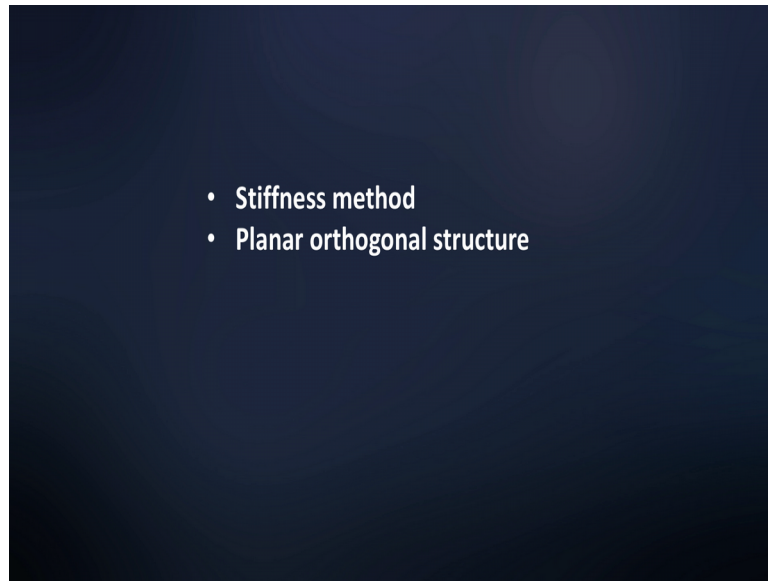


**Computer Methods of Analysis of Offshore Structures**  
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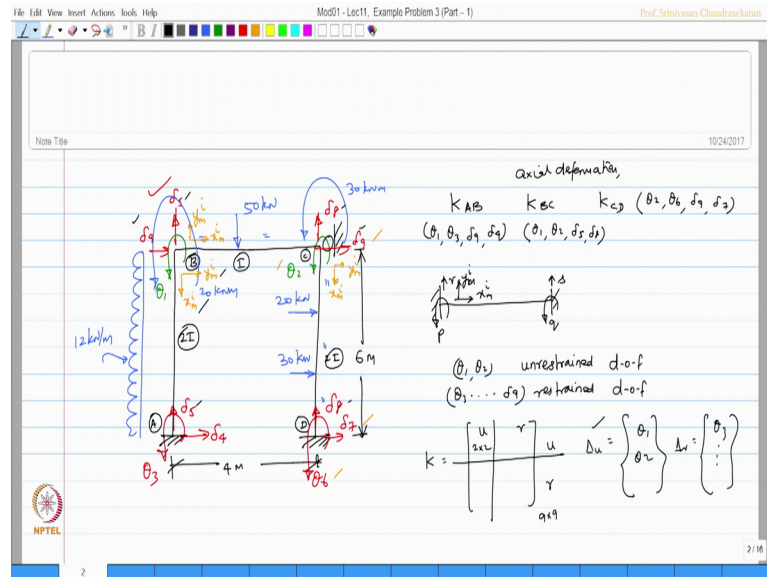
**Module – 01**  
**Lecture – 11**  
**Example Problem 3 (Part – 1)**

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Friends, let us continue with the 11th lecture in module 1. Here, we will discuss another example problem of orthogonal structures planar. Before we move on to the general principle applied to problem 3, let us try quickly revisit what we are seen in the last example.

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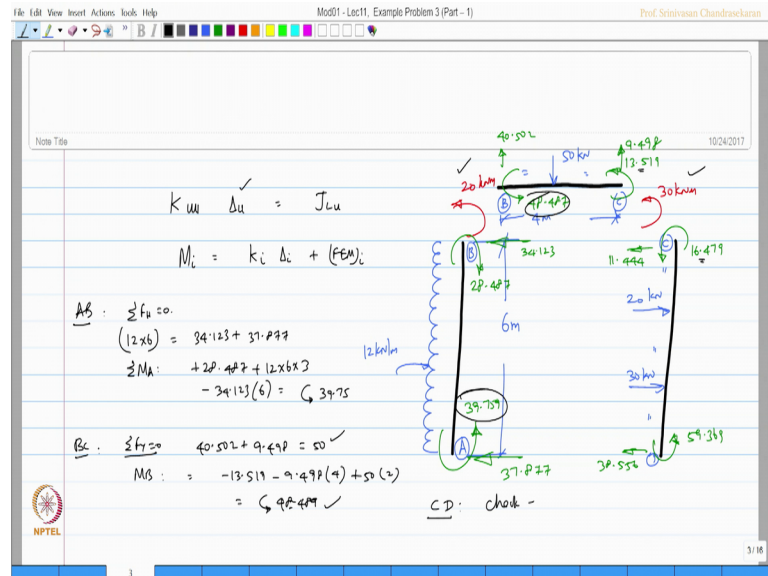
The last example is actually a single bay single story frame as you see in the figure here the 3 members, A B, B C and C D of different EI values subjected to variety of loads on the span A B, B C and C D; if you look at the results; what we have obtained? We obtained the stiffness matrix for the member A B stiffness matrix for the member B C and stiffness matrix for the member C D considering fixed beam as a standard model.

For example, while deriving the stiffness matrix for A B, the labels where theta 1, theta 3, delta 9 and delta 4, this is simple to a fixed beam of p q r and s. So, 2 rotations and 2 translations where this is my x axes and this is my y axes. So, for every member, I have to orient the axes and accordingly, I must select the label similarly for K B C, we should have labels as theta 1, theta 2 and delta 5 and delta 8 for K C D, the labels are going to be theta 2, theta 6, delta 9 and delta 7.

So, the whole derivation we have neglected the axial deformation friends, it is because of this reason you will see delta 9 and delta 9 are same at the point B and C similarly delta five is same at the points are the nodes a and B by the same reason delta 8 is same at the nodes D and C. So, among this where very well known that theta one and theta 2 are unrestrained degrees and remaining theta 3 through delta 9 are restrained degrees. So, we found the stiffness matrix with u and r we partitioned the stiffness matrix the total matrix is 9 by 9 we partitioned it with 2 by 2 and further, then we also partition the displacements delta u as theta 1, and theta 2 and delta restrained will be actually the

remaining theta 3 onwards which is actually 0 because all are restrained degrees of freedom.

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We obtained delta u by inverting  $K u$ , we use this relationship  $K u = J u$  into delta u is actually equal to  $J^{-1} L u$  we found that delta u by inverting this matrix and we got delta u, then we found out fixed end moments in end reaction beam using this relationship fixed end moments of the ith member. So, the finite solution what we have looked like this, this is subjected to a UDL of 12 kilo Newton per meter for a span of 6 meters which is A B.

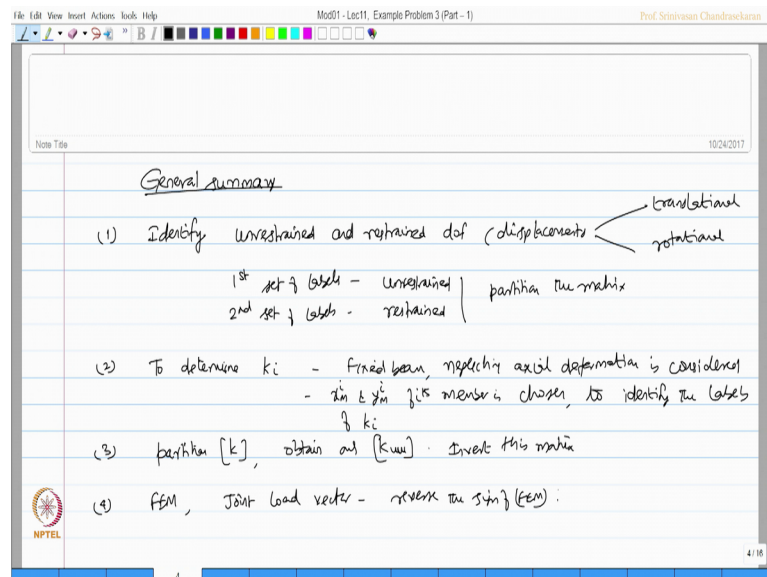
Similarly, for B C, we had a point load of 50 kilo Newton which is equally spaced and this distance is 4 meters and for C D, we had again loads which are 20 kilo Newton and 30 kilo Newton which are equally spaced; in addition, we also had joint moments here as 20 and here as 30. So, for this we found out the results; the results are as follows this reaction end moment is 39.759 and this is 28.9487 and this reaction was 34.123 and this value was 37.877.

And for this beam, this reaction was 48.487 and this was 13.519 and this became 16.479 and this became 59.369; this reaction was 40.502 and this reaction is 9.498 and this reaction is 11.444 and this reaction shear is 38.556.

So, one can apply it; check and see let us take this span A B. So, let us say  $\sigma_{FH}$  should be 0. So, one can see here  $12 \times 6$  is actually equal to  $34.123 + 37.877 \sigma_{MA}$ . let us try to find out which is going to be plus  $28.9487 + 12 \times 6 \times 3 - 34.123 \times 6$  which will be counter clockwise of  $39.75$  which is as same as this value.

If we look at span B C,  $\sigma_{Fy}$  should be set to 0. So,  $40.502 + 9.498$  amounts to  $50$ . Similarly want to compute  $M_B$  will be actually equal to  $-13.519 - 9.498 \times 4 + 50 \times 2$  which amounts to anticlockwise of  $48.489$  which is same as this in addition you can see the net moment join B is  $20$  and the net moment at joint C is  $30$ .

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So, by this logic, one can check the section C D, I leave it to you; check for any error in the solution. So, let us try to find here general summary what we have so far applied to these 2 problems. So, the first step is identify unrestrained and restrained degrees of freedom essentially displacements, they can be translational or rotational.

But please note, first set of labels will be unrestrained, the second set of labels will be restrained, this is required if we really want to partition the matrix for analysis; the second step was to determine the stiffness matrix of every member, fixed beam neglecting axial deformation is considered  $x_m$  and  $y_m$  of the  $i$ th member is chosen to identify the labels of stiffness matrix of the  $i$ th member.

So, once  $K_i$  of all the members are found out then we partition the  $k$  matrix of the entire system and obtain only  $K$  unrestrained matrix then invert this matrix from the fixed end moment, we need to identify the joint load vector, joint load vector is nothing but reverse the sign of fixed end moment.

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The slide contains the following handwritten content:

$$[J] = \begin{Bmatrix} J_u \\ J_r \end{Bmatrix} \quad [k_{uu}] \{\Delta_u\} = \{F\}_u \quad \Delta_u = k_{uu}^{-1} \{F\}_u$$

$$\Delta_r = \begin{Bmatrix} 0 \end{Bmatrix}$$

-  $\{M_i\} = K_i \Delta_i + \{FEM\}_i$

- Check this solution also with the following relationship

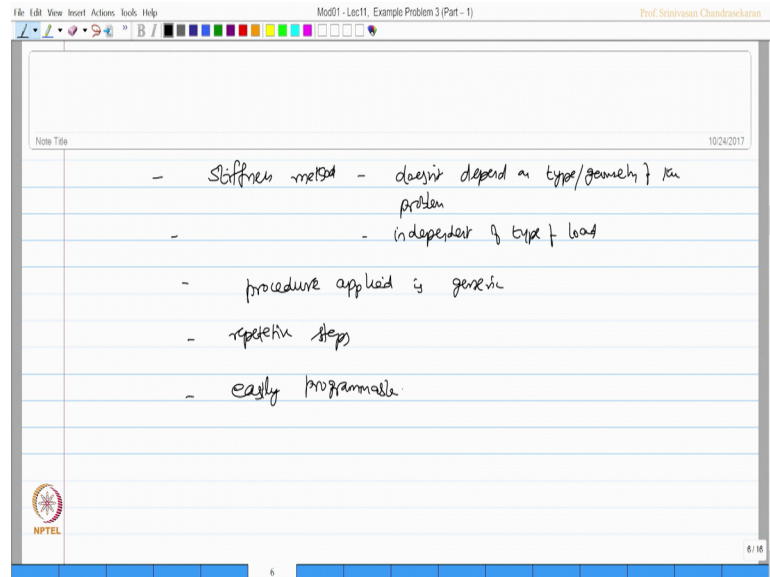
$$[K]_{ru} \{\Delta\}_u - \{J\}_r = \{R\}_r \quad \checkmark$$

And identify the joint vector, the joint load matrix is also partitioned as unrestrained and restrained, then  $K$  unrestrained with joint load unrestrained is  $K$  unrestrained with displacements, unrestrained is joint load unrestrained.

So, then we obtain  $\Delta_u$  which is actually  $K_{uu}^{-1}$  of joint load unrestrained. So, we can get unrestrained displacements and finally we all now remember the restrained displacements will be 0. Then we identified the end moments and end reactions of each member by using this algorithm,  $L_i \Delta_i$  plus fixed end moments of the  $i$ th member and found out the  $M_i$  vector for every member and then one can check this solution also with the following relationship  $K_{ru} \Delta_u - J_r$  should be actually also equal to the reaction of.

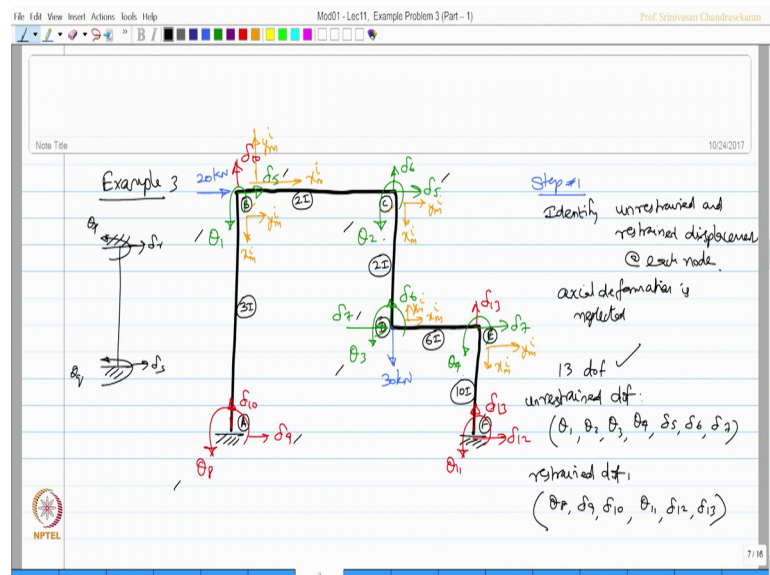
So, substitute this examine the results obtained from the previous steps and then compare and validate.

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So, interestingly friends, in this case, the method proposed which is actually the stiffness method does not depend on the type or geometry of the problem, it is independent of the type of load the procedure applied is generic it has got repetitive steps and therefore, easily programmable. Let us apply this for another problem. In fact, we will not solve this problem, but we will see how solution can be obtained for these problems.

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So, this is example 3 which is slightly a complicated problems; let us say have a frame like this which is fixed at both the ends. Now the moment of inertia of the column

section and the beam sections are marked as shown in the screen; let us also name the joints A, B, C, D, E and F. Let us see the loading applied to the system, there is a joint load here is about 20 kilo Newton, there is a joint load here which is about 30 kilo Newton; that is all.

There are unknown member loads there are directly applied on the joints. So, what is the first step the first step is identify unrestrained and restrained displacements at each node; is it not; let us do that let us mark unrestrained in green and restrained in red. So, this is theta 1, theta 2, theta 3, theta 4, delta 5; since delta 5 is same at nodes B and C, we can say that axial deformations are neglected.

So, delta 5, then delta 6, then delta 7, then we have theta 8, delta 9 and delta 10, theta 11, delta 12 and delta 13. So, now, friends there are 13 degrees of freedom; let us now say what are unrestrained degrees; we can name them theta 1, theta 2, theta 3, theta 4, delta 5, delta 6 and delta 7; what all the restrained degrees of freedom, we can name them theta 8, delta 9, delta 10, theta 11, delta 12, delta 13.

So, that makes total 30 degrees out of which seven are unrestrained the remaining 6 are restrained. So, there is no difficulty in doing.

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Member	j <sup>th</sup> end	k <sup>th</sup> end	DOF labels
BA	B	A	( $\theta_1, \theta_2, \delta_5, \delta_6$ )
BC	B	C	( $\theta_1, \theta_2, \delta_{10}, \delta_4$ )
CD	C	D	( $\theta_3, \theta_4, \delta_5, \delta_7$ )
DE	D	E	( $\theta_3, \theta_4, \delta_6, \delta_{13}$ )
EF	E	F	( $\theta_4, \theta_1, \delta_7, \delta_{12}$ )

The first step which is identifying this degrees of freedom and next step is for a member this is step number 2 let us identify what should be the jth end and kth end that is

orientation of the member and what all the degrees of freedom labels let us take for example, B A the moment, I say this is the orientation. So, this is going to be my  $x_m$  and  $y_m$  for B A. So, I should write here B A jth end; jth end is at B kth end is at A and what are the degrees of freedom compare this with the standard fixed beam.

Let us say the fixed beam is marked parallel for our understanding; let us call this as  $\theta_p$ , this as  $\theta_q$ , this as  $\delta_r$  and  $\delta_s$ , that is a standard beam, we have neglecting axis deformation. So, what are the labels compared to that as for as the member B A is concerned, so  $\theta_1$ ,  $\theta_8$ ,  $\delta_5$  and  $\delta_9$ . So, let us do that. So, the labels are  $\theta_1$ ,  $\theta_8$ ,  $\delta_5$  and  $\delta_9$ .

Similarly, let us do for the member B C. So, the B C will get aligned this way this is  $x_m$  and this is going to be  $y_m$ , right. So, jth end is at B and kth end is at C. So, for the member B C jth end is at B and this is at C let see; what are the labels. So, you correctly pointed out rotation left rotation, right, normal displacements left normal displacements right. So,  $\theta_1$ ,  $\theta_2$  and this is  $\delta_{10}$  and this is  $\delta_6$ . So, that is right. So, let us write down that  $\theta_1$ ,  $\theta_2$ ,  $\delta_{10}$  and  $\delta_6$ .

Let us do it for C D. So, C D will be oriented in this fashion. So, this is going to be my  $x_i$ , this is going to be my  $y_i$ . So, the labels are going to be  $\theta_2$   $\theta_3$   $\delta_5$  and  $\delta_7$ . So, for the member C D, C is the jth end and D is the kth end and the labels are  $\theta_2$ ,  $\theta_3$ ,  $\delta_5$  and  $\delta_7$ , correct;  $\theta_2$ ,  $\theta_3$ ,  $\delta_5$  and  $\delta_7$ , let us do for the member D E, the is the orientation which is  $x_m$  and this is  $y_m$ . So, D and E  $\theta_3$ ,  $\theta_4$  and  $\delta_6$  and  $\delta_{13}$  correct.

So, let us do that here. So, for the member D E, D and E are the ends in the labels are  $\theta_3$ ,  $\theta_4$ ,  $\delta_6$  and  $\delta_{13}$ , you can see here  $\theta_3$ ,  $\theta_4$ ,  $\delta_6$  and  $\delta_{13}$ ; then for the member E F; let us do the orientation for E F; E is the jth end and F is the kth end and the labels are rotation at j rotation at k displacements at j displacements at k, so 4, 11, 7, 12 correct. So, 4, 11, 7 and 12 thetas are rotations and deltas are translations. So, this step number 2. So, there is no confusion step number 2; we can do it for these problems, correct. Let us next step say step number 3.