

**Computer Methods of Analysis of Offshore Structures**  
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**Module - 01**  
**Lecture - 14**  
**Non-Orthogonal Structures 3 (Part – 1)**

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- Stiffness method
- Planar non-orthogonal structure

So, friends let us continue with the discussions on **planar** Non-orthogonal structural members, This is lecture fourteen and lecture one. Where we will explain and continue to discuss about solving non orthogonal structural members. We already said that  $K$  bar of the  $i$ -th member can be given by  $T$  transpose of the  $i$ -th member  $k$  local of the  $i$ -th member and  $t$  of the  $i$ -th member, where  $k$  bar is the stiffness matrix of  $i$ -th member in the reference axes system.

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Module 1  
Lecture 14  
plane, non-orthogonal structures - III

$$[\bar{k}]_i = [T]_i^T [k]_i [T]_i$$

$\bar{k}$  is stiffness matrix of a member in the reference axes (x-y)

Hence,  $\{\bar{m}\}_i = [k]_i \{\bar{d}\}_i$  - valid x-y sys

Hence the conventional expression  $\bar{m}_i$  will be equal to  $\bar{k}_i$  of  $\bar{d}_i$  is also valid, where all of them are being expressed in the reference axes system.

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$$[T]^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_x & 0 & c_y & 0 \\ 0 & 0 & 0 & c_x & 0 & c_y \\ 0 & 0 & -c_y & 0 & c_x & 0 \\ 0 & 0 & 0 & -c_y & 0 & c_x \end{bmatrix}$$

$$[T] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_x & 0 & -c_y & 0 \\ 0 & 0 & 0 & c_x & 0 & -c_y \\ 0 & 0 & c_y & 0 & c_x & 0 \\ 0 & 0 & 0 & c_y & 0 & c_x \end{bmatrix}$$

$$[k] = \begin{bmatrix} \frac{AE}{L} & & & & & \\ & \frac{AE}{L} & & & & \\ & & \frac{6EJ}{L^2} & & & \\ & & & \frac{6EJ}{L^2} & & \\ & & & & \frac{AE}{L} & \\ & & & & & \frac{AE}{L} \end{bmatrix}$$

So, we already have this expression, where T transpose or T of arbitrarily oriented member for all the six degrees is given by this. And T matrix that is the transformation

matrix is also known to us, which is interchange of this rows and columns, and the k matrix of local for six degrees of freedom namely p q r s t and h is also known to us is it not.

So, I hope you will be able to fill up the remaining values by looking at this except that in the last case is going to be AE by l minus AE by l, AE by l and AE by l. Now look at this expression, if you want to find k global I have to do T transpose pre multiplier to my k i, and then post multiply this at the transformation matrix, I have all the three here I think you can do a simple arithmetic and try to find k bar as an interesting exercise for any i-th matrix, there is no difficulty in getting this.

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Further,

$$[K] [\Delta] = \{JL\} + \{R\} \quad (1)$$

$$[k_{uu}] \{\Delta_u\} = \{JL\}_u \quad (2)$$

$$[k_{ru}] \{\Delta_u\} - \{JL\}_r = \{R\}_r \quad (3)$$

Once we get k bar then, we know the following equations are valid K of delta is joint load plus restraints call this is equation 1 k uu that is un restrain degree of delta u will be again valid with delta u J L u and k r u with delta u is again minus J L r will give you the end moments and shear of the given member.

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The image shows a screenshot of a presentation slide with a white background and blue horizontal lines. At the top, there is a header bar with the text "Mod01 - Lec14, Non-Orthogonal Structures 3 (Part - 1)" and "Prof. Srinivasan Chandrasekaran". Below the header, there is a title bar that says "Note Title" and a date "10/26/2017". The main content of the slide is handwritten text in blue ink. It starts with a red heading: "- Important steps, which make the analysis of non-orthogonal structures." followed by two numbered steps: (1) "Choosing  $j^{\text{th}}$  end of a member, will position  $x_m, y_m$  axis of the member" and (2) "locate position of reference axes  $(x-y)$ ". Each step has several bullet points explaining the details. The slide also features a small NPTEL logo in the bottom left corner and a page number "4/24" in the bottom right corner.

- Important steps, which make the analysis of non-orthogonal structures.

(1) Choosing  $j^{\text{th}}$  end of a member, will position  $x_m, y_m$  axis of the member

- Origin of the axes ( $x_m, y_m$ ) is @  $j^{\text{th}}$  end
- $x_m$  axis should be oriented towards  $k^{\text{th}}$  end making length of the member in the +ve side of  $x_m$
- $y_m$  axis is  $90^\circ$  anti-clockwise to  $x_m$
- This will fix  $k^{\text{th}}$  end.

(2) locate position of reference axes  $(x-y)$

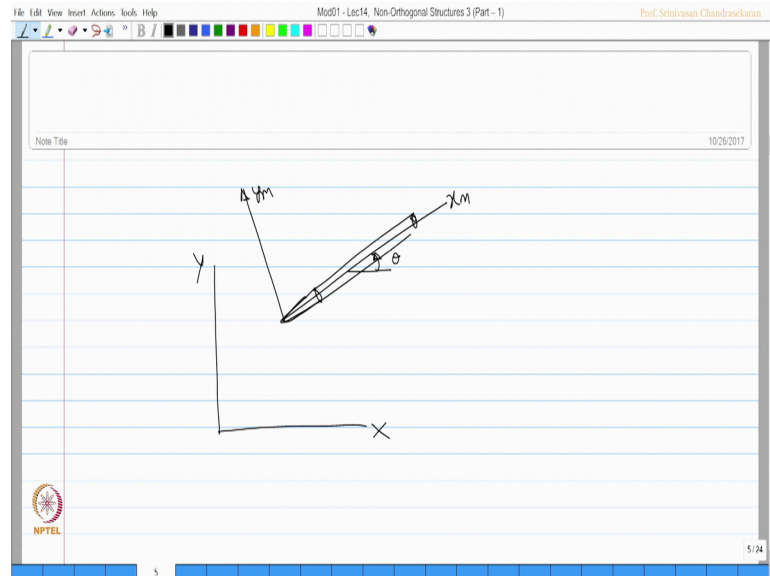
- find  $\theta$ , which is the anticlockwise angle b/w  $(x-y)$  and  $x_m, y_m$

Let us now find out what are those important steps, which make the analysis of non orthogonal structures special let us see this.

First step could be choosing the  $j$ -th end member of a member, will position the  $x_m, y_m$  axes of the member, how to choose this we already have an idea, origin of the axes  $x_m, y_m$  is at  $j$ -th end,  $x_m$  axes should be oriented towards  $k$ -th end making length of the member in the positive side of  $x_m$  and  $y_m$  axes is 90 degrees anticlockwise to  $x_m$  that is how you will choose this. So, now this will fix the  $k$ -th end. So, for a given member  $x_m, y_m$  is fixed, now locate position of the reference axes that is  $x, y$ , now find theta which is the anticlockwise angle between  $x, y$  and  $x_m, y_m$ .

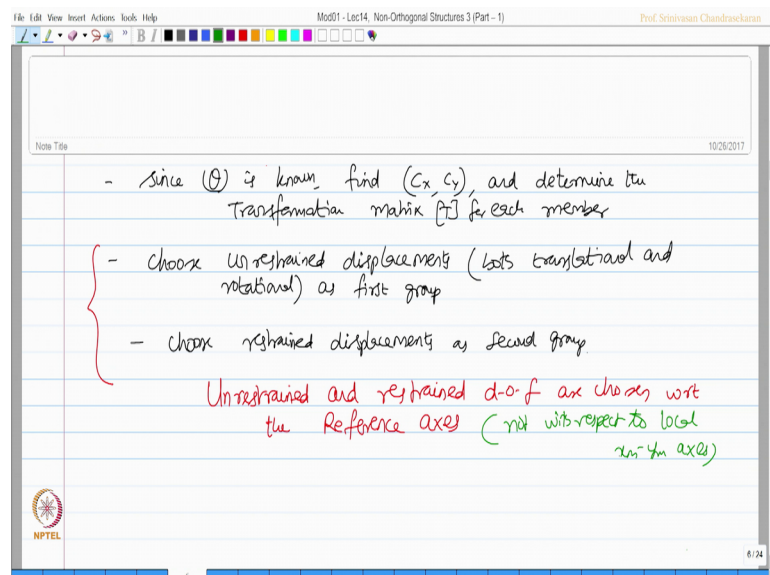
Friends let us elaborate this; this is my  $x_m, y_m$  of a given member. Let us say this is my  $x_m$  and  $y_m$  of a given member.

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And let us say this is my x y axes of the whole structure, I am looking for the angle theta which is anticlockwise inclination of x m with respect to x.

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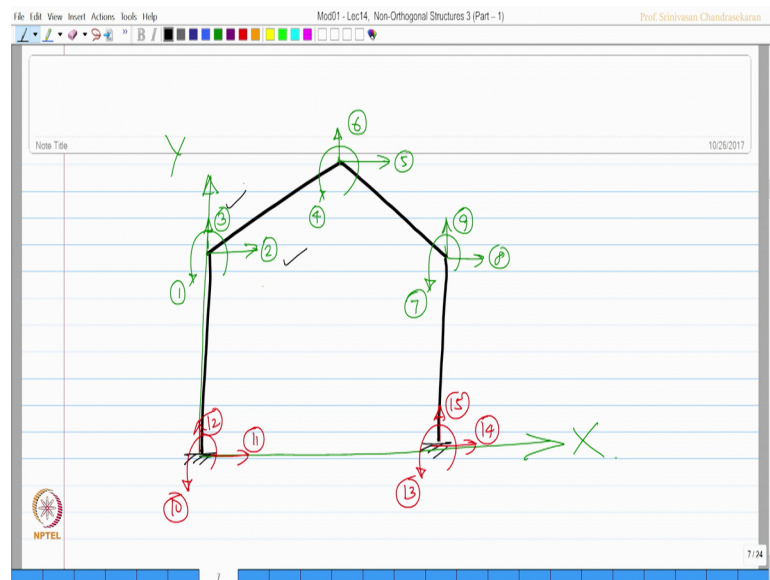


Then next step could be since theta is known, find c x and c y and determine the transformation matrix T for each member. After that choose unrestrained displacements

both translational and rotational, as first group then choose restrained displacements as second group friends, when we do this we have to do an important step here, we have to take care that the unrestrained and restrained degrees of freedom are chosen with respect to the reference axes, I will explain this with the problem.

So, I should write here not with respect to local  $x_m$ ,  $y_m$  axes its very important step. Lets explain this with an example, let us take a problem very quickly just explain this, I have got a frame which has non orthogonal members as you see here.

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Now I want to mark the reference axes exactly here,  $x$   $y$  axes. Now I want to mark the degrees of freedom right, so labelling is different but degrees of freedom alone let us discuss.

So, I am marking the understand degrees so, 1, 2 and let us say 3, 4, 5, 6, 7, 8 and 9 the restrained degrees could be, 10, 11, 12, 13, 14 and 15. So, we have got the group the unrestrained degrees first, and then the restrained degree levels while marking them please note my  $x$  and  $y$  displacements, which are unrestrained displacements, and restrained displacements are all oriented with reference to the reference axes, but not with reference to the local  $x_m$  or  $y_m$  axes, this is very important. You must choose the

unrestrained and restrained degrees of freedom in such a manner that they are marked with respect to the reference axes that is how it is done.

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Joint loads should be computed  
 - compute the joint loads wrt reference axes

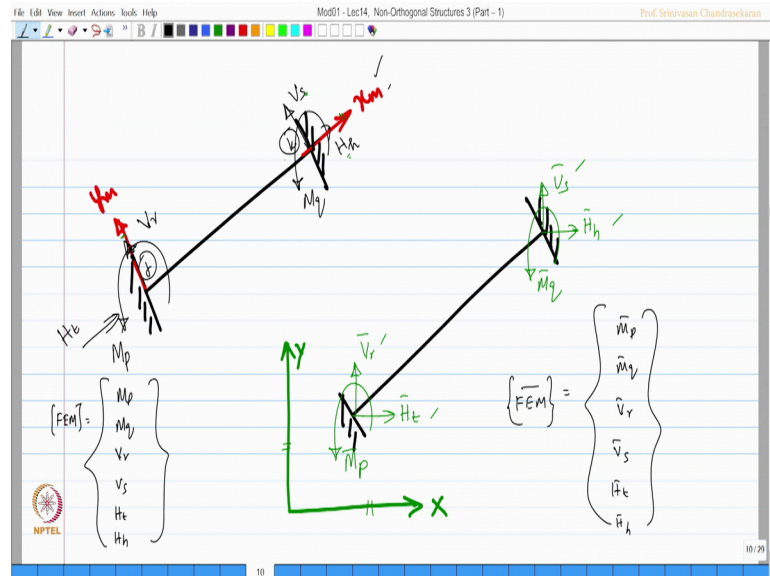
$$[FEM]_i = [T]_i^T [FEM]_i \quad \text{--- (4)}$$

$$[FEM]_i = \begin{Bmatrix} M_A \\ M_B \\ V_A \\ V_B \\ H_A \\ H_B \end{Bmatrix} \begin{matrix} p \\ q \\ r \\ s \\ t \\ h \end{matrix}$$

Joint load should be computed again, you should compute the joint loads, with respect to reference axes. So, you know joint loads are actually reversal of signs of the fixed end moments, so fixed end moments of any i-th member of bar that is global will be actually; T transpose, of the i-th member of the fixed end moment of the i-th member.

Let us call this equation number 4 now for our understanding let us recollect fixed end moments of the i-th member is simply for the values of p q r s t and h, so I should say this is going to be m maybe M A, M B, V A, V B, H A and H B.

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So, that is how we can compute for each member, let us try to explain this in a graphical form. Let us make two statements I am drawing them individually, they are parallel of the same length one is for the local axes, other is the reference axes. We know this is my j-th end; this is my k-th end, because the member should be oriented towards the positive side of x m and y m should be 90 degrees to x m ok.

So, I want to mark the degrees of freedom, let us say this is going to be my M p. This is my M q, that is the moments this is my V r this is my V s and this is going to be my H t and this is my H h, please note the degrees of freedom, in x m y m axes are marked parallel and normal to this axes. When you move this to the global axes system, then I mark in a different manner this is going to be my M bar p this is my M bar q, M bar refers to the reference axes, this is going to be my V bar r and V bar s, and this is going to be my H bar t and H bar h. Please note the sign conventions of these they are related to this axes, please note the sign conventional of these they are related to these two axes, so there is a difference ok.

Now, if I say FEM of the local which can be simply M p, M q, V r, V s, H t and H h. If I say FEM bar of the i-th member, then that should be equal to m bar p, m bar q, v bar r, v bar s, H bar t, and H bar h.



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$\{\bar{FEM}\}$  is the transformed  $\{FEM\}$  of the member in the reference axes system

$$\{\bar{FEM}\}_i = [T]_i^T \{FEM\}_i$$

Reverse the sign of  $\bar{FEM}$  to get joint load  $[JL]$   
 $[JL]$  similar

Now  $\bar{FEM}$  bar actually is a transformed value of  $FEM$  so, therefore, the fixed end moments bar is the transformed  $FEM$  vector of the member in the reference axes, how to get this we already gave this equation, let us once again write the equation there is no problem.  $FEM$  in the global of the  $i$ -th member will be given by  $T$  transpose of the  $i$ -th member of  $FEM$  of local, so I can get  $\bar{FEM}$  bar, now once I get  $\bar{FEM}$  bar reverse the sign of  $\bar{FEM}$  bar to get the joint load ok.

So, now I can get joint load bar of every member, but I want the joint load of the entire structure.

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The slide shows two equations. The first equation is:

$$\begin{Bmatrix} M \end{Bmatrix}_i = [K] \begin{Bmatrix} \delta_i \end{Bmatrix} + \{FEM\}_i$$

The second equation expands the first, showing the components of the moment vector and displacement vector:

$$\begin{Bmatrix} M_p \\ M_q \\ V_r \\ V_s \\ H_t \\ H_h \end{Bmatrix} = [K]_i \begin{Bmatrix} \delta \end{Bmatrix}_i + \{FEM\}_i$$

Once I get this then the end moments  $M_i$  of the member will be equal to the  $k$  local of the member, multiplied by  $\delta_i$  of the member, plus fixed end moment vector of the  $i$ -th member whereas, I can expand this and write (Refer Time: 20:54)  $M_p, M_q, V_r, V_s, H_t, H_h$  will be actually equal to the  $K$  matrix of the  $i$ -th member in local multiply by  $\delta$  of the  $i$ -th member plus fixed end moments of the  $i$ -th member, but I am not interested in knowing  $m$ , I want to know  $\bar{M}$ .

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The slide shows the same equation as the previous slide, but with a checkmark next to the expanded version:

$$\begin{Bmatrix} \bar{M}_p \\ \bar{M}_q \\ \bar{V}_r \\ \bar{V}_s \\ \bar{H}_t \\ \bar{H}_h \end{Bmatrix} = [k]_i \begin{Bmatrix} \bar{\delta} \end{Bmatrix}_i + \{FEM\}_i$$

So,  $\bar{M}$  vector of the  $i$ -th member is given by  $\bar{K}$  vector sorry matrix of the  $i$ -th member, multiplied by  $\bar{\delta}$  of the  $i$ -th member plus fixed end moments of the  $i$ -th member. It is very easy to get the end reactions of the member as  $\bar{M}_p$ ,  $\bar{M}_q$ ,  $\bar{V}_r$ ,  $\bar{V}_s$ ,  $\bar{H}_t$ , and  $\bar{H}_h$  I can get this. So now, we have to apply this discussion to a specific problem, and solve the problem using non orthogonal matrix knowledge and try to find out this problem.