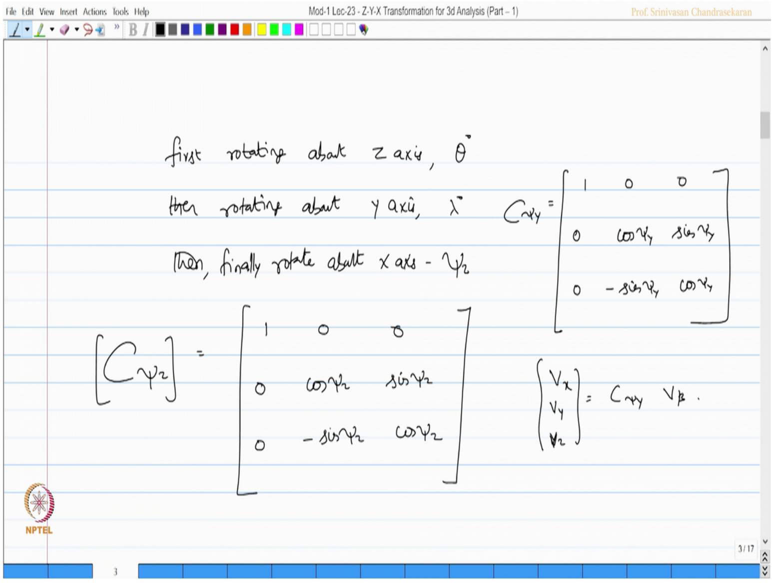

So, we are first rotating about Z axes by angle theta, then rotating about Y axes lambda then finally, rotate about X axes to get psi Z.

So, now the matrix C psi Z is directly written as can look into the previous derivation of C psi Y; I can recollect it for you from the previous lecture C psi Y is actually 1 0 0 0 cos psi y sin psi y 0 minus sin psi y cos psi y which connects V x, V y, V z to V beta in the same style, I can now write this as 1 0 0 0 cos psi z sin psi z 0 minus sin psi z cos psi z stated which connects V x, V y and V z to that of V.

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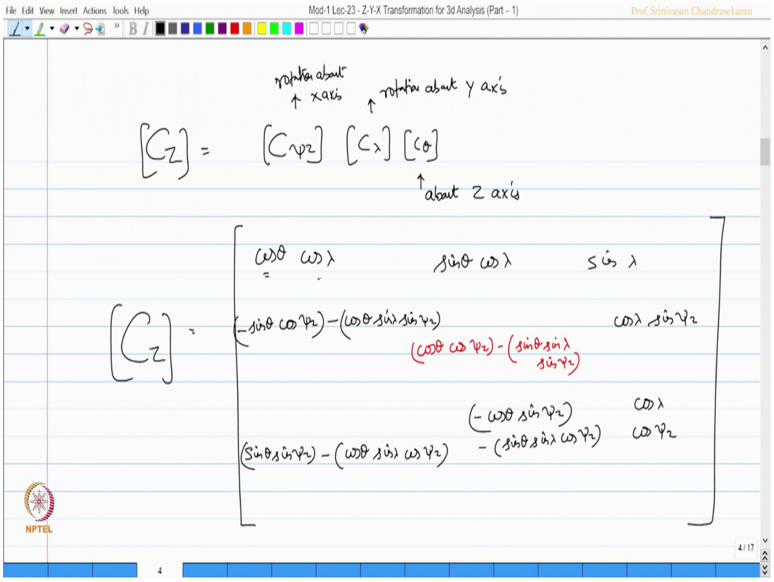

So, now we also know that C z can be now said as C psi z C lambda and C theta because this is rotation about z axes this is rotation about y axis this is rotation about x axis. So, let us substitute this and try to find C z C z actually is given by cos theta cos lambda sin theta cos lambda and sin lambda minus sin theta cos psi z minus cos theta sin lambda sin psi z.

This term I am writing it here cos theta cos psi z minus sin, theta sin lambda sin psi z, the third term is cos lambda sin psi z the last row sin theta sin psi z minus cos theta sin lambda cos sin z, this term second column will be minus cos theta sin psi z minus sin theta sin lambda cos psi z the last term will be cos lambda cos psi z.

Now, what would be the value of for example, these trigonometry relationships etcetera let us look at this figure the angle theta is marked cos theta or let say sin theta will be essentially L C y by C x square plus C z square this is actually this is actually C x, this is C x and this value is C z; is it not and this value is C y. So, C x square plus C x z square or the value of this will be L i this is the original vector L i therefore, from this figure.

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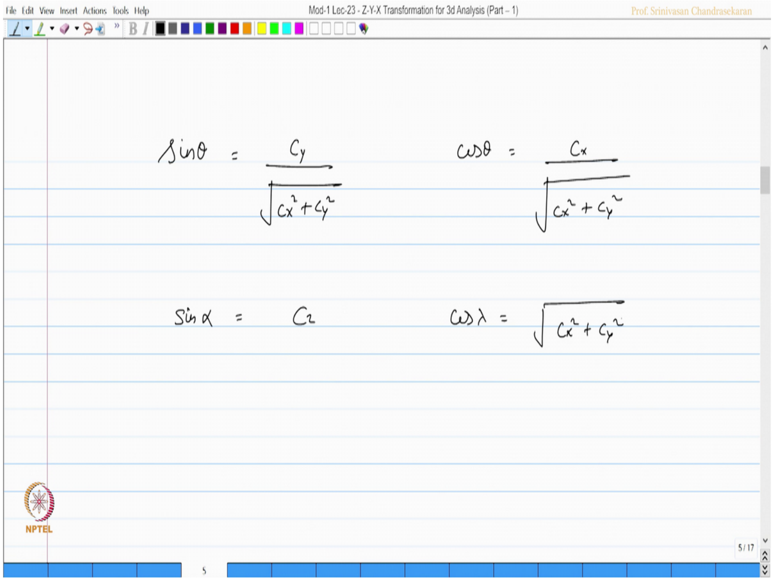

I can simply say sin theta will be C y by root of C x square plus C y square and cos theta will be C x by root of C x square plus C y square sin alpha we can see the alpha as angle is here lambda angle we are transporting transferring it through lambda angle.

So, one can always say the lambda angle sin lambda is C z and cos lambda is root of C x square plus C y square. Now in this equation I have got cos and sins; let us substitute them back and write here full equation for C z.

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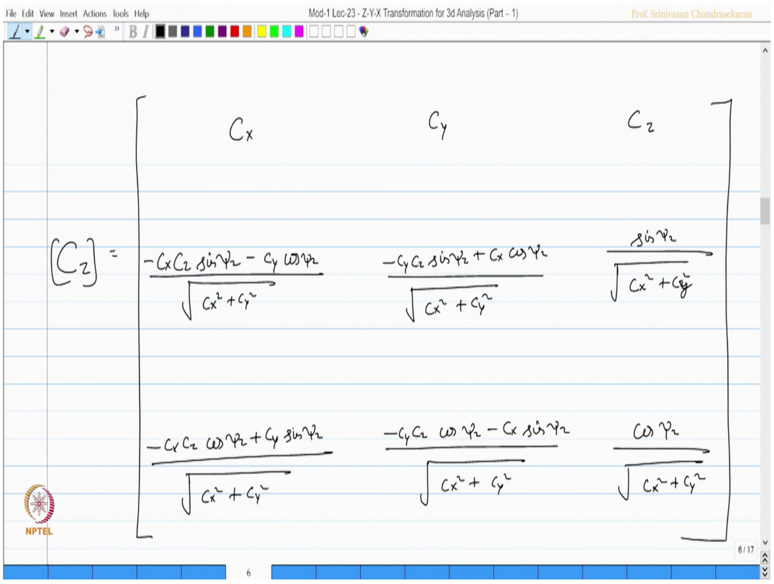

Which is this value which can be given by C x, C y, C z minus C x, C y sin psi z minus C y cos psi z by root of C x square plus C y square; this value will be minus C y, C z sin psi z plus C x cos psi z by root of C x square plus C y square.

This will be sin psi z by root of C x square plus C y square the third row minus C x, C z cos psi z plus C y sin psi z by root of C x square plus C y square minus C y, C z cos psi z minus C x sin psi z by root of C x square plus C y square; this value will be cos psi z by root of C x square plus C y square that is my C z matrix.

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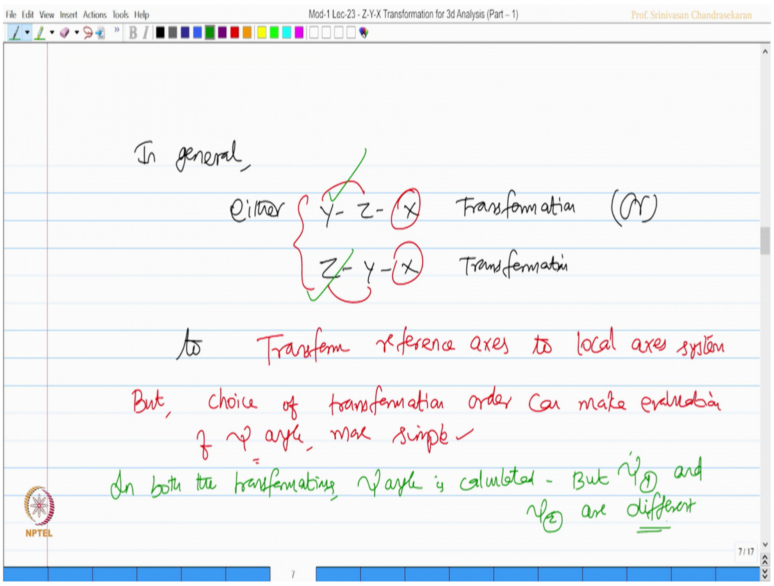

So, friends in general one can use either Y-Z-X transformation or Z-Y-X transformation to transform the reference axes system to local axes system, but the choice of transformation order can make the evaluation of psi angle more simple in both transformation the rotation about the last step is about the x axes only is only the variation between Y Z and Z Y; the last rotation is always about the x axes.

So, choice of the transformation order can make the computation of psi angle more simple most importantly in both transformations psi angles are calculated; is it not, but psi angle calculated from y transformation and psi angle calculated from z transformation are completely different. So, this subscript y stands for which are the first rotation this subscripts z stands for which is the first rotation both angles are called as psi, but the values will be different.

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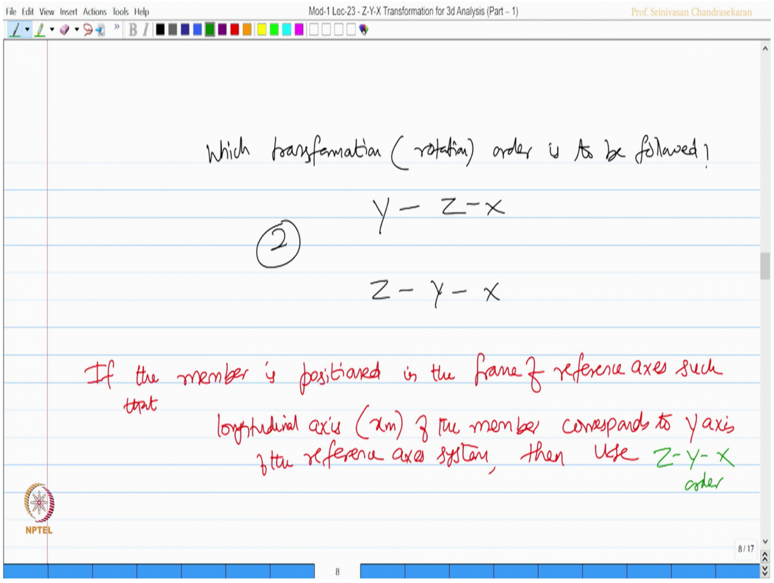

So, you have to very clear which transformation order I should follow. So, let us ask this question for a given member which transformation order or let us rotation order is to be followed how many orders are there; there are 2 orders one Y-Z-X; other is Z-Y-X there are 2 types. So, interestingly if the member is positioned in the frame of reference axes such that the longitudinal axes of the member corresponds to Y axes of the reference system, then use Z-Y-X.

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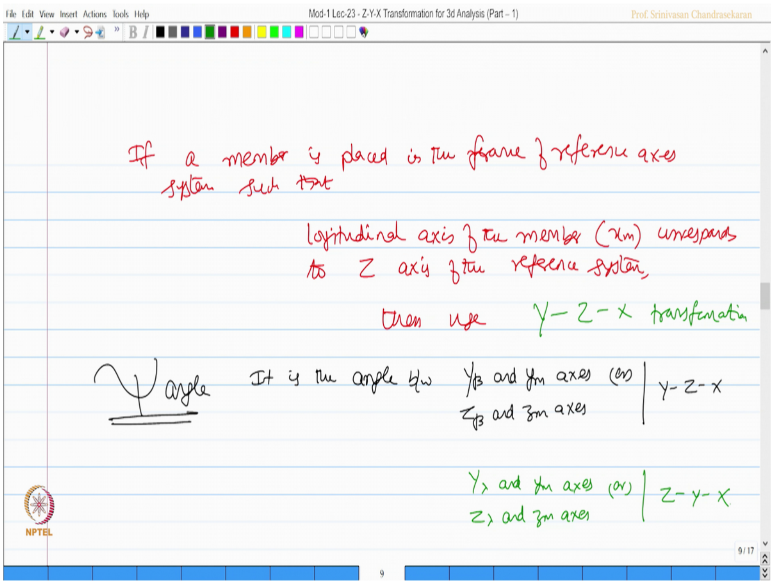

Similarly, if a member is placed in the frame of reference axes system such that a longitudinal ax of the member corresponds to z axes of the reference system, then use Y-Z-X transformation. Let us talk slightly about the psi angle where that seems to be very important actually, it is the angle between y beta and y m axes are z beta and z m axes if it is Y-Z-X transformation; it is between y lambda and y m axes or z lambda and z m axes if it is say y x order that is a psi angle. Let us try to explain this graphically; let us take I have.