Computer Methods of Analysis of Offshore Structures
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Module - 01
Lecture - 21
3d structures: Transformation matrix

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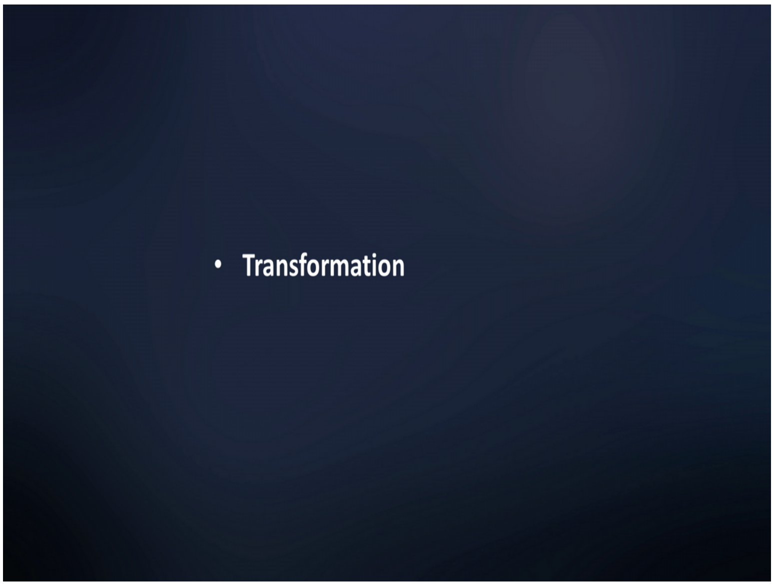

Friends, welcome to the 21st lecture in module 1, we have been discussing about the computer methods of structural analysis applied to offshore structures, we have been taking examples of beam element truss element with planar and non planar members involved in a structural system on a 2 dimensional analysis. In the last lecture, we introduced the 3 dimensional analysis and the member stiffness matrix of size 12 by 12, we derived the stiffness matrix for the member which is on a space frame.

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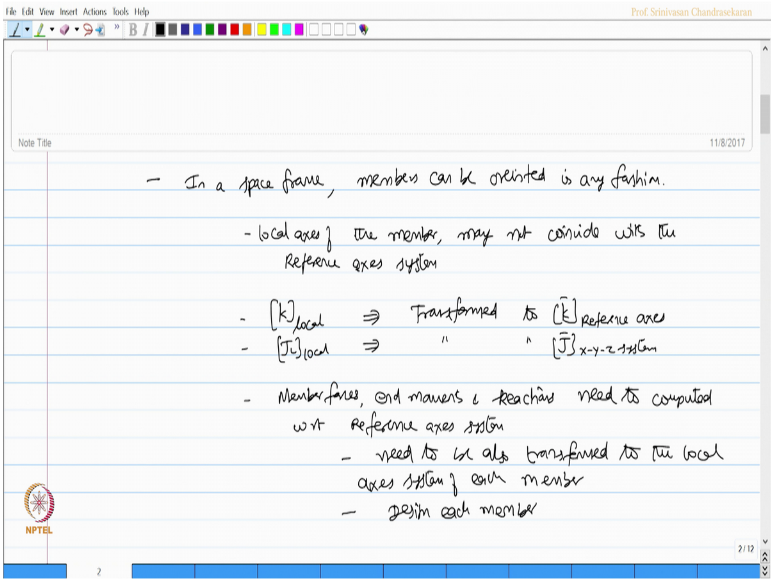

Now, in this lecture we are going to discuss about how do we obtain or derive the transformation matrix. Now the 1st question comes, why transformation matrix is important. Very simple reason we all know that, in a space frame members can be oriented in any fashion to be very specific; the local axes of the member may not coincide with the reference axes system. In such situation whatever stiffness matrix we derived in the last lecture they are considered to be local and they need to be transformed which is with respect to the reference axes system.

Secondly, whatever load which is applied on the local need to be also transformed to get J bar with reference to the x-y-z system. Most importantly, the member forces end moments and reactions need to be computed with respect to the reference axes system, but need to be also transformed to the local axes system of each member the question is why this is required this is required because to design the member .

So, now the question is in a space frame members oriented in different formats and alignments, how do we transform them to match the alignment with the top the reference axes system. So, that is what we are going to discuss today in this lecture which we call as a transformation matrix.

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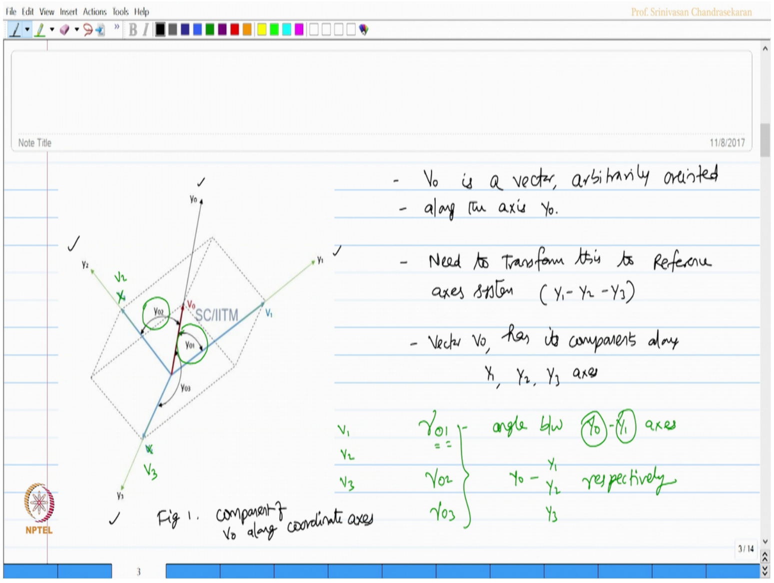

One can see here, V0 is a vector which is arbitrarily oriented; this V0 vector is oriented along the axis Y0. Now I want to transform this, this to the reference axes system for example, we have considered the reference axes here has Y1, Y2 and Y3 axis which also has an orientation with reference to X Y Z plane, but let us say I want to transformed this to Y1, Y2 axis we can see here Y1, Y2 and Y3 axis are different from that of the Y0 axis along which the vector or the member is oriented.

Now, let us defined the angles V0 is a vector which is oriented with respect to some set of orthogonal axis Y1, Y2, Y3. Now vector V0 has its components, along Y1, Y2 and Y3 axis. To know it is component we should like to know the inclination of this vector or position of this vector with reference to these 3 axis.

Let us, look at this symbol this is gamma 01. So, for example, gamma 01 is the angle between the Y0 and Y1 axis. So, the first let us stands for Y0 and the second subscript stands for Y1, with that algorithm please look at this angle Y 02 this is an inclination with respect to Y0 and Y2. Similarly, Y0 and Y3 will give gamma 03. So, gamma 01, gamma 02, gamma 03 are now the angles between the Y0 axis and Y1, Y2 or Y3 respectively.

Now, the corresponding components of them along Y1, Y2, Y3 are marked in blue color and they are V1, V2 and V3. So this should be V3, this is V3 and this is V2 along V1, V2 and V3. So one can easily find the following relationship valid V1, V2 and V3 will be

V0 cos gamma 01, V0 cos gamma 02 and V0 cos gamma 03, I call this equation number as 1.

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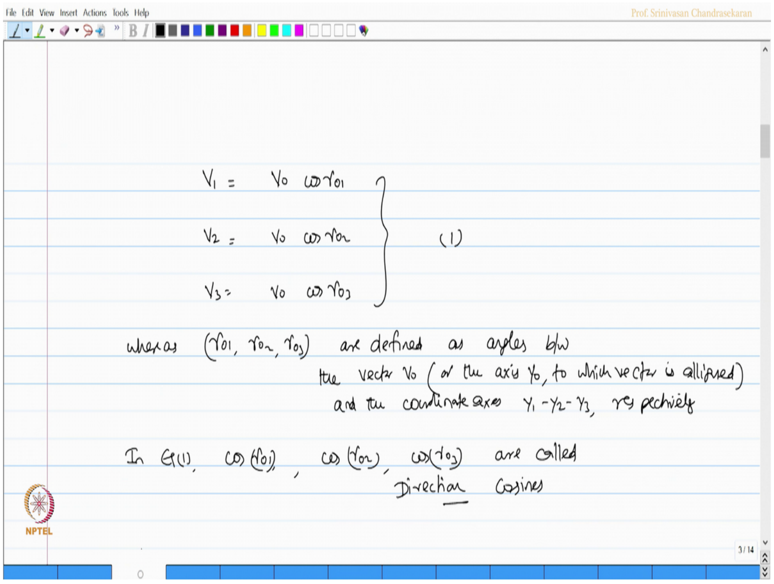

Whereas gamma 01, gamma 02, and gamma 03 are defined as angles between the vector V0 or the axis Y0 to which vector is placed and the coordinate axes Y1, Y2 and Y3 respectively, now the above equation 1 the terms cos gamma 1, cos gamma 02, cos gamma 03 are called direction cosines.

Now we have resolved the vector V0 along Y1, Y2, Y3 which are some set of coordinate axes. Let us, now resolved this or transform this to a standard reference axes system which is xyz. So, let us now draw a figure where the standard reference axes let X1, X2 and X3 be the reference axes as shown in the figure.

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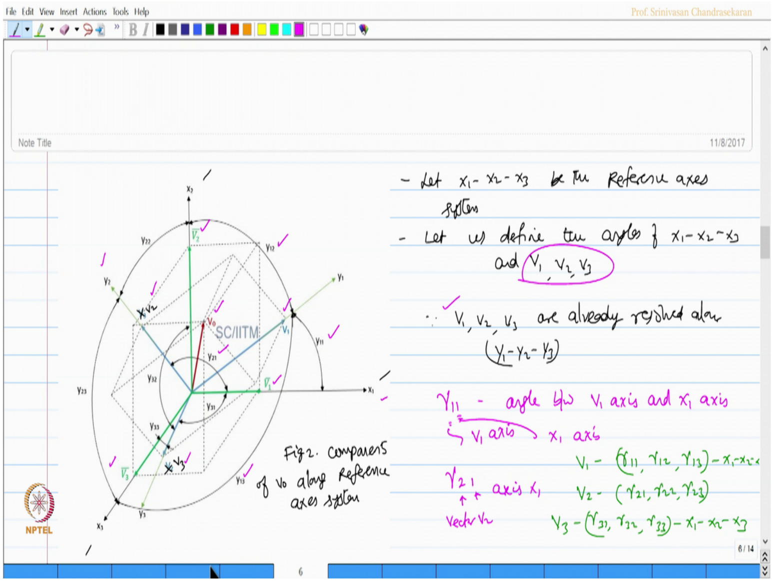

So, let us name the figures this will be figure 1 which has component of vector V0 along coordinate axes; let this be figure 2, where we are looking at components of V0 along reference axes ok.

Let X1, X2 X1, X2 and X3 be the reference axis; obviously, this vector as a mistake this was V3 and this was V2, I borrowed this figure from the previous. Now let us define the angles, now let us define the angles of X1, X2 and X3 and the vector V0 or vector V1, V2 and V3 because vectors V1, V2, V3 are already resolved along Y1, Y2, Y3 is it not we already have this we can see here, V1 we have V1 we have, V 2 we have along Y2 and V3 we have along Y3.

So, now I am interested in knowing, what is the angle between this vectors and the reference axes system X1, X2, X3. So, let us mark those angles let us say gamma 11 is the angle between the V1 axis and X1 axis. So, the first subscript stands for V1 axis the second subscript stands for the X1 axis. So, you can see here gamma 11. Similarly, the angle between V1 and X2 should be gamma 12.

Similarly, the angle between V1 and X3there is this angle it will be gamma 13 by that logic, I can now find or defined angles between the vector V2 with that of X1 X2 and X3 like for example, V2 vector is here, X1 is here the angle between V2 and X1 is here. So, the first subscript in gamma 21 this stands for vector V2 and this stands for axis X1 and so on.

So, this figure explains or defines all angles between the respective vectors V1, V2 and V3 which were resolved along some coordinate axes system Y1, Y2, Y3 when a push in vector V0 is placed in space. Now I am transforming that vector or those vectors V1, V2 and V3 which were initially transformed through direction cosines along some reference axes Y1, Y2, Y3 are now being transformed to the standard set of reference axes X1, X2, X3, so you will like to do this.

So, now we can still write a statement that V1 vector makes an angle gamma 11 gamma 12 and gamma 13 with reference to X1, X2, X3. Similarly, V2 vector makes an angle gamma 21, gamma 22, gamma 23 with reference to X1, X2, X3. Similarly, V3 makes an angle gamma 31, gamma 32, gamma 33 with reference to X1, X2, X3 is that clear, so this figure is clear.

Now, let us write this connecting equation or the matrix which connects the transformed components of theses vectors. So, what is the transformed component of this vector along X1, X2, X3 that is very clear here the transformed components are V1 bar, V2 bar and V3 bar. So, now I want to connect V1 bar, V2 bar and V3 bar with that of V1, V2, V3. Because V1, V2, V3 we already know, we can see from this previous equation V1, V2 and V3 are known provided the direction cosines are measurable and the vector is known to us ok.

So, now having now V1, V2, V3 we are interested in finding out the components of those vectors resolved or transformed along the standard reference axes system X1, X2, X3 which I call as V1 bar, V2 bar and V3 bar.

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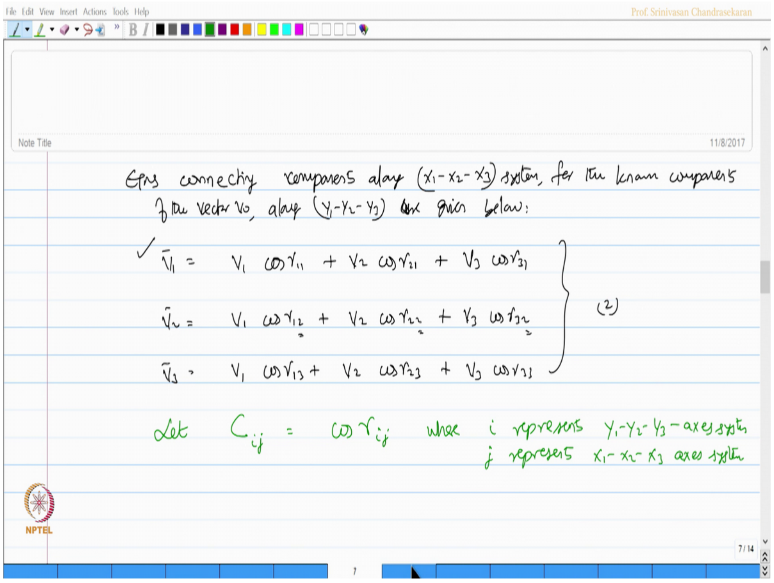

So, now we can write the governing equation which connects these 2 as below. So, the set of equations connecting components along X1, X2, X3 system for the known components of the vector V0 along Y1, Y2, Y3 are given below.

So, I should say V1 bar, V2 bar, V3 bar or simply V1 cos gamma 11 plus V2 cos gamma 21 plus V3 cos gamma 31, because these angles are defined with respect to X1 axis where V1 is the component similarly with reference to X2 axis I can still write V1 cos gamma 12 plus V2 cos gamma 22 plus V3 cos gamma 32, because these are all with reference to axis 2 that is X2. Similarly, with reference to axis X3, 13, 23 and 33 I call this as equation set 2.

Now, let us replace this cos term by a letter C. So, let C i j is actually equal to cos gamma i j where, i represents the Y1, Y2, Y3 axes system and j represents X1, X2, X3 axis system; by that logic I can convert equation 2 in much a simpler form as given here.

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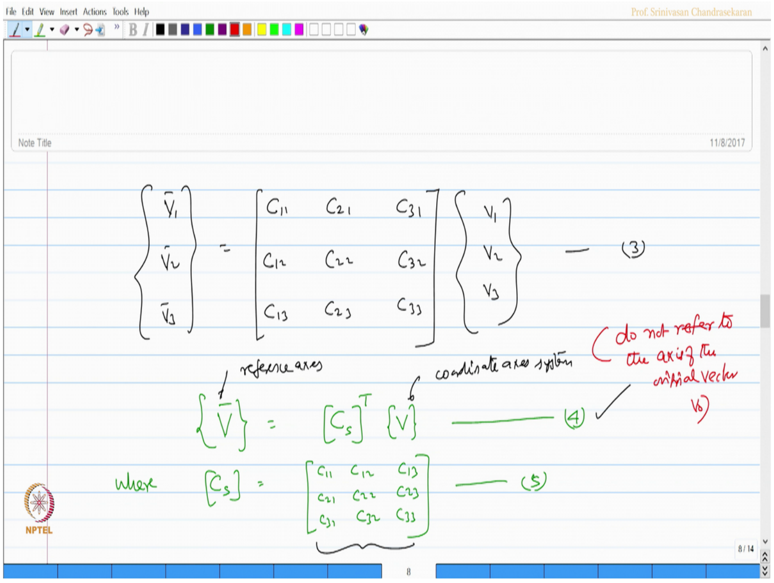

So, I can now write V1 bar, V2 bar and V3 bar in a matrix form as C 11, C 21, C 31, C 12, C 22, C 32 and C 31, C 32, C 13, C 23 and C 33. Connecting these two V1, V2 and V3 call equation number 3 I can now write a comprehensive equation of this as saying V bar is expressed as C of the space transpose of that of V where Cs matrix refers to C 11, C 12, C 13, C 21, C 22, C23, C31, C32, C 33.

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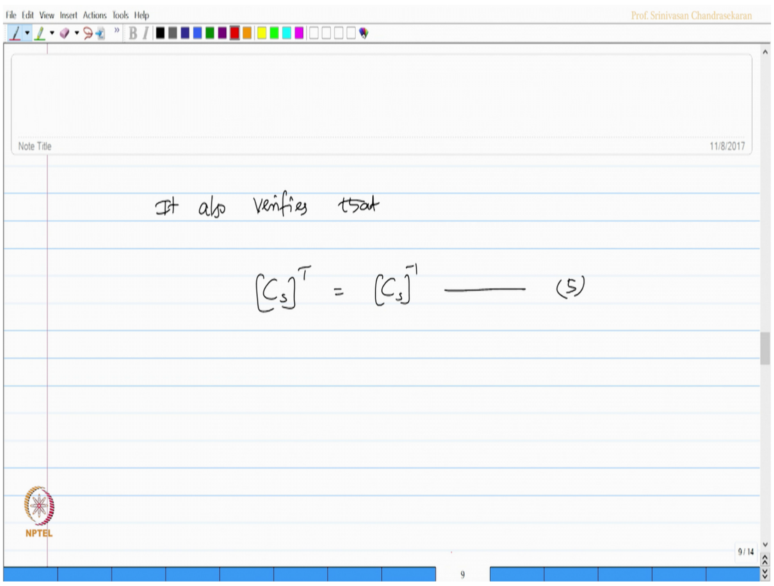

Friends, interestingly it also verifies that you can verify this individually verifies that C space frame transpose matrix is as same as C space frame inverse I call this as equation number 4 5.

So, now we have very comprehensive equation which is equation 4, V bar the component of the vector, V0 along reference axes system X1, X2, X3 is simply given by the transformation matrix for the direction cosine matrix; the direction cosine matrix Cs, if I know the comprehensive of this vector along any predefined coordinate axes system Y1, Y2, Y3. Please understand here one important statement this refers to a reference axes system, this refers to some coordinate axes system.

The important point to note that is both of them do not refer to the axis of the original vector V0 is it not that is very important. Having said this instead of a vector let us considered as a member. So, I want to now find the member transformation matrix.