

**Computer Methods of Analysis of Offshore Structures**  
**Prof. Srinivasan Chandrasekaran**  
**Department of Ocean Engineering**  
**Indian Institute of Technology, Madras**

**Module - 01**  
**Lecture – 13**  
**Non-Orthogonal Structures 2 (Part – 2)**

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- Stiffness method
- Planar non-orthogonal structure
- Transformation matrix

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File Edit View Insert Actions Tools Help      Mod01 - Lec13, Non-Orthogonal Structures 2 (Part - 2)      Prof. Srinivasan Chandrasekaran

Note Title      10/25/2017

$$m_i = [T] [\bar{m}] \quad \checkmark \quad T^T = T^{-1} \text{ orthogonal}$$

$$\begin{Bmatrix} m_p \\ m_q \\ v_r \\ v_s \\ H_e \\ H_h \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 0 & 0 & \cos\theta & 0 & -\sin\theta \\ 0 & 0 & \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{Bmatrix} \bar{m}_p \\ \bar{m}_q \\ \bar{v}_r \\ \bar{v}_s \\ \bar{H}_e \\ \bar{H}_h \end{Bmatrix} \quad \checkmark$$

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So, I can now find  $m$  I will be actually  $t$  matrix of  $m$  bar. So, I can write that simply we say  $m$  p,  $m$  q,  $v$  r,  $v$  s,  $H$  t and  $H$  h, should be equal to the transpose of the matrix, so  $1$  multiplied by  $m$  bar  $v$  bar and  $H$  t bar.

So, I can now say this information is true. We also know the  $T$  transpose is as same as  $T$  inverse and  $T$  is supposed to be orthogonal, this is also true.

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We can now write the following eqn.

$$\{\bar{\delta}\}_i = [T]_i^T \{\delta\}_i \quad \text{--- (3a)}$$

and

$$\{\delta\}_i = [T]_i \{\bar{\delta}\}_i \quad \text{--- (3b)}$$

$$\{\delta\}_i = \begin{Bmatrix} \delta_p \\ \delta_q \\ \delta_r \\ \delta_s \\ \delta_t \\ \delta_h \end{Bmatrix} \quad \{\bar{\delta}\}_i = \begin{Bmatrix} \bar{\delta}_p \\ \bar{\delta}_q \\ \bar{\delta}_r \\ \bar{\delta}_s \\ \bar{\delta}_t \\ \bar{\delta}_h \end{Bmatrix}$$

The diagram shows a vector in a 2D coordinate system with axes  $X$  and  $Y$ . The vector is rotated counter-clockwise from the  $X$ -axis by an angle  $\theta$ . The components of the vector are labeled as  $\cos(\theta)$  along the  $X$ -axis and  $\sin(\theta)$  along the  $Y$ -axis.

Having said this, we can now write the following set of equations delta bar on the  $i$ -th member a simply  $T$  transpose of the  $i$ -th member of delta of the  $i$ -th member and delta of the  $i$ -th member is  $T$  of the  $i$ -th member of delta bar. So, I call this as 3 a and 3 b where delta  $i$  is simply theta p, theta q, delta r, delta s, delta t, delta h, delta bar  $i$  will be theta bar p q, delta r s, delta bar s, delta bar t, delta bar h ok.

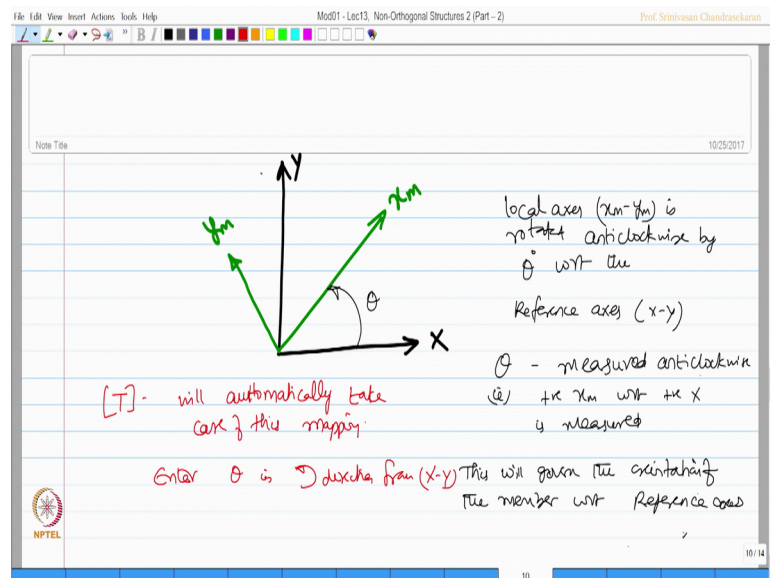
Now, if you look at the **transpose of** transformation matrix there are **cos** and **sine** values. So, if the members arbitrarily oriented if this becomes theta and this is my  $x$   $m$  and this is my  $X$  and  $x$   $m$  is measured from  $X$  anticlockwise, you know this component is actually  $\cos$  theta and this component a  $\sin$  theta so I can call this component as  $c$   $x$  because, I am resolving this along  $x$  axis, and I can call this component as  $c$   $y$  because I am resolving this along the  $y$  axis. So, now having said this my transformation matrix  $T$  can be said slightly in a different manner ok.

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$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_x & 0 & -c_y & 0 \\ 0 & 0 & 0 & c_x & 0 & -c_y \\ 0 & 0 & c_y & 0 & c_x & 0 \\ 0 & 0 & 0 & c_y & 0 & c_x \end{bmatrix}$$

Look at this matrix cos is replaced with  $c_x$ , and sin is replace with  $c_y$  and look at this matrix now and so on. So, as we said let us re insist is fact for solving the problem there are two axes.

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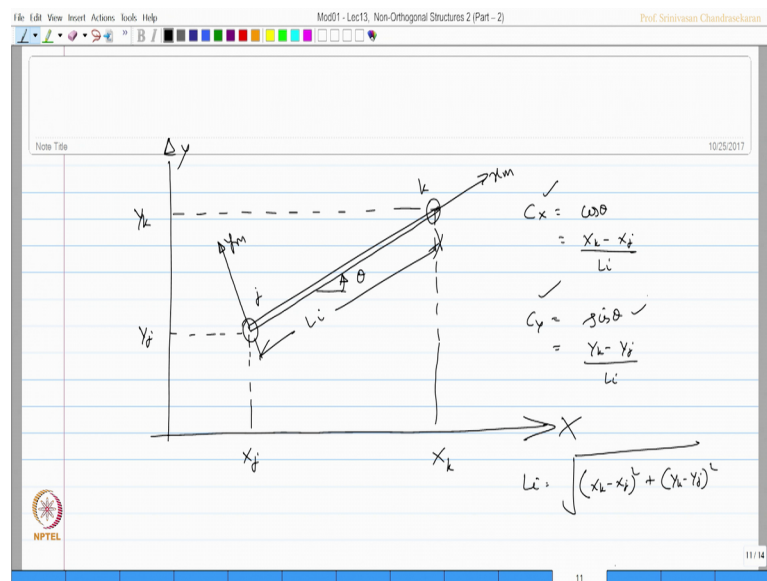


The reference axes which is X Y the member axes which is  $x_m$  and  $y_m$ . Now it is interesting to note that local axes  $x_m, y_m$  is rotated anticlockwise by theta degrees with respect to the reference axes is not.

So, I call this angle as theta. So, theta is measured anticlockwise that is positive x m with respect to positive x is measured correct. Now, this will govern the orientation of the member with respect to reference axes the most interesting feature is the T matrix will automatically take care of this mapping, it means just enter theta in counterclockwise direction measured from x y that is all.

So, whatever maybe the value if the value is more than 90 more than 180, it is automatically taken care of in the T matrix. Let us take an arbitrary oriented member.

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This becomes my x m and normal to this, becomes my y m axes and this is my global axes or reference axes x and y. Now this is my j-th end, this is my k-th end and the member has an orientation theta, the member has a length which is actually equal to  $L_i$  which I want to map, so that can be easily done. So, let us project these values on the x y plane, so this is  $X_k$ , this is  $X_j$ , this is  $Y_j$ ,  $Y_k$  is not.

So, now I can find  $C_x$  which is  $\cos \theta$  which is actually  $X_k$  minus  $X_j$  by  $L_i$ ,  $C_y$  which is  $\sin \theta$  can be  $Y_k$  minus  $Y_j$  by  $L_i$  where  $L_i$  is square root of  $X_k$  minus  $X_j$  square plus  $Y_k$  minus  $Y_j$  square. So, once I know  $C_x$ ,  $C_y$  and  $\theta$ , I can always define the matrix T for a given oriented member of  $\theta$  with respect to x y axes ok.

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for known orientation of  $(x_m - y_m)$  axes with to  $(x - y)$  axes,

- $[T]$  is completely known
- $[k]_i$  is the local axes

$[K]$  global axes

Diagram: A member bar with nodes p, q, r, s, t, h. A checkmark is drawn next to the bar.

So, now for known orientation of  $x_m, y_m$  axes with reference to the  $x, y$  axes transformation matrix is completely known.

Now, we also know the stiffness matrix of the  $i$ -th member in the local axes is not, that is this matrix which is  $p, q, r, s, t, h$   $4EI$  by  $1, 2EI$  by  $1$  we know this matrix, this is for the local axes. Now I want to find the stiffness matrix of this member bar with respect to the global axes ok.

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we know,

$$\{m\}_i = [k]_i \{d\}_i \quad (1)$$

we also know

$$\begin{cases} \{m\}_i = [T]_i \{\bar{m}\}_i \\ \{d\}_i = [T]_i \{\bar{d}\}_i \end{cases} \quad (2)$$

Sub (2) in (1)

$$[T]_i \{\bar{m}\}_i = [k]_i [T]_i \{\bar{d}\}_i$$

$$\{\bar{m}\}_i = [T]_i^{-1} [k]_i [T]_i \{\bar{d}\}_i$$

$$\{\bar{m}\}_i = \underbrace{[T]_i^{-1} [k]_i [T]_i}_{[K]_i} \{\bar{d}\}_i$$

$[K]_i = T^T k T$

We know  $m_i$  is equal to  $k_i$  of  $\delta_i$ , let us call equation number one. We also know  $m_i$  is transformation matrix of  $i$ -th member with respect to  $m_{bar}_i$ , and  $\delta_i$  local is transformation matrix of  $i$ -th member to  $\delta$  global, I call this as equation number two.

Substituting this we can very well say, now substituting 2 in 1. So, let us replace the left hand side with  $T$  of  $m_{bar}_i$ , is not will be equal to  $k$   $\delta_i$  is again replace thus  $T$   $\delta_{bar}_i$ , I want to convert this into reference axes system. So, pre multiply with a  $T$  end bars I get  $m_{bar}_i$ , will be  $T_i$  inverse,  $k_i T_i$  of  $\delta_{bar}_i$ , we already know  $T$  is an orthogonal matrix therefore,  $T$  inverse is a same as same as  $T$  transpose. So,  $k_i$  of  $T$  of  $\delta_{bar}_i$ .

So, friends I have a relationship now  $m_{bar}_i$  is sum value of  $\delta_i$ , so I can now say this is what we call as  $k_{bar}$ . So,  $k_{bar}_i$  which is the global stiffness matrix of the member is nothing but  $T$  transpose  $k$   $t$ .

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$[K]_i = [T]_i^T [k]_i [T]_i \quad \text{--- (3)}$   
 global stiffness matrix of  $i^{\text{th}}$  member, which is  
 arbitrarily oriented with respect to reference axes  
 $\{m\}_i = [K]_i \{\delta\}_i \quad \text{--- (4)}$   
 (4) gives relationship b/w component end displacements ( $\delta_i$ )  
 and end actions for member ( $m_i$ ) in X-Y axes system

Now let us write that here, so  $k_{bar}$  of any member is  $T$  transpose of that member  $k$  local of that member and again post multiply by the transpose matrix, I mean transformation matrix. So, we have established a relationship to find the global stiffness matrix of  $i$ -th member, which is arbitrarily oriented, is not with respect to the reference axes correct.

Now, I can write  $m_{bar}_i$  is  $k_{bar}_i$  of  $\delta_{bar}_i$ , so the equation this is 3, this equation 4 gives the relationship between component end displacements that is  $\delta_{bar}_i$  and end actions of the member that is  $m_{bar}_i$  in  $x$   $y$  axes system is not.

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$$[T] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_x & 0 & -c_y & 0 \\ 0 & 0 & 0 & c_x & 0 & -c_y \\ 0 & 0 & c_y & 0 & c_x & 0 \\ 0 & 0 & 0 & c_y & 0 & c_x \end{bmatrix}$$

$T^T$  ✓

$$[\bar{K}] = [T]^T [K_i] [T]$$

$[K_i]$  - is known for i<sup>th</sup> member

Now, we can say T matrix is the transformation matrix which is given by and I can write T transpose as well. So, once I know T and T transpose, I can always find K global with the simple equation T transpose k local with T where k i is known value for i-th member is not.

We already have this matrix with us, which need not be change which is completely valid it is only transform with this equation that is what we want to emphasize.

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Summary

- $(x_m, y_m) \Rightarrow$  transformed to  $(x, y)$
- $[k_i]_{n \times n}$  is valid, provided it is transformed to the reference axis system

$$\begin{cases} [m]_i = [T]_i [m]_i \\ [m]_i = T^T [m]_i \end{cases}$$

$$\bar{K}_i = [T]^T [K_i] [T]$$

So, friends in the summary we understood that for an arbitrarily oriented member whose local axes are  $x$   $m$   $y$   $m$  need to be transformed to the reference axes system  $x$   $y$ ; however, the stiffness matrix what you have worked out, for the local axes is valid provided it is transformed in the reference axes system.

So, all relationships like moment local is  $T$  of moment global or moment global is  $T$  transpose of moment local similarly,  $k$  global is  $T$  transpose  $k$   $T$  and so on, which we discussed in this lecture we will continue discussion and apply it on a problem, and show you how this can be solved easily for a nonorthogonal structures.

Thank you very much.