

Computer Methods of Analysis of Offshore Structures
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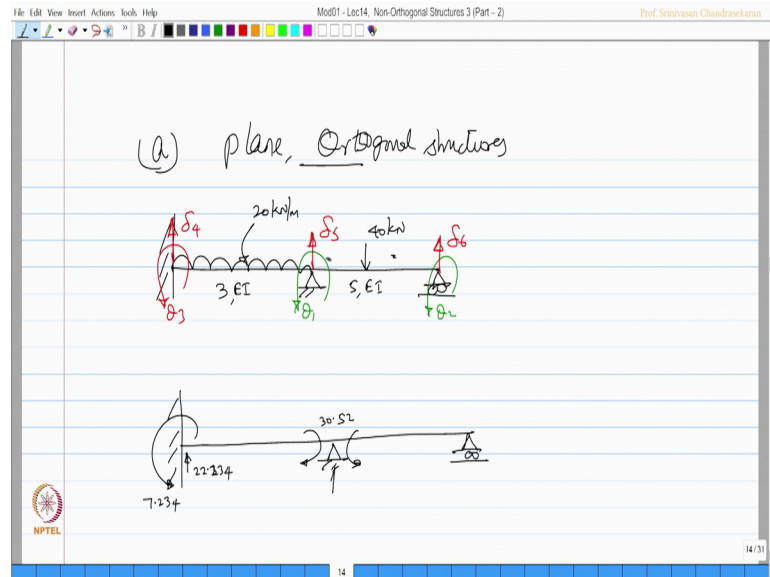
Module - 01
Lecture - 14
Non-Orthogonal Structures 3 (Part - 2)

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- Stiffness method
- Planar orthogonal structure
- MATLAB code

Let us try to apply this problem. But before that I want to show you a very interesting computer code which has been used to solve the problem in the previous 2 examples of orthogonal structures.

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So, remember in **planar** orthogonal structures, we have 2 examples; the first example was a problem of this kind which had loading of this kind and this is 3 comma EI, this is 5 comma EI and these were the unrestrained and restrained degrees of freedom which we have marked to solve the problem.

Now, I want to show you how a computer program can help you to solve this problem; we already have the solution for this which is; so, this value was 7.234 and this is 30.52 and this reaction 22.234 and this reaction is 63.871 in total and this value is 13.895 that is what we got; we can solve this problem using the computer code which is being shown here.

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Mod01 - Lec14, Non-Orthogonal Structures 3 (Part - 2) Prof. Srinivasan Chandrasekaran
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Planar Orthogonal Problem 1:
% stiffness matrix method
% input
clear;
n = 2; % number of member
i = [1 2]; % Moment-of-inertia in e4
L = [3 5]; % length in m
m = [1 2]; % member number
uo = 2; % Number of unrestrained degrees of freedom
ur = 4; % Number of restrained degrees of freedom
uul = [1 2]; % global labels of unrestrained dof
url = [3 4 5 6]; % global labels of restrained dof
l1 = [3 1 4 5]; % Global labels for member 1
l2 = [1 2 5 6]; % Global labels for member 2
I = [11; 12];
Ktotal = zeros(6);
fem1 = [15 -15 30 -30]; % Local Fixed-end moments of member 1
fem2 = [25 -25 50 20]; % Local Fixed-end moments of member 2

% rotation coefficients for each member
rc1 = 4./L;
rc2 = 2./L;

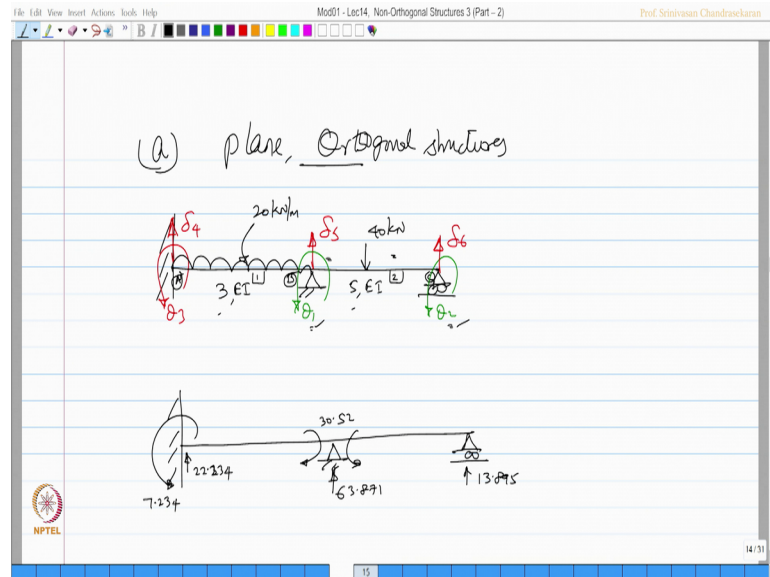
% stiffness matrix 4 by 4 (axial deformation neglected)
for i = 1:n
    Krow = zeros(6);
    K1 = [rc1(i); rc2(i); rc1(i)+rc2(i)/L(i); -(rc1(i)+rc2(i))/L(i)];
    K2 = [rc2(i); rc1(i); rc1(i)+rc2(i)/L(i); -(rc1(i)+rc2(i))/L(i)];
    K3 = [(rc1(i)+rc2(i))/L(i); rc1(i)+rc2(i)/L(i); (2*(rc1(i)+rc2(i))/L(i)^2); -(2*(rc1(i)+rc2(i))/L(i)^2)];
end
```

You can see here; this problem solves a plane orthogonal structure using stiffness matrix method, we need identify the number of members, here we are given 2 because there are 2 members that is AB and BC; let us mark them here AB and BC; there are 2 members. So, I have given 2 here is in input for simplicity, since EI is constant, I have said I is and E is not varying length of the members in input which is 3 meter and 5 meter 3 meter and 5 meter I can say here.

And the member numbers are 1 and 2 member number; this is member 1 and this is member 2. Now unrestrained degrees are 2 was an input, you can see here marked in green theta 1 and theta 2 restrained degrees are 4 that is 3, 4, 5, 6. So, they are 4 and labels of unrestrained degrees of 1 and 2; you can see here theta 1 and theta 2. So, theta 1 and theta 2 unrestrained degrees are 3, 4, 5, 6.

Let us see; what is a global labels of member 1; let us look at member one what are the local labels member one is member AB the local labels will be 3 1 4 and 5 for the member BC the labels are 1 2 5 and 6.

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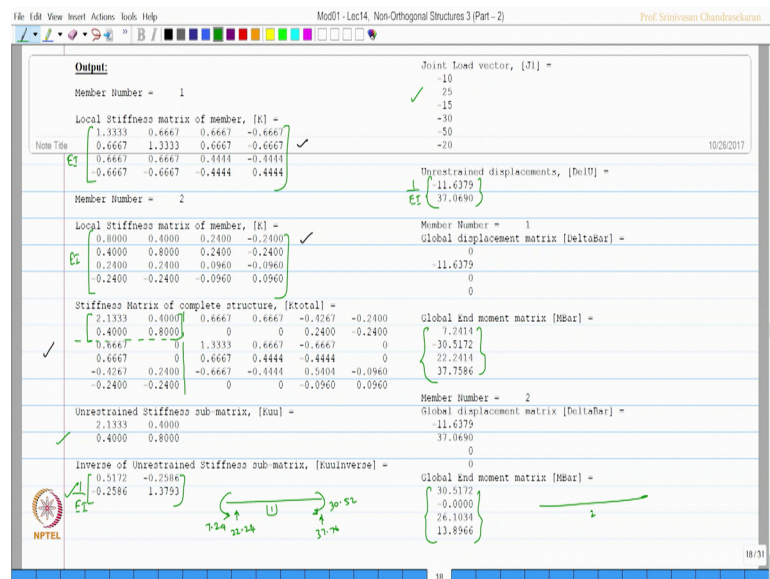
So, that is what I entered here. Then let us talk about the fixed end moments, if you look at the problem, the fixed end moments has been computed like this, you already have their value the previous lecture. Please see that then we compute the rotation coefficients, then we get the stiffness matrix, once we get the matrix, we print the matrix, we do the calculation, map them for different labels of each member, then we get the stiffness matrix of the entire structure, then we try to get the unrestrained stiffness matrix alone; unrestrained stiffness matrix alone.

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File Edit View Insert Actions Tools Help Mod01 - Lec14, Non-Orthogonal Structures 3 (Part - 2) Prof. Srinivasan Chandrasekaran
k4 = -k3;
E = [k1 k2 k3 k4];
fprintf('Member Number =');
disp(i);
fprintf('Local Stiffness matrix of member, [K] = \n');
disp(K);
for p = 1:4
    for q = 1:4
        Knew(i(i),p), (i(i),q)) = K(p,q);
    end
end
Ktotal = Ktotal + Knew;
if i == 1
    Eyz = E;
elseif i == 2
    Eyz = E;
end
end
fprintf('Stiffness Matrix of complete structure, [Ktotal] = \n');
disp(Ktotal);
Kunr = zeros(7);
for x = 1:7
    for y = 1:7
        Kunr(x,y) = Ktotal(x,y);
    end
end
fprintf('Unrestrained Stiffness sub-matrix, [Kunr] = \n');
disp(Kunr);
KunrInv = inv(Kunr);
fprintf('Inverse of Unrestrained Stiffness sub-matrix, [KunrInverse] = \n');
disp(KunrInv);
```

Then we get the inverse of that then subsequently, we give the joint loads look at the joint loads, we already computed; thus from the last lecture amongst there, the first 2 values are unrestrained joint loads then we use a relationship to compute the delta u; delta u which is k inverse of joint load unrestrained, then once we get delta u, we can find the end moments of each member; member 1 and member 2. Let us see the results you know the local stiffness matrix of the first member, if you look at the previous problem solution, you will see exactly it is same; it is for the member 2, this is for the complete structure.

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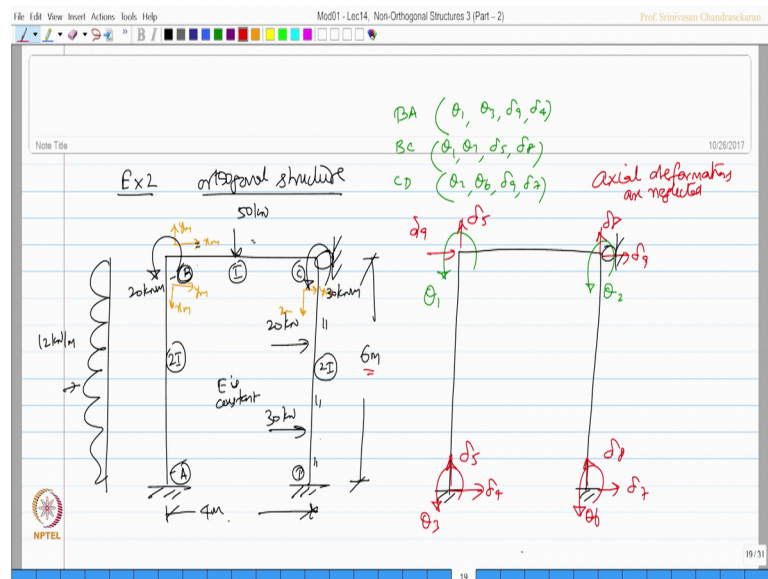
Now, from this I cut out only these values partition this matrix; is it not. So, I get only this which I went out here we get the inverse of this, whereas 1 by EI is available here. In fact, in all this cases you know EI is constant; is it not. So, here also EI is constant and so on; we get the joint load vector, then we get the unrestrained displacement which is again 1 by EI, then we get the member end reactions which is here this is for the member 1, this is for the 2.

Let us try to plot this you will see for member 1; member 1, let us draw this, again this is anticlockwise 7.24 and this is clockwise because negative. So, 30.52, this is upper 22.24, this is also upper 37.76 for the member 2, this for the member 1, for the member 2, you will see that this becomes 30.52 which matches with this the answer, we have which we already solved and this is 0 because this simply supported end for the problem see here.

The simply supported end for the problem, therefore, the moment is 0 there which we get here then the reactions are 26.1 and 13.89 which exactly matches with this value. So, the sum of these 2; the sum of this and this will amount to this. So, the code exactly solves the same problem and the manual solution; what we had matches with the results; what we have in the code this is the code what we have they given the full code for your understanding you can rerun the program again and try to get the results.

Similarly, we can also solve the second problem which we did we remember the second problem was example 2 of orthogonal structure. So, this is the example which we solved by hand, we applied a joint load here, but just 20 kilo Newton meter and here 30 kilo Newton meter and this was 4 meters.

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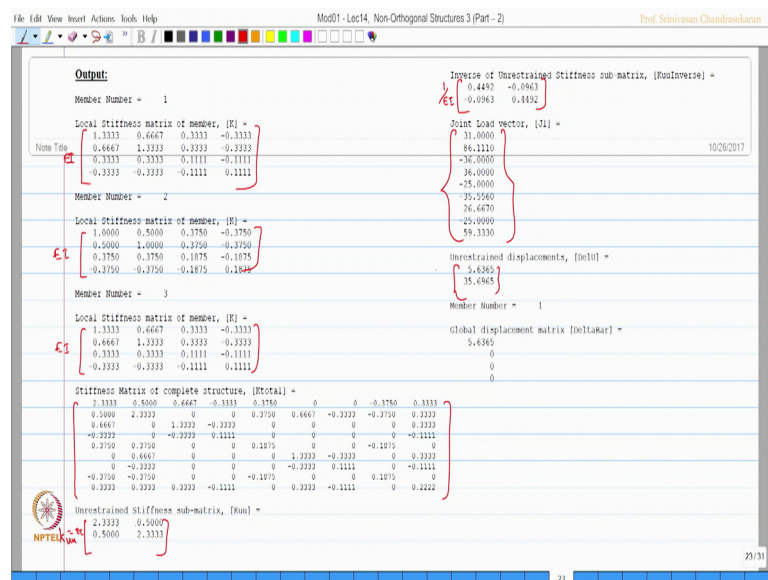
And the height of the frame was 6 meters right and this is subjected to a member load of 12 kilo Newton per meter and there were 2 more loads of 20 kilo Newton and 30 kilo Newton here which are equally spaced and this is a central load here of 50 kilo Newton this was the structure; is it not.

And this member is 2 I, this member had I this member had 2 I and E is constant correct, this was the problem, then we mark the unrestrained and restrained degrees of freedom for this problem correct. So, the unrestrained degrees where theta 1 and theta 2, the restrained degrees where marked as theta 3, delta 4, delta 5, theta 6, delta 7, delta 8 and delta 9 this also delta 5 this delta 8 because axial deformations are neglected; is it not.

So, for this we have the member labels let us write down the labels here is easy to write down here this is A, this is B, this is C and D. So, we say member B A, this was my x m y m for this member; for this member, this is x m and y m for this member this is x m and this is y m correct. So, for the member B A the labels where I write here for the B A the labels are theta 1, theta 3 and delta 9 and delta 4 for the member BC the labels where theta 1, theta 2, delta 5 and delta 8 and for the member C D, the labels where theta 2 theta 6 delta 9 and delta 7 correct, we add this values which we did in the earlier problem please see that we have a computer coding.

Now there are 3 members I am marking them with red color for our clearance 3 members I is varying we can see here 2 I. So, I can say 2 1 2 fine member lengths in meters 6 meter this is 4 meter this is again 6 meter. So, 6 4 6 member numbers 1 2 1 3; how many unrestrained degrees are there marked in green 2 1 and 2. So, unrestrained degrees remaining restrained because there are total 9; there are total 9 is it not. So, labels are one and 2 for unrestrained; unrestrained it is 3 to 9, correct. So, let us talk about member 1 the global members 1 3 9 4, you see here 1 3 9 4 for member 2 1 2 5 8 for member 2 one 2 5 8; is it not this is 2 actually right, this is let us write it clearly this is 2 right m then 26 9 7 26 9, we have it here.

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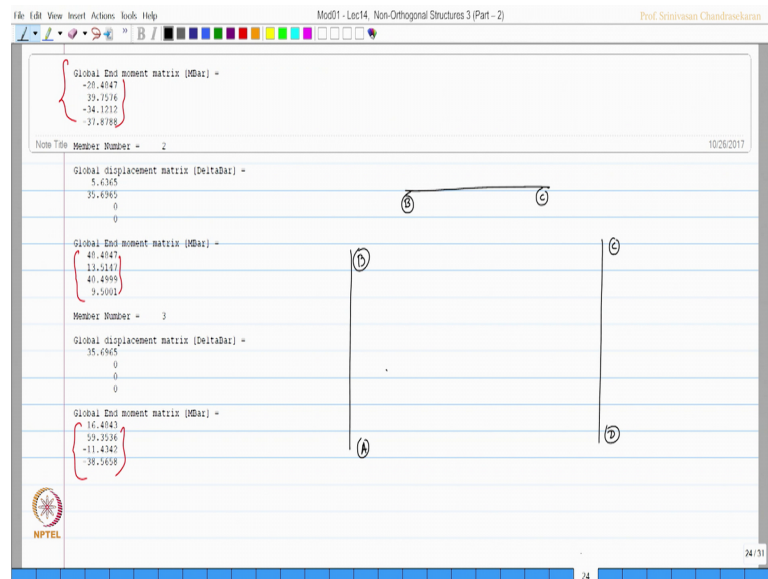
Once we do this, we get the stiffness matrix, then we apply the loads, the program is continuing from here it continues here; that is how the program continues and then we

get the global end moment matrix for 3 members member 1, member 2 and member 3; let us write down the values which I am having here; if you compare the results; what we already had shown in the previous lecture this is my stiffness matrix for the member 1 of course, there is a multiplier of E out I is included in this E is out.

Let us say EI an multiplier this is EI and multiplier you can compare this results are exactly same and this is for the member 3 EI, this my total stiffness matrix out of which unrestrained is this value this is nothing, but K_{uu} I have EI out here also and this is same as that this is K_{uu} inverse which has 1 by EI here, this is my joint load vector which we computed this is my delta u the delta u value is exactly same what we had. So, there is 1 by EI here.

So, these are the results of end moments for first member second member and third member.

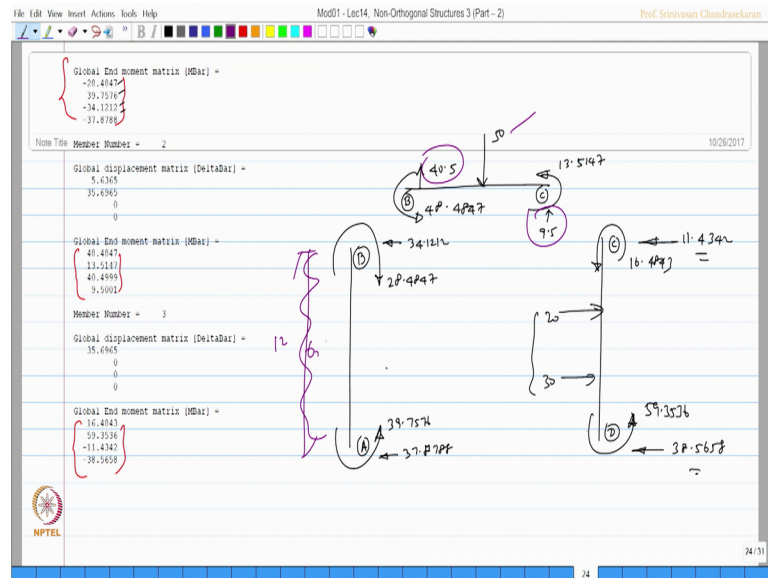
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So, let us mark them separately and compare the results for our convenience, I am drawing member wise this is the first member this is the second member this is the third member. So, this is A and B, this is B and C, this is C and D, right. So, let us mark the value. So, let us A and B; so, we know a has got theta one here you can see that the labels of AB, we have to transformed in the same style the labels of AB is 1 3 9 4.

Comparing that one 3 9 4 that is how we have to write. So, this is going to be minus. So, this is 28.4847, this is anticlockwise which is 39.7576; this is minus. Therefore, is going to be 34.1212; this is 37.8788; correct.

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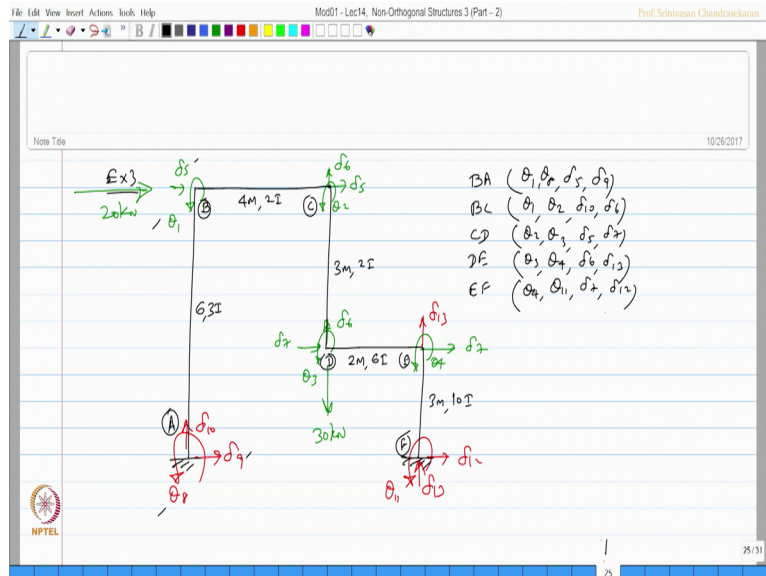
Similarly, BC you will see this is going to be positive, so, 48.4847 and this is going to be 13.5.47 and this value is 40 point; let us say 5 and this value is 9.5.

Similarly, when you come here this is 16.4843 and this is 59.3536 and this is negative therefore, 11.4342 and this is 38.5658 just for the check the load applied here is 20 and 30; you know these 2 will amount to 50. Similarly the load applied here was 50, these 2 amounts to 50; the load applied here was 12 for length of 6 meters 72. So, these 2 will amount to 72.

So, it is very clear that the problem is examined and check we get the same answers as we are solved by hand using the computer method which is stiffness matrix method. So, friends we have given the coding we have run the problem we have given the results which are exactly matching with the same methodology what we solved by hand. We have one more problem which I gave you which we only marked which I want to you to solve. So, the problem was example 3 which I will spend few minutes in explaining the solution of this problem as well this is the problem given, right.

So, this is 6 meters and 3 I, this is 4 meters and 2 I, this is 3 meters and 2 I, this is 2 meters and 6 I, this is 3 meters and 10 I, that is a problem.

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We mark the degrees of freedom unrestrained theta 1, theta 2, theta 3, theta 4, delta 5, delta 6.

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Output:

Member Number = 1

Local Stiffness matrix of member, [K] =

$$EI \begin{bmatrix} 2.0000 & 1.0000 & 0.5000 & -0.5000 \\ 1.0000 & 2.0000 & 0.5000 & -0.5000 \\ 0.5000 & 0.5000 & 0.1667 & -0.1667 \\ -0.5000 & -0.5000 & -0.1667 & 0.1667 \end{bmatrix}$$

Member Number = 2

Local Stiffness matrix of member, [K] =

$$EI \begin{bmatrix} 2.0000 & 1.0000 & 0.7500 & -0.7500 \\ 1.0000 & 2.0000 & 0.7500 & -0.7500 \\ 0.7500 & 0.7500 & 0.3750 & -0.3750 \\ -0.7500 & -0.7500 & -0.3750 & 0.3750 \end{bmatrix}$$

Member Number = 3

Local Stiffness matrix of member, [K] =

$$EI \begin{bmatrix} 2.6667 & 1.3333 & 1.3333 & -1.3333 \\ 1.3333 & 2.6667 & 1.3333 & -1.3333 \\ 1.3333 & 1.3333 & 0.8889 & -0.8889 \\ -1.3333 & -1.3333 & -0.8889 & 0.8889 \end{bmatrix}$$

Member Number = 4

Local Stiffness matrix of member, [K] =

$$EI \begin{bmatrix} 12 & 6 & 9 & -9 \\ 6 & 12 & 9 & -9 \\ 9 & 9 & 9 & -9 \\ -9 & -9 & -9 & 9 \end{bmatrix}$$

Member Number = 5

Local Stiffness matrix of member, [K] =

$$EI \begin{bmatrix} 13.3333 & 6.6667 & 6.6667 & -6.6667 \\ 6.6667 & 13.3333 & 6.6667 & -6.6667 \\ 6.6667 & 6.6667 & 4.4444 & -4.4444 \\ 6.6667 & -6.6667 & -4.4444 & 4.4444 \end{bmatrix}$$

Unrestrained Stiffness sub-matrix, [Kuu] =

$$EI \begin{bmatrix} 4.0000 & 1.0000 & 0 & 0 & 0.5000 & -0.7500 & 0 \\ 1.0000 & 4.6667 & 1.3333 & 0 & 1.3333 & -0.7500 & -1.3333 \\ 0 & 1.3333 & 14.6667 & 6.0000 & 1.3333 & 9.0000 & -1.3333 \\ 0 & 0 & 6.0000 & 25.3333 & 0 & 9.0000 & 6.6667 \\ 0.5000 & 1.3333 & 1.3333 & 0 & 1.0556 & 0 & -0.8889 \\ -0.7500 & -0.7500 & 9.0000 & 9.0000 & 0 & 9.7500 & 0 \\ 0 & -1.3333 & -1.3333 & -6.6667 & -0.8889 & 0 & 5.3333 \end{bmatrix}$$

Inverse of Unrestrained Stiffness sub-matrix, [KuuInverse] =

$$EI \begin{bmatrix} 0.2750 & -0.0296 & -0.0053 & -0.0016 & -0.1078 & 0.2063 & 0 \\ -0.0296 & 0.2843 & -0.0747 & -0.0808 & -0.2639 & 0.1776 & 0.1366 \\ -0.0053 & -0.0747 & 0.2963 & 0.1090 & -0.4029 & -0.3962 & -0.1490 \\ -0.0016 & -0.0808 & 0.1090 & 0.1914 & -0.2690 & -0.2997 & -0.2748 \\ -0.1078 & -0.2639 & -0.4029 & -0.2690 & 2.3060 & 0.4154 & 0.5540 \\ 0.2063 & 0.1776 & -0.3962 & -0.2997 & 0.4154 & 0.7873 & 0.4175 \\ -0.0246 & 0.1344 & -0.1490 & -0.2760 & 0.5540 & 0.4175 & 0.6221 \end{bmatrix}$$

Joint Load vector, [F1] =

$$\begin{bmatrix} 20 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Unrestrained displacements, [dE10] =

$$EI \begin{bmatrix} -2.5291 \\ -10.6047 \\ 3.8284 \\ 1.4551 \\ 21.4762 \\ -11.3104 \\ -1.4465 \end{bmatrix}$$

And then the restrained degrees theta 8, delta 9, delta 10, theta 11, delta 12 and delta 13. So, there are 13 degrees of freedom. So, this is member nomenclature A B C D E and F I

the loading where like this actually this frame is subjected to 20 kilo Newton here and 30 kilo Newton here that is what is the problem given, correct.

We already said that the member B A, B A, BC, C D, D E and E F has the following labels which is theta 1, theta 8, theta 1, theta 8, then delta 5 and delta 9 is it not. Similarly, BC theta 1, theta 2, 10 and 6: similarly C D 2 3 5 and 7: similarly D E 3 4 6 and 13 E F 4 11 7 and 12 that is how we got.

So, we wrote the program; I mean use the same program, **give** the input appropriately and develop the answers. So, this is the stiffness matrix of the first member there is a multiplier of EI to this the second member multiplier of EI to this which is exactly matching what we derived in the previous lecture, EI to this member 3 member 4 and member 5, these are a stiffness matrixes then based on this we found out the unrestrained stiffness matrix 7 because there are 7 unrestrained degrees you can see here. So, it will be 7 by 7 3 7 by 7 which is going to be EI again.

Which is actually K_{uu} we inverted this matrix 1 by EI is there B substitute of joint vector and got δu where 1 by EI is again there once we get this we have obtain the end reactions of member 1, member 2, member 3, member 4 and member 5 which we can plot and give you the answers I am plotting them here member 1, member 2, member 3, member 4 and member 5. So, the values are very simple, I am looking for member 1. So, this is 7.98, 10.9091.

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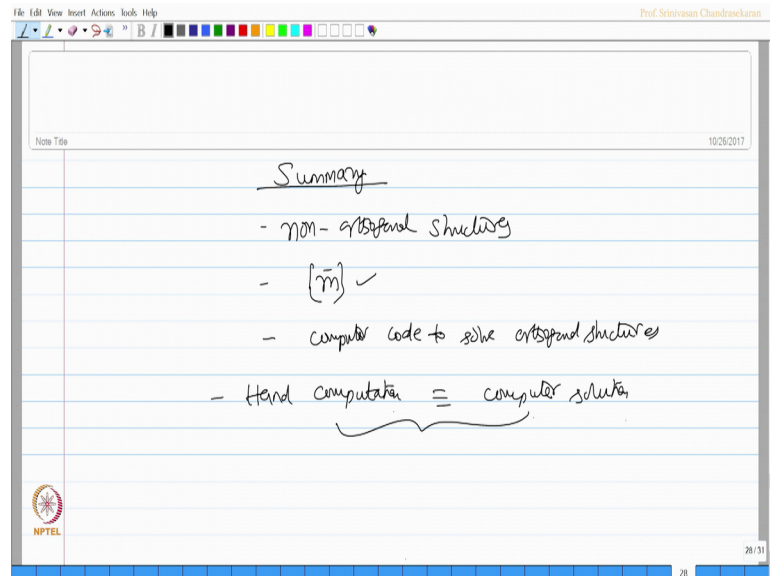
The screenshot displays a software window titled "Mod01 - Lec14, Non-Orthogonal Structures 3 (Part - 2)" with a toolbar and a menu bar. The main content area shows the stiffness matrices for five members, arranged in a grid. Each member's data includes a global displacement matrix [deltaKar] and a global end moment matrix [MKar].

Member Number	Global displacement matrix [deltaKar]	Global end moment matrix [MKar]
1	$\begin{bmatrix} -2.9221 & 0 & 27.6762 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 7.9800 & 10.9091 & 3.1462 \\ 0 & 0 & 0 \\ -3.1462 & 0 & 0 \end{bmatrix}$
2	$\begin{bmatrix} -2.9221 & -10.6047 & 0 \\ 0 & 0 & -11.3104 \end{bmatrix}$	$\begin{bmatrix} -7.9800 & -15.6556 & -5.9089 \\ 0 & 0 & 5.9089 \\ 5.9089 & 0 & 0 \end{bmatrix}$
3	$\begin{bmatrix} 10.6047 & 0 & 27.6762 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 15.6556 & 34.8950 & -16.8518 \\ 0 & 0 & 0 \\ -16.8518 & 0 & 0 \end{bmatrix}$
4	$\begin{bmatrix} 3.4921 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 34.8950 & -36.9180 & 39.3089 \\ 0 & 0 & 0 \\ 39.3089 & 0 & 0 \end{bmatrix}$
5	$\begin{bmatrix} 3.4921 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 34.8950 & -36.9180 & 39.3089 \\ 0 & 0 & 0 \\ 39.3089 & 0 & 0 \end{bmatrix}$

Handwritten annotations in blue ink are present on the right side of the matrices for members 3, 4, and 5. For member 3, a vertical line is drawn next to the value 15.6556, with an arrow pointing to it from the label "15.6556". For member 4, a vertical line is drawn next to the value 3.142, with an arrow pointing to it from the label "3.142". For member 5, a vertical line is drawn next to the value 3.142, with an arrow pointing to it from the label "3.142".

So this is 3.148 and this is 3.148; similarly member 2 below cat these are the values. So, is going to be clockwise 7.98; you can see there is a perfect match this anticlockwise and this is clockwise is it not. So, this is going to be again clockwise 15.6556; this is minus 5.91; this is plus 5.91 and so on. So, you can plot them and interpret them like this.

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So, friends let us look at the summary very quickly we have discussed about the procedure to solve non orthogonal members, we have explained how the global end reactions can be obtained we have also demonstrated the full computer code to solve the orthogonal structures. And we said that the results what we had by hand computation or exactly matching with the computer solution; is it not. So, we have discussed about the computer methods of solving the problem we also gave you the computer code and as a perfect compatibility of the solution for the update example problems.

I hope you have enjoyed the lecture. And I am sure you will key in this code in MATLAB and try to solve the problem once again and verify the solutions. And then you will enjoy solving more and more problems in your classrooms.

Thank you very much and bye.