

**Computer Methods of Analysis of Offshore Structures**  
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**Module – 01**  
**Lecture – 11**  
**Example Problem 3 (Part – 2)**

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- Stiffness method
- Planar orthogonal structure

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Step #3 [K]<sub>i</sub>

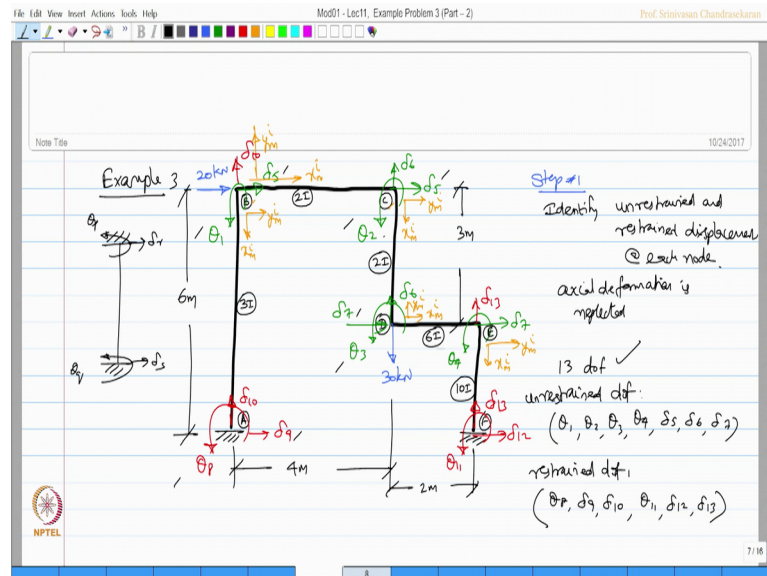
$k_{AB} = \frac{2EI}{L}$ , span = 6m,  $\frac{4EI}{L} \Rightarrow \frac{4E(3I)}{6} = 2EI$   
 $\frac{2EI}{L} \Rightarrow EI$

$k_{BC} = \frac{2EI}{L}$ , span = 4m,  $\frac{4EI}{L} \Rightarrow \frac{4E(2I)}{4} = 2EI$   
 $\frac{2EI}{L} \Rightarrow EI$

$k_{CD} = \frac{2EI}{L}$ , span = 3m,  $\frac{4EI}{L} \Rightarrow \frac{4E(2I)}{3} = 2.667$   
 $\frac{2EI}{L} \Rightarrow 1.333$

In step number 3, I want to find the stiffness matrix of any ith member. So, let us do it for k A B, if we want to do it for k A B, k A B has 3 I and the span is 6 meters. Let us say what is 4 E I by l for this which is going to be 4 E into 3 I by 6 meters.

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This distance is 6 meters and this distance is 4 meters and this distance is 2 meters and this distance is 3 meters, right. So, l is 6 meters. So, that becomes 2 E I and 2 E I by l will be half of that which is simply E I.

Similarly, required for span B C we need to also find the E I is 2 I in this case and the span of B C is 4 meters; therefore, 4 E I by l for this problem could be 4 E into 2 I by 4 which is 2 E I and 2 E I by l is actually E I; is it not is E I; let us do this for K C D. So, K C D again has 2 I and the span of C D is 3 meters; therefore, the rotational coefficient 4 E I by l could be 4 E into 2 I by 3 which is 2.667 and 2 E I by l will be half of this which is 1.333; let us do this for K D E.

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The image shows a software application window titled "Mod01 - Lec11, Example Problem 3 (Part - 2)" with a user name "Prof. Srinivasan Chandrasekaran". The window contains handwritten mathematical work on a grid background. The work is divided into two sections. The first section is for member DE, where the stiffness coefficient is given as  $K_{DE} : EI = 6EI$  and the span is  $2m$ . Calculations show  $\frac{4EI}{l} \Rightarrow \frac{4E(6I)}{2} = 12EI$  and  $\frac{2EI}{l} \Rightarrow 6EI$ . The second section is for member EF, where the stiffness coefficient is given as  $K_{EF} : EI = 10EI$  and the span is  $3m$ . Calculations show  $\frac{4EI}{l} \Rightarrow \frac{4E(10I)}{3} = 13.333EI$  and  $\frac{2EI}{l} \Rightarrow 6.667EI$ . An NPTEL logo is visible in the bottom left corner of the window.

In this case, the  $E I$  value actually is  $6 E I$  and the span is 2 meters; therefore,  $4 E I$  by  $l$  of this span would be  $4 E$  into  $6 I$  by  $2$  which is going to be  $12 E I$  and  $2 E I$  by  $l$  will be half of that which is  $6 E I$  that is the rotational coefficient for this member  $D E$ ; let us do this for  $E F$ ; here  $E I$  is going to be  $10 E I$  and the span is going to be 3 meters; therefore,  $4 E I$  by  $l$  would be  $4 E$  into  $10 I$  by  $3$  is going to be  $13.333 E I$  and  $2 E I$  by  $l$  is going to be half of that which is  $6.667 E I$ .

So, now we have the rotational coefficients for  $K A B$ ,  $K B C$   $E I$  again  $C D$ ,  $D E$  and  $E F$ . So, we can now write the stiffness coefficient on the stiffness matrix for the member  $K A B$  for the general member  $K i$ .

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$k_i = \begin{bmatrix} \frac{4EI}{l} & \frac{4EI}{l} & \frac{6EI}{l} & -\frac{6EI}{l} \\ \frac{2EI}{l} & \frac{2EI}{l} & \frac{6EI}{l} & -\frac{6EI}{l} \\ \frac{6EI}{l} & \frac{6EI}{l} & \frac{12EI}{l^2} & -\frac{12EI}{l^2} \\ -\frac{6EI}{l} & -\frac{6EI}{l} & -\frac{12EI}{l^2} & \frac{12EI}{l^2} \end{bmatrix}$

We should say  $E I$  of the rotational coefficients  $E I$  by  $l$   $2 E I$  by  $l$ ; remember, this is  $2 E I$  by  $l$ , this is  $4 E I$  by  $l$ , once I have this these sum of these  $2$  by  $l$  that is  $6 E I$  by  $l$  square minus  $6 E I$  by  $l$  square; similarly sum of these  $2$  by  $l$  which will be  $6 E I$  by  $l$  square minus  $6 E I$  by  $l$  square.

Similarly, sum of these  $2$  by  $l$  again; so,  $6 E I$  by  $l$  square sum of these  $2$  which is  $6 E I$  by  $l$  square, then sum of these  $2$   $12 E I$  by  $l$  cube minus  $12 E I$  by  $l$  cube; the last column will be the minus of the third column which is minus  $6 E I$  by  $l$  square minus  $6 E I$  by  $l$  square minus  $12 E I$  by  $l$  cube  $12 E I$  by  $l$  cube; use this now for every member and write down the stiffness matrix.

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The slide shows a stiffness matrix  $K_{AB} = EI$  and a matrix equation. The matrix equation is:

$$\begin{bmatrix} 2.0 & 1.0 & 0.5 & -0.5 \\ 1.0 & 2.0 & 0.5 & -0.5 \\ 0.5 & 0.5 & 0.167 & -0.167 \\ 0.5 & -0.5 & -0.167 & 0.167 \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \delta_5 \\ \delta_9 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

The nodes are labeled 1, 2, 5, and 9. The value 2.0 is circled in green. The value 2.0 is also written to the right of the matrix.

Let us do it for K A B faster K A B is going to be E I times of what are the labels of A B we already said the labels of A B could be 1, 8, 5, 9.

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Member	Joint	End	dof labels
BA	B	A	$(\theta_1, \theta_2, \delta_5, \delta_9)$
BC	B	C	$(\theta_1, \theta_2, \delta_5, \delta_4)$
CD	C	D	$(\theta_2, \theta_3, \delta_5, \delta_7)$
DE	D	E	$(\theta_3, \theta_4, \delta_6, \delta_{13})$
EF	E	F	$(\theta_4, \theta_1, \delta_3, \delta_{12})$

Let us do that labels here 1, 8, 5, 9. So, 2 rotations and then translations that is order 1, 8, 5 and 9 rotations and then translations. So, in that case, we already know the rotation coefficient of A B is 2 and 1; we can do that here. So, 2.0 minus 0.5; so, 2.0 minus 0.5; so, 0.5, 0.5, 0.167 minus 0.167; so, minus 0.5 minus 0.5 minus 0.167; 0.167 K A B.

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$K_{BC} = EI$

	1	2	10	6	
1	2.0	1.0	0.75	-0.75	1
2	1.0	2.0	0.75	-0.75	2
10	0.75	0.75	0.375	-0.375	10
6	-0.75	-0.75	-0.375	0.375	6

Similarly, I can do for K B C which is E I times of. So, let us see what are the labels of B C, B C labels are rotations and translations 1, 2, 10, 6 and B C values are 2 A in E I. So, let us do that here 1, 2, 10, 6. So, 1, 2, 10, 6; the values are 2.0, 1.0, 0.5, 0.75 minus 0.75, 2, 1, 0.75 minus 0.75, 0.75, 0.75, 0.375 and 375 negative minus 0.75, 0.75, 375 and 375 that is K B C.

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$K_{CD} = EI$

	2	3	5	7	
2	2.667	1.333	1.333	-1.333	2
3	1.333	2.667	1.333	-1.333	3
5	1.333	1.333	0.889	-0.889	5
7	-1.333	-1.333	-0.889	0.889	7

Let us do it for K C D which is again going to be E I times of C D labels are 2, 3, 5, 7. So, 2, 3, 5, 7 and the values for K C D is 2.667, 1.333, 1.333 minus 1.333, 2.667, 1.333,

1.333 minus 1.333, 1.333, 1.333, 0.889 minus 0.889. So, this is going to be the negative of the third column.

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$$K_{DE} = EI \begin{bmatrix} 12.0 & 6.0 & 9.0 & -9.0 \\ 6.0 & 12.0 & 9.0 & -9.0 \\ 9.0 & 9.0 & 9.0 & -9.0 \\ -9.0 & -9.0 & -9.0 & 9.0 \end{bmatrix}$$

So, we now obtain K C D with the labels; let us now do K D E which is again E I times of if you look at the labels of D E, they are going to be 3, 4, 6 and 13 values are going to be 12, 6, 9 and minus 9 I get K D E.

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$$K_{EF} = EI \begin{bmatrix} 13.333 & 6.667 & 6.667 & -6.667 \\ 6.667 & 13.333 & 6.667 & -6.667 \\ 6.667 & 6.667 & 4.445 & -4.445 \\ -6.667 & -6.667 & -4.445 & 4.445 \end{bmatrix}$$

Let us do for last span K E F is also going to be E I times of I want you to label the degrees of freedom of K E F 2 rotations and 2 translations of 4, 11, 7, 12 . So, 4, 11, 7,

12; 4, 11, 7, 12 the values are 13.333, 6.667; the remaining can be obtain as we have just now explain.

Let us write down this matrix a forth column is actually a negative of the third column; now we have a stiffness matrix of A B, B C, C D, D E and E F; 5 members, we want to combine them and then form the stiffness matrix which is unrestrained degree.

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	①	②	③	④	⑤	⑥	⑦	
①	4.0	1.0	-	-	0.5	-0.75	-	①
②	1.0	4.667	1.333	-	1.333	-0.75	-1.333	②
③	-	1.333	14.667	6.0	1.333	9.0	-1.333	③
④	-	-	6.0	25.333	-	9.0	6.667	④
⑤	0.5	1.333	1.333	-	1.056	-	-0.889	⑤
⑥	-0.75	-0.75	9.0	9.0	-	9.375	-	⑥
⑦	-	-1.333	-1.333	6.667	-0.889	-	5.334	⑦

So, can you tell me; how many unrestrained degrees are there in this problem. So, go back to the figure unrestrained degrees are about seven. So, the  $K_u u$  matrix will be square of size 7 by 7.

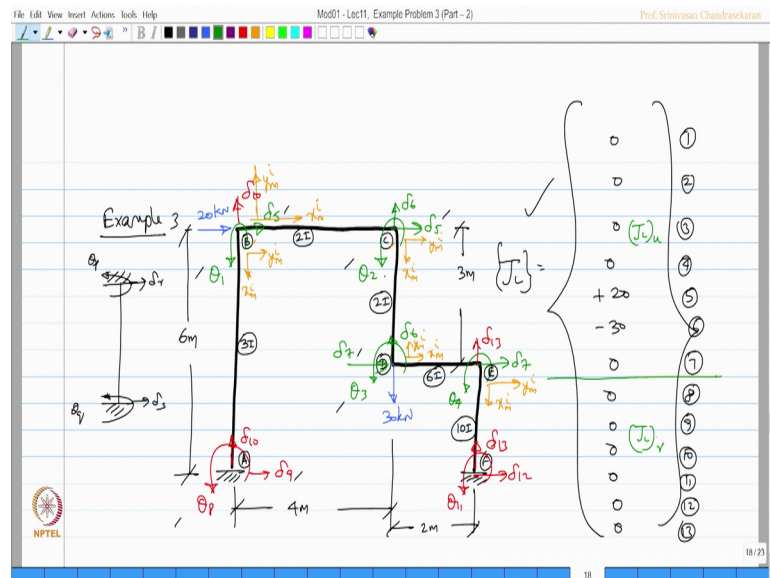
So, let us say this is going to be 7 by 7; 1, 2, 3, 4, 5, 6 and 7. So, 1, 2, 3, 4, 5, 6 and 7; so, pick up the values of these rows and columns from all the matrices and then enter the values if you try to do your exercise, I am entering the values here you can please check. So, 4.0, 1.0, 0.5 minus 0.75, 1.0, 4.667, 1.333, 1.333 minus 0.75 minus 1.333 and third row will be I will enter it here 0, 1.333, 14.667, 6.0, 1.333, 9.0 minus 1.333.

Forth will be 0, 0, 6.0, 25.333, 0, 9.0, 6.667, fifth row values will be 0.5, 1.333, 1.333, 0, 1.056, 0 minus 0.889; sixth row values will be minus 0.75 minus 0.75, 9.0, 9.0, 0, 9.375 and 0; the last row values are minus 1.333, minus 1.333, 6.667 minus 0.889, 0, 5.334, I get  $K_u u$  which is nothing, but choosing the values of these for example,  $K_{11}$ ; this value will be obtained from how can you check this, it is very easy please see the labels.



K 1 1 or anything related to 1 can be obtained from 2 matrices; one is this matrix because there is 1 here other is this matrix because there is 1 here, let us go to this matrix K B A, let us search for 1 1. So, I get this let us write down the value. So, 2.0, 1, 1, then let us go to this value again 2.0. So, we have 2.0 and 2.0 which will ultimately become 4. So, that is how one can check all the values chosen from these matrices and get K u u once I get K u u, then I have to go for the fixed end moments of all the members let us look at the loading diagram of this problem.

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The members do not have any load only the loads are the joints. So, I can directly write the joint loads directly let us copy this figure let us copy this and put it here. So, the joint load vector will be let say there is no load on 3 1 0 1. So, let say 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 and 13 and 13 degrees of freedom. So, no load on 1 0, no load on 2, you can see here 0 and at 3; there is no moment. So, 0 at 4; no moment, 0 at 5 which is a translation there is a force. So, plus 20, then 6; there is a load which is down. So, minus 30 and 7 is translational no load 0 8; no load 9 and 10 again, no load, 11 no load, 12 and 13; no load that becomes my joint load vector.

So, out of which the unrestrained degrees of freedom are 7 in number. So, I will partition this matrix at 7. So, this becomes J L u and this becomes J L r. So, I want to write the J L u separately I can write J L u.

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The image shows a digital whiteboard with handwritten mathematical notes. At the top, the text reads 'Mod01 - Lec11, Example Problem 3 (Part - 2)' and 'Prof. Srinivasan Chandrasekaran'. The main content includes the following equations and notes:

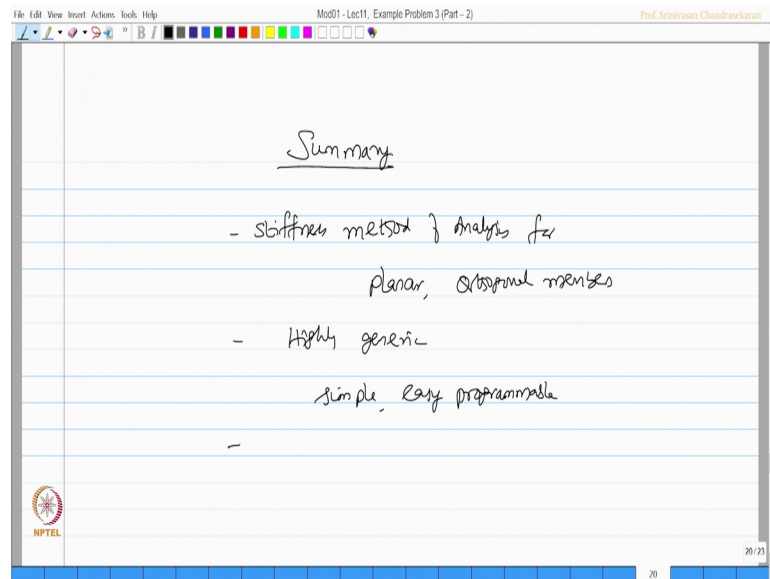
- $$K_{uu} \Delta u = J_{L_u} \quad , \quad \Delta u = K_{uu}^{-1} (J_{L_u})$$
- $$M_i = k_i \Delta_i + (FEM)_i$$
- A list of nodal moments:  $M_{AB}$ ,  $M_{BC}$ ,  $M_{CD}$ ,  $M_{DE}$ , and  $M_{EF}$ .
- In the center, there is a note:  $(K_{uu})_{7 \times 7}$  followed by  $\underline{K_{uu}^{-1}}$  and the word 'invert' with an arrow pointing to the inverse matrix.

The whiteboard interface includes a menu bar at the top with options like 'File', 'Edit', 'View', 'Insert', 'Actions', 'Tools', and 'Help'. A toolbar with various drawing tools is visible below the menu. The NPTEL logo is in the bottom left corner, and the slide number '19' is in the bottom right corner.

Once I get  $J L u$   $K u u$  of  $\Delta u$  is  $J L u$ . So, I can find  $\Delta u$  as  $K u u$  inverse of  $J L u$ , once I get  $\Delta u$ , I can apply this equation and solve for  $M_i$  that is  $M_{AB}$ ,  $M_{BC}$ ,  $M_{CD}$ ,  $M_{DE}$  and  $M_{EF}$ ; I can solve this value and check the moment. So, I am leaving this solution to you because of a simple reason  $K u u$  as you see here. So, 7 by 7 matrix. So, you need to invert this matrix.

So, I will give you a computer program using MATLAB which can invert this matrix and then solve in the mean time you can also try and try to solve this problem and check. So, friends we have discussed and mostly completed the discussion on stiffness method of analysis.

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For planar orthogonal members, we found the systems or the method used is highly generic and I should say it is simple easily programmable we have shown 3 examples to solve the problem and understand. I hope you have realized and understood and enjoyed the method. In the next lecture, we will discuss about the stiffness method applied to planar non orthogonal members.

Thank you very much.