Computer Methods of Analysis of Offshore Structures
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Module - 01
Lecture - 13
Non-Orthogonal Structures 2 (Part - 1)

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Friends, let us continue with the 13th lecture in module 1, where we are going to talk about the continuation of analysis of non-orthogonal structures.

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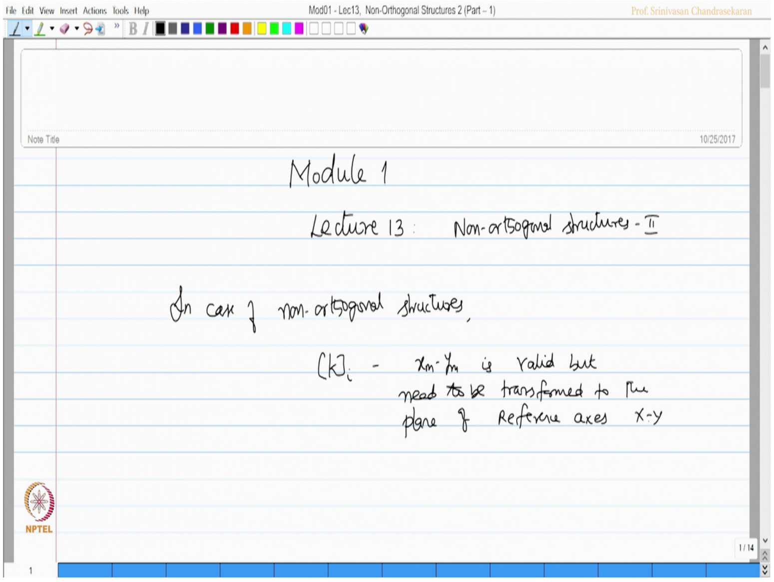

We already said that in case of non-orthogonal structures, that is members which are not intersecting at 90 degrees to each other k i of the member derived based on the local axes x m y m is valid, but need to be transformed to the plane of reference axes x-y. We will talk about how this transformation is going to happen.

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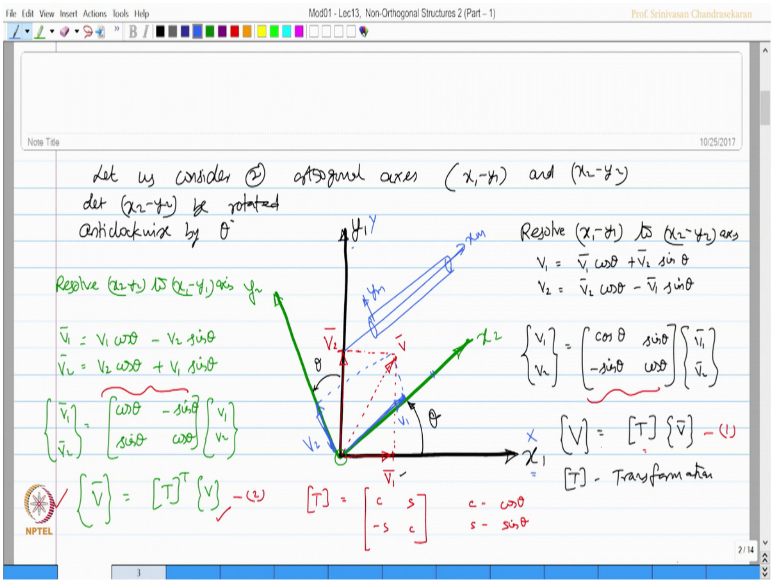

Let us consider 2 orthogonal set of axes, let us say x 1 y 1 and x 2 y 2, let us consider this; say this is x 1 y 1 and this is x 2 and y 2, you will basically observe that y 1 is anticlockwise 90 degree to x 1. Similarly y 2 is anticlockwise 90 degree to y 2. So, we are maintaining that relationship between y 1 and x 1 be it x 1 y 1 axes or other reference axes x to it, we have a common point origin O. Let x 2 y 2 be rotated anticlockwise by theta degrees; let us say this is theta degree. Now I want to find out the components of these respective in the other coordinate. So, let us say this is going to be V 1, I call this as V 1 bar and I call this component that is this component as V 2. Let us say this is our V, let us now mark V 1 and V 2 such that they are mapped as shown in the figure.

One can resolve x 1 y 1 to x 2 y 2 axes. So, one can say that V 1 is going to be that is this component is going to be V 1 bar cos theta plus V 2 bar. Similarly, V 2 can be said as V 2 bar cos theta because this angle will also be theta minus V 1 bar sin theta because the component of V 1 bar will be opposite to it. So, I can now express this in a simple matrix form V 1; V 2 can be said as cos theta sin theta minus sin theta and cos theta of V 1 bar V 2 bar; we can express this as V is some matrix T of V bar.

Now, I say T matrix is called the transformation; alternatively I can also resolve x 2 y 2 to x 1 y 1 axes. So, by that logic; I should now find out V 1 bar and V 2 bar is it not which will be V 1 cos theta minus V 2 sin theta and V 2 bar will be V 2 cos theta plus V 1 sin theta expressing this in a matrix form cos theta minus sin theta sin theta cos theta of V 1 V 2. So, I should say V bar is T transpose of V.

Please see this matrix please see this matrix with this matrix you will see that this is actually a transpose of T the rows and columns are interchanged. Now, two expressions expression one and expression 2 both are valid where the T matrix is actually equal to cos sin minus sin cos where c stands for cos and s stands for sin.

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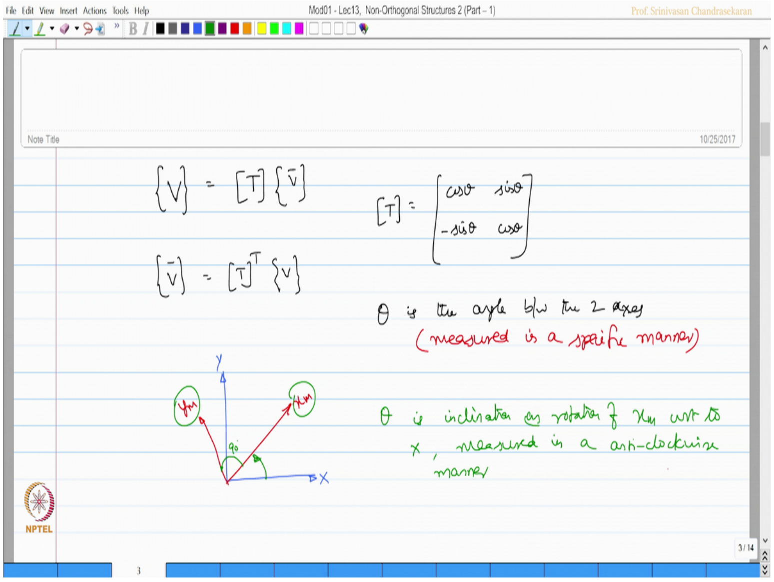

So, we have 2 relationships now which is simply V is actually equal to the transformation matrix into V bar or V bar is transformation matrix transpose into V where the transformation matrix is given by cos theta sin theta minus sin theta and cos theta where theta is the angle between the 2 axes measured in a specific style I will come to the point; what I want you to pay attention is some specific properties. Let us look at this figure whenever we are connecting V bar to V.

We are saying T transpose whenever we are connecting V to V bar we say T and further V bars are x 1 y 1 Vs are x 2 y 2. So, for any number which is arbitrarily oriented let us take I have a beam or I have a column member I have a shaft member arbitrarily oriented which is similar to or parallel to this. So, now, this becomes my local axes x m and y m and this becomes my reference axes x and y.

So, with this argument that is say this is my x y axes and this could be my x m y m axes and therefore, theta is inclination or rotation of x m with respect to x measured in a anticlockwise manner that is very important in the last lecture we already seen and understood that how x m and y m are given for our mark for a given section or member. So, x m should be considered in such a manner that the length of the member should be on the positive side of x m and y m is 90 degrees anticlockwise to x m is it not we already know how to x m and y m for a given member which arbitrarily oriented with respect to the reference axes x and y, right.

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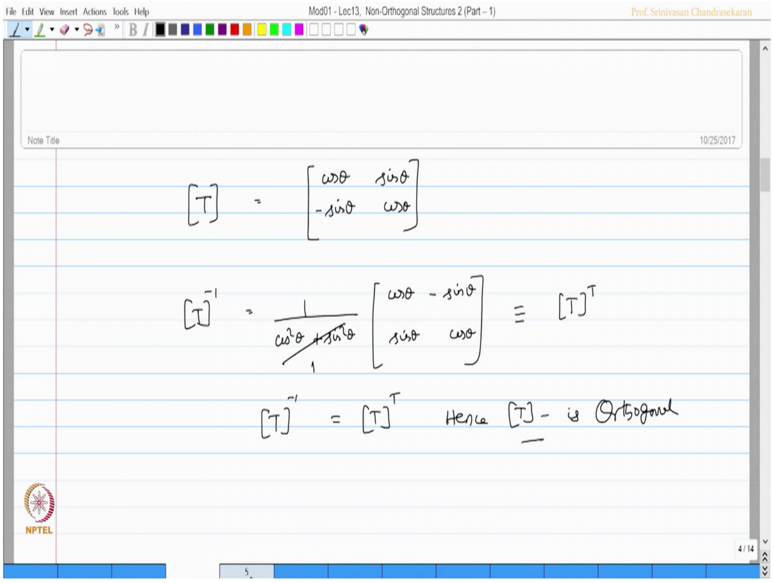

Let us now look at the specific property of this T matrix which is the transformation matrix we know T matrix is actually cos theta sin theta minus sin theta and cos theta; let us try to find the inverse of this matrix which will be 1 by cos square theta plus sin square theta of cos theta minus sin theta sin theta and cos theta because this is now equal to 1 which will be as same as the transpose of this matrix is it not change of rows and columns. So, T inverse is actually equal to T transpose hence the transformation matrix is orthogonal having said this, let us now talk about transformation of the end moments and reactions of an arbitrarily beam with respect to the reference axes systems.

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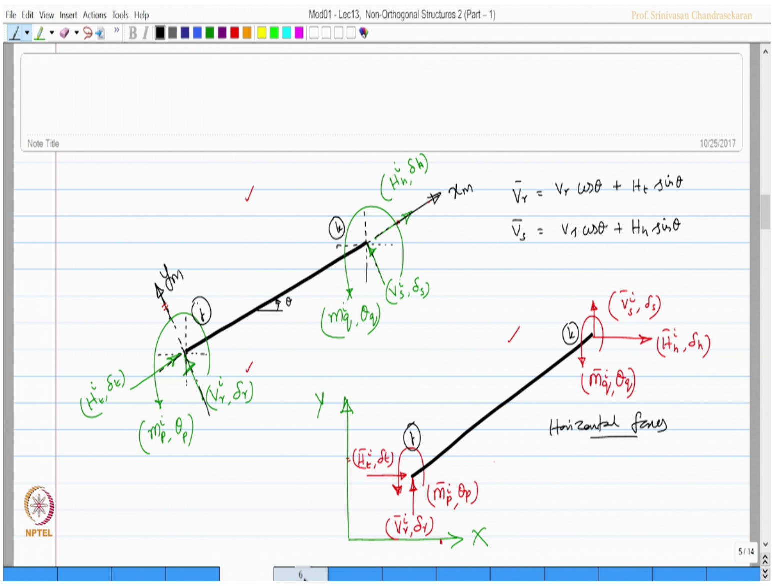

Let us mark a beam which is arbitrarily oriented let us also mark the degrees of freedom, let us say this is inclined by an angle theta let us mark these values let us now mark the degrees of freedom; let us say this is we know it is theta p equal and end moment is m p of the i-th member and we know this is theta q equalling end moment is m q of the i-th member.

And this reminds my x m and y m axes then let us mark the vertical reaction along y which is V r similar to delta r and we also have here V s similar to delta s of the i-th member let us also mark the axial deformations which will be H t of the i-th member causing delta T and this will be h; H of the i-th member causing delta H please note that all the symbols used here does not have a bar on the top it means they are local this is similar to a fixed beam which is arbitrarily oriented.

Let us now try to map this try to map this with reference to the reference axes which is x y Let us mark all of them back again here. So, this is going to be m p bar theta p, I am using bar of the i-th member and this is going to be m q bar with theta q of the i-th member and now this reaction along y will be parallel to y this was parallel to y m this is now parallel to y this value is going to be V r bar delta r.

Similarly, will mark it here this is going to be V bar s delta s and this reaction like this was parallel to x m. So, this will be parallel to x. So, this is going to be H bar T delta T and this is going to be H bar H I delta H the difference between these 2 figures are the following all degrees of freedom displacement translations are marked without a bar on the top whereas, there are bar in; here the degrees of freedom are marked along with x and y plane here they are marked on x y plane which is the reference axes. Now I want to see; how I can map this on to this or the reference axes to the local axes.

Let us do that the m p and m p bar m q and m q bar whatever may be the angle of inclination has no deference. So, let us write down that.

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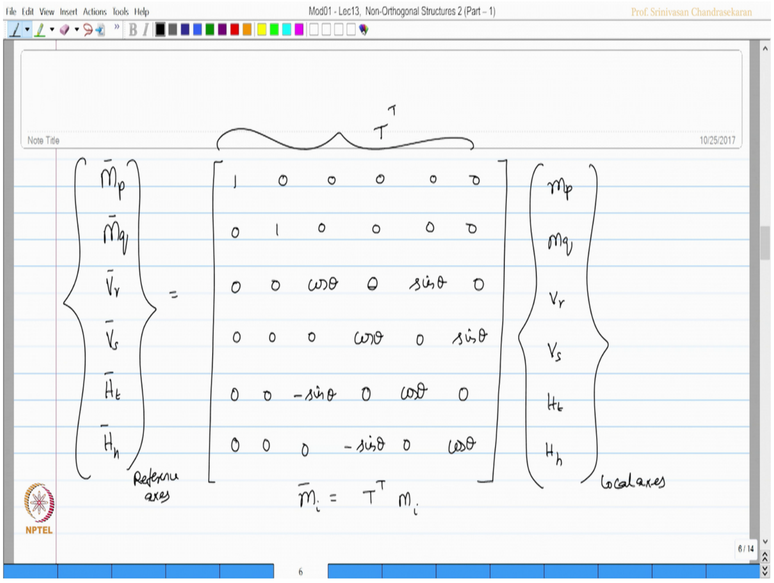

I should say m p bar m q bar V r bar V s bar H t bar and H h bar. So, all these are displacements along the reference axes this is reference axes this should be equal to some transformation matrix and connect this to the local axes I can equally mark this is going to be simply m p m q V r V s H t and H h, there is no bar let us go back to this figure m p bar is as same as m p, I should say one and there is no contribution from anything else. Similarly m q and m q bar are exactly mapped. So, 0, 1, 0, 0, 0, 0, let us talk about V bar r; we just now did this transformation of transforming any 2 value of V 1 horizontal vertical. So, we will use that logic now and say that V r bar will be actually equal to we can write it here I can write here.

So, V r bar will be actually equal to V r cos theta is it not plus H t sin theta is it not if we want to find V s that is write hand side that is the k-th end this is my j-th end this is my k-th end similarly the j-th end and the k-th end and the k-th end you know V s bar will be actually equal to V s cos theta and H h sin theta. So, one can write this similarly for the horizontal forces then one can generate the matrix as you see here. So, it is going to be 0, 0 cos theta 0 sin theta 0 and 0, 0, 0 cos theta sin theta this will be 0 0 minus sin theta 0 cos theta 0 and this is 0, 0, 0 minus sin theta 0 cos theta.

So now, I can say m bar is actually equal to T transpose of m of i-th member. So, this matrix actually the T transpose matrix. So, by transposing this once again I will get T.