

Computer Methods of Analysis of Offshore Structures
Prof. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Module - 01
Lecture – 15
Example problem - Planar non-orthogonal structure

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- Solved Examples on Planar Orthogonal Structures
- Computer Code

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Example 1

$10kN$

$M_0 I = 0.0016 m^4$
 $E = \text{constant}$
 $A = 0.120 m^2$

$\theta = \tan^{-1}(4/3) = 63.435^\circ$

member no	Ends	l_i	θ	C_x	C_y	global labels
1	(A) (B)	$\sqrt{4^2+4^2} = 4\sqrt{2} = 5.6568$	$+63.435^\circ$	0.447	0.894	(1, 2, 3)
2	(B) (C)	4m	0	1	0	(4, 5, 6)

Let us estimate the joint load vector because if you look at the figure the structure does not have any member loading it has only one load in the joint. So, directly I can write the joint load vector because there are no fixed end moments generated by the loads on the member. So, one carefully can observe that this 10 kilo Newton load is applied along with the degree of freedom which is 3 remaining all joint loads are practically 0.

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The image shows handwritten notes on a digital whiteboard. The title is "Transformation matrix".

The transformation matrix $[T]_{AB}$ is given as:

$$[T]_{AB} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_x & 0 & -c_y & 0 \\ 0 & 0 & 0 & c_x & 0 & -c_y \\ 0 & 0 & c_y & 0 & c_x & 0 \\ 0 & 0 & 0 & c_y & 0 & c_x \end{bmatrix}$$

The joint load vector $\{J_u\}$ is given as:

$$\{J_u\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 10 \end{Bmatrix}$$

The matrix is partitioned into unrestrained and restrained parts:

$$\{J_u\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 10 \end{Bmatrix} = \begin{Bmatrix} J_{Lu} \\ J_{Lr} \end{Bmatrix}$$

The unrestrained part J_{Lu} has 3 degrees of freedom, and the restrained part J_{Lr} has 6 degrees of freedom.

So, with this logic the joint load vector can be entered as 0 0 plus 10. So, there are 12, 1 2, 3, 6; I mean 9 values; is it not there is a 9 degree of freedom, here we have total 9 degrees of freedom; is it not. So, we have 9 the vector is 9 by 1. So, one can make a sub matrix of this. So, this is J L unrestrained and this is J L restrained why unrestrained there are 3 unrestrained degrees of freedom for this problem, there are 3 green unrestrained degrees of freedom; therefore, I partition this matrix at 3 let me J L u.

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Handwritten notes on a digital whiteboard showing matrix transformations for stiffness matrices. The notes include equations for local to global transformations for a general member, member AB, and member BC. To the right, two 6x6 matrices are shown with global labels 1-6 and 7-12. The top matrix is labeled K_{AB} and the bottom one K_{BC} .

$$[\bar{k}]_i = [T]_i^T [k]_i [T]_i$$

$$[\bar{k}_{AB}]_i = [T]_{AB}^T [k]_{AB} [T]_{AB}$$

$$[\bar{k}_{BC}]_i = [T]_{BC}^T [k]_{BC} [T]_{BC}$$

So, J L u is actually 0 0 plus 10. In fact, I can call this a J L u bar; is it not because these degrees of freedom are related to the reference axes system x y, I get the J L bar. Once I get that I can now find the total stiffness matrix. First let me find K bar of each member as T transpose of the ith member with K of the ith member with T of the ith member. So, now, I can do this for K A B which will be T transpose of A B K of A B and T of A B, then I do this for B C which will be T transpose of B C; K of B C and then T of B C.

Now, K A B will have labels K A B K bar; A B will have now labels you can be identify this labels 7, 1, 9, 2, 8, 3; is it not. So, 7, 1, 9, 2, 8, 3 these are global labels. So, 7, 1, 9, 2, 8 and 3 there are global labels. Similarly I can also find for K B C bar which will again be a 6 by 6 the labels will be 1, 4, 2, 5, 3, 6, 1, 4, 2, 5, 3, 6. So, what I do; I assemble this matrix and get K total.

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$\bar{K}_{total} = 9 \times 9$

\bar{K}_{uu} 3x3	\bar{K}_{ur}
\bar{K}_{ru}	\bar{K}_{vv}

$\bar{K}_{AB} = E \times 10^{-4}$

14	7	2	-2	-4	4	7	1	9
7	14	2	-2	-4	4	1	9	2
2	2	215	-215	106	-106	9	2	8
-2	-2	-215	215	-106	106	7	1	9
-4	-4	106	-106	55	-55	8	3	7
4	4	-106	106	-55	55	7	1	9

I get K bar total which will be a 9 by 9 matrix which will contain a partition sub matrices this will be K u u; this is K u r; this is K r u; this is K r r; all will be bars because this is K bar all will be with reference to the reference axes x y system now.

K u u will be of size 3 by 3; I plug out K u u and K u u bar for this structure can be worked out from the program which we get this as E into 10 power minus 4. Before that let us write down the values of let us write down for simplicity write down the values of K bar A B which is E into 10 power minus 4; 14, 7, 2 minus 2 minus 4, 4, 7, 14, 2 minus 2 minus 4, 4, 2, 2, 2, 1, 5 minus 2, 1, 5, 1, 0, 6 minus 1, 0, 6 minus 2 minus 2 minus 2 1, 5, 1, 5 minus 1, 0, 6, 1, 0, 6 minus 4 minus 4, 1, 0, 6 minus 1, 0, 6, 55 minus 55. 4, 4 minus 1, 0, 6, 1, 0, 6 minus 55; 55 the labels will be 7, 1, 9, 2, 8, 3 7, 1, 9, 2, 8, 3 just for a K A B.

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$$\bar{K}_{BC} = E \times 10^{-4} \begin{bmatrix} 16 & 8 & 6 & -6 & 0 & 0 \\ 8 & 16 & 6 & -6 & 0 & 0 \\ 6 & 6 & 3 & -3 & 0 & 0 \\ -6 & -6 & -3 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 300 & -300 \\ 0 & 0 & 0 & 0 & -300 & 300 \end{bmatrix}$$

Let us do it for K bar B C which will be E 10 power minus 4, 16, 8, 6 minus 6, 0, 0, 8, 16, 6 minus 6, 0, 0, 6, 6, 3 minus 3, 0, 0 minus 6 minus 6 minus 3, 3, 0, 0, 300 and minus 300; the labels will be 1, 4, 2, 5, 3, 6, 1, 4, 2, 5, 3, 6.

Now, assemble this matrix get K total then plug out K u u and we get K u u; K unrestrained bar.

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$$\bar{K}_{uu} = E \times 10^{-4} \begin{bmatrix} 30 & 4 & 4 \\ 4 & 218 & 126 \\ 4 & 106 & 355 \end{bmatrix}$$

$$[\bar{d}_u] = [\bar{K}_{uu}]^{-1} [\bar{F}_u]$$

$$[\bar{K}_{uu}]^{-1} = \frac{1}{E} \begin{bmatrix} 330.863 & -4.561 & -2.631 \\ -4.561 & 53.769 & -16.055 \\ -2.631 & -16.055 & 32.984 \end{bmatrix}$$

$$[\bar{d}_u] = \frac{1}{E} \begin{bmatrix} -26.307 \\ -160.545 \\ 329.804 \end{bmatrix}$$

Which is 3 by 3 matrix which will be equal to E into 10 power minus 4, 34, 4, 4, 2, 18, 1, 0, 6, 4, 1, 0, 6, 3, 55 and the degrees of freedom would be 1, 2 and 3 which are

unrestrained as degrees of freedom for the given problem. Let us try to find $K u u$ inverse of this problem which will be one by E times of 330.863 minus 4.561 minus 2.631, 53.769, 32.980.

Now, I do the operation of delta u bar which will be $K u u$ bar inverse multiplied by J L u. Now I have both of them. So, I get delta u bar as 1 by E minus 26.307 minus 160.545 and 329.804.

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$$[\delta]_{AB} = \frac{1}{E} \begin{Bmatrix} 0 \\ -26.307 \\ 0 \\ -160.545 \\ 0 \\ 329.804 \end{Bmatrix} \quad [\delta]_{BC} = \frac{1}{E} \begin{Bmatrix} -26.307 \\ 0 \\ -160.545 \\ 0 \\ 329.804 \\ 0 \end{Bmatrix}$$

$$\bar{M}_i = \bar{k}_i \bar{\delta}_i + [FEM]_i$$

Now, I can interpret delta bar A B as A B as 6 degrees of freedom. So, the values will be 0 minus 26.307 0 minus 160.545, 0, 329.804, delta bar B C will be is 1 by E of 1 by E of minus 26.307 0 minus 160.545, 0, 329.804 and 0.

Now, I want to find M bar that is end moment and reactions of the member A B in simple terms, I can use if you want to find M bar of any ith member, I should say K bar; K bar of the ith member and delta bar of the ith member plus fixed end moments of the ith member; is it not?

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$$\bar{M}_{AB} = \begin{Bmatrix} \bar{M}_2 \\ \bar{M}_1 \\ \bar{V}_9 \\ \bar{V}_2 \\ \bar{H}_8 \\ \bar{H}_3 \end{Bmatrix} = \begin{Bmatrix} 0.1572 \\ 0.1384 \\ -0.0639 \\ 0.0639 \\ -0.1059 \\ 0.1059 \end{Bmatrix}$$

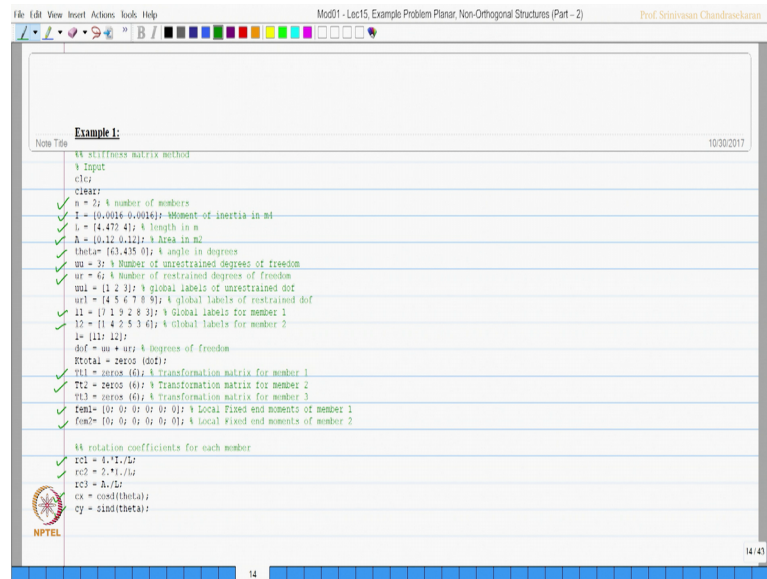
$$\bar{M}_{BC} = \begin{Bmatrix} \bar{M}_1 \\ \bar{M}_4 \\ \bar{V}_2 \\ \bar{V}_5 \\ \bar{H}_3 \\ \bar{H}_6 \end{Bmatrix} = \begin{Bmatrix} -0.1384 \\ -0.1174 \\ -0.0639 \\ +0.0639 \\ 9.8941 \\ -9.8941 \end{Bmatrix}$$

So, by this logic, I can now find \bar{M}_{AB} which will be looked at the degrees of freedom of A B; 7, 1, 9, 2, 8, 3; 7, 1, 9, 2, 8, 3. So, 7; rotation 1, rotation 9 and 2 are vertical reactions 8 and 3 are horizontal reactions keeping that in mind, I can say now this is \bar{M}_{AB} \bar{M}_{AB} 1 rotations that is end moments.

Then vertical reactions that is along y, then horizontal reactions along x at different labels; is it not which is computed as 0.1572, 0.1384 minus 0.0639, 0.0639 minus 0.1059; 0.1059, this for \bar{M}_{AB} ; let us do it for \bar{M}_{BC} which will be look at the labels of \bar{M}_{BC} \bar{M}_{BC} as 1, 4, 2, 5, 3, 6. So, 1 and 2 are 1 and 4 are rotations that is end moments 2 and 5 are vertical reactions 3 and 6 are horizontal reactions keeping that in mind, I can now write \bar{M}_{BC} 1, \bar{M}_{BC} 4, \bar{V}_{BC} 2, \bar{V}_{BC} 5, \bar{H}_{BC} 3 and \bar{H}_{BC} 6; is it not.

So, which will be actually equal to minus 0.1384 minus 0.1174 minus 0.0639 plus 0.0639, 9.8941 minus 9.8941; let us see the computer program how this works.

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Example 1:
% Stiffness matrix method
% Input
clear;
n = 2; % Number of members
I = [0.0016 0.0016]; % Moment of inertia in m^4
L = [4.472 4]; % Length in m
A = [0.12 0.12]; % Area in m^2
theta = [63.435 0]; % Angle in degrees
ur = 3; % Number of unrestrained degrees of freedom
ur = 6; % Number of restrained degrees of freedom
u1 = [1 2 3]; % Global labels of unrestrained dof
u2 = [4 5 6 7 8 9]; % Global labels of restrained dof
l1 = [1 9 2 8 3]; % Global labels for member 1
l2 = [1 4 2 5 3 6]; % Global labels for member 2
i = [11 12];
dof = ur + ur; % Degrees of freedom
Ktotal = zeros (dof);
% K1 = zeros (6); % Transformation matrix for member 1
% K2 = zeros (6); % Transformation matrix for member 2
% K3 = zeros (6); % Transformation matrix for member 3
fem1 = [0 0 0 0 0 0]; % Local fixed end moments of member 1
fem2 = [0 0 0 0 0 0]; % Local fixed end moments of member 2

% Relation coefficients for each member
rc1 = 4.*I./L;
rc2 = 2.*I./L;
rc3 = A./L;
cx = cos(theta);
cy = sind(theta);
```

So, this is the problem there are 2 members in the problem; is it not; let us use the green color, there are 2 members in the problem, I values are given as per the input length as the members are given area of cross sections given in square meters thetas computer for each member.

So, we need C_x , C_y to calculate. So, that has been done here C_x , C_y ; there are 3 unrestrained degrees and 6 restrained degrees the labels of unrestrained of 1, 2, 3 and labels of restrained are 4 to 9 as you see here. So, then we have the local member labels the global member labels for 1, 7, 1, 9, 2, 8, 3 which is as same as this see here 7, 1, 9, 2, 8, 3 which is same as this you can see here 7, 1, 9, 2, 8, 3; similarly 1, 4, 2, 5, 3, 6, then obtain the transformation matrix for member 1 and member 2.

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for i = 1:n
    % know = zeros (6x6);
    k1 = [rc1(i); rc2(i); (rc1(i)+rc2(i))/L(i); -(rc1(i)+rc2(i))/L(i); 0; 0];
    k2 = [rc2(i); rc1(i); (rc1(i)+rc2(i))/L(i); -(rc1(i)+rc2(i))/L(i); 0; 0];
    k3 = [(rc1(i)+rc2(i))/L(i); (rc1(i)+rc2(i))/L(i); (2*(rc1(i)+rc2(i))/L(i)^2); -(2*(rc1(i)+rc2(i))/L(i)^2); 0; 0];
    ki = -k3;
    k5 = [0; 0; 0; 0; rc3(i); -rc3(i)];
    k6 = [0; 0; 0; 0; -rc3(i); rc3(i)];
    K = [k1 k2 k3 k4 k5 k6];
    fprintf('member number =');
    disp(i);
    fprintf('local stiffness matrix of member, [k] = \n');
    disp(K);
    r1 = [1; 0; 0; 0; 0; 0];
    r2 = [0; 1; 0; 0; 0; 0];
    r3 = [0; 0; cx(i); 0; cy(i); 0];
    r4 = [0; 0; 0; cx(i); 0; cy(i)];
    r5 = [0; 0; -cy(i); 0; cx(i); 0];
    r6 = [0; 0; 0; -cy(i); 0; cx(i)];
    T = [r1 r2 r3 r4 r5 r6];
    fprintf('Transformation matrix of member, [T] = \n');
    disp(T);
    Tr = T';
    fprintf('Transformation matrix transpose, [T] = \n');
    disp(Tr);
    K_bar_i = Tr'*K;
    fprintf('Global Matrix, [K global] = \n');
    disp(K_bar_i);
end

```

$\bar{k}_i = [T]^T [K_i] [T]$

Once this is completed; find the fixed end moments in this problem, they are 0, find the rotation coefficients and the axes stiffness coefficient, they got the values, once this is completed, obtain the stiffness matrix for each member. So, obtain K_i local. So, print the local stiffness matrix, then find the transformation matrix and transpose matrix and obtain K_i global. So, now, here you get \bar{K}_i which is nothing, but \bar{K}_i is $T^T K_i T$ of i ; is it not. So, $T^T T^T K_i T$ of i , we got \bar{K}_i of each member, once I get that we compute the FEM and find the stiffness matrix as total structure plug out unrestrained stiffness matrix and get the inverse that is we got \bar{K}_u and we got \bar{K}_u inverse.

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Mod01 - Lec15, Example Problem Planar, Non-Orthogonal Structures (Part - 2) Prof. Srinivasan Chandrasekaran
for q = 1:6
    Knew(i(i,p),i(i,q)) = Kqp,q;
end
Ktotal = Ktotal + Knew;
if i == 1
    r1 = 7;
    Kp1 = Kp;
    Kmember = r11**Kp1;
elseif i == 2
    r2 = 7;
    Kp2 = Kp;
    Kmember = r22**Kp2;
end
fprintf('Stiffness Matrix of complete structure, (Ktotal) = \n');
disp(Ktotal);
Kunr = zeros(6);
for x=1:unr
    for y=1:unr
        Kunr(x,y) = Ktotal(x,y);
    end
end
fprintf('Unrestrained Stiffness sub-matrix, (Kunr) = \n');
disp(Kunr);
[Kunr\ -r1r2];
fprintf('Inverse of Unrestrained stiffness sub-matrix, (Kunr\inverse) = \n');
disp(Kunr\);

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K_{unr}
 $[K_{unr}^{-1}]$

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Mod01 - Lec15, Example Problem Planar, Non-Orthogonal Structures (Part - 2) Prof. Srinivasan Chandrasekaran
% Creation of joint load vector
j1 = 10; 0; 10; 0; 0; 0; 0; 0; 0; 0; % values given in kN or kNm
j2 = 0; 0; 10; 0; 0; 0; 0; 0; 0; % load vector in unrestrained dof
delu = Ktotal\j2;
fprintf('Joint load vector, (J1) = \n');
disp(j1);
fprintf('Unrestrained displacements, (delU) = \n');
disp(delu);
delr = [0; 0; 0; 0; 0; 0];
del = zeros(dof,1);
del = [del; delr];
for i = 1:n
    for p = 1:6
        deli(p,1) = del(i(i,p),1);
    end
    if i == 1
        delbar1 = del1;
        mbar1 = (Kp1 * delbar1) + fmember1;
        fprintf('Member Number = ');
        disp(i);
        fprintf('Global displacement matrix [DeltaBar] = \n');
        disp(delbar1);
        fprintf('Global End moment matrix [MBar] = \n');
        disp(mbar1);
    elseif i == 2
        delbar2 = del2;
        mbar2 = (Kp2 * delbar2) + fmember2;
        fprintf('Member Number = ');
        disp(i);
        fprintf('Global displacement matrix [DeltaBar] = \n');
        disp(delbar2);
        fprintf('Global end moment matrix [MBar] = \n');
        disp(mbar2);
    end
end

```

$\{ \bar{d}_u \} = [K_u]^{-1} \{ J_u \}$
 $[M] = [K] \{ \bar{d} \} + [FEM]$

Once I get that I create the load vector create the joint load vector unplug out the unrestrained joint load vector, then I find the unrestrained displacement which will be unrestrained displacement in global degree will be $K_{unr}^{-1} u_{unr}^{-1} J L u$ we got this, once I obtain; I can find M_{bar} of i th member as K_{bar} of the i th member with Δ_{bar} of the i th member plus $f E M_{bar}$ of the i th member which is the equation here look at the results.

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Output: Member Number = 1

Local Stiffness matrix of member, [K] =

$$\begin{bmatrix}
 0.0014 & 0.0007 & 0.0005 & -0.0005 & 0 & 0 \\
 0.0007 & 0.0014 & 0.0005 & -0.0005 & 0 & 0 \\
 0.0005 & 0.0005 & 0.0002 & -0.0002 & 0 & 0 \\
 -0.0005 & -0.0005 & -0.0002 & 0.0002 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0.0268 & -0.0268 \\
 0 & 0 & 0 & 0 & -0.0268 & 0.0268
 \end{bmatrix}$$

Transformation matrix of member, [T] =

$$\begin{bmatrix}
 1.0000 & 0 & 0 & 0 & 0 & 0 \\
 0 & 1.0000 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0.4472 & 0 & -0.8944 & 0 \\
 0 & 0 & 0 & 0.4472 & 0 & -0.8944 \\
 0 & 0 & 0.8944 & 0 & 0.4472 & 0 \\
 0 & 0 & 0 & 0.8944 & 0 & 0.4472
 \end{bmatrix}$$

Transformation matrix transpose, [T] =

$$\begin{bmatrix}
 1.0000 & 0 & 0 & 0 & 0 & 0 \\
 0 & 1.0000 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0.4472 & 0 & 0.8944 & 0 \\
 0 & 0 & 0 & 0.4472 & 0 & 0.8944 \\
 0 & 0 & -0.8944 & 0 & -0.4472 & 0 \\
 0 & 0 & 0 & -0.8944 & 0 & -0.4472
 \end{bmatrix}$$

Global Matrix, [K global] =

$$\begin{bmatrix}
 0.0014 & 0.0007 & 0.0006 & -0.0006 & 0 & 0 \\
 0.0007 & 0.0016 & 0.0006 & -0.0006 & 0 & 0 \\
 0.0006 & 0.0006 & 0.0003 & -0.0003 & 0 & 0 \\
 -0.0006 & -0.0006 & -0.0003 & 0.0003 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0.0300 & -0.0300 \\
 0 & 0 & 0 & 0 & -0.0300 & 0.0300
 \end{bmatrix}$$

So, this is my member 1, this is my stiffness matrix local of the member 1, this is my transformation matrix of the member 1, this is transpose of the matrix, this is K bar that is K global of member 1.

Similarly, we get local matrix for member 2 transformation matrix. So, there is a multiplier here E, there is a multiplier here E, there is a multiplier here E, again there is a multiplier here in K bar E. So, this is actually K bar A B. Now this is actually K bar B C.

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Stiffness Matrix of complete structure, [K global] =

$$\begin{bmatrix}
 0.0020 & 0.0004 & 0.0004 & 0.0002 & -0.0006 & 0 & 0.0007 & -0.0004 & 0.0002 \\
 0.0004 & 0.0020 & 0.0004 & 0.0006 & -0.0003 & 0 & -0.0002 & -0.0106 & -0.0215 \\
 0.0004 & 0.0004 & 0.0020 & 0 & 0 & -0.0009 & 0.0004 & -0.0055 & -0.0106 \\
 0.0002 & 0 & 0 & 0.0016 & -0.0006 & 0 & 0 & 0 & 0 \\
 -0.0006 & -0.0003 & 0 & -0.0006 & 0.0003 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0.0300 & 0 & 0 & 0 \\
 0.0007 & -0.0002 & 0.0004 & 0 & 0 & 0 & 0.0014 & -0.0004 & 0.0002 \\
 -0.0002 & -0.0106 & -0.0055 & 0 & 0 & 0 & -0.0004 & 0.0055 & 0.0106 \\
 0.0002 & -0.0215 & -0.0106 & 0 & 0 & 0 & 0.0002 & 0.0106 & 0.0215
 \end{bmatrix}$$

Member Number = 1

Global displacement matrix [Delta Bar] =

$$\begin{bmatrix}
 0 \\
 0 \\
 -26.3069 \\
 -160.5452 \\
 0 \\
 0 \\
 329.8036
 \end{bmatrix}$$

Unrestrained Stiffness sub-matrix, [K un] =

$$\begin{bmatrix}
 0.0020 & 0.0004 & 0.0004 \\
 0.0004 & 0.0020 & 0.0004 \\
 0.0004 & 0.0004 & 0.0020
 \end{bmatrix}$$

Inverse of Unrestrained Stiffness sub-matrix, [K un inverse] =

$$\begin{bmatrix}
 330.8428 & -4.5612 & -2.4307 \\
 -4.5612 & 53.7632 & 16.0545 \\
 -2.4307 & 16.0545 & 32.9804
 \end{bmatrix}$$

Joint Load vector, [F] =

$$\begin{bmatrix}
 0 \\
 0 \\
 20 \\
 0 \\
 0 \\
 0 \\
 0
 \end{bmatrix}$$

Unrestrained displacements, [Delta U] =

$$\begin{bmatrix}
 -26.3069 \\
 -160.5452 \\
 329.8036
 \end{bmatrix}$$

Member Number = 2

Global displacement matrix [Delta Bar] =

$$\begin{bmatrix}
 -26.3069 \\
 0 \\
 -160.5452 \\
 0 \\
 0 \\
 329.8036
 \end{bmatrix}$$

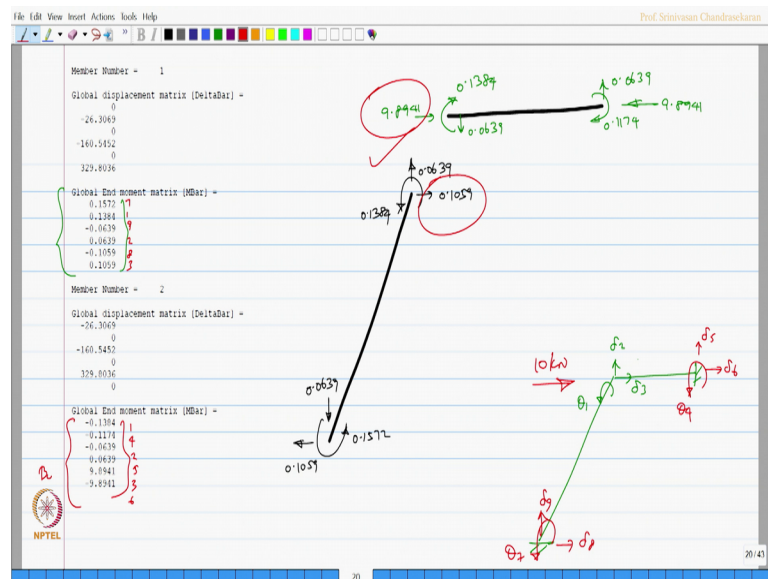
Global End moment matrix [M Bar] =

$$\begin{bmatrix}
 0.1572 \\
 0.1391 \\
 -0.0639 \\
 0.0639 \\
 -0.1059 \\
 0.1059
 \end{bmatrix}$$

Once I have this, I assemble this and get the total stiffness matrix which is 9 by 9 which is K_{total} where E is constant out; from this I plug out only the sub matrix which is u which is 3 by 3. So, I plug out only this matrix and write this as K_{uu} with E outside; is it not.

Then I invert this matrix, I get K_{uu}^{-1} with E out, then I get the joint load vector I partition this, I get JL_u , then I find Δ_u which is nothing, but Δ_{bar} u this K_{uu}^{-1} of JL_u , I get this which will be 1 by E times of this, then we find Δ_{bar} of the member AB , then we find M_{bar} of the member AB , similarly Δ_{bar} of the member BC and M_{bar} of the member BC , let us plot these results and show how do they match.

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So, let us show these results. This is my original structural system. So, you know this is my M_{bar} of member 1; the degrees of freedom of the structure if you remember, I am just marking them again; this was $\theta_1, \delta_2, \delta_3$ and $\theta_4, \delta_5, \delta_6, \theta_7, \delta_8$ and δ_9 . So, for the member 1, the labels could be you know the labels are 7, 1, 9, 2, 8, 3 rotations, then along y and along x , similarly for the member BC will be 1, 4, 2, 5, 3, 6, correct; 1, 4, 2, 5, 3, 6.

Let us now interpret the results 0.1572; let me write down the values this is positive. So, anticlockwise 0.1572 is plus again. So, 0.1384, then 9; 9 is here is minus. So, downward 0.0639, then 2 which is plus positive 0.0639, then 8 is minus. So, this way which is

0.1059 and 3 which is positive. So, let us say 0.1059 that is the E multiplier; in all this cases, E is constant this for member 1.

Let us do it for member 2. So, member 2 the labels are 1. So, let us say 1 minus. So, it is going to be clockwise which is 0.1384 and 2 is also negative. So, clockwise 0.1174, then 2 is negative. So, downward 0.0639 and 5 is positive. So, upward 0.0639, then the degree of freedom 3 is positive. So, 9.8941 and this is negative which is 9.8941.

So, friends; if you look at this problem, this problem was applied to load of 10 kilo Newton here which actually amounts to 9.841 plus point. So, 10 it is exactly matching that is how the problem is solved. So, it is very interesting friends in this lecture, we understood the computer program for solving a non-orthogonal structure, we have marked that labels and degrees of freedom unrestrained restrained followed. The exact procedure what we discussed in the derivation and did the coding exactly on the same line and got the answers which is being solved for this problem.

I hope you will understand and practice this particular problem again with the coding given on the screen, I will you obtain the same set of results.

Thank you very much.