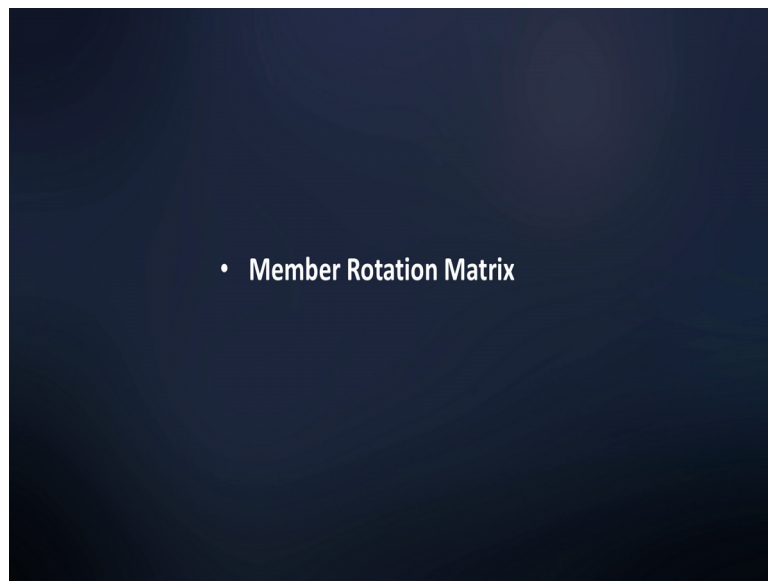


Computer Methods of Analysis of Offshore Structures
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Module - 01
Lecture - 21
3d structures: Transformation matrix

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Fig 3. Member rotation definition

member is aligned arbitrarily in space

(b) Member rotation matrix

- Consider a beam element whose length is L_i (marked in red)
- member has 2 nodes
 - (P)¹⁵ node
 - (Q)¹⁵ node
- Member is oriented along its local axis system ($x_m - y_m - z_m$)

(This local axis system ($x_m - y_m - z_m$) is placed arbitrarily in space, w.r.t reference axes ($x - y - z$))

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So let us say, I want to now know the member; we know the member say transformation, we say member rotation matrix, there is a reason why I am called this as rotation, I will explain this, later member rotation matrix. Let us consider the beam element which is whose length is L i marked in red color, you can see a L i the member has 2 nodes j-th node and k-th node; is it not; you can see here j-th node and k-th node.

Now, let us say the member is oriented along its local axes system which we all know it is X_m Y_m and Z_m , you can see here X_m Y_m and Z_m there is a local axes. So, the member is along aligned along X Y is 90 degree anticlockwise, Z is further 90 degree anticlockwise. So, X_m , Y_m , Z_m is a standard local axes system along which the member is aligned which a very common principle what we also followed in the 2 dimensional analysis, we borrow the same logic back again here.

Now, this local axes system that is X_m Y_m Z_m is placed arbitrarily in space with reference to the standard reference axes system X , Y , Z you can see. So, reference axes system is xyz local axes system is X_m Y_m Z_m . So, on the other hand, we try to say by making the statement, we are making an indirect statement that the member is aligned arbitrarily it is placed arbitrarily in space correct because X_m , Y_m , Z_m is aligned to the member you can see that. So, the member is arbitrarily place now.

So, now let us try to find out the angle of that axes system X_m Y_m Z_m with reference to X , Y , Z frame.

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$\left. \begin{matrix} \checkmark r_x \\ \checkmark r_y \\ \checkmark r_z \end{matrix} \right\}$ angles of X_m axis w.r.t x - y - z respectively

$$L_i = \sqrt{(x_k - x_j)^2 + (y_k - y_j)^2 + (z_k - z_j)^2}$$

det $\checkmark C_x = \cos r_x$
 $\checkmark C_y = \cos r_y$
 $\checkmark C_z = \cos r_z$

As (A) & (B) coordinates of the member, positioned in space are known, the direction cosines can be written as follows:

$$C_x = \frac{x_k - x_j}{L_i} \quad C_z = \frac{z_k - z_j}{L_i}$$

$$C_y = \frac{y_k - y_j}{L_i}$$

coordinates of the beam element placed in space

So, γ_x , γ_y , γ_z are actually the angles of X m axis with respect to X Y and Z respectively, you can see that in this figure X m axis with respect to X, let us say γ_x ; x m axis with respect to Y γ_y , X m axis with respect to Z γ_z ; one can always find the projection of this which will be actually equal to $L \cos \gamma_x$; this distance; this value will be $L \cos \gamma_x$ is it not.

This value will be $L \cos \gamma_z$ and this value is $L \cos \gamma_y$ so; obviously, this values also equal to $L \cos \gamma_x$ is or not and this value is also equal to $L \cos \gamma_z$. Therefore, the diagonal length can be simply root of sum of this squares. So, this value now is actually equal to $L \sqrt{\cos^2 \gamma_x + \cos^2 \gamma_z}$ this can be easily computed no confusion in this. So, I call this as figure 3 which I am going to use for member rotation.

So, the figure is clear and these are the angles made by the X m axis with respect to the reference axes xyz respectively now you can also write $\cos \gamma_x$ as $\frac{X_k - X_j}{L}$ let $\cos \gamma_y$ be $\frac{Y_k - Y_j}{L}$ and $\cos \gamma_z$ be $\frac{Z_k - Z_j}{L}$ which are direction cosines now interestingly has the j and k coordinates of the member positioned in space or no because you know the position of this member or no; the direction cosines can be written as follows.

$\cos \gamma_x$ will be $\frac{X_k - X_j}{L}$ $\cos \gamma_y$ will be $\frac{Y_k - Y_j}{L}$ and $\cos \gamma_z$ is $\frac{Z_k - Z_j}{L}$ and L can be simply said as $\sqrt{(X_k - X_j)^2 + (Y_k - Y_j)^2 + (Z_k - Z_j)^2}$. So, now, we know $\cos \gamma_x$, $\cos \gamma_y$ and $\cos \gamma_z$; we know of course, the angles between them. So, direction cosines can be easily estimated based upon these set of equations written on the screen now interestingly X_k , Y_k and Z_k .

Similarly, X_j , Y_j and Z_j or actually coordinates of the member or the beam element placed in space is it not placed in space having said this.

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It is important to note that

Direction cosines give components of beam element only along the reference axes system

- But, an important information of orientation of the local axes system wrt Reference axes system is

NOT KNOWN

It is clear that

- beam element is oriented along the local axes ($x_m - y_m - z_m$)

It is now important to note that the direction cosines; give the components of beam element only along the reference axes system, but an important information of orientation of the local axes system with reference to the reference axes system is not known.

That is very important we need to find out this also therefore, it is clear that beam element is oriented along the local axes which is in my case X_m, Y_m, Z_m hence.

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Hence, orientation of ($x_m - y_m - z_m$) axes local axes system wrt Reference axes system ($x - y - z$) is called as

ψ angle

This is to be estimated

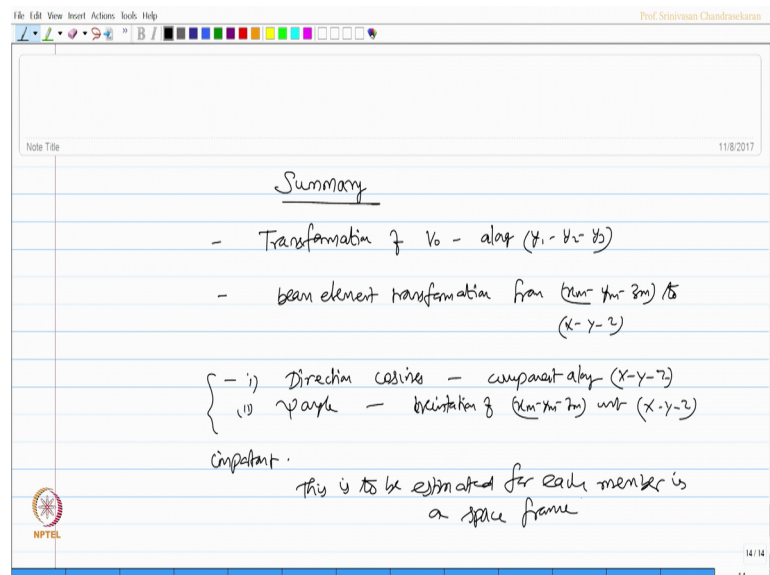
- orientation of local axes system wrt Reference axes system

i) Direction cosines
ii) ψ angle

Hence orientation of X_m , Y_m , Z_m axes that is the local axes system with reference to the reference axes system which is X , Y , Z is called as the psi angle, we need to compute this; this is to be estimated. So, what the time to estimate is the orientation of local axes system with respect to the reference axes system that is what we are interested.

To do this, we need estimate 2 things one the direction cosines, 2 the psi angle. So, we need to know both of these to do this, right. So, importantly let us also try to understand the angle of inclination of alpha and beta the angle of alpha and beta is also important which will be using in the transformation.

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So, we need to actually now, transform the beam element vectors or the beam element matrices to the reference axes system.

So, friends in this lecture we learnt about the transformation of vector v zero along any axis y_1 , y_2 , y_3 , we have learnt a beam element transformation from local axes to the global axes we understood that direction cosines which gives me the component along reference axes and the psi angle which gives me the orientation of local axes with respect to reference axes system are important. This is to be estimated for each member in a space frame.

I hope you understood and the figures are of good clarity you will be able to redraw them and understand the conventions and notations which are try to **explain** during the lecture.

Give a reading once again and try to understand, if you have any difficulties please post it in the discussion forum, we will try to help you out.

Thank you very much.