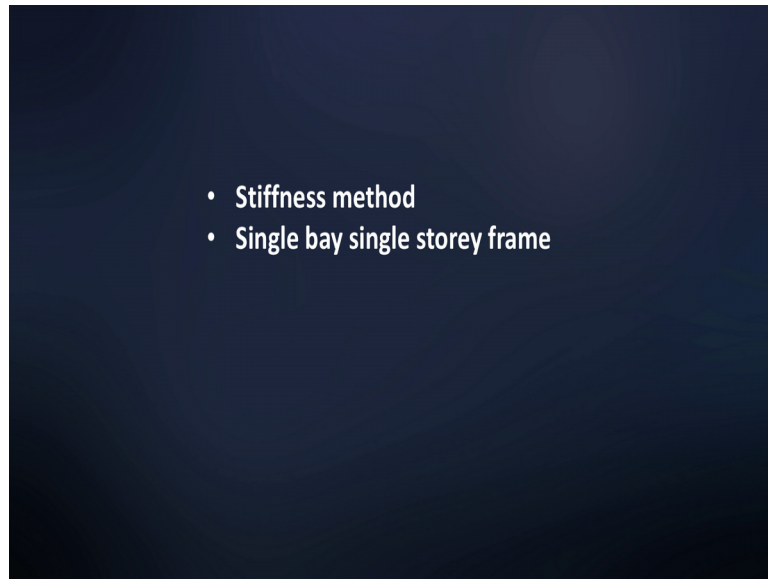


Computer Methods of Analysis of Offshore Structures
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Module - 01
Lecture - 10
Example Problem 2 Continued...

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Friends, let us continue with the example 2 in the 10th lecture, in module 1. This is the example; what we are working out we have a single **bay** single story frame, we have marked the degrees of freedom for the members AB, BC and CD by identified the size of each stiffness matrices and the labels we worked out the stiffness matrices of each member K_{AB} , K_{BC} and K_{CD} .

(Refer Slide Time: 00:54)

Handwritten matrix K_{uu} on a slide. The matrix is a 4x4 block matrix:

$$K_{uu} = EI \begin{bmatrix} 1.333 & 0.667 & 0.333 & -0.333 \\ 0.667 & 1.333 & 0.333 & -0.333 \\ 0.333 & 0.333 & 0.111 & -0.111 \\ -0.333 & -0.333 & -0.111 & 0.111 \end{bmatrix}$$

The matrix is partitioned into four quadrants labeled 1, 2, 3, and 4. The top-left quadrant (1) contains the value 1.333. The top-right quadrant (2) contains the value 0.667. The bottom-left quadrant (3) contains the value 0.333. The bottom-right quadrant (4) contains the value 0.111. A checkmark is next to the matrix. To the right of the matrix, the text $K_{11} = 1.333$ is written with a checkmark. The slide also shows a toolbar at the top and an NPTEL logo at the bottom left.

(Refer Slide Time: 00:55)

Handwritten matrix K_{uu} on a slide. The matrix is a 4x4 block matrix:

$$K_{uu} = EI \begin{bmatrix} 1 & 0.5 & 0.375 & -0.375 \\ 0.5 & 1 & 0.375 & -0.375 \\ 0.375 & 0.375 & 0.188 & -0.188 \\ -0.375 & -0.375 & -0.188 & 0.188 \end{bmatrix}$$

The matrix is partitioned into four quadrants labeled 1, 2, 3, and 4. The top-left quadrant (1) contains the value 1. The top-right quadrant (2) contains the value 0.5. The bottom-left quadrant (3) contains the value 0.375. The bottom-right quadrant (4) contains the value 0.188. A checkmark is next to the matrix. The slide also shows a toolbar at the top and an NPTEL logo at the bottom left.

We combine them and obtain partition matrix K_{uu} .

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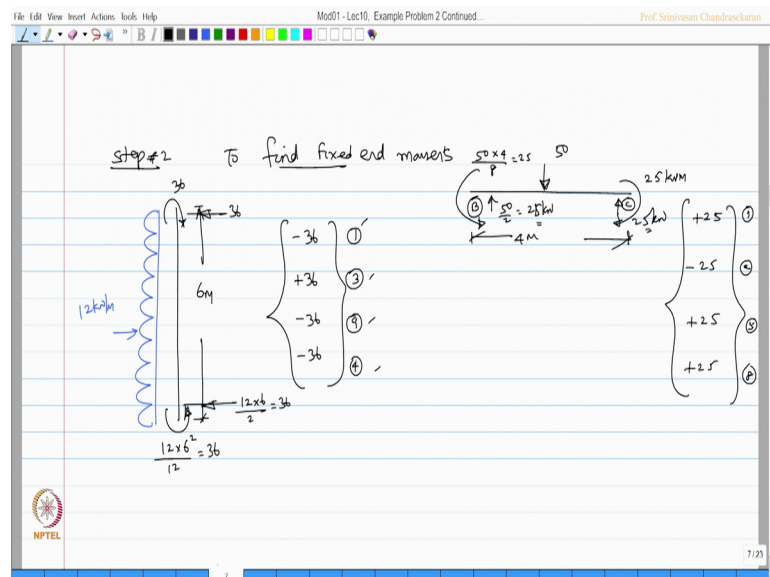
Handwritten equations for the stiffness matrix $[K]_{uu}$ and its inverse $[K]_{uu}^{-1}$.

$$[K]_{uu} = EI \begin{bmatrix} 1.333 & 0.5 \\ 1.000 & 2.333 \\ 0.5 & 1.000 \\ 2.333 & 2.333 \end{bmatrix}$$

$$[K]_{uu}^{-1} = \frac{1}{5.193 EI} \begin{bmatrix} 2.333 & -0.5 \\ -0.5 & 2.333 \end{bmatrix}$$

We also got the inverse of this the next step.

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We have to determine, to find the fixed end moments, we did for the span AB, this is for the span AB, we did this also for the span BC, we did this also for the span CD, we are going to do this.

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Example 2

$[k]_{ABC D} = \text{size } 9 \times 9$

$\{\theta_1, \theta_2\}$ unrestrained d.o.f ✓

$\{\theta_3, \delta_4, \delta_5, \theta_6, \delta_7, \delta_8, \delta_9\}$ - restrained d.o.f

Member	1 st end	2 nd end	d.o.f (local)
BA	B	A	$(\theta_1, \theta_3, \delta_7, \delta_4)$
BC	B	C	$(\theta_1, \theta_5, \delta_5, \delta_8)$
CD	C	D	$(\theta_2, \theta_6, \delta_9, \delta_7)$

Now, for CD, let us say as for as 2 is concerned, let us mark the degrees of freedom for let say member CD member CD has theta 2. So, let us look at the degrees of freedom of the member CD, then say theta 2 theta 6, then delta 9 and delta 7 that is what we have here.

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Takin moments about (D)

$20 \times 4 = V_c \times 6 \Rightarrow V_c = 13.333$

$V_b = 20 - 13.333 = 6.667$

$30 \times 4 = V_c \times 6 \Rightarrow V_c = 20$

$V_c = 10 \text{ kN}$

$20 \times 2 = 2.6667$

$20 \times 4 = 17.778$

$30 \times 4 = 13.333$

$10 \times 6 = 30 \times 2$

$V_c = 10 \text{ kN}$

2	-17.778	-31.111
6	-13.333	+35.556
9	-13.333	-23.333
7	-6.667	-26.667
CD	-20.000	

Let us mark it here for our convenience, this is C end, this is D end, this is theta 2, this is theta 6, this is delta 9 and delta 7.

So, now I have to compare theta 2 with this value and this value let us enter this here. So, this is anticlockwise; clockwise. So, I should say minus 17.778, then minus 13.333 that is my theta 2 that is M 2. Similarly for 6, I have to compare this value and these value both are anticlockwise; so, plus 8.889 plus 26.667. Similarly for 9, I should compare this value and this value both are towards left. So, minus this value is minus 13.333 and minus 10.000.

Similarly, for 7, I should compare this value and V D which will be V D is also acting this way. So, minus 6.667 and minus 20; so, which amounts to minus 31.111 plus 35.556 minus 23.333 and minus 26.667; now I am interest in estimating the joint loads.

Lets us go back to the problem; have a small addition. In the problem, the addition is this frame is also subjected to an external moment; an external moment of 20 kilo Newton meter and 30 kilo Newton meter at the joints B and C respectively. So, let us now work out the joint load vector. So, I have the FEM vectors, let us write down those values here, once again fixed end moments vectors. So, for AB and BC, we have this values.

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The image shows handwritten mathematical work on a digital whiteboard. It details the assembly of a global joint load vector $[J]$ from fixed end moment (FEM) vectors for three members: AB, BC, and CD.

- FEM Vectors:**
 - Member AB: $\begin{Bmatrix} -36 \\ +36 \\ -36 \\ -36 \end{Bmatrix}$ (Degrees of Freedom 1, 3, 9, 4)
 - Member BC: $\begin{Bmatrix} +25 \\ -25 \\ +25 \\ +25 \end{Bmatrix}$ (Degrees of Freedom 2, 5, 8, 7)
 - Member CD: $\begin{Bmatrix} -31.111 \\ 35.556 \\ -23.333 \\ -26.667 \end{Bmatrix}$ (Degrees of Freedom 6, 9, 7, 2)
- Global Joint Load Vector $[J]$:**

$$[J] = \begin{Bmatrix} +36 - 25 + 20 \\ +36 + 25 + 30 \\ -36 \\ -25 \\ -35.556 \\ +26.667 \\ -25.000 \\ +36 + 23.333 \\ -45 + 33.333 \end{Bmatrix}$$
- Resulting Joint Load Vector J_u :**

$$J_u = \begin{Bmatrix} 31 \\ 86.111 \end{Bmatrix}$$

Let us write down that fixed end moment vectors for AB, I have which will be the labels of 1, 3, 9 and 4 minus 36 plus 36 minus 36 minus 36 and for BC, we have the labels are 1, 2, 5 and 8; the values are plus 25 minus 25 plus 25 and plus 25 and for CD, we have the fixed end moment vector as minus 31.111, 35.556 minus 23.333 and minus 26.667 the labels are 2 6 9 and 7.

So, now I want to estimate the joint load vector let us do the joint load vector which is going to be 9 rows and 1 column; let me do that here. So, 1, 2, 3, 4, 5, 6, 7, 8 and 9; let us see the one; I have to pick up this value, I have to pick up this value, I have to reverse them because I am talking about joint load. So, plus 36 minus 25 in addition; if we look at the problem, I also have a load of plus 20 at theta 1; is it not.

So, let us include that also. So, I going to be plus twenty which makes this as plus 31. Similarly 2 I have looked into this value, this value and there is one more 30 kilo Newton meter applied. So, plus 30 that is anticlockwise and plus 25 and plus 31.111 which makes it as 86.111, if you look at 3, it will be this value; there is no other 3. So, reverse the sign minus 36.

Look at 4; this value reverse the sign plus 36 why reverse the sign; we are looking for the joint load vector then the fifth one is this value reverse the sign minus 25, sixth value this reverse the sign minus 35.556, seventh value will be this reverse the sign. So, plus 26.667 eighth row is this value; so, minus 25.000 and ninth value will be this plus this. So, let us reverse the sign; so plus 36 plus 23.333; which makes it as plus 59.333.

Now, we partition this matrix at 2 that is the unrestrained degree of freedom this is restrained. So, I can say J L will have now 2 values. This is J L unrestrained, this is J L restrained, this is 9 by 1. So, this is going to be 2 by 1, this is 7 by 1. So, J L u if you really want to write J L u alone which is going to be 31 and 86.111, these 2 values having said this that is used this value K comprises of unrestrained restrained unrestrained restrained.

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$$\begin{bmatrix} k_{uu} & k_{ur} \\ k_{ru} & k_{rr} \end{bmatrix} \begin{Bmatrix} u \\ r \end{Bmatrix} = \begin{Bmatrix} \Delta u \\ \Delta r \end{Bmatrix} = \begin{Bmatrix} J_{Lu} \\ J_{Lr} \end{Bmatrix}$$

$$k_{uu} \Delta u = J_{Lu}$$

$$\Delta u = k_{uu}^{-1} J_{Lu}$$

Let say these are the partition this is k u u this is k u r this is k r u this is k r r multiplied by this delta u delta r which will be actually equal to the joint load vector.

So, now I can expand this k u u into delta u is J L u therefore, delta u is actually k u inverse of J L u I have k u u I have J L u, I can find delta u.

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$$\Delta u = \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \frac{1}{5.193 EI} \begin{bmatrix} 2.333 & -0.5 \\ -0.5 & 2.333 \end{bmatrix} \begin{Bmatrix} 31.0 \\ 86.11 \end{Bmatrix}$$

$$= \frac{1}{5.193 EI} \begin{Bmatrix} 29.268 \\ 185.377 \end{Bmatrix} \text{ rad}$$

Let us do that. So, delta u which will be the unrestrained degrees theta 1 and theta 2 you can see the problem these are unrestrained degrees marked in green theta 1 and theta 2 which will be given by 1 by 5.193 EI multiplied by k u u inverse, we already have it

here, we already have it here. So, I am just writing it here 2.333 minus 0.5 minus 0.5; 2.333 multiplied by this vector which is 31 and 86.111.

So, 31 and 86.111; so, I must get this value as 1 by 5.193 EI of 29.268 and 185.397, so many radians because they both are rotations; that is my delta u.

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$$M_i = K_i \Delta_i + (FEM)_i$$

$$M_{AB} = \begin{Bmatrix} M_1 \\ M_3 \\ V_9 \\ V_4 \end{Bmatrix}_{4 \times 1} = EI \begin{bmatrix} 1.333 & 0.667 & 0.333 & -0.333 \\ 0.667 & 1.333 & 0.333 & -0.333 \\ 0.333 & 0.333 & 0.111 & -0.111 \\ -0.333 & -0.333 & -0.111 & 0.111 \end{bmatrix}_{4 \times 4} \begin{Bmatrix} \theta_1 \\ \theta_3 \\ \delta_9 \\ \delta_4 \end{Bmatrix}_{4 \times 1} + \begin{Bmatrix} -36 \\ 36 \\ -36 \\ -36 \end{Bmatrix}$$

$$\theta_i = \frac{29.268}{5.193 EI} \begin{Bmatrix} M_1 \\ M_3 \\ V_9 \\ V_4 \end{Bmatrix} = \begin{Bmatrix} -28.487 \\ 39.757 \\ 34.123 \\ \dots \end{Bmatrix}$$

Now I am interested in finding end moment and the end shears. So, the equations is $M_i = K_i \Delta_i + FEM$ will be $K_i \Delta_i + FEM$ of the member i . So, let us expand this for M_{AB} which will be the labels of AB or $1\ 3\ 9\ 4$ therefore, M_1, M_3, V_9 and V_4 which is going to be EI times of K_{AB} which I am writing here $1.333, 0.667, 0.333$ minus $0.333, 1.333, 0.667, 0.333$, minus 0.333 . This is again $0.333; 0.333, 0.111$ and minus 0.111 that is K_{AB}, K_{AB} multiplied by.

So, this is going to be 4 by 1 , this is 4 by 4 expected to be 4 by 1 which is going to be the displacements for the first degree for member AB . So, it is going to be $\theta_1, \theta_3, \delta_9$ and δ_4 . So, out of which you; now these 3 are 0 you can look at the problem θ_3, θ_3 , then δ_9, δ_9 and δ_4, δ_4 , they are restrained therefore, they are 0 .

So, they are 0 which will be added to the fixed end moments of the member AB which is here which is fixed end moments of the member AB I am just adding this vector back again so minus 36 plus 36 minus $36; 36$. So, one can work it out and try to find out the

moments where theta one actually is equal to 29.268 by 5.193 EI substitute that and one can find the moments n moments and n shears of this.

So, it is very simple; 1.33 of this minus 36 will be able to get. For example, I will find out the first vector let us say M 1, M 3, V 9, V 4 which will be equal to minus 28.487, 39.759, 34.143, and so on. So, I think you should be able to fill up this and check that is for AB.

(Refer Slide Time: 15:20)

$$M_{BC} = \begin{Bmatrix} M_1 \\ M_2 \\ V_5 \\ V_8 \end{Bmatrix} = EI \begin{bmatrix} 1 & 0.5 & 0.375 & -0.375 \\ 0.5 & 1 & 0.375 & -0.375 \\ 0.375 & 0.375 & 0.188 & -0.188 \\ -0.375 & -0.375 & -0.188 & 0.188 \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \delta_5 \\ \delta_8 \end{Bmatrix} + \begin{Bmatrix} 2.5 \\ -2.5 \\ 2.5 \\ 2.5 \end{Bmatrix}$$

$$\begin{Bmatrix} M_1 \\ M_2 \\ V_5 \\ V_8 \end{Bmatrix} = \begin{Bmatrix} 48.487 \\ 13.519 \\ 40.502 \\ 9.498 \end{Bmatrix} \quad BC$$

Let us do for M BC which is again the BC labels are 1 2 5 and 8 1 2 5 and 8; that is how the labels of M BC which will be EI times of the stiffness matrix 1.5 0.375 minus 0.375 multiplied by theta 1, theta 2, delta 5 and delta 8 plus fixed end moments of the member BC which is available here and see here this vector 25 minus 25, 25, 25. So, let us do that here 25 minus 25 and both are 25 and we all know that delta 5 and delta 8 are 0 because they are restrained degrees of freedom.

Substituting, I may get M 1, M 2, V 5, V 8 as 48.487, 13.519, 40.502 and 9.498 that is for the member BC.

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$$M_{CD} = \begin{Bmatrix} M_2 \\ M_6 \\ V_9 \\ V_7 \end{Bmatrix} = EI \begin{bmatrix} 1.333 & 0.667 & 0.333 & -0.333 \\ 0.667 & 1.333 & 0.333 & -0.333 \\ 0.333 & 0.333 & 0.111 & -0.111 \\ -0.333 & -0.333 & -0.111 & 0.111 \end{bmatrix} \begin{Bmatrix} \theta_2 \\ \theta_6 \\ \delta_9 \\ \delta_7 \end{Bmatrix} + \begin{Bmatrix} -31.111 \\ 35.556 \\ -23.333 \\ -26.667 \end{Bmatrix}$$

$$\begin{Bmatrix} M_2 \\ M_6 \\ V_9 \\ V_7 \end{Bmatrix} = \begin{Bmatrix} 16.479 \\ 59.369 \\ -11.444 \\ -38.556 \end{Bmatrix} \quad \theta_2 = \frac{1}{5.193 EI} (185.397)$$

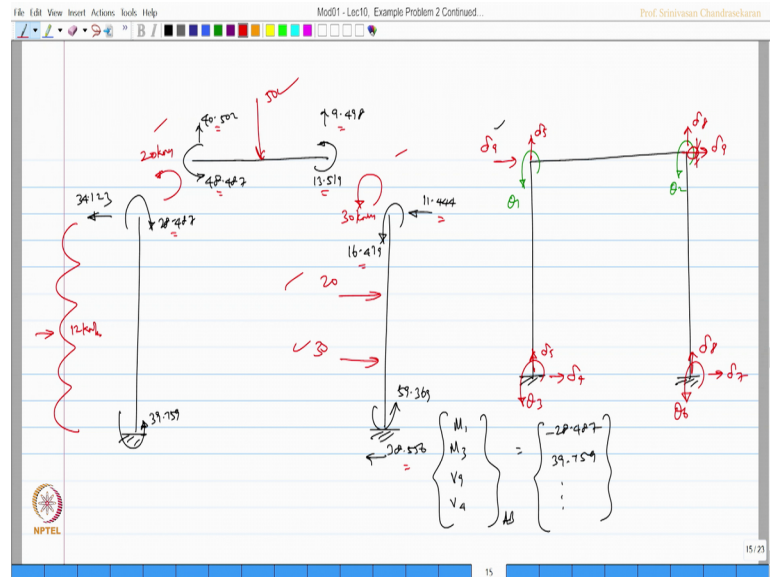
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Similarly, we can do for the member CD for M_{CD} ; we have the vector as M_2, M_6, V_9 and V_7 that is the label for the member CD which will be equal to EI times of the stiffness matrix of the member CD multiplied by $\theta_2, \theta_6, \delta_9$ and δ_7 plus fixed end moments of the member CD which is minus 31.111 that is the fixed end moment you can see here of the member CD and this copying it there.

35.556 minus 23.333 minus 26.667; once I do this because if we look at these values you. Now θ_6, θ_9 and θ_7 will be 0; they are restrained degrees and θ_2 value is known to us θ_2 is $1 / 5.193 EI$ of 185.397; substituting I will get M_2, M_6, V_9 and V_7 as 16.479, 59.369 minus 11.444 minus 38.556.

I request you to please check these values yourself and verify them. So, now, we have computed the end moments in all the cases let us check them now.

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So, I have 3 member of the beam and the frame M 1, M 3, let us mark the degrees of freedom in the given frame the degrees of freedom in the given frame are marked again for our comparison which is theta 1, theta 2, theta 3, delta 4, delta 5, theta 6, delta 7, delta 8 and delta 9.

So, let us find out the values we will enter this values now here I am writing this values this is for AB which is M 1, M 3, V 9 and V 4; this is for the member AB which is actually equal to minus 28.487, 39.759 and so on; let us mark it here for the member AB. So, this corresponds to first degree. So, minus clockwise 28.487, M 3 is plus. So, anticlockwise 39.759 and V 9 is to the right positive. So, the V 9 is negative; so 34.123 and so on.

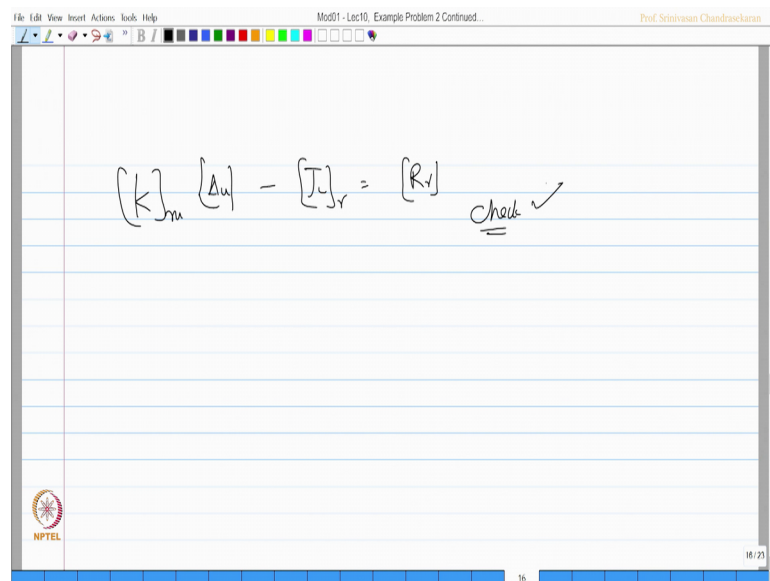
Similarly, one can plot for M BC, I am plotting the value directly. So, which is 48.487 and this is 13.511 and this is 9.498 and this is 40.502 and this value for CD 16.479, I want you to check this values 11.444 and this is 59.369 and 38.556; let us check this values.

So, this is subjected to some lateral load of 12 kilo Newton per meter, this should be actually equal to the reactions if you look at the sum of these moments 28 and 48; the net balance is 20 which is acting and disjoint. Similarly, if you look at 16 and 15, 13 and 16, you get the next value moment as 30 is available in the frame here. So, these 2 are

matched similarly if you look at the load 50, if you look at add these reactions nearly to 50.

Similarly, if you look at these loads twenty and thirty and add these reactions they will add up to 50. So, one can check the fixed end moments and end shears as you computed and one can really solve the like this.

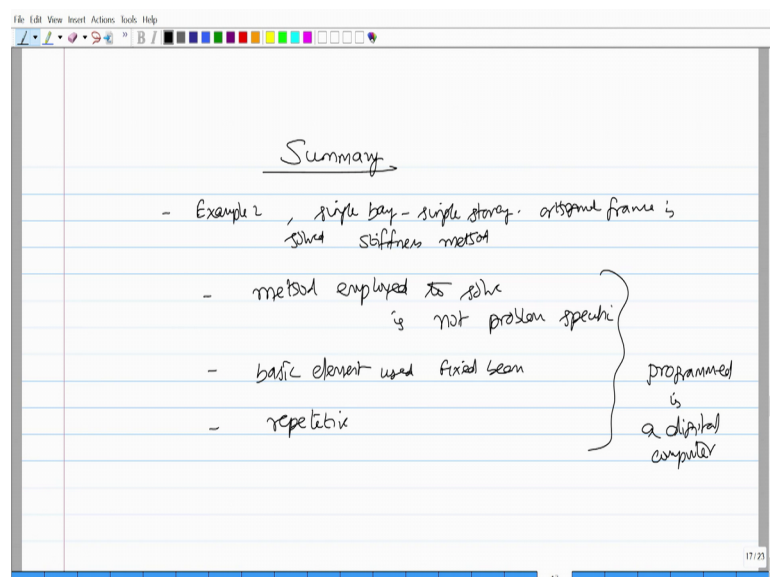
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The screenshot shows a digital whiteboard interface with a menu bar at the top (File, Edit, View, Insert, Actions, Tools, Help) and a toolbar with various drawing tools. The main area contains the handwritten equation $[K]_m [\Delta] - [J]_r = [R]$. To the right of the equation, the word "check" is written with a checkmark. The NPTEL logo is visible in the bottom left corner, and the slide number "16" is in the bottom right corner.

Further you can also used this relationship $K \Delta - J_r$ should be the end reactions r .

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- The screenshot shows a digital whiteboard interface with a menu bar at the top (File, Edit, View, Insert, Actions, Tools, Help) and a toolbar with various drawing tools. The main area contains the handwritten word "Summary" followed by a list of bullet points:
- Example 2, single bay - single story portal frame is solved stiffness method
 - method employed to solve is not problem specific
 - basic element used fixed beam
 - repetitive
- A large curly bracket on the right side of the list groups the last three items, with the text "programmed is a digital computer" written next to it. The NPTEL logo is visible in the bottom left corner, and the slide number "17" is in the bottom right corner.

I want you to do as a simple assignment and check and show the values are as same as this. So, friends let us write down the summary for this problem.

We have understood that the example 2 which is a single bay single story orthogonal structure is solved using stiffness method. So, we have seen that the method employed to solve this problem is not problem specific, it is very general; is it not and the basic element used is a fixed beam; is it not and further the process is repetitive. So, keeping this in mind this can be easily programmed in a digital computer.

So, I will also give you the computer **codes** to solve this problem. Later, I want you to first do it by hand and try to have a comfortable solution. Then we can compare these answers by writing computer program run the same problem in the program and compare the results.

Thank you very much.