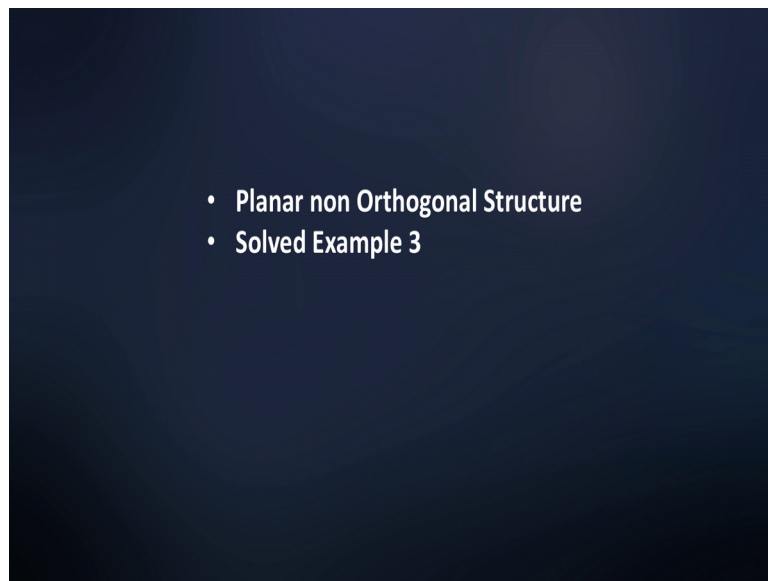


Computer Methods of Analysis of Offshore Structures
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Module - 01
Lecture - 17
Non-orthogonal frame - Example 3

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Friends, welcome to the 17th lecture in module 1. We will discuss one more example problem on planar non orthogonal frame. We will solve this problem on the fundamental steps and then use the computer **codes**.

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Unrestrained dof : $\{\theta_1, \theta_2, \delta_3, \delta_4, \delta_5, \delta_6\}$
 restrained dof : $\{\delta_7, \delta_8, \delta_9, \theta_{10}, \delta_{11}, \delta_{12}\}$

| Member | (j) | (k) | θ | A (m ²) | I (m ⁴) | L _i | C _x | C _y | global labels |
|--------|-----|-----|----------|---------------------|---------------------|----------------|----------------|----------------|---|
| AB | (A) | (B) | 0° | 0.135 | 2.28m ⁴ | 4m | 0 | 1 | $\{ \delta_3, \delta_4, \delta_5 \}$ |
| BC | (B) | (C) | 0° | 0.150 | 3.125m ⁴ | 6m | 1 | 0 | $\{ \delta_7, \delta_8, \delta_9 \}$ |
| CD | (C) | (D) | -53.13° | 0.135 | 2.28m ⁴ | 5m | 0.6 | -0.8 | $\{ \delta_{10}, \delta_{11}, \delta_{12} \}$ |

K_{AB}, k_{BC}, k_{CD}

Which we discussed in the last lecture to solve this problem the input frame which is going to be analyzed is shown in the screen now. So, if the dimensions of the frame are given, 6 meter and 3 meters and this is 4 meters. The frame is subjected to you know uniform distributed load on the member, vertical column of intensity 30 kilo Newton per meter and a vertical load of 100 kilo Newton at a distance of 2 meters and 4 meters as marked in the problem.

Let us now mark the degrees of freedom **unrestrained** and **restrained** for the given problem. So, unrestrained degrees are marked in green. So, theta 1, theta 2 delta 3 and delta 4 5 6 now the restrained degrees are marked in red 7 8 and 9 10 11 and 12. So, these are actually global degrees of freedom. So, let us **put** the bars on them for our clear understanding these are global degrees of freedom.

So, the global axis is indicated here, this is my x axis and y axis this is the reference axis system and of course, each members will have a local axis which are going to identify. So, now, let us say the unrestrained degrees are 6, in number that is theta 1 theta 2 delta 3 4 5 and 6 and restrained degrees of freedom are 7 8 and 9 10 11 and 12. Let us now mark the local axes, let us take a table let us say a member the joint which is going to be the j-th end and the k-th end then the corresponding theta of the member.

Then the area and moment of inertia of the member length of the member then C x and C y and then the global labels let us make this table, let us take the member a b this is

member 1 this is member 2 this is member 3; let us say member 1 which is at A, B, C and D. So, remember AB which is the first member.

The local axis the joint is at A and this is at B. So, the local axis is marked here. So, this is going to be my x m and y m. So, the direction of rotation of the local with respect to global is this angle anti clockwise positive therefore, I should say this is 90 degrees.

Now, for the member BC which is member number 2, now local axis and global axis have marked as x m and y m. So, the angle is 0 degrees and the members are j th node z B this is z c now the member CD member indication 3, the local axis is xm and normal to that is y m you know this is my corresponding reference axis and this is my angle which is actually as same as this angle this angle is actually can be computed as 53.123 degrees from the geometry.

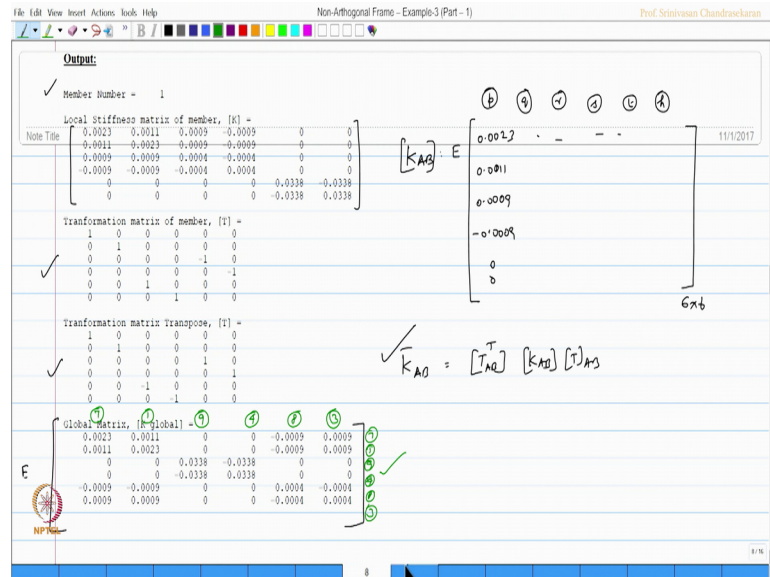
So, now this is clock wise. So, I am saying the ends are at C and D. So, the angle is going to be 53.123 degrees. So, one can work out the cos and sin of values of them respectively yt let us say what is the area and moment of inertia of this sections, let say the i of this section is going to be 2.28 into 10 power minus 3, we are entering it in meter 4.

I 2 is 3.125 into 10 power minus 3 and this is same as 2.28 into 10 power minus 3; area in square meters this member has area of 0.135 this has 0.150 this has 0.135 length of the member AB is 4 meters BC is 6 meters.

And CD is 5 meters sum of root of sum of squares let us enter the global labels for the member AB, global labels should always say the order as moments in the i j and k end then along y in j and k end and along x in j and k end. So, by that logic this label will be 7 1 9 4 8 3 is it not.

Similarly, for member BC 1 2 4 6 3 5 for member CD its going to be 2 10 6 12 5 11; so we have the labels entered. Now, one can then compute the local stiffness matrix of the member AB member BC and member CD you have seen the program let us find out those values.

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Let us say the inputs are as given in the standard program this is local member number 1.

The local stiffness matrix which will be entered here is what I am writing here this is K_{AB} which will have an E multiplier outside 0.0023 0.011, 0.0009 minus 000900 and so, on which will be a 6 by 6 matrix and labels will be p q r s t and h there is no numbering for this label because this is a local stiffness matrix ok.

Now, we also worked out the transformation matrix and the transpose of this. So, I can find K_{AB} which is the global stiffness matrix which is T for AB transpose K local AB then T for AB I get K global say K global is entered here with a multiplier of E outside and we get K global now. So, I have K global AB .

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Member Number = 2

Local Stiffness matrix of member, [K] =

| | | | | | |
|---------|---------|---------|---------|---------|---------|
| 0.0021 | 0.0010 | 0.0005 | -0.0005 | 0 | 0 |
| 0.0010 | 0.0021 | 0.0005 | -0.0005 | 0 | 0 |
| 0.0005 | 0.0005 | 0.0002 | -0.0002 | 0 | 0 |
| -0.0005 | -0.0005 | -0.0002 | 0.0002 | 0 | 0 |
| 0 | 0 | 0 | 0 | -0.0250 | -0.0250 |
| 0 | 0 | 0 | 0 | -0.0250 | 0.0250 |

Note Title

Transformation matrix of member, [T] =

| | | | | | |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 |

Transformation matrix Transpose, [T]^T =

| | | | | | |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 |

Global Matrix, [K_{global}]

| | | | | | |
|---------|---------|---------|---------|---------|---------|
| 0.0021 | 0.0010 | 0.0005 | -0.0005 | 0 | 0 |
| 0.0010 | 0.0021 | 0.0005 | -0.0005 | 0 | 0 |
| 0.0005 | 0.0005 | 0.0002 | -0.0002 | 0 | 0 |
| -0.0005 | -0.0005 | -0.0002 | 0.0002 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0.0250 | -0.0250 |
| 0 | 0 | 0 | 0 | -0.0250 | 0.0250 |

$K_{BC} = E$

| | | | | | |
|---------|---------|---------|---------|---------|---------|
| 0.0021 | 0.0010 | 0.0005 | -0.0005 | 0 | 0 |
| 0.0010 | 0.0021 | 0.0005 | -0.0005 | 0 | 0 |
| 0.0005 | 0.0005 | 0.0002 | -0.0002 | 0 | 0 |
| -0.0005 | -0.0005 | -0.0002 | 0.0002 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0.0250 | -0.0250 |
| 0 | 0 | 0 | 0 | -0.0250 | 0.0250 |

Handwritten labels: p, q, r, s, t, h

6x6 matrix:

| | | | | | |
|---------|---------|---------|---------|---------|---------|
| 0.0021 | 0.0010 | 0.0005 | -0.0005 | 0 | 0 |
| 0.0010 | 0.0021 | 0.0005 | -0.0005 | 0 | 0 |
| 0.0005 | 0.0005 | 0.0002 | -0.0002 | 0 | 0 |
| -0.0005 | -0.0005 | -0.0002 | 0.0002 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0.0250 | -0.0250 |
| 0 | 0 | 0 | 0 | -0.0250 | 0.0250 |

Similarly, for member number 2 I have K local for the member AB; so K local for the member BC again a multiplier E with 6 by 6 ok.

So, the values are 0.0021 0.0010, 0 point triple 05 minus triple 05 and 0 and 0 and so, on. So, the labels could be again p q r s t h, which are local labels, which cannot be assigned to a global labels until you do this for transformation. So, when I say now the k global for AB is available then I can enter the labels as 7 1 9 4 8 3. So, this is because these are global labels.

Similarly, for the member BC the labels could be 1 2 3 sorry 4 6 3 1 2 3 1 and 5 we can do this.

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Member Number = 3

Local Stiffness matrix of member, [K] =

| | | | | | |
|---------|---------|---------|---------|---------|---------|
| 0.0018 | 0.0009 | 0.0005 | -0.0005 | 0 | 0 |
| 0.0009 | 0.0018 | 0.0005 | -0.0005 | 0 | 0 |
| 0.0005 | 0.0005 | 0.0002 | -0.0002 | 0 | 0 |
| -0.0005 | -0.0005 | -0.0002 | 0.0002 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0.0270 | -0.0270 |
| 0 | 0 | 0 | 0 | -0.0270 | 0.0270 |

Transformation matrix of member, [T] =

| | | | | | |
|--------|--------|---------|---------|--------|--------|
| 1.0000 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1.0000 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0.6001 | 0 | 0.7999 | 0 |
| 0 | 0 | 0 | 0.6001 | 0 | 0.7999 |
| 0 | 0 | -0.7999 | 0 | 0.6001 | 0 |
| 0 | 0 | 0 | -0.7999 | 0 | 0.6001 |

Transformation matrix Transpose, [T]^T =

| | | | | | |
|--------|--------|--------|--------|---------|---------|
| 1.0000 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1.0000 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0.6001 | 0 | -0.7999 | 0 |
| 0 | 0 | 0 | 0.6001 | 0 | -0.7999 |
| 0 | 0 | 0.7999 | 0 | 0.6001 | 0 |
| 0 | 0 | 0 | 0.7999 | 0 | 0.6001 |

Global Matrix, [K_g] = E [K] [T]^T [T]

| | | | | | |
|---------|---------|---------|---------|---------|---------|
| 0.0018 | 0.0009 | 0.0003 | -0.0003 | -0.0004 | -0.0004 |
| 0.0009 | 0.0018 | 0.0003 | -0.0003 | 0.0004 | -0.0004 |
| -0.0003 | -0.0003 | -0.0174 | -0.0174 | -0.0129 | -0.0129 |
| -0.0003 | -0.0003 | -0.0174 | -0.0174 | 0.0129 | 0.0129 |
| 0.0004 | 0.0004 | 0.0129 | 0.0129 | 0.0099 | -0.0099 |
| -0.0004 | -0.0004 | 0.0129 | -0.0129 | -0.0099 | 0.0099 |

Handwritten notes: K_{AB} , K_{BC} , K_{CD} are grouped and labeled. A note says "Assemble them to form the total stiffness matrix". The global matrix is labeled $K_g = E$ and has nodes 2, 10, 6, 12, 5, 11 circled.

We also found for the member number three the local stiffness matrix with a multiplier E outside we did transformation and we got k_g for member CD; with an E multiplier outside and the labels could be 2 10 6 12 5 11. Now, friends we have K global AB, K global BC and K global CD with us with the labels ok.

We can assemble them to form the total stiffness matrix. So, we use the program which I gave you in the last lecture and we got K total which is here.

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Stiffness Matrix of complete structure, [K_{total}] =

| | | | | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|--------|---------|---------|---------|---------|
| 0.0044 | 0.0019 | 0.0019 | 0.0005 | -0.0005 | 0.0011 | -0.0009 | 0 | 0 | 0 | 0 | 0 |
| 0.0019 | 0.0037 | 0 | 0.0009 | -0.0004 | -0.0004 | 0 | 0 | 0 | 0.0009 | -0.0004 | -0.0003 |
| 0.0019 | 0 | 0.0034 | -0.0020 | 0 | 0.0009 | -0.0004 | 0 | 0 | 0 | 0 | 0 |
| 0.0005 | 0.0009 | 0 | 0.0027 | 0 | -0.0002 | 0 | 0 | -0.0002 | 0 | 0 | 0 |
| 0 | 0.0004 | -0.0020 | 0 | 0.0048 | -0.0119 | 0 | 0 | 0 | 0.0004 | -0.0018 | 0.0179 |
| -0.0005 | -0.0004 | 0.0020 | -0.0002 | 0.0129 | 0.0129 | 0 | 0 | 0.0005 | 0.0017 | -0.0174 | 0 |
| 0.0011 | 0 | 0.0004 | 0 | 0 | 0.0033 | -0.0004 | 0 | 0 | 0 | 0 | 0 |
| -0.0009 | 0 | 0.0004 | 0 | -0.0030 | 0 | 0.0004 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | -0.0030 | 0 | 0 | 0.0030 | 0 | 0 | 0 | 0 |
| 0 | -0.0004 | 0 | -0.0004 | -0.0004 | 0 | 0 | 0 | -0.0004 | -0.0004 | -0.0119 | 0 |
| 0 | -0.0004 | 0 | 0 | -0.0004 | 0 | 0 | 0 | 0 | 0 | -0.0119 | 0.0174 |

Unrestrained Stiffness sub-matrix, [K_{uu}] =

| | | | | | |
|---------|---------|---------|---------|---------|---------|
| 0.0044 | 0.0010 | 0.0009 | 0.0005 | 0 | -0.0005 |
| 0.0010 | 0.0039 | 0 | 0.0005 | 0 | -0.0004 |
| 0.0009 | 0 | 0.0254 | 0 | 0.0250 | 0 |
| 0.0005 | 0.0005 | 0 | 0.0339 | 0 | 0.0002 |
| 0 | 0.0004 | -0.0250 | 0 | 0.0349 | -0.0129 |
| -0.0005 | -0.0002 | 0 | -0.0002 | -0.0129 | 0.0175 |

Inverse of Unrestrained Stiffness sub-matrix, [K_{uu}⁻¹] =

| | | | | | |
|---------|---------|---------|---------|---------|---------|
| 0.2587 | -0.0592 | -0.1255 | -0.0035 | -0.1188 | -0.0801 |
| -0.0592 | 0.2772 | -0.0618 | -0.0036 | -0.0649 | -0.0664 |
| -0.1255 | -0.0618 | 1.2623 | 0.0075 | 1.2396 | 0.9046 |
| -0.0035 | -0.0036 | 0.0075 | 0.0296 | 0.0075 | 0.0057 |
| -0.1188 | -0.0649 | 1.2396 | 0.0075 | 1.2567 | 0.9175 |
| -0.0801 | -0.0664 | 0.9046 | 0.0057 | 0.9175 | 0.7271 |

Handwritten notes: K_{uu} is circled in red. K_{uu}^{-1} is also circled in red. A note says "Assemble them to form the total stiffness matrix".

So, again this is K bar total which is global which has a multiplier E outside and now the labels are continuous the labels are continuous 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12.

Similarly, the first row second row third row and forth row and so on till 12 we already know this program or this problem has 6 **unrestrained** degrees of freedom, we said that there are six **unrestrained** degrees of freedom right. So, therefore, we do the partition at this level and we pick up only this matrix this is nothing but K **unrestrained** degree which we write here.

So, again there is an E multiplier which is actually K bar uu , we have pick up that then found the inverse of this matrix which will be 1 by E of this which is K **unrestrained** inverse which is given by this equation and the solution is here. So, now we have K uu bar.