

Computer Methods of Analysis of Offshore Structures
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Module - 01
Lecture - 19
Planar Truss System Examples (Part – 1)

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- Solved Example 2 Using Computer Program

Friends, let us continue with the discussion on planar truss system which we had in the last lecture. We are talking about the computer method of structural analysis **applied** to planar truss system examples. We **had** this problem in the last lecture that there are 5 member truss as shown in the example here. We arrived at the unrestrained; unrestrained degrees of freedom, then we made this table and calculated the transformation coefficient C_x and C_y .

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Handwritten mathematical derivations for stiffness matrices k_{AB} and k_{CD} .

For $k_{AB} = E \times 10^3$, the global stiffness matrix is shown as:

$$k_{AB} = E \times 10^3 \begin{bmatrix} \gamma & \delta & \epsilon & \eta \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1.25 & -1.25 \\ 0 & 0 & -1.25 & 1.25 \end{bmatrix}$$

The local stiffness matrix is:

$$k_{AB} = E \times 10^3 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

For $k_{CD} = E \times 10^3$, the global stiffness matrix is:

$$k_{CD} = E \times 10^3 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1.25 & -1.25 \\ 0 & 0 & -1.25 & 1.25 \end{bmatrix}$$

The local stiffness matrix is:

$$k_{CD} = E \times 10^3 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1.061 & -1.061 \\ 0 & 0 & -1.061 & 1.061 \end{bmatrix}$$

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Handwritten derivations for transformation matrices T_i and k_{Ti} .

The local stiffness matrix k_{Ac} is:

$$k_{Ac} = E \times 10^3 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1.061 & -1.061 \\ 0 & 0 & -1.061 & 1.061 \end{bmatrix}$$

The transformation matrix T_i is:

$$T_i$$

where $i = 5$ member.

The transformed stiffness matrix k_{Ti} is:

$$k_{Ti} = T_i^T k_{Ac} T_i$$

where $i = 5$ member.

The global stiffness matrix k_{AB} is:

$$k_{AB} = T_{AS}^T k_{AD} T_{AS}$$

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$k_{Ac} = E \times 10^{-3}$

$$[T_i] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1.061 & -1.061 \\ 0 & 0 & -1.061 & 1.061 \end{bmatrix}$$

$[K_T]_i = [T_i]^T [k_{Ac}] [T_i]$

$[K_T]_i$

$K_{AB} = T_{AB}^T k_{AB} T_{AB}$

Then we estimated the stiffness matrices and transformation coefficients matrices for all the members and worked out the global stiffness matrix of each member as A B, B C and so on.

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$K_{AS} = E x$

$$\begin{bmatrix} 0.0013 & -0.0013 & - & - \\ -0.0013 & 0.0013 & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix}$$

$K_{BC} = E x$

$$\begin{bmatrix} - & - & - & - \\ - & - & - & - \\ - & - & 1 & -1 \\ - & - & -1 & 1 \end{bmatrix}$$

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$K_{CD} = E \begin{bmatrix} 0.0013 & -0.0013 & 0 & 0 \\ -0.0013 & 0.0013 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

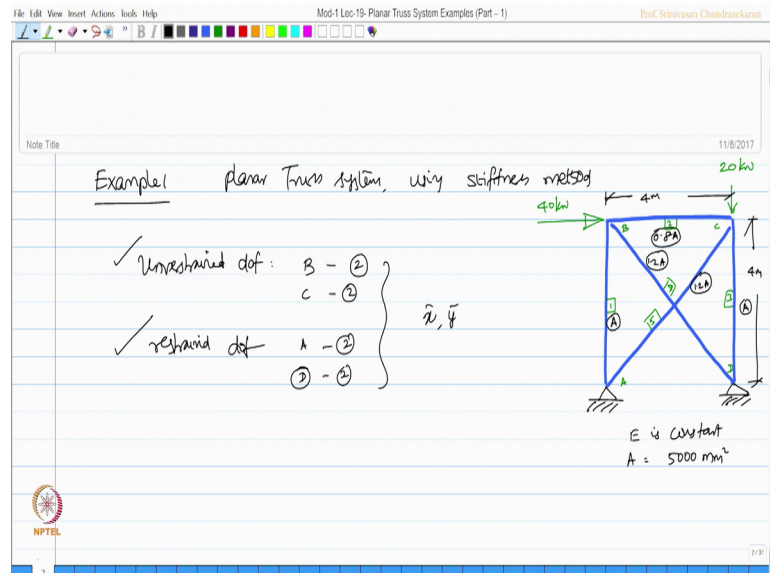
$K_{AB} = E \times 10^{-3} \begin{bmatrix} 0.5304 & -0.5304 & -0.5304 & 0.5304 \\ 0.5304 & 0.5304 & -0.5304 & -0.5304 \\ S_{y4} & 0.5304 & -0.5304 & 0 \\ 0.5304 & 0.5304 & 0 & 0 \end{bmatrix}$

$K_{AC} = E \times 10^{-3} \begin{bmatrix} 0.5304 & -0.5304 & 0.5304 & -0.5304 \\ 0.5304 & -0.5304 & -0.5304 & 0.5304 \\ S_{y4} & 0.5304 & -0.5304 & 0 \\ 0.5304 & -0.5304 & 0.5304 & -0.5304 \end{bmatrix}$

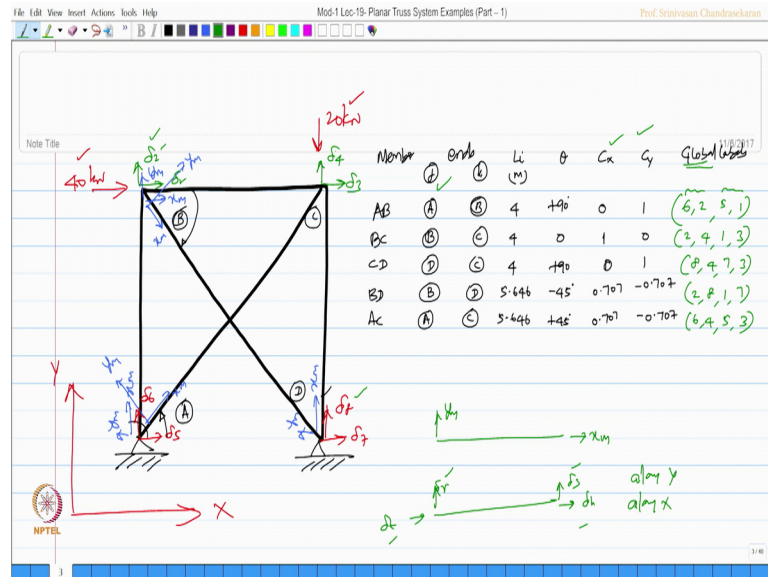
So we have got A B, B C, C D, B D and A C all the members.

Let us continue with the discussion on estimating the joint loads for this problem.

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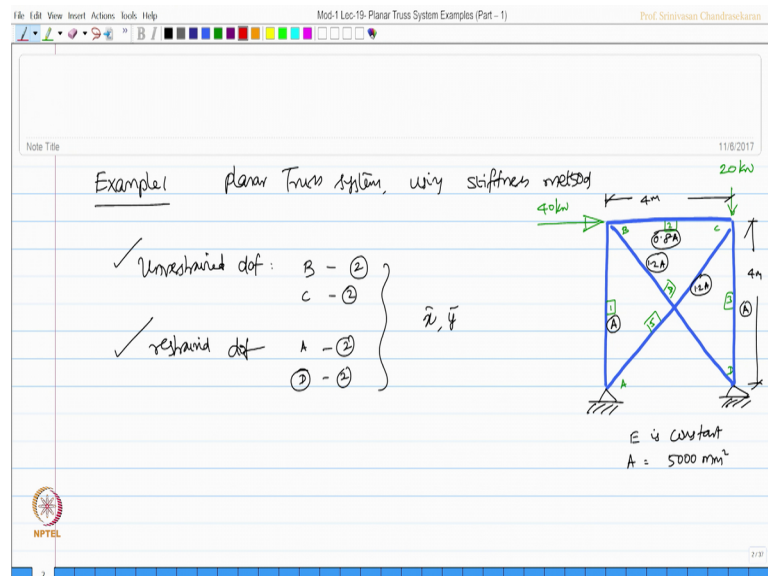


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So, looking at this example you know the joint forces are applied at degrees of freedom you can see here delta one the problem the forces are applied at 40 and 20 here.

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That is do that at this is 40 kilo Newton and this is 20 kilo Newton. So, now, the joint load vector at delta one you get 40 and delta 2 0, delta 3 0, delta 4 is again 20. So, let us do the joint load vector in the simple term like this.

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$$\{J_L\} = \begin{Bmatrix} +40 \\ 0 \\ 0 \\ -20 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \begin{matrix} ① \\ ② \\ ③ \\ ④ \\ ⑤ \\ ⑥ \\ ⑦ \\ ⑧ \end{matrix}$$

$$\{d\} = [K_{uu}]^{-1} \{J_{Lu}\}$$

$$[K_{uu}] = E \begin{bmatrix} ① & ② & ③ & ④ \\ 0.0015 & -0.0005 & -0.0010 & 0 \\ & 0.0018 & 0 & 0 \\ \text{Sym} & 0.0015 & 0.0005 & \\ & & & 0.0018 \\ & & & & ④ \end{bmatrix} \begin{matrix} ① \\ ② \\ ③ \\ ④ \end{matrix}$$

So, the joint load vector bar that is a global is plus 40, 0, 0, minus 20, the labels could be 1, 2, 3, 4 and 5, 6, 7, 8. So, there is a partition here. Now I can apply this equation delta bar that is the reference axes displacement will be given by K uu inverse multiplied by J L u. Now I have K uu inverse I have to assembling this stiffness matrix we get K uu inverse as let us first find K uu which will be actually E times of 0.0015 minus 0.0005 minus 0.0010 and 0.

0.0018, 0, 0, 0.0015 and 0.0005 and 0018 with symmetry here and the labels could be 1, 2, 3 and 4 for the unrestrained degrees of freedom.

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$$[k_{uu}]^{-1} = \frac{1}{E} \times 10^3 \begin{bmatrix} 1.5534 & 0.4628 & 1.1319 & -0.3372 \\ 0.6995 & 0.3372 & -0.1005 & 0 \\ 1.5534 & -0.4628 & 0 & 0 \\ 0.6995 & 0 & 0 & 0 \end{bmatrix}$$

$$\{\bar{\delta}\} = [k_{uu}]^{-1} \{J_{uu}\}$$

Now, I find K_{uu} inverse which will be one by E 10 power 3 1.5534 , 0.4628 , 1.1319 minus 0.3372 , 0.6995 , 0.3372 minus 0.1005 , 1.5534 minus 0.4628 and 0.6995 .

That is the labels could be again 1 , 2 , 3 and 4 , this is K_{uu} inverse; now I apply this equation to find the displacement in reference axes system as k_{uu} inverse multiplied by J_{uu} I substitute this.

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$$\{\bar{\delta}\} = \frac{1 \times 10^4}{E} \begin{bmatrix} 6.8881 \\ 2.0521 \\ 5.4532 \\ -2.7477 \end{bmatrix}$$

Now, I will get delta bar with reference to the reference axes system as 1 into 10 power 4 by E 6.8881, 2.0521, 5.4532 and minus 2.7479, the degrees of freedom are going to be 1, 2 and 3 and four.

So, that is my global displacement vector in the reference axes system, once I get this, I can always find the member forces in each member.

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Member forces

$$\bar{M}_{AB} = \begin{Bmatrix} \bar{V}_6 \\ \bar{V}_2 \\ \bar{H}_5 \\ \bar{H}_1 \end{Bmatrix} = \begin{Bmatrix} -25.6509 \\ 25.6509 \\ 0 \\ 0 \end{Bmatrix}$$

$$\bar{M}_{BC} = \begin{Bmatrix} \bar{V}_2 \\ \bar{V}_4 \\ \bar{H}_1 \\ \bar{H}_5 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 14.3491 \\ -14.3491 \end{Bmatrix}$$

So, for A B, let us say I want to find the member forces for A B which will be V bar 6, V bar 2, H bar 5 and H bar 1 which will amount to minus 25.6509 plus 25.6509 and 0s, similarly M B C these are all M bars will be V bar 2, V bar 4, H bar 1 and H bar 5 which will be 0, 0, 14.3491 and minus 14.3491.

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The image shows two matrix equations written on a digital notepad. The first equation is for member CD:

$$\bar{M}_{CD} = \begin{Bmatrix} \bar{V}_8 \\ \bar{V}_4 \\ \bar{H}_7 \\ \bar{H}_3 \end{Bmatrix} = \begin{Bmatrix} 34.3491 \\ -34.3491 \\ 0 \\ 0 \end{Bmatrix}$$

The second equation is for member BD:

$$\bar{M}_{BD} = \begin{Bmatrix} \bar{V}_2 \\ \bar{V}_8 \\ \bar{H}_1 \\ \bar{H}_7 \end{Bmatrix} = \begin{Bmatrix} -25.6509 \\ 25.6509 \\ 25.6509 \\ -25.6509 \end{Bmatrix}$$

The notepad interface includes a menu bar (File, Edit, View, Insert, Actions, Tools, Help), a toolbar with drawing tools, and a status bar at the bottom showing the slide number 14 and the NPTEL logo.

Let us do it for M C D which will be labeled as V bar 8, V bar 4, H bar 7 and H bar 3 which will be 34.3491 minus 34.3491, 0, 0. Similarly M bar B D which will be labeled as V bar 2, V bar 8, H bar 1 and H bar 7 which will be minus 25.6509, 25.6509, 25.6509, again 25.6509.

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The image shows a matrix equation written on a digital notepad for member AC:

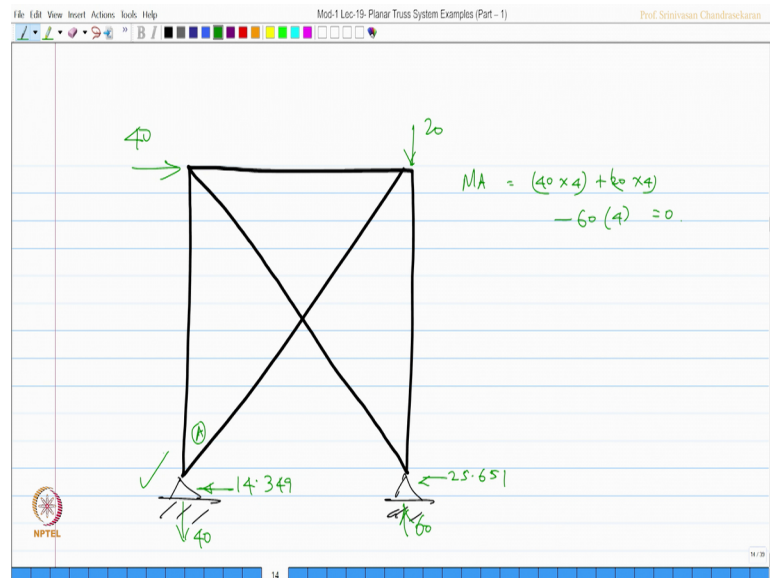
$$\bar{M}_{AC} = \begin{Bmatrix} \bar{V}_6 \\ \bar{V}_4 \\ \bar{H}_5 \\ \bar{H}_3 \end{Bmatrix} = \begin{Bmatrix} -14.3491 \\ 14.3491 \\ -14.3491 \\ 14.3491 \end{Bmatrix}$$

The notepad interface includes a menu bar (File, Edit, View, Insert, Actions, Tools, Help), a toolbar with drawing tools, and a status bar at the bottom showing the slide number 11 and the NPTEL logo.

Let us work out M bar A C which will have labels has V bar 6, V bar 4, H bar 5 and H bar 3 which will amount to minus 14.3491, 14.3491 minus 14.3491 and 14.3491.

Assembling all these M bar values of all the members they can always find the end reaction has I show you here now.

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So, this is my given truss system with these are my supports; now if you look at the final reactions the final reactions are going to be plus 40, then this is minus 20, then this is minus 40, then this is plus 60 and this is 14.349 and 25.651 checking that you know if you take moment about the point A. So, you know as going to be 40 into 4 plus 20 into 4 again minus 60 into 4 which will be 0 which is conformed.

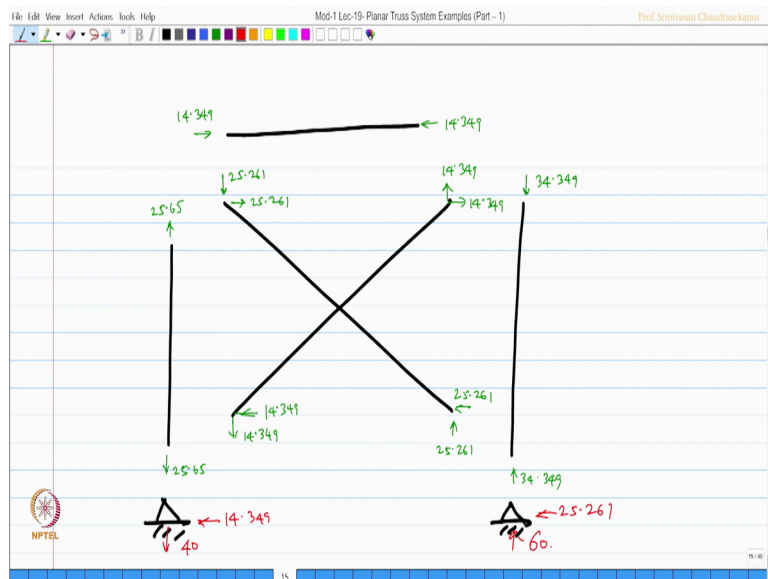
So, that is how we get the final reaction one can also find the forces in the members as we can take it from each member.

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```

MATLAB CODE
% Stiffness matrix method for planar truss
% Input
clear;
n = 5; % Number of members
L = [4 4 5.656 5.656]; % Length in m
A = [58-3 48-3 58-3 48-3]; % Area in m2
theta = [90 0 90 -45 45]; % Angle in degrees
uu = 4; % Number of unrestrained degrees of freedom
ur = 4; % Number of restrained degrees of freedom
uul = [1 2 3 4]; % Global labels of unrestrained dof
url = [5 6 7 8]; % Global labels of restrained dof
l1 = [6 2 5 1]; % Global labels for member 1
l2 = [2 4 1 3]; % Global labels for member 2
l3 = [18 4 7 3]; % Global labels for member 3
l4 = [2 8 1 7]; % Global labels for member 4
l5 = [16 4 5 3]; % Global labels for member 5
l = [l1; l2; l3; l4; l5];
dof = uu + ur; % Degrees of freedom
Rfixal = zeros(4*dof);
Tl1 = zeros(4); % Transformation matrix for member 1
Tl2 = zeros(4); % Transformation matrix for member 2
Tl3 = zeros(4); % Transformation matrix for member 3
Tl4 = zeros(4); % Transformation matrix for member 4
Tl5 = zeros(4); % Transformation matrix for member 5
feal = [0; 0; 0; 0]; % Local Fixed end forces of member 1
feal2 = [0; 0; 0; 0]; % Local Fixed end forces of member 2
feal3 = [0; 0; 0; 0]; % Local Fixed end forces of member 3
feal4 = [0; 0; 0; 0]; % Local Fixed end forces of member 4
feal5 = [0; 0; 0; 0]; % Local Fixed end forces of member 5
    
```

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So, member 1, member 2, member 3, member 4 and member 5; let us do it separately for each member. So, this is 25.65 and 25.65; you can always find this from this vector, if you say MAB it is $V6$, if you look at the degrees of freedom then always the mark the degrees of freedom. So, this is going to be 1, 2, 3, 4, 5, 6; the labels are 6 2 and then 5 and 1.

So, conforming that look at $AB62511$. So, 6 minus 2 is plus I am marking it as minus and plus other is 0; similarly for this member this is going to be 14.349 and 14.349 and

for this member; it is 34.349 and 34.349 and for this member, it is 25.261; 25.261. Similarly 25.261 and 25.261 and for this member, it is 14.349 and 14.349, this is also 14.349 and this is 14.349 positive.

So, I got the reactions. Now I convert that into the system. So, I get now the values as minus 40 plus 60 and 14.349 and 25.261 which is as same as what you get here. So, the problem is now solved. We will do one more example. Now in this case, first we will discuss the computer program for this problem you know there are 5 members in this problem. So, there are 5 members we entered the length of the each member we entered area of cross section of each member.

Then we entered the values of theta please understand, if you member is arbitrarily oriented this is my x M and this is my y M, if this is my reference axes theta is always measured anti clockwise positive entered the values of theta then labels of every member is entered which we already have with us for example; L 1 member 1 has 6 2 5 1, see here member 1 has 6 2 5 1. So, similarly we can entered these members labels, then we find the transformation matrix for each member.

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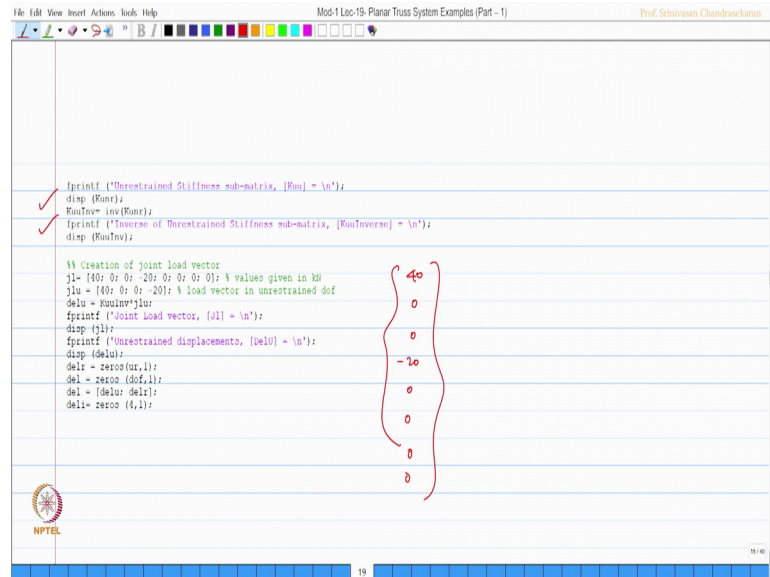
```

file Edit View Insert Actions Tools Help
Mod-1 Lec-19- Planar Truss System Examples (Part - 1) Prof. Srinivasan Chandrasekaran
for p = 1:4
    for q = 1:4
        Kew = (1/(i,p), 1/(i,q)) * Kg(p,q);
    end
    Ktotal = Ktotal + Kew;
end
if i == 1
    Tt1 = T;
    Kq1 = Kg;
    fembar1 = Tt1*fem1;
elseif i == 2
    Tt2 = T;
    Kq2 = Kg;
    fembar2 = Tt2*fem2;
elseif i == 3
    Tt3 = T;
    Kq3 = Kg;
    fembar3 = Tt3*fem3;
elseif i == 4
    Tt4 = T;
    Kq4 = Kg;
    fembar4 = Tt4*fem4;
else
    Tt5 = T;
    Kq5 = Kg;
    fembar5 = Tt5*fem5;
end
end
fprintf('Stiffness Matrix of complete structure, [Ktotal] = \n');
disp(Ktotal);
sumr = zeros(sum);
for x=1:sum
    for y=1:sum
        sumr(x,y) = Ktotal(x,y);
    end
end

```

Then we obtain the assembly of the local stiffness matrix of each member and find the global stiffness matrix of each member \bar{K} , then we assemble them.

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```
fprintf('Unrestrained Stiffness sub-matrix, [Kuu] = \n');
disp(Kuu);
KuuInv= inv(Kuu);
fprintf('Inverse of Unrestrained Stiffness sub-matrix, [KuuInverse] = \n');
disp(KuuInv);

%% Creation of joint load vector
j1= [40; 0; 0; -20; 0; 0; 0; 0]; % value given in kN
j1u = [40; 0; 0; -20]; % load vector in unrestrained dof
delu = KuuInv*j1u;
fprintf('Joint Load vector, [j1] = \n');
disp(j1);
fprintf('Unrestrained displacement, [delu] = \n');
disp(delu);
deli = zeros(4,1);
del = zeros(8,1);
del = [delu; deli];
deli = zeros(4,1);
```

And get the stiffness matrix completely partition them and get the unrestrained stiffness matrix and get the inverse of the stiffness matrix entered the joint load vector the joint load vector we can say from here I write down is going to be 40, 0, 0, minus 20, 0, 0, 0, 0, that is the value I have; let us compare this with the joint load vector here which is exactly the same see 40, 0, 0 and minus 20, 4 0s. So, this is exactly the same here you have is or not 40, 0, 0, minus 20, 0 0 0 0.

So, we have the joint load vector then we found out the unrestrained displacement values.

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```
file Edit View Insert Actions Tools Help Mod-1 Lec-19- Planar Truss System Examples (Part - 1) Prof. Srinivasan Chandrasekaran
for i = 1:n
    for p = 1:4
        deli(p,1) = del((1,i,p),1);
    end
    if i == 1
        delbar1 = deli;
        mbar1 = (K3 * delbar1)+fembar1;
        fprintf('Member Number =');
        disp(i);
        fprintf('Global displacement matrix [DeltaBar] = \n');
        disp(delbar1);
        fprintf('Global End moment matrix [MBar] = \n');
        disp(mbar1);
    elseif i == 2
        delbar2 = deli;
        mbar2 = (K2 * delbar2)+fembar2;
        fprintf('Member Number =');
        disp(i);
        fprintf('Global displacement matrix [DeltaBar] = \n');
        disp(delbar2);
        fprintf('Global End moment matrix [MBar] = \n');
        disp(mbar2);
    elseif i == 3
        delbar3 = deli;
        mbar3 = (K3 * delbar3)+fembar3;
        fprintf('Member Number =');
        disp(i);
        fprintf('Global displacement matrix [DeltaBar] = \n');
        disp(delbar3);
        fprintf('Global End moment matrix [MBar] = \n');
        disp(mbar3);
    end
end
```

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```
file Edit View Insert Actions Tools Help Mod-1 Lec-19- Planar Truss System Examples (Part - 1) Prof. Srinivasan Chandrasekaran
elseif i == 4
    delbar4 = deli;
    mbar4 = (K4 * delbar4)+fembar4;
    fprintf('Member Number =');
    disp(i);
    fprintf('Global displacement matrix [DeltaBar] = \n');
    disp(delbar4);
    fprintf('Global End moment matrix [MBar] = \n');
    disp(mbar4);
elseif i == 5
    delbar5 = deli;
    mbar5 = (K5 * delbar5)+fembar5;
    fprintf('Member Number =');
    disp(i);
    fprintf('Global displacement matrix [DeltaBar] = \n');
    disp(delbar5);
    fprintf('Global End moment matrix [MBar] = \n');
    disp(mbar5);
end
end
```

Then we find the member forces then we check the member forces endly for each member that is the computer program we have which we have used; solved this problem for simple planar truss with 5 member.

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```
File Edit View Insert Actions Tools Help Mod-1 Loc-19-Planar Truss System Examples (Part - 1) Prof. Srinivasan Chandrasekaran
Local Stiffness matrix of member, [K] =
Member Number = 1
Local Stiffness matrix of member, [K] =
1.0e-03 *
0 0 0 0 0 0
0 0 0 0 0 0
0 0 0.0013 -0.0013 0 0
0 0 -0.0013 0.0013 0 0
Transformation matrix of member, [T] =
0 0 -1 0 0 0
0 0 0 -1 0 0
1 0 0 0 0 0
0 1 0 0 0 0
Transformation matrix of member, [T] =
1 0 0 0 0 0
0 1 0 0 0 0
0 0 1 0 0 0
0 0 0 -1 0 0
Transformation matrix transpose, [T] =
0 0 1 0 0 0
0 0 0 1 0 0
-1 0 0 0 0 0
0 -1 0 0 0 0
Transformation matrix Transpose, [T] =
1 0 0 0 0 0
0 1 0 0 0 0
0 0 1 0 0 0
0 0 0 1 0 0
Global Matrix, [K global] =
0.0013 -0.0013 0 0 0 0
-0.0013 0.0013 0 0 0 0
0 0 0 0 0 0
0 0 0 0 0 0
Global Matrix, [K global] =
1.0e-03 *
0 0 0 0 0 0
0 0 0 0 0 0
0 0 1.0000 -1.0000
0 0 -1.0000 1.0000
22
```

Let us do one more problem; these are typical output which we have the answers for this member. I wish you should go through them thoroughly and try to compare the values what we have obtain for this member.