Computer Methods of Analysis of Offshore Structures
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Module - 01
Lecture - 20
3d Structures : Analysis by Stiffness method (Part - 1)

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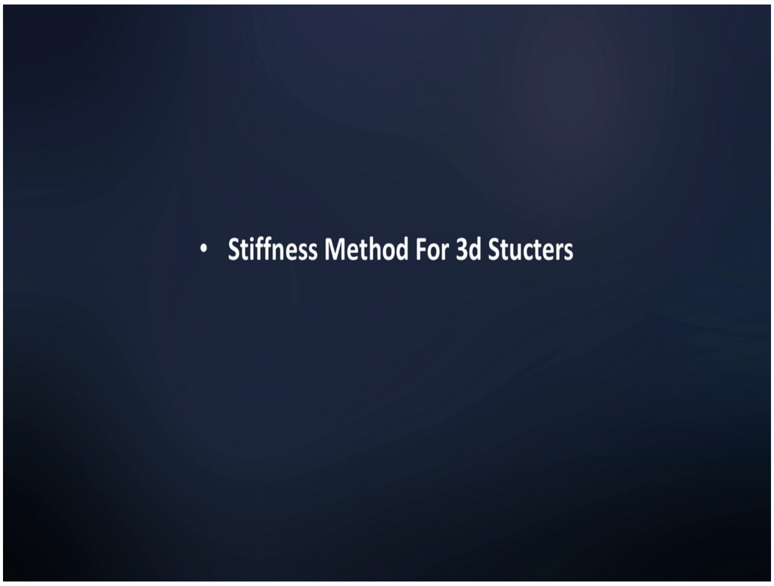

Friends, welcome to the 20th lecture and module 1. In this lecture, we are going to discuss about the stiffness method of analysis applied to 3 dimensional structures. In the previous set of lectures, we have explained; how to use computer methods of structural analysis for analyzing planar orthogonal and non-orthogonal members with beam elements, then planar truss members using computer program. We have solved couple of examples; explain the computer codes in detail and also solved them by hand and compared the answers what you get from the computer problem.

We will extend the same algorithm, same logic, same sign conventions to the 3 dimensional structure to make this analysis very very simple and very compatibly easy. So, let us brush; what we had in the beam element of a planar orthogonal structural system 2 dimensional. Let us borrow the basics from there and extent slightly for 3 dimensional structures.

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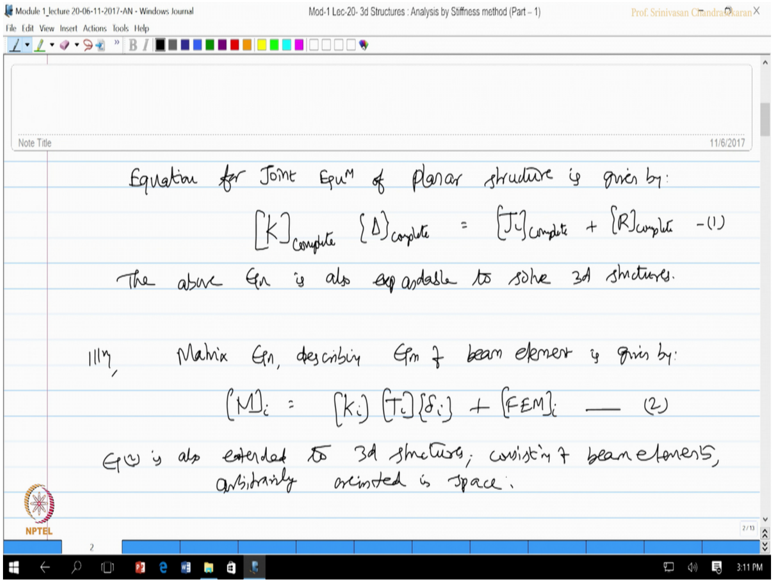

We know that equation for joint equilibrium of planar structure is given by the stiffness matrix complete multiplied by delta of the complete structure will be the joint load of the complete structure plus any additional reaction of the complete structure equation 1.

Now, the above equation is also expandable to solve 3 dimensional structures similarly the matrix equation describing equilibrium of beam element is given by M i is actually K i T i delta i plus FEM of i equation 2; equation 2 is also extended to analyze 3 dimensional structures consisting of beam elements arbitrarily oriented in space having said this, the first task in 3 dimensional analysis is to develop the stiffness matrix of the complete structure which can be simply done by summation of member stiffness matrices of individual elements.

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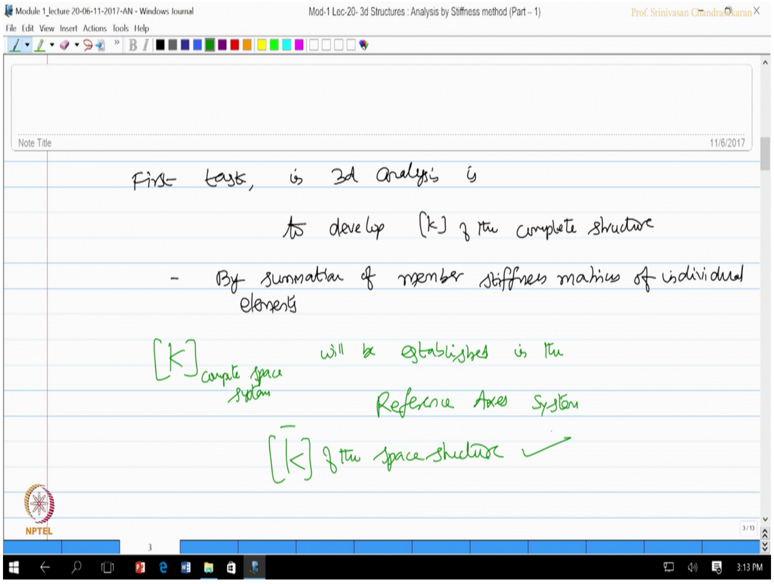

So, it is very important to note that the complete stiffness matrix of the space system will be established in the reference axes system. So, what we are trying to say is we need to find K bar of the space system that is what we want. Having said this, let us talk about the sign convention.

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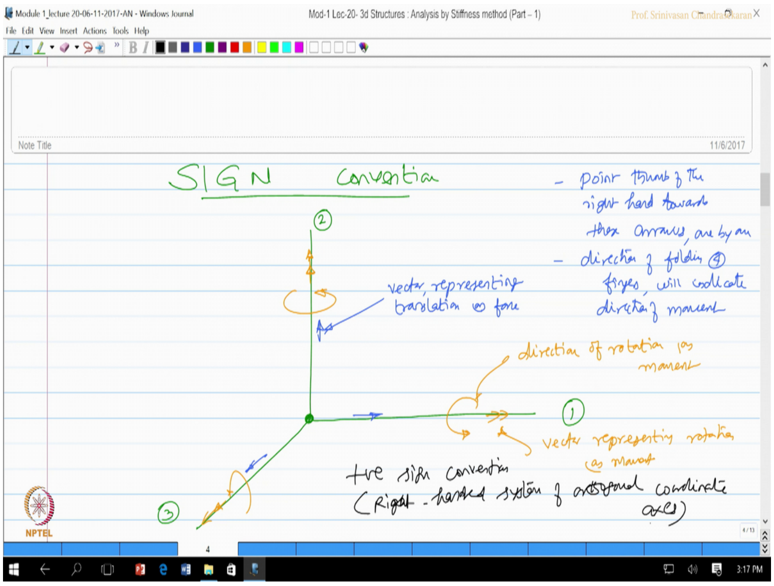

Let us say have 3 axis, we calls axis in 1, axis 2 and axis 3; let us mark directed representing translation forces like this along x along y and along z and the single arrow; let us say this is vector representing translation or force, then let us put the thumb towards this direction of the right hand. So, point thumb of the right hand towards these arrows 1 by 1.

So, if you put it towards one; remain 4 fingers the direction of folding 4 fingers will indicate the direction of moment. Let us mark that which is also marked the double arrow similarly is also marked with double arrow. Similarly is also marked with double arrow. So, these indicate the direction of rotation or moment and this indicate the vector representing rotation or moment.

So, all these are considered to be positive established using right hand system of orthogonal coordinate axes now.

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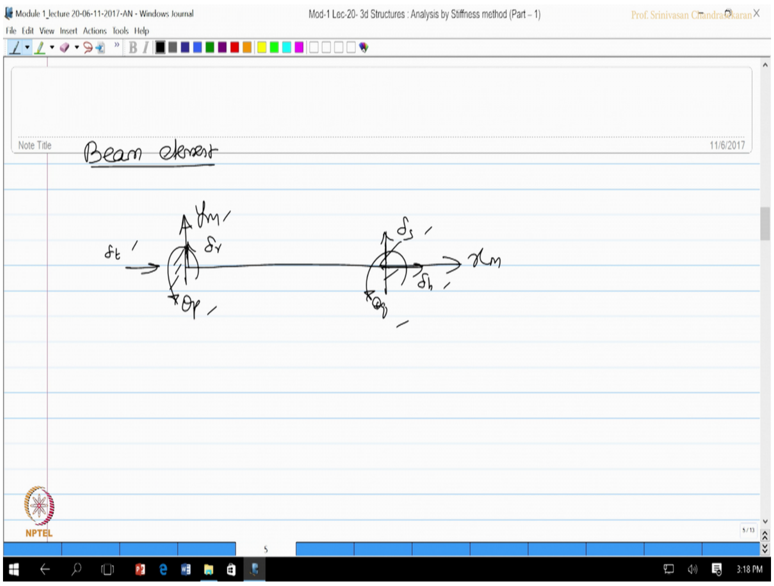

Let us consider a beam element; let us track mark what we had in the last planar orthogonal systems, we had a beam element fixed at both the ends, this is considered to be x m and y m of the beam element and the beam had theta p theta q delta r delta s and delta p and delta h as a degrees of freedom. So, 2 rotations 2 translations along y and 2 translation long x, correct; let us extent this algorithm for a 3 dimensional member arbitrarily oriented in space.

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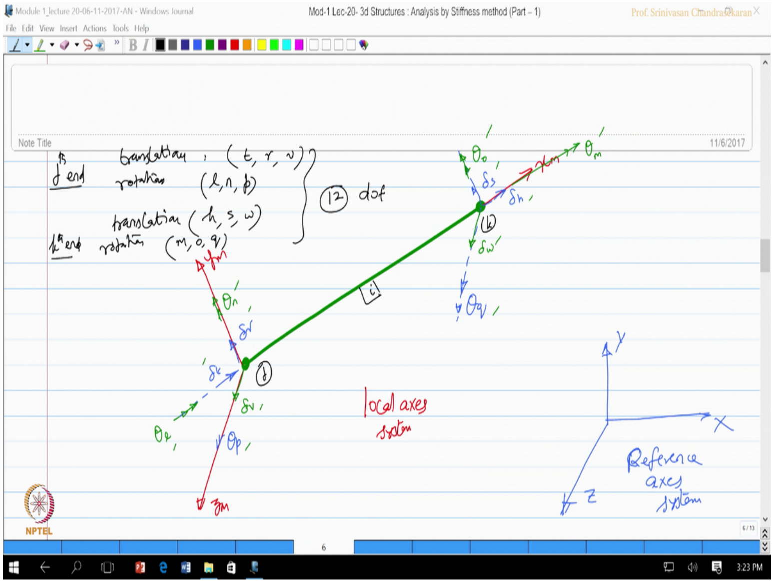

So, let us mark the member with 2 joints j and k or the i-th member with 3 axis local are going to be x axis normal to that is y axis and z m. So, this is local axes system which is orthogonal to each other comparatively; let us have the global axes system like this x y and z; this is let us say reference axes system. Now let us mark the degrees of freedom which we had similar to that of what you had in this.

So, now let us mark theta p and theta q, then similarly along y, it was delta r is in r; delta r and delta s and along this; this is going to be delta T and delta H which is similar to what we had in the 2 dimensional truss states; is it not; in addition, let us also mark displacements along y and rotations, etcetera. So, let us mark the additional degrees now.

So, this is going to be displacement along z I call this as v and correspondingly along this I call as w rotation, I called this as theta l rotation, I called this as theta M and rotations I call this as theta n and this rotation I call as theta o. So, now, for every end, I have 3 translations that is e t r n v and 3 rotations that is p l and l; similarly for the k joint 3 translations X, Y and Z; 3 rotations about X, about Y and about Z. So, there are 12 degrees of freedom. Now 3 translations and 3 rotations at each end makes it 12.

So, I can always write at the j-th end translations or t r and v correct X, Y and Z rotations r l that is about X, then n about Y and p about Z. Similarly let us write for the k-th end translations are along x h, along y s, along z w and rotations are about x m about y o and about z q. So, there are 12 degrees of freedom. So, my stiffness matrix member stiffness matrix will be of size 12 by 12 we would like to derive them now.

So, now let us take unit displacement along t unit displacement along t z m.

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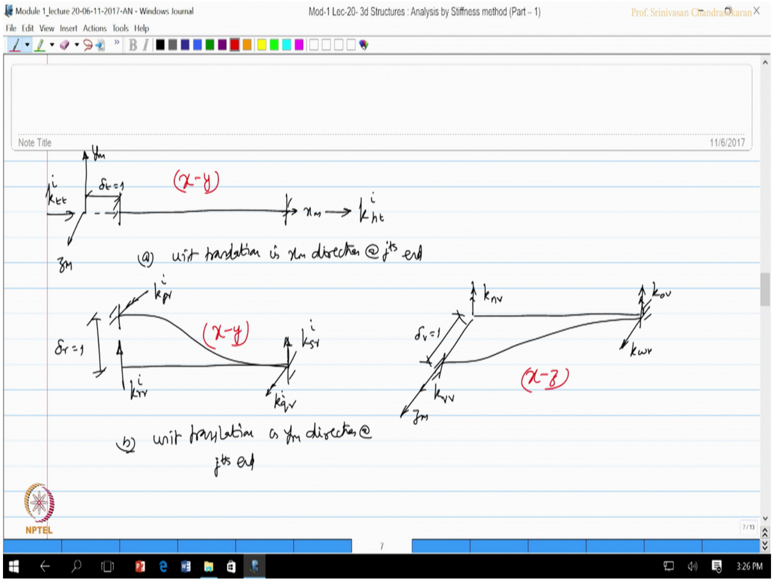

So, this displacement is along t which is unity this will give you the forces K tt of the i-th member and this is again b k h t of the i-th member. So, I should say this is unique translation in x m direction at j-th n, then I should give unit translation in the y-th direction, I should say this has unity which will now cause K pr, then K qr then K rr of the i-th member then K s r of the i-th member.

So, this is unit translation in y m direction at j-th end, I can also give this translation along z axis which I draw. So, we gave this as delta V as unity. So, this will create K rr, this will create K vv and K wv and then the moment which will be K nv and K ov; let us identify the planes where you have marking them. So, this is happening in x y plane; this is happening in x y plane; this happening in x z plane.

Let us find unit rotations as well for these members.

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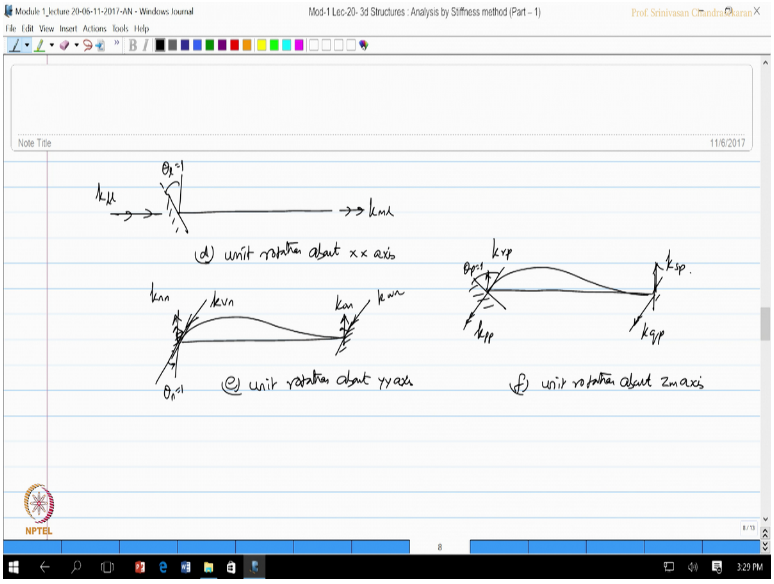

So, let us say we give unit rotation as one. So, we get the moments as K l l and K m l. So, this is going to give me unit rotation about x; x axis, then this is going to cause a rotation which is going to theta n is unity, this will give me K vn, K wn, K nn and K o n. So, this is going to be unit rotation about y y axis, let us have one more figure which is going to mark unit rotation as K pp; K q p which is resisted by K rp and K sp and this value is going to be unity.

So, this is unit rotation about z m axis. So, we have got you know displacements along x, y, z unit rotations about x, y, z. So, we have 6 on the z-th n. Similarly, one can draw the 6 for the k-th end as well which I am not doing it here.