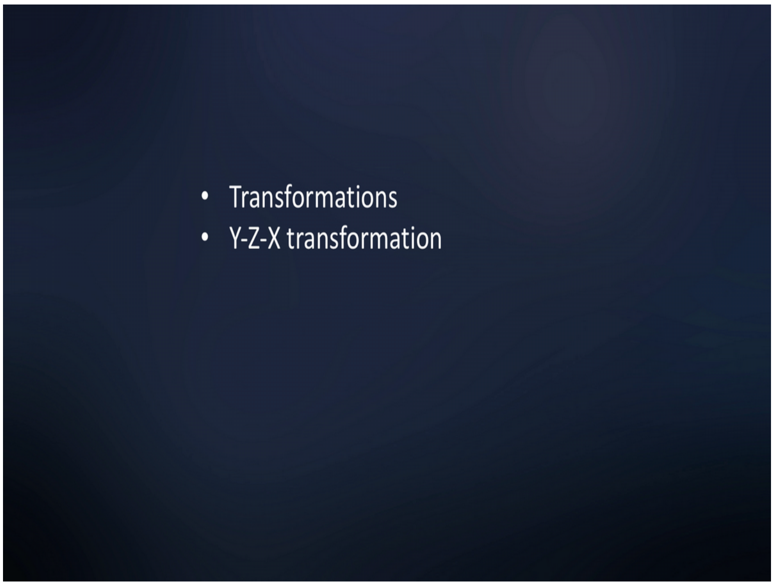
Computer Methods of Analysis of Offshore Structures

Prof. Srinivasan Chandrashekarn
Department of Ocean Engineering
Indian Institute of Technology, Madras

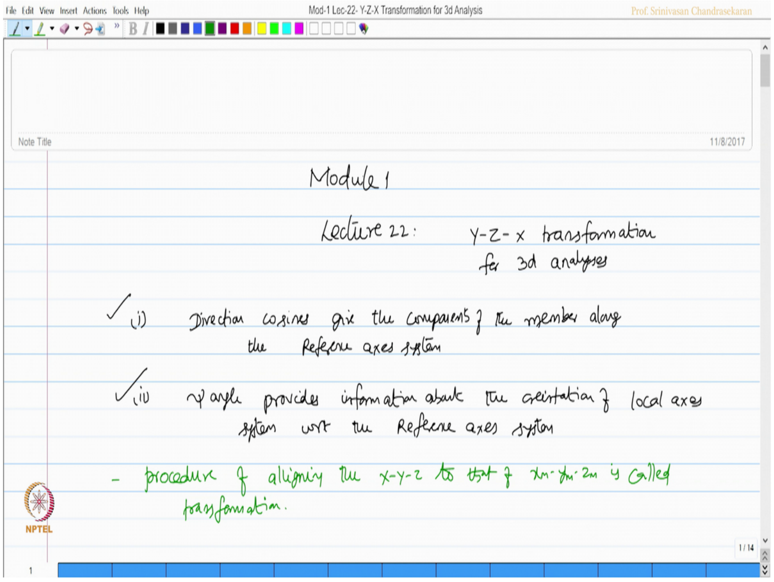
Module - 01
Lecture - 22
Y-Z-X Transformation for 3d Analysis

(Refer Slide Time: 00:17)



Friends, welcome to the 22nd lecture, module 1. Where, we are continuing to discuss about the 3 d analysis of structural members using stiffness method. In this lecture, we are going to discuss about one important transformation procedure which is called Y-Z-X transformation. We will talk about that in detail, we will derive the transformation matrix for this particular sequence of transformation; let us first ask the question in 3 d analysis by doing transformation what did we gain.

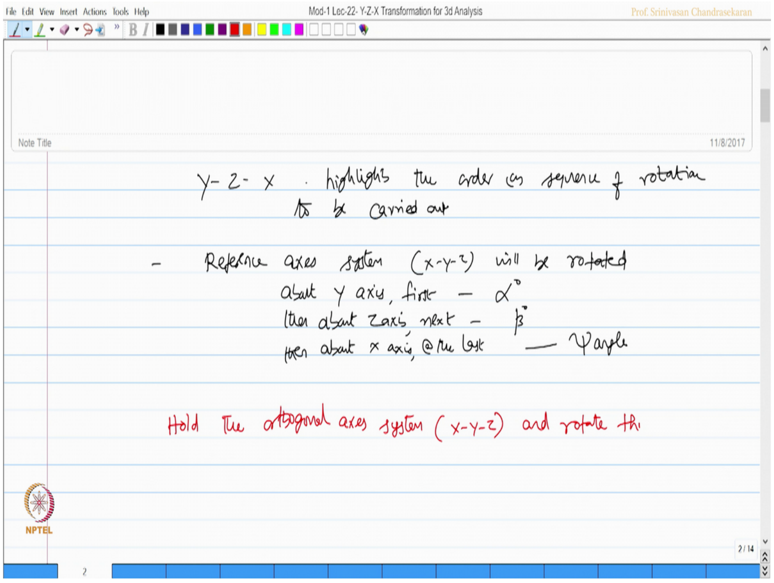
(Refer Slide Time: 01:10)



Number one direction cosines give the components of the member along the reference axes system, further the psi angle provides information about the orientation of local axes system with respect to the reference axes system which we discussed in the last lecture. We have already said how to obtain the direction cosines for arbitrarily oriented beam element. We will now discuss how to get this psi angle in detail while doing so; it is very important to note that the procedure of aligning the reference axes system to that of local axes system is actually called transformation.

So, there are various schemes, steps involved in doing this one such scheme is called the Y-Z-X transformation. It means rotate the reference axes about Y axis first, then Z axis next and then about X axis at the last; that is called Y-Z-X. So, Y X highlights the procedure or in fact, highlights the order or sequence of rotation to be carried out.

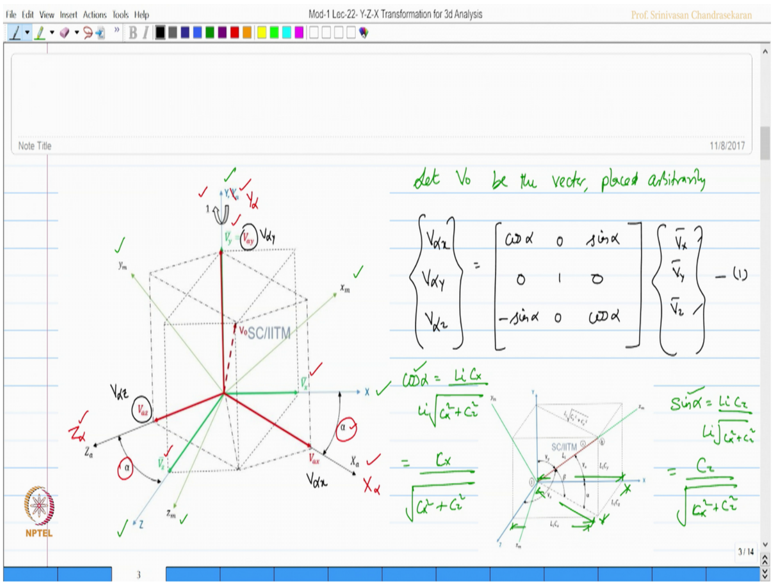
(Refer Slide Time: 03:25)



Now, the reference axes system which is x, y, z will be rotated about Y axis first, then about Z axis next and then about X axis at the last.

The amount of rotation which will happen about Y axis will be alpha degrees about Z axis will we be beta degrees and then ultimately. We will get the psi angle. So, the procedure is very simple, hold the orthogonal axis system that is x, y, z and rotate this axes system about Y axis by alpha degrees.

(Refer Slide Time: 05:29)



Look at this figure carefully, this is my reference axes system x, y, z; this is our local axes system x m, y m, z m, we have arbitrarily placed vector V 0.

So, let V 0 be the vector placed arbitrarily what we do now is we rotate this by about Y axis and that is a first level of rotation and the angle of rotation is alpha degrees. So, since a rotating about Y axis; y and y alpha will be same, whereas, the x will move to x alpha and z will move to z alpha; this is x alpha, this is x alpha, this is z alpha. So, x will move to x alpha by alpha and by the same amount z will move to z alpha by alpha, we have understood that how to compute this V x bar V y bar and V z bar which are nothing, but the components of this vector which are with reference to some standard axes in my case it is x, y, z.

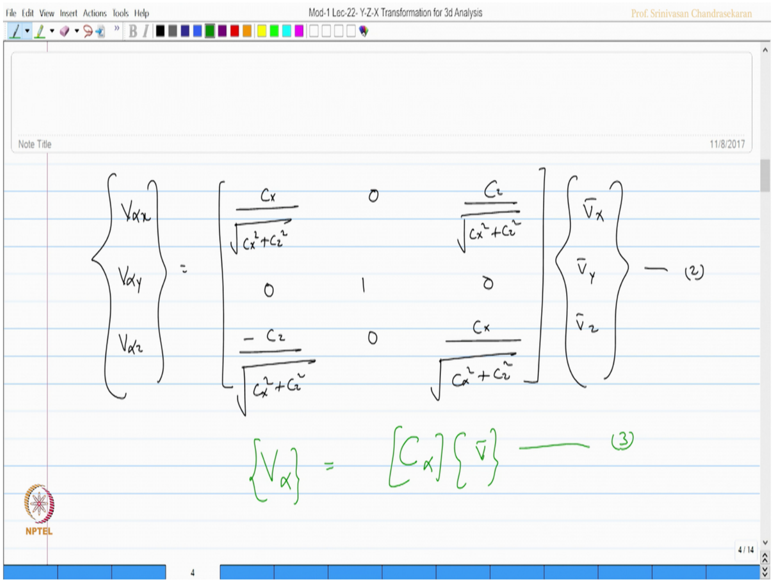
So, now I want to connect V alpha x, V beta x and V beta y alpha y sorry V alpha y and alpha z by a transformation matrix which is the direction cosine matrix and I want to connect this to V x bar V y bar and V z bar. So, if you look at this figure from this figure it is very easy to compute the angle sin alpha. So, if you look at this figures let us say sin alpha will be this dimension by this dimension I can write here sin alpha is L i C z by L i root of C x square plus C z square which essentially C z by root of C x square plus C z square.

One can also find cos alpha from this figure which will be this dimension this dimension which will be equal to the this dimension which is L i C x by square root of C x square plus C x square L i which ultimately is C x by root of C x square plus z square. So, sin alpha and curve alpha are known to me and alpha actually is amount of rotation what we are doing with respect to the Y axis to move the x, y, z axis to get align about the Y axis now by doing so; y and y alpha this is y alpha y and y alpha will be same whereas, x alpha and z alpha will rotate by an angle alpha.

So, the technologies relationship of this angle sin and cos are available on the screen now. So, using this relationship, I want to connect the projection of V x bar on x alpha etcetera. So, I want to find out V alpha x V alpha y this is V alpha x this is V alpha x this is V alpha y and this is b alpha z. So, I am connecting this 2 from a simple relationship, we can very well see here is going to be cos alpha 0 and sin alpha because this will have components of x and z; only V y will have no component from anywhere because its rotated about the Y axis and this going to be minus sin alpha 0 cos alpha where sin alpha and cos alpha are available on the screen as you see here, I call this as equation number let us say on.

Now I can substitute this and write down expression for V alpha x.

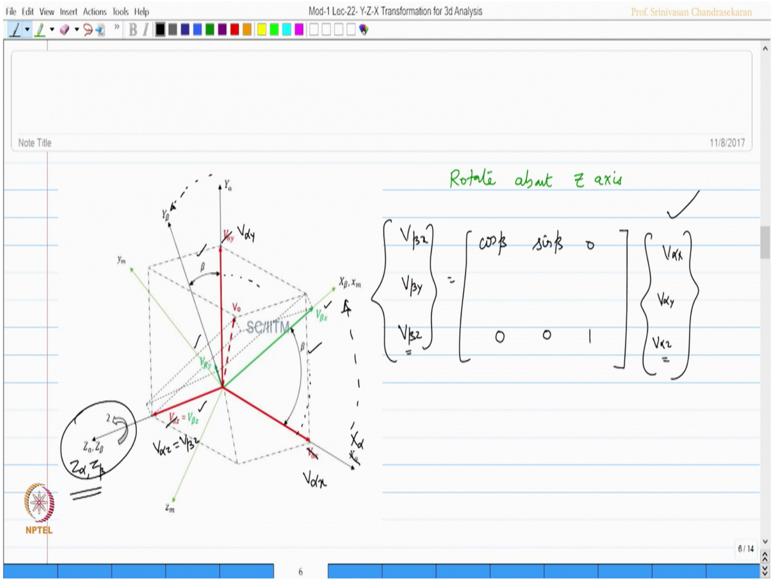
(Refer Slide Time: 11:12)



So, V alpha x V alpha y V alpha z will be C x by root of C x square plus C z square 0 C z by root of C x square plus C z square this is 0 1 0, this is minus C z by root of C x square plus C z square, this is C x by root of C x square plus C z square and this connects V alpha x to V bar x V bar y and V bar z the equation number 2. So, now, I can write the simple expression saying V alpha is actually equal to C alpha multiplied by V bar where V bar. Already, we know this value from the previous lecture, if we see any arbitrarily oriented vector, you can have the V bar components of other vector to any reference axes system using a transformation matrix which we already discussed in the last lecture.

Having said this we are looking for Y-Z-X transformation we have now rotated about y; let us now rotate this about z axis. So, step 2 rotate about.

(Refer Slide Time: 13:05)



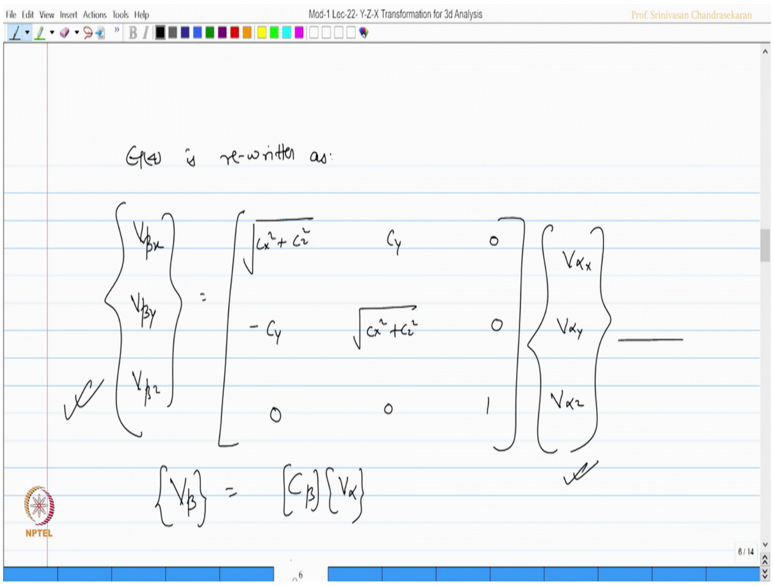
Z axis by beta angle; let us look at the figure . So, now, we are rotating we are attempting to rotate about z axis. So, rotating about Z axis by an angle beta; so, x alpha which originally, he was president; this is x alpha which was originally present has now become x beta and this is V alpha x which we have already know from here, this is V alpha y this is V alpha z. So, step number 2; we are rotating about Z axis when we do this z alpha and z beta will be same axis because I am rotating about z axis.

When you do that; obviously, this axis will shift 2 x beta and this axis will now shift 2 y beta making an angle beta and beta respectively now the components along them will be respectively marked as V beta x V beta y and V beta z. Since I am rotated about the Z axis V alpha z and V beta z will miss him is or not, let us say I want to now develop relationship between V beta x V beta y and V beta z knowingly connecting this with V alpha x V alpha y V alpha z which will be no because you see here this is known.

So, looking at the same algorithm cos sin etcetera, I want express this in terms of the matrix connecting this 2 as you correctly guest V beta said is same as V z therefore, 0 0 1; this is going to be cos beta sin beta, I am resolving V alpha x and V alpha y about V beta x. So, this is cos beta and that is sin beta, we can see here anything swift along the angle is cos anything swift away from the angle is sin by that logic this is minus sin beta plus cos beta 0.

Now, from this figure look at the angle beta angle beta will be look at this triangle, I am marking this triangle in different color or I am hatching this triangle in a different color; hatching this triangle; look at this triangle. So, beta sin beta will be this value by the diagonal is it not. So, l C y by square root of this value which is some of squares of this which is L i essentially this is L i is it not this is L i essentially. So, let us use that and write down sin beta will be L i C y by L i and cos beta will be L i square root of C x square plus C z square by L i which amounts to C y this amounts to C x square plus C z square substituting this in the previous equation.

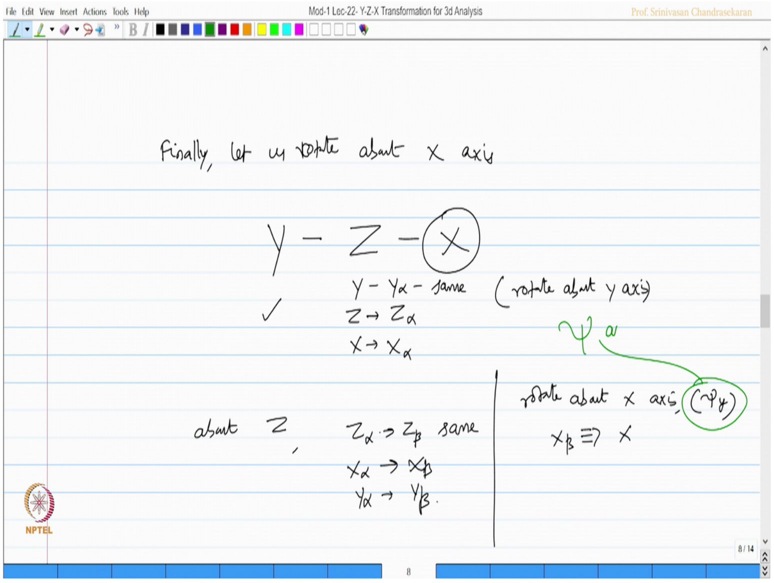
(Refer Slide Time: 18:10)



So, substituting five we can modify 4; see equation 4 is rewritten as V beta x V beta y V beta z is connected by root of C x square plus C z square C y 0 minus C y root of C x square plus C z square 0 0 0 1 and connect this 2 V alpha x V alpha y and V alpha z.

So, we are doing step by step this factory already know you see here this vector is known now this vector also be known because C x C z is are known to us is or not having said this we can write a relationship V beta is actually equal to C beta with V alpha equation 6, equation 7; one we did this.

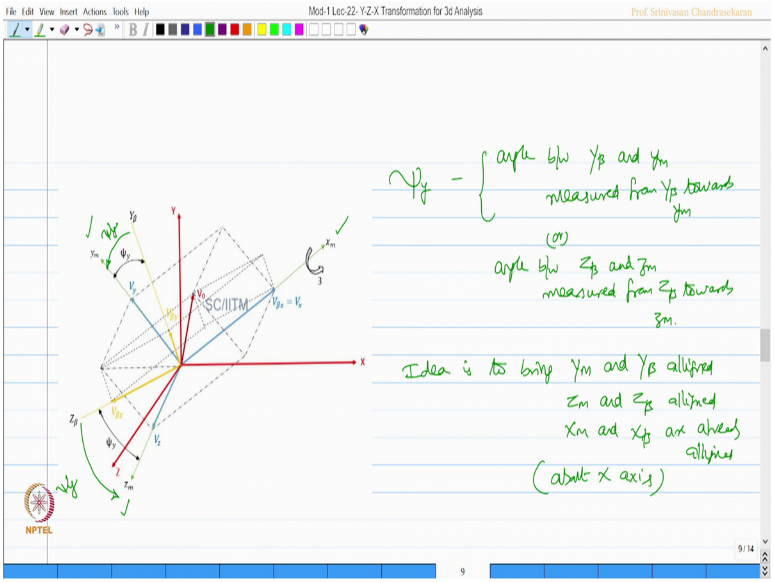
(Refer Slide Time: 19:40)



Finally, let us rotate about x axis. So, what we did is we did about y, then we did it about z, then we are doing with x when we did rotation about y only z change to z alpha x change to x alpha.

When you did rotation about z z alpha remained z beta same whereas, x alpha change to x beta and y alpha change to y beta in this case y remains y alpha same because I am rotating about y axis; so, by this logic; when I rotate about z x axis. So, when I rotate about X axis is expected that x beta now will remain same as I am rotating this by an angle psi y. So, please note psi is angle which we want and this letter y indicates the transformation order is Y-Z-X. So, this y is what is indicated here.

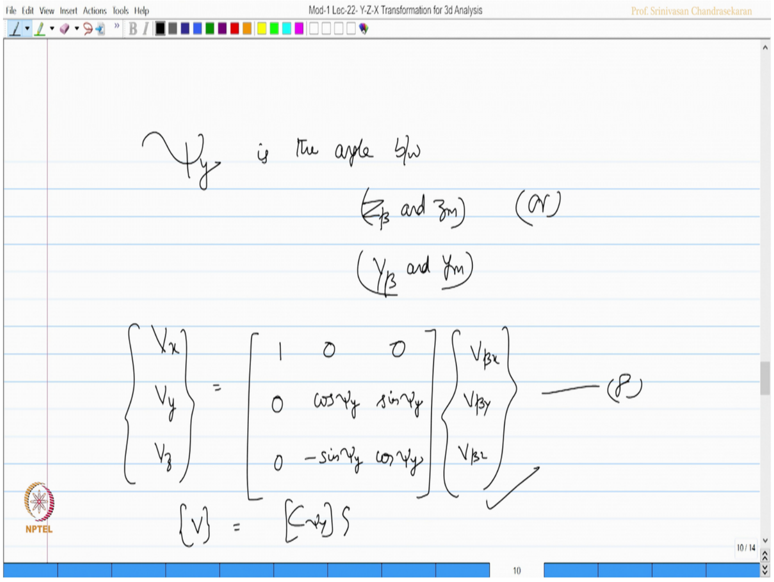
(Refer Slide Time: 21:24)



Let us see the figure; how do we rotate this. So, I rotate about X axis that is x beta. So, then I mark you know; x will not change the change will happen from y beta and z beta my idea is principally to align this axis with local axis x m, y m and z m. So, I am aligning with x m and finding the angle between y beta and y m or z beta and z m. So, the angle psi is actually the angle between y beta and y m that is measured from y beta towards y m. So, it is measured this way.

Similarly, it is measured this way or angle between z beta and z m measured from z beta towards z m. So, the idea is the idea is to bring y m and y beta aligned z m and z beta aligned and of course, x m and x beta are already aligned because I am rotating about X axis. So, now, psi y is the angle between z beta and z m or y beta and y m therefore, I want really find V x, V y and V z which are the values of the components along z y, n x y, n z.

(Refer Slide Time: 24:13)

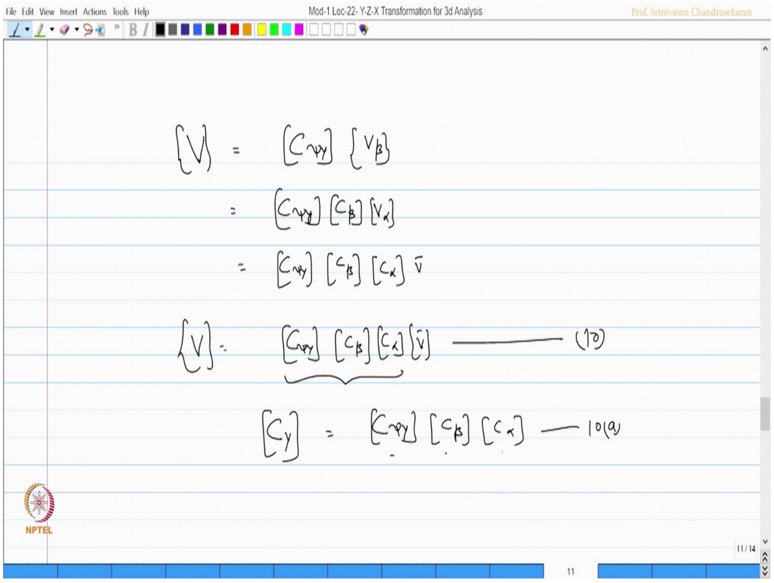


You can see here small x small y and small z, I want to find that which will be now connected by a matrix again and it will be connected to V beta x V beta y V beta z; please understand, this vector is known to us see here this vector is known is it not. So, this is known; I want to connect as you correctly observed we are rotating about X axis therefore, it will be 1 0 0. So, no contribution from y and z look at this figure I am interested in finding out V y; V y is this value as you mark in blue.

So, this will be essentially contributed from V beta y which is the yellow 1 cos of psi y this angle is psi y is it not. So, V beta y psi by and V beta z and V beta y cos by and V beta z sin y; so, simply I can say 0 cos psi y and sin psi y similarly if you look for V z which is this value the blue one this will have a component from V beta z which will be cos of psi y whereas, look at V beta y it will opposite. So, sin of psi y, but minus. So, 0 minus sin psi y cos psi y; so, I can express this is in a vectorial form; the equation number could be 8 which is expressed as C psi by matrix connected to V beta by equation 9.

Now, let us look at the all the equations V connects V beta and V beta is connected to V alpha and V alpha is connected to V x is or not. So, ultimately, we are interested in estimating this value in terms of V bar V bar is a component of the vector or the member along any reference axes. So, let us establish that relationship now.

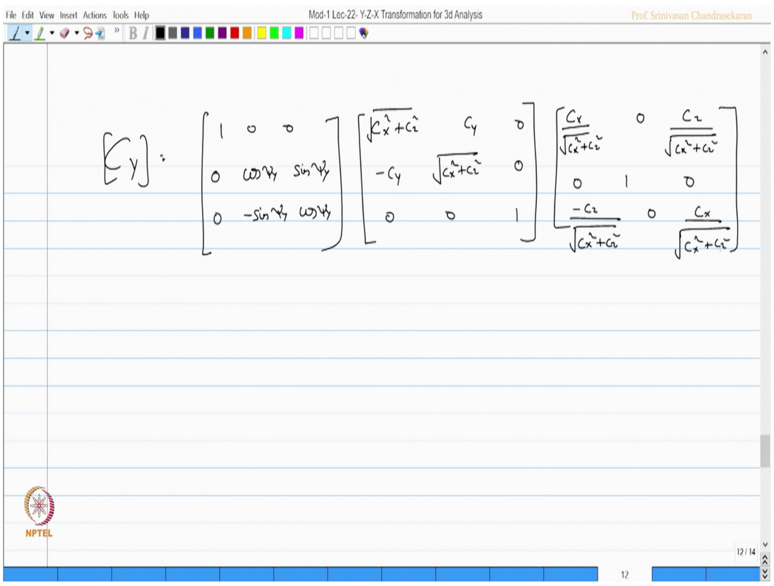
(Refer Slide Time: 27:23)



We can now say that V is actually equal to C psi y of V beta which can be extended as C psi y V beta is C beta of V alpha is or not let us extended this further as C psi y C beta which is C alpha of V bar.

So, v; so, actually equal to C psi y C beta C alpha of V bar the equation number 10; so, now, I call this as C y which is C psi y C beta C alpha 10 a.

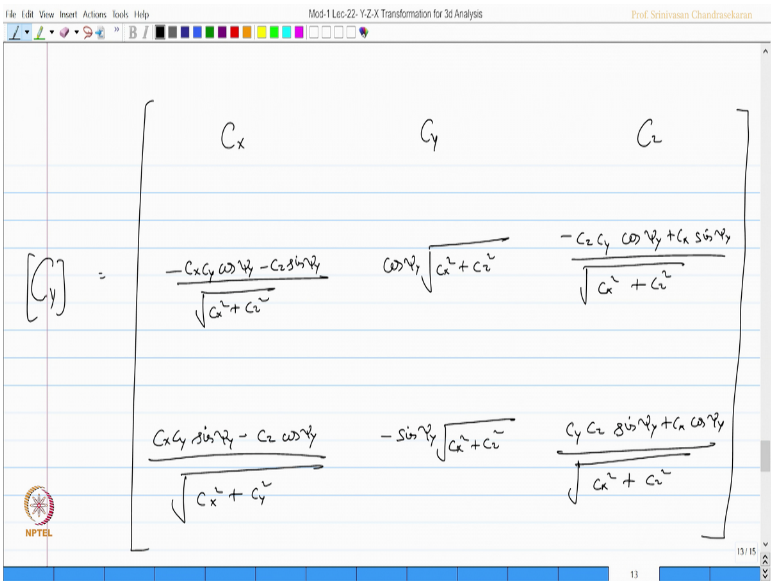
(Refer Slide Time: 28:45)



So, let us try to find out this value C y. So, let us write down these matrices separately again and multiply them. So, C y is actually equal to C psi y C psi y is 1 0 0 and this value let us enter this matrix 1 0 0 0 cos psi y, sin psi y 0 minus sin psi y cos psi y, then what do you want is C beta; let us see C beta C beta is this; this matrix. Let us enter this matrix multiplied by C x square plus C z square root C y 0 minus C y root of C x square plus C z square root 0 0 0 1.

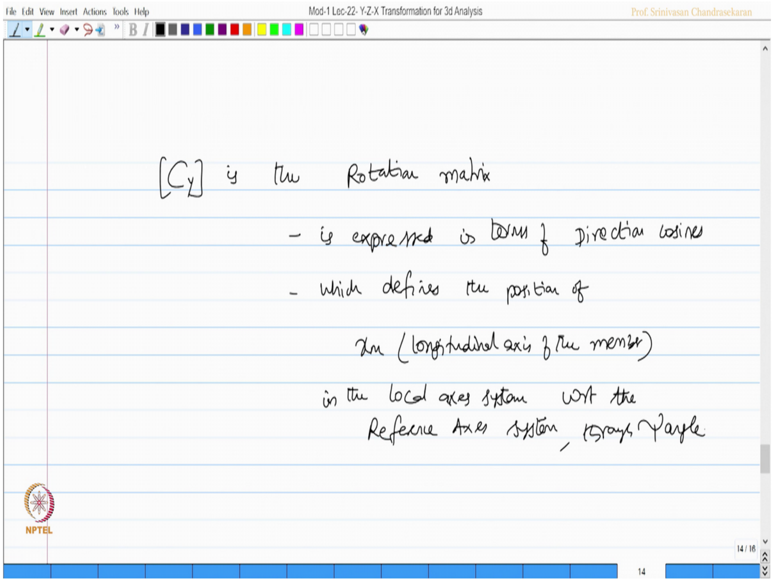
Further multiplied by I want C alpha let search for C alpha in this screen. So, this is my C alpha matrix see here. So, let us enter that value here which is C x by root of C x square plus C z square 0 C z by root of C x square plus z square 0 1 0 minus C z by root of C x square C z square 0 C x by root of C x square plus C z square I will get C y. So, multiply and my C y is ultimately going to be is C x C y C z minus C x C y cos psi y minus C z sin psi y by root of C x square plus C z square cos psi y root of C x square plus C z square minus C z C y cos psi by plus C x sin psi y by root of C x square plus C z square.

(Refer Slide Time: 31:02)



The third row will be C x C y sin psi y minus C z cos psi y by root of C x square plus C y square minus sin psi y into C x square plus C z square C y C z sin psi y plus C x cos psi y by root of C x square plus C z square I get C by matrix like this.

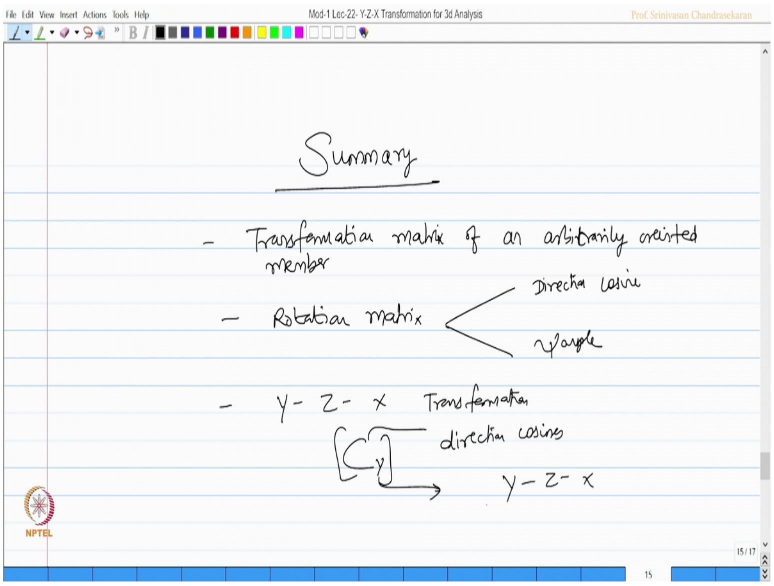
(Refer Slide Time: 32:51)



So, interestingly C y is called as the rotation matrix which is expressed in terms of direction cosines which in fact, defines the position of x m that is the longitudinal axis of the member in the local axes system with respect to the reference axes system and through the psi angle.

So, friends let us look at the summary, we understood the importance of transformation matrix of an arbitrarily oriented member we call this as rotation matrix.

(Refer Slide Time: 34:16)



So, to explain the procedure and position of the vector in a given reference axes system we need to issues one is a direction cosine other one is a psi angle octane the psi angle we did Y-Z-X transformation and we obtain the rotation matrix C y C because the rotation matrix contains direction cosines y because the transformation is of an order Y-Z-X it means the first rotation was about Y axis.

Friends, I hope you understand slowly how we have done this transformation the idea is to orient X, Y, Z axis to X, Y, Z m axis of the member that is the idea. So, we did this. So, for a given member arbitrarily oriented, if we are able to explain the direction cosines one can calculate the psi angle if we know the psi angle, I can easily estimate the C y value for a given system. So, there is one more method by which we can obtain the psi angle that is Z-Y-X transformation. We will discuss that in the next lecture.

Thank you very much.