Computer Methods of Analysis of Offshore Structures
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Module - 01
Lecture – 02
System of Linear Equations (Part - 2)

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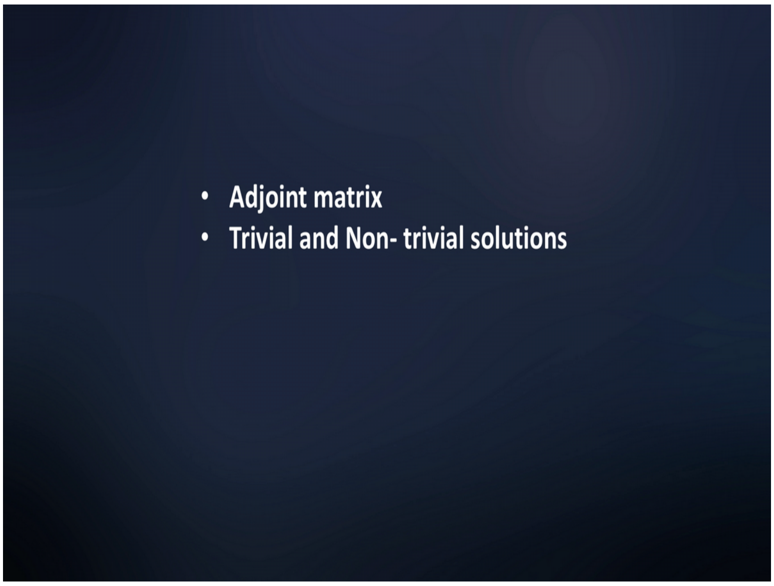

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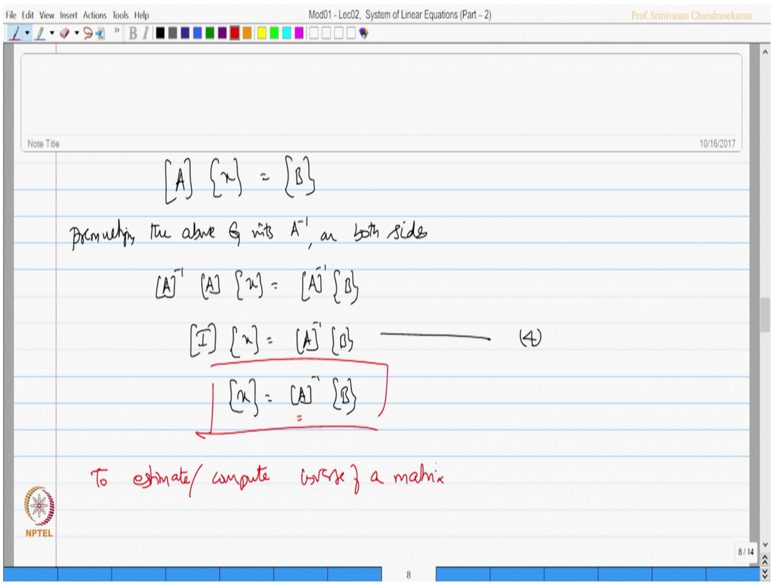

So, A x is equal to B this my typical equation, I have pre multiply this equation with A inverse on both sides. So, A inverse A of x is A inverse B, A inverse A will give me identity matrix which is x, which is A inverse of B equation number 4. So, the above equation is able to actually give me the unknown value x, if I can compute the inverse matrix of A and multiply this with the B vector. So, that is what my idea is.

Now, I would be interested in estimating the inverse of a given matrix. So, now, the problem is to estimate or to compute inverse of a matrix because if I know how to estimate the inverse of a matrix, I can always find the variable vector x from this equation to estimate the inverse of a matrix.

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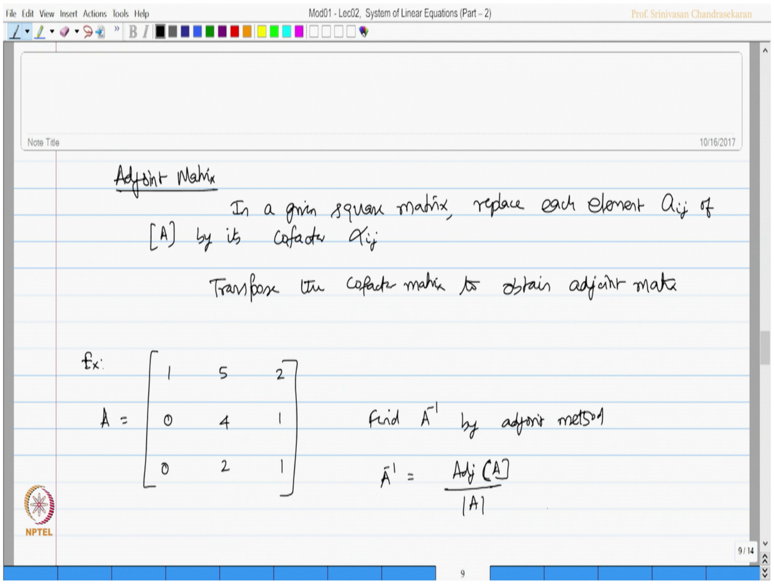

Let us talk about adjoint matrix; what is an adjoint matrix? In a given square matrix replace each element that is A ij of the matrix A by its cofactor alpha ij transpose, the cofactor matrix to obtain adjoint matrix.

Let us quickly find out the procedure for finding A inverse for given problem, let us say we will take a simple example, let us say I have a matrix which is 1, 5, 2, 0, 4, 1, 0, 2, 1; I call this as a matrix; what I wanted to know is find A inverse by adjoint method A inverse can be given by adjoint of A by determinant of A.

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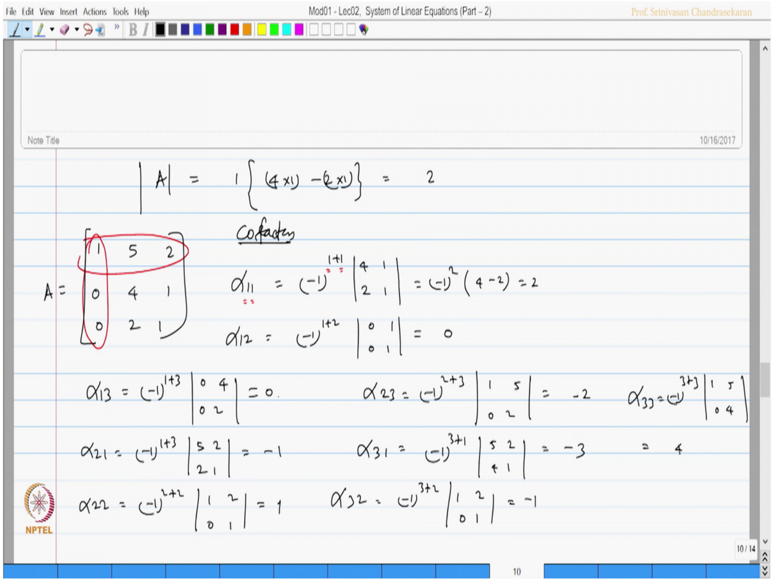

Let us first find determinant of A which will given by 1 into 4 into 1 minus 2 into 1 which will be actually equal to 2.

So, my matrix is 1, 5, 2, 0, 4, 1, 0, 2, 1; this is my A matrix; let us find; try to find the cofactors; let us say I want to find alpha 1 1 which will be equal to minus 1 1 plus 1; that is if this is 1 and 1 this becomes 1 and 1. So, minus 1 to the root of power of this and then find determinant of eliminate that row and that column and find determinant of 4 1 2 1 which is minus 1 square 4 minus 2 which is 2 .

So, I want to find alpha 1 2 which is minus 1 1 plus 2 determinant of 0 1 0 1 which gives me 0. Similarly alpha 1 3 minus 1 1 plus 3 determinant of 0 4 0 2 which is 0 alpha 2 1 minus 1 1 plus 3 5 2 2 1 which will be minus 1 alpha 2 2 minus 1 to the power 2 plus 2 determinant of 1 2 0 1 which will be 1.

Alpha 2 3 minus 1 2 plus 3 determinant of 1 5 and 0 2 which will be minus 2 alpha 3 3 3 1 will be minus 1 3 plus 1 5 2 4 1 which will be minus 3 alpha 3 2 minus 1 3 plus 2 1 2 0 1 which will be minus 1 and alpha 3 3 will be minus 1 3 plus 3 1 5 and 0 4 which will be 4.

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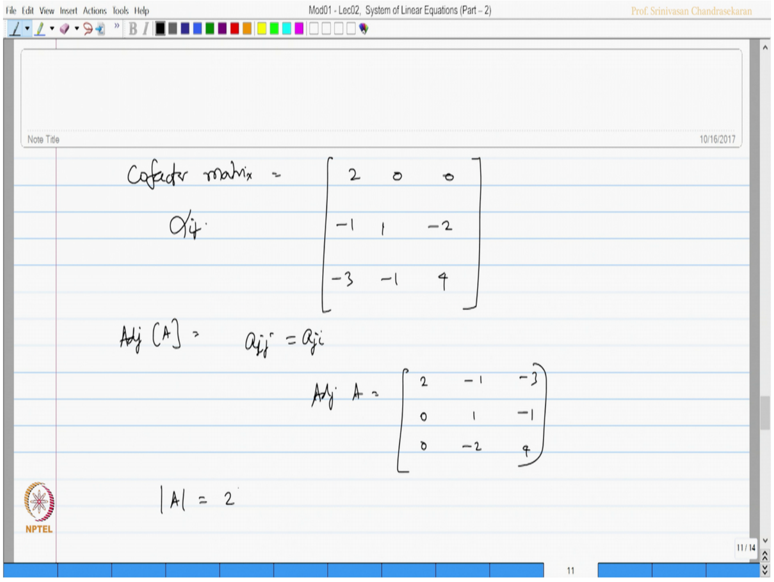

So, now I have the cofactor matrix which I am writing here which are alpha ij. So, that is going to be alpha 1 1 is 2 1 2 0 1 3 is 0. So, the first row is 2 0 0. So, I am writing here 2 0 0, similarly minus 1 1 minus 2 minus 3 minus 1 4; you can see here minus 1 1 minus 2 minus 3 minus 1 4 I written that.

So, now I want to write the adjoint of matrix A. So, transpose this matrix that is A ij should now become A ji. So, adjoint of A is actually given by you have to change the rows and columns. So, this becomes 2 0 0 minus 1 1 minus 2 and minus 3 minus 1 4; we already know determinant of A is actually 2.

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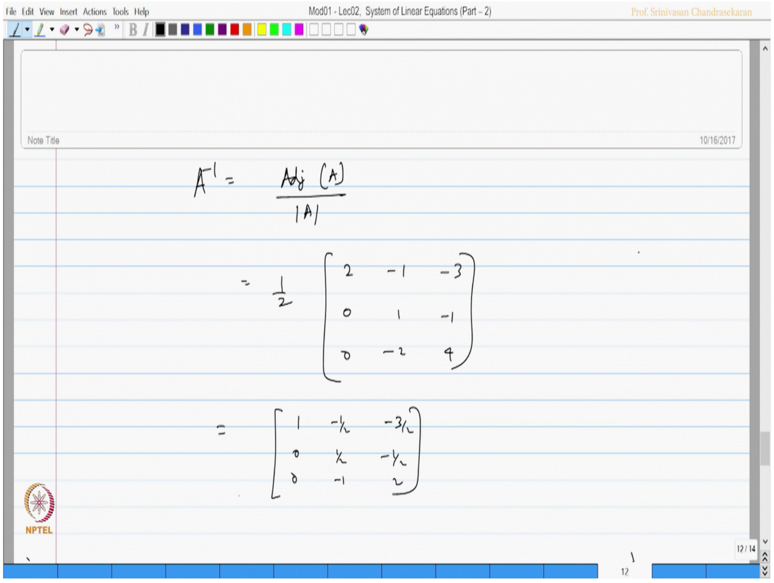

And hence A inverse is adjoint of A by determinant of a which can be 1 by 2 of 2 0 0 minus 1 1 minus 2 minus 3 minus 1 4 multiplying it I will get this value as 1 0 0 minus half; half minus 1 minus 3 by 2 minus half 2.

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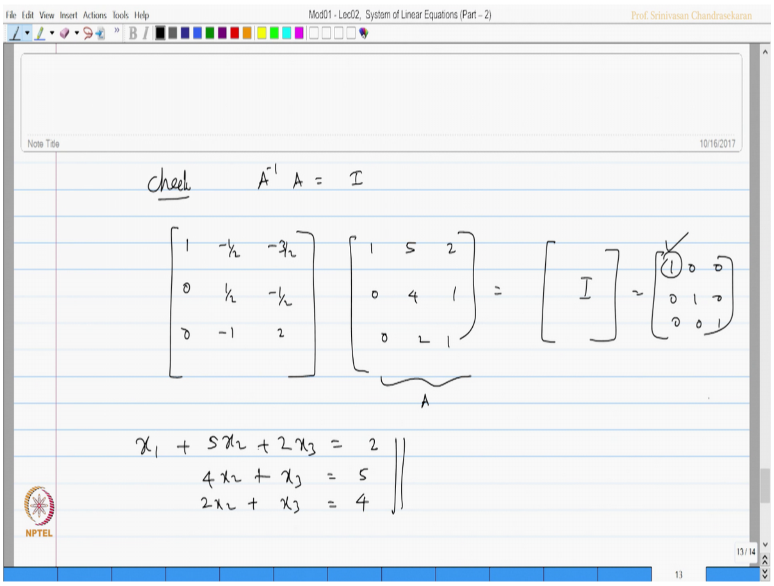

Now, I want to check this. So, we know that A inverse A should be I. So, let say A inverse is 1 0 0 minus half half minus 1 minus 3 by 2 minus half 2 that is the matrix, we have multiply this with the original a matrix which is 1, 5, 2, 0, 4, 1, 0, 2, 1 that is the matrix the original matrix is 1, 5, 2, 0, 4, 1, 0, 2, 1; 1, 5, 2, 0, 4, 1, 0, 2, 1 when you multiply you will see the you will be getting an identity matrix which will be actually 1 0 0 0 1 0 0 0 1, then check this particular value as one into one you will get identity and so on and so forth.

So, now let us express this equation a this my matrix A as system of equations; let say I can express this as x 1 plus 5 x 2 plus 2 x 3 as some number 2; let say 4 x 2 plus x 3 is 5 2 x 2 plus x 3 is 4; let say I have this equations in the system of equations which I have to solve to find out the variables x 1, x 2, x 3.

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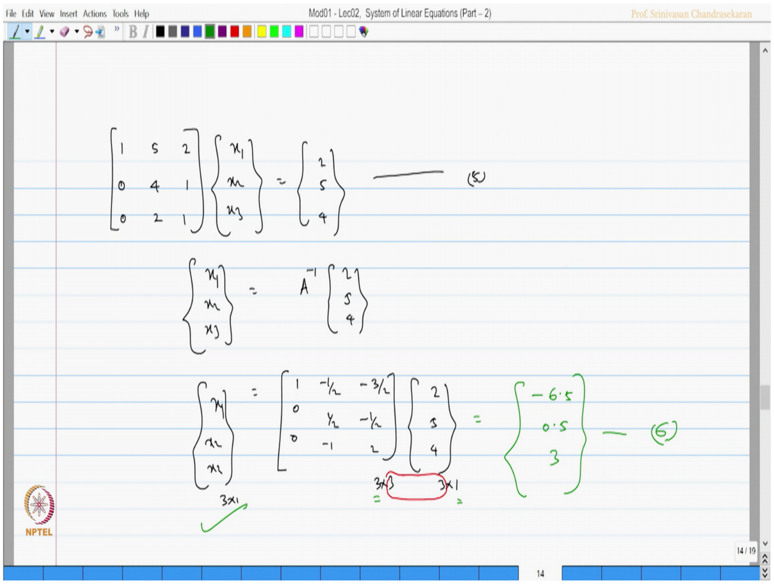

Therefore I can write this in matrix form 1, 5, 2, 0, 4, 1, 0, 2, 1 of x 1, x 2, x 3 as 2 5 and 4.

So, we know from this equation this equation number will be we know this equation x 1 x 2 x 3 can be simply given as A inverse of this matrix or this vector. So, A inverse we already have with us which will be 1 0 0 minus half half minus 1 minus 3 by 2 minus half 2, I multiply this with this vector 2 5 4 to get my variable vector x 1 x 2 x 3; you can see the compatibility this 3 row and 1 column this is 3 by 3 matrix. This 3 row and 1 column; you know the columns and the rows of the multipliers should be identical; I will get ultimately 3 row and 1 column which I am getting here. So, by solving this I will be able to get these value as minus 6.5 0.5 and 3 the solution for the system of equation shown in 1. So, that is my equation 6.

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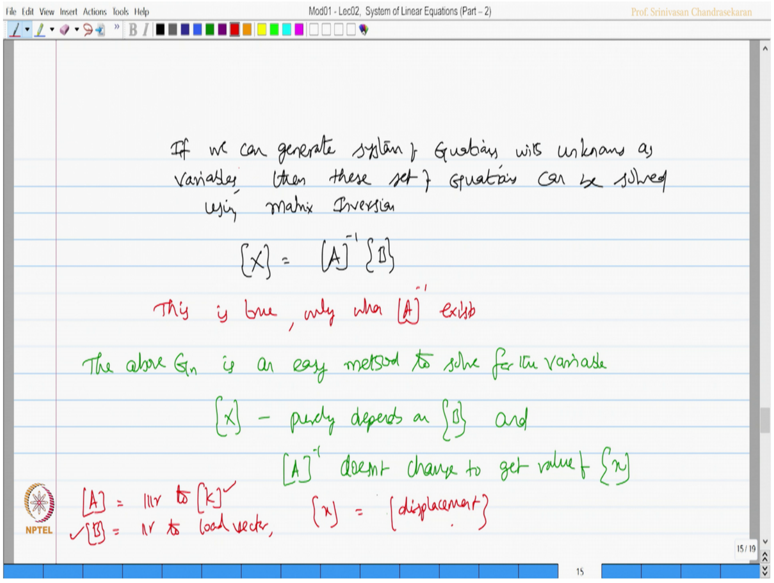

So, friends if I have generated, if we can generate system of equations with unknowns as variables, then these set of equations can be solved using matrix inversion that is x vector can be simply inverse of a matrix multiplied by A vector; of course, this is true only when A inverse exists the above equation is an easy method to solve for the variables, it is also important to note that the variable x purely depends on B and A inverse does not change to get the value of x.

Now, this is the very classical problem; let us compare that A is similar to a stiffness matrix of A given system and B vector is similar to a load vector and x vector is the displacement vector; by this comparison. You can see here without changing the k inverse 1 can easily find the value of displacement vector for a change load vector that is very important and very interesting observation we have.

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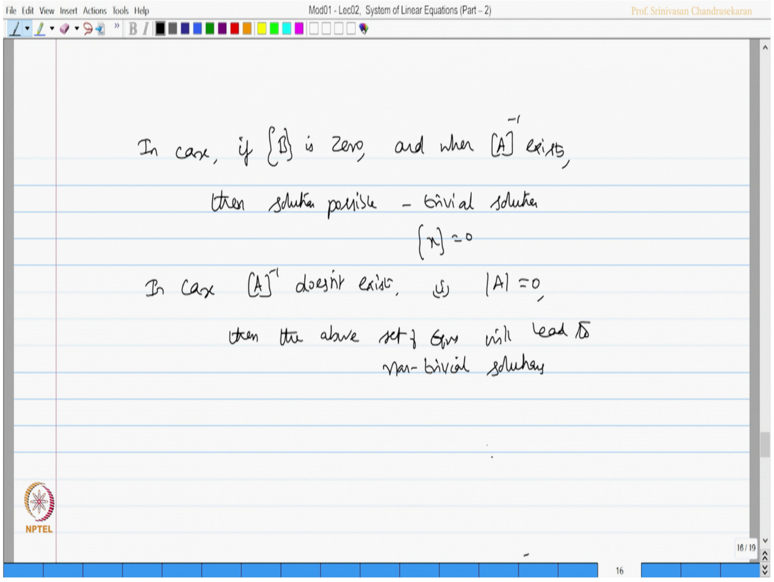

In case if the B vector is 0 and when A inverse exists then solution possible of saying it is a trivial solution that is x will be 0; in case A inverse does not exist that is determinant A is 0, then the above set of equations will lead to non trivial solution.

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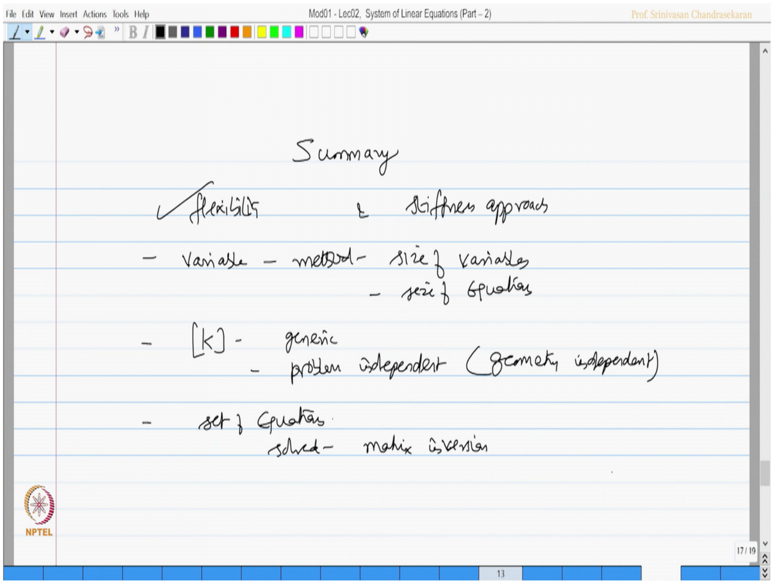

So, friends let us look at the summary what we learnt in this lecture, we learnt the comparison between flexibility and stiffness approach we understood how selection or choice of the method affects the size of variables and therefore, the size of set of equations we also understood that stiffness method is more or less generic it is problem independent I should say rather geometry independent.

We also studied an example of set of equations and solved this using matrix inverse method; we now extend this knowledge to other set of problems to solve classical problems in structural analysis. In the next lecture, we will discuss more about basics on partitioning of matrices and we will see how they can be beneficial in doing such analysis of large size problems.

Thank you very much.