Computer Methods of Analysis of Offshore Structures
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Module – 01
Lecture – 09
Example Problem 2 (Part – 1)

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So, friends, we will continue with the discussion on module 1. This lecture will be lecture 9 where we will solve another example problem of a frame. Before that let us add slightly more discussions to the last problem what we solved in the previous lecture. The last problem; what we had is an example one of a 2 span continuous beam as you see in the screen theta one theta 2 are unrestrained, there is a freedom marked in green color, theta 3, delta 4, 5 and 6 are restrained; there is a freedom marked in red color keeping fixed beam as a basic module; span A B will have degrees of freedom labels as theta 3 theta 1, delta 4 and delta 5 taking A as the jth end and B as the kth end.

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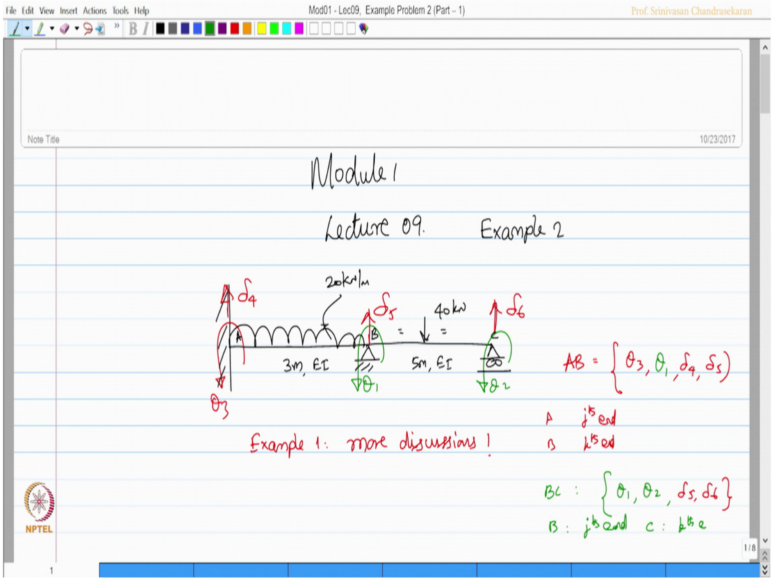

Similarly, span B C will have degrees of freedom theta 1, theta 2, delta 5 and delta 6, both will be the jth end and C will be the kth end taking this at the module.

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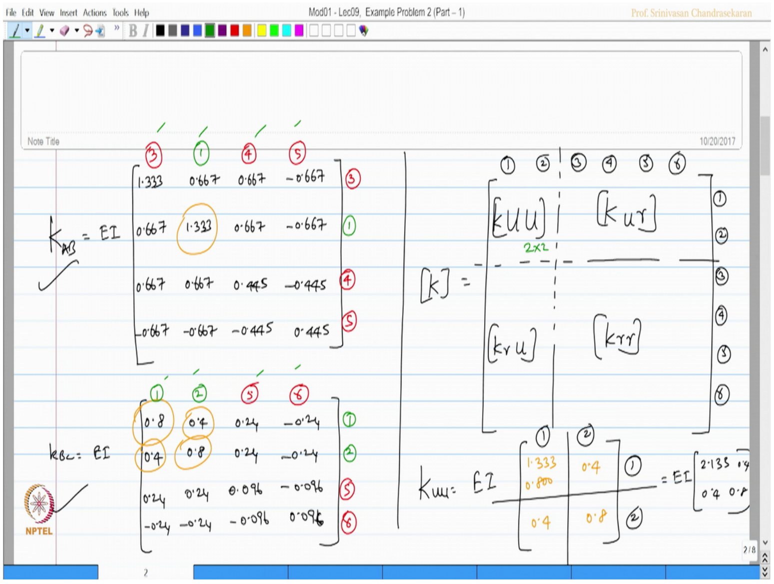

We have generated for example, 3 1 4 5; 3 1 4 5; we generated this stiffness matrix similarly for K B C 1 2 5 6; 1 2 5 6 generated, we partitioned the matrix and got K u u as 2 by 2 and we picked up K u u founded K u inverse and applied the joint loads computed the joint load vector partitioned the matrix for unrestrained restrained and unrestrained vector is picked up.

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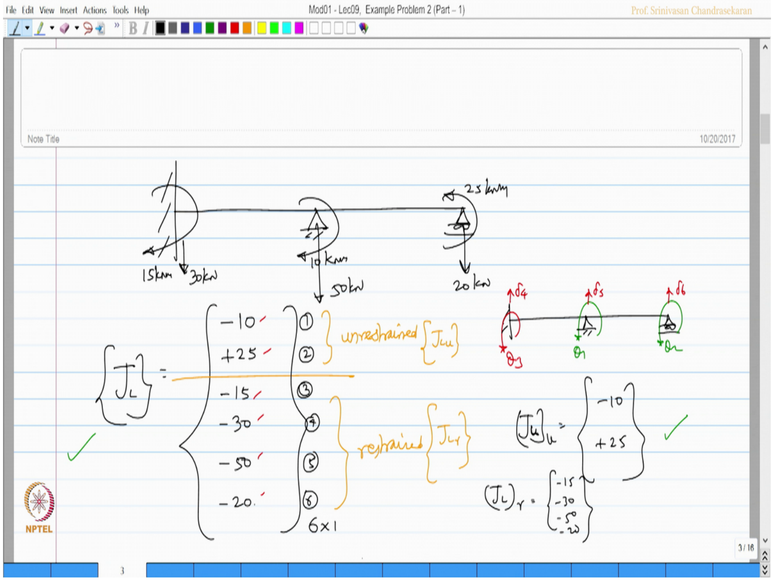

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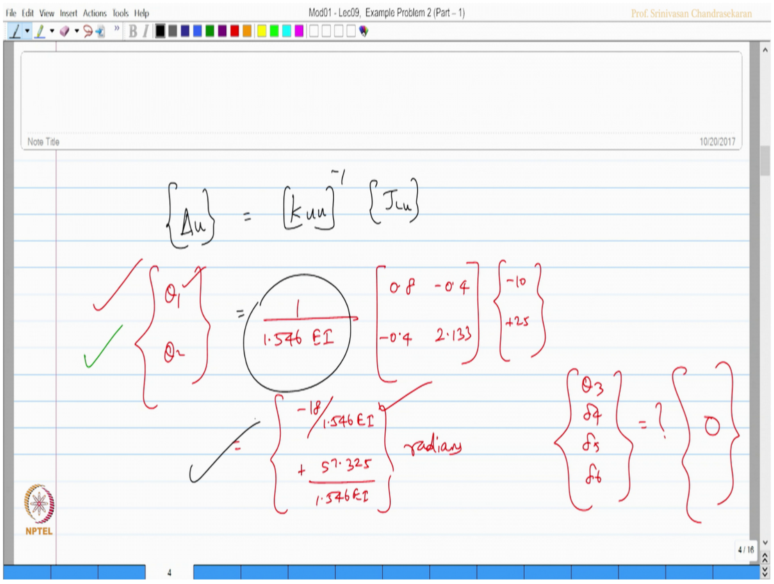

Then we solved this for delta u.

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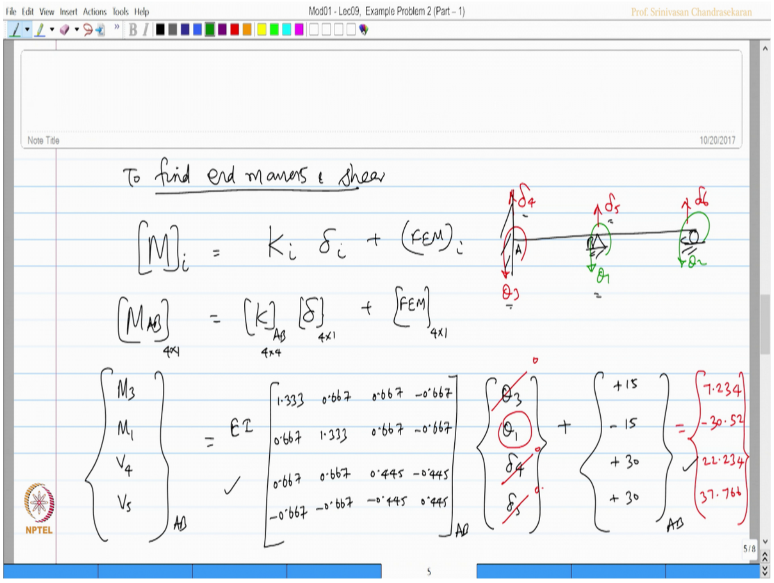

Then we apply and found out the moments; M moments and shears in both the spans A B and B C and we have the results; here A B is this value and B C is this value any way from the free body diagram we checked and showed that the values what you have in a and B C are matching; what we adhere is another way of checking this which will now discuss.

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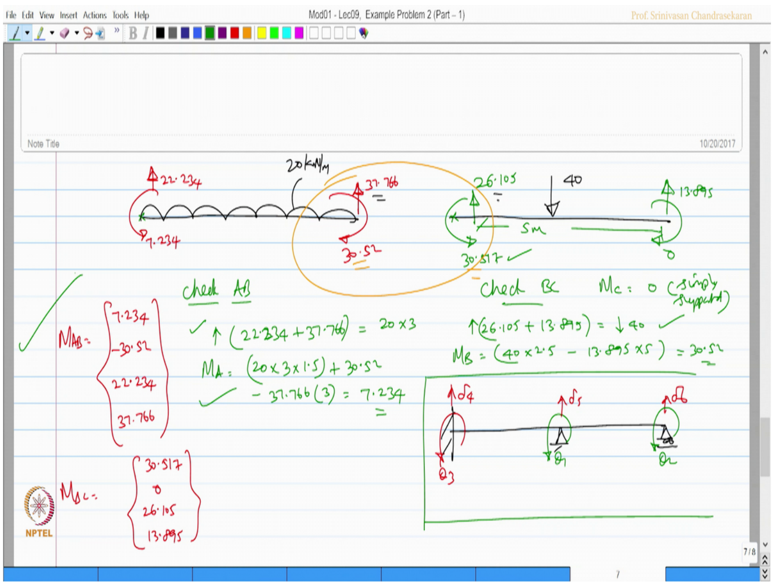

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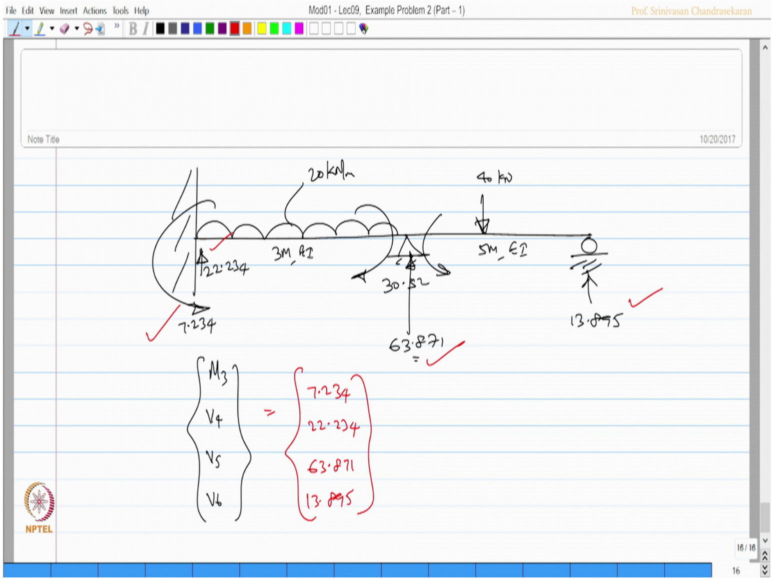

So, this was the solution we had now we want to check this once again if you look at the matrix the K matrix.

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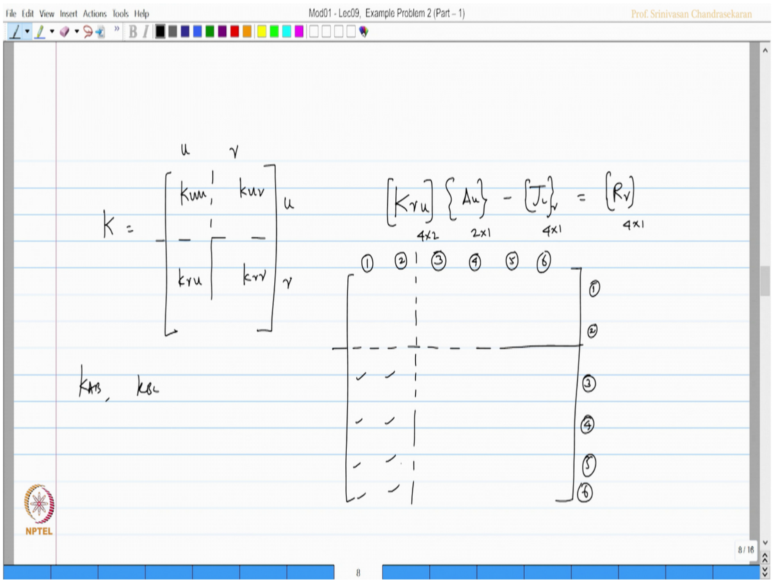

We said that K has got sub matrices K u u, K u r, K r u; K r r which we have partitioned the matrix. So, this is unrestrained restrained unrestrained restrained.

So, we can also say K r u multiplied by delta u minus J L r will be actually equal to the restrained vector r. So, you can see this going to be a 4 by 2 matrix; if you look at the full stiffness matrix of size the full stiffness matrix will be a size 6 by 6. So, this will be 4 by

2, this will be 2 by 1; this again J L r if you see the vector this going to be 4 by 1; it is going to be 4 by 1 and I will get ultimately 4 by 1 as my result.

So, from the given matrix K A B from the given matrix K A B and K B C can always pick up the labels for K r u. So, let us write down this value as a K matrix will have labels 1, 2, 3, 4, 5, 6; similarly 1, 2, 3, 4, 5 and 6. So, I have a partition at 2 by 2 because that is my unrestrained degree of freedom. So, now, these values can be picked up from the respective K matrices K A B and K B C and this can be filled up.

So, I leave this exercise to you.

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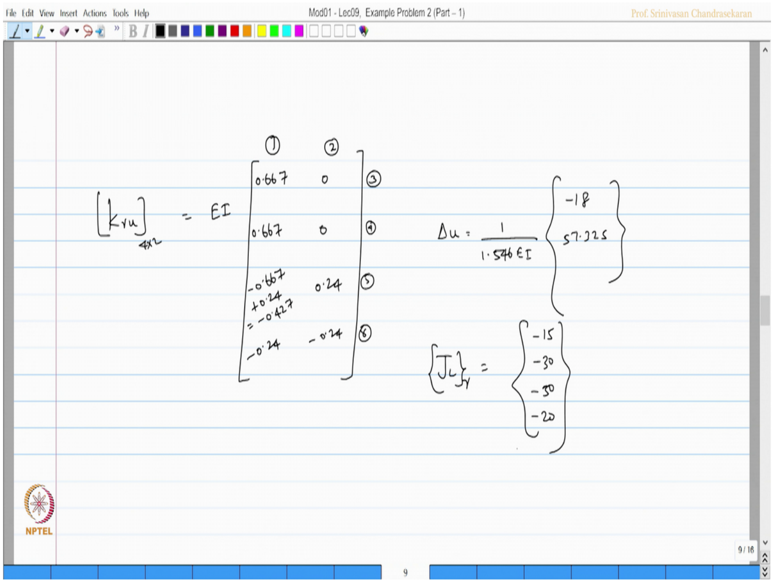

So, I fill up this matrix and I get K r u matrix 4 by 2 as below which is going to be E I of this is first row first column second column r. So, it was going to be 3, 4, 5 and 6. So, this value is 0.667; this value is 0.667 and this value is going to be minus 0.667 plus 0.24 which will be minus 0.427 in this is minus 0.24. So, this will be 0; this will be 0.24; this is minus 0.24.

So, that is going to be my K r u matrix delta u v already have I am rewriting it again 1.546 E I. So, we have this value here delta u which is 1.546 E I; 0.8 minus 0.4 and so on and I get delta u as this value; let us rewrite it here minus 18 and 57.325 that is what the value we have here 1.546 common 18 and 3.25. So, this; what we are having here.

Now, we look at the J L r vector this what I am looking at J L r, I can write it here; J L r is going to be minus 15 minus 30 minus 50 and minus 20; let me rewrite this matrix back, again here J L r is going to be minus 15 minus 30 minus 50 and minus 20.

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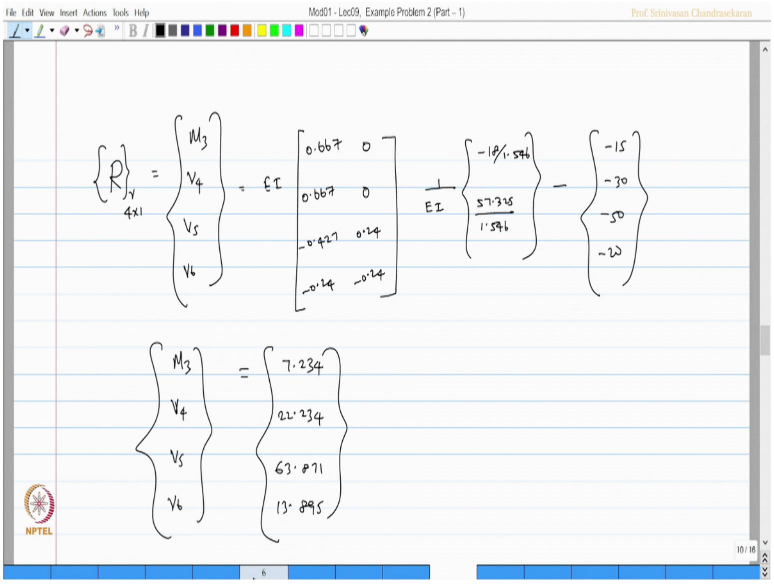

Let us substitute in that equation to get R r which is the restrained vector of 4 by 1 which actually V equal to look at the original problem a restrained degrees of freedom are theta 3, delta 4 the restrained degrees are theta 3, delta 4, delta 5 and delta 6 correspondingly; the reactions will be M 3, V 4, V 5 and V 6.

So, let us go back here and do that which will be M 3, V 4, V 5, V 6 which can be obtained as E I times of J L r. So, 0.667, 0, 0.667 that is my K r u minus 0.427, 0.24 minus 0.24 minus 0.24 multiplied by 1 by E I minus 18 by 1.546, 57.325 by 1.546 and subtract this with my J L r which is minus 15 minus 30 minus 50 and minus 20. If I do this simplification, I get M 3, V 4, V 5 and V 6 as 7.234, 22.234, 63.871 and 13.895.

So, let us compare the results; what we have with this here. So, I can say the M 3, V 4, V 5 and v 6 will be essentially this value 7.234 and this value 22.234 and this value which is 63.871 and this value which is 13.895. So, if you compare this value with what we have here is exactly the same. So, one can check the solution by this method also. So, that the solution obtained by the previous steps and the final step are compared and verified.

Having said this, let us move on to the second example problem to solve a single bay single story frame. So, let us take example 2.

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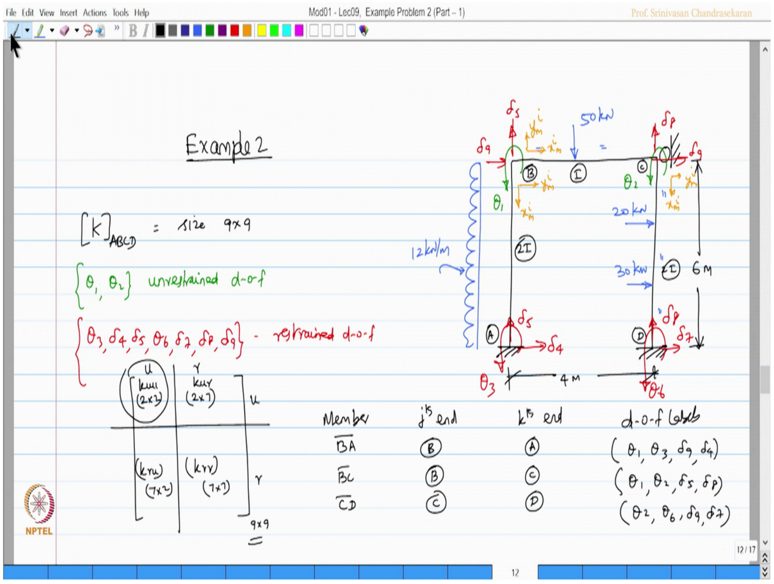

A single bay single story frame and try to solve this problem. So, let both the ends be fixed, let us also put the restrained here let us apply a loading to this part of the frame which is given to be 12 kilo Newton per meter. Let us apply a point load here which is at the center of intensity 50 kilo Newton. Let us put a 2 point loads here of intensity 20 kilo Newton and 30 kilo Newton of equal space.

Let the dimensions of the frame be height of 6 meters and width of 4 meters. Let us have E I constant throughout. Now the first step as you see will be to mark the degrees of freedom. So, let us mark unrestrained degree then restrained degree unrestrained degree we will use green color. So, there will be rotation here which is theta 1; there can be rotation here which is theta 2, these are the 2 unrestrained degrees what we have and the restrained degrees are going to be theta 3, delta 4, delta 5 and no accelerate deformation therefore, delta 5, theta 6 delta 7 and delta 8, no accelerate deformation therefore, delta 8 and delta 9 and delta 10.

So, we have for in information from this the stiffness matrix of the complete frame A B C D that is this is A, this is B, this is C, this is D will be of size 9 by 9 theta 1 and theta 2 are unrestrained degrees and theta 3, delta 4, delta 5 theta 6 delta 7, delta 8 and delta 9 all restrained degrees therefore, the matrix will have a partition the size of the matrix will be unrestrained and restrained this is unrestrained restrained unrestrained restrained, this will be 2 by 2 the total size is 9 by 9.

Therefore this is going to be 2 rows and 7 columns that is K u u, this is K u r, this K r u which will be 5 rows and 2 columns K r r will be 5 rows and 5 columns. So, we have to invert only a 2 by 2 matrix for this problem that is number 1. Let us also write another statement. From this figure let us talk about the member let say what is my jth end and kth end this is the orientation member and what are the degrees of freedom labels let us talk about member A B. So, I am writing this as member B A. So, jth end is A B kth end is A A. So, becomes my orientation.

So, this is my x M i, this is my y M i for this member. So, degrees of labels could be theta 1, theta 3, delta 9 and delta 4. Similarly member B C B end is J and C end is K degrees of freedom are going to be let say this is x M i; this is y M i for this member B C. So, the degrees of freedom labels are going to be theta 1, theta 2, delta 5 and delta 8. Similarly for the member C, D C is a jth end, D is a kth end. So, I am going to mark this again as x M i and y M i therefore, labels are going to be theta 2 theta 6 delta 9 delta 7.

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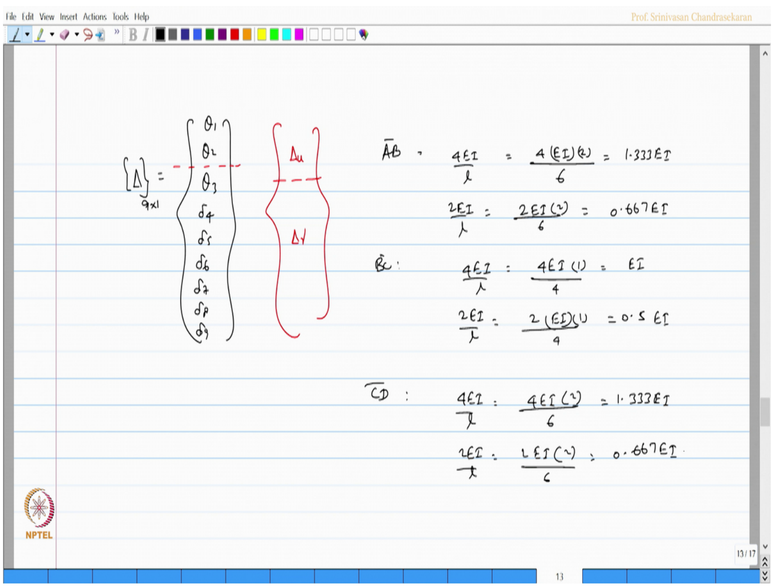

Having said this let us mark the displacement vector rotational and translational both which is going to be this is going to be 7 by 2 and this is going to be 7 by 7 total is 9. So, this is going to be a vector of 9 by one which will have theta 1, theta 2, then theta 3, delta 4, delta 5, 6, 7, 8 and 9 where I am going to do a partition at this level. So, this is also written as delta u and delta r also for the member A B let us try to find this 4 E I by L value; we need the rotational constants which is going to be 4 E I; let us take the E I value for this member is 2 I, this member is I and this member is 2 I.

Let us not take E I as constant; let us vary the E I. So, in that case twice by L is 4 meters, it will give me 1.333 E I. Similarly 2 E I by L for the beam will be 2 E I into 2 by 6 which is 0.667 E I for the member B C 4 E I by L could be 4 E I into 1 by 4 which is E I 2 E I by L could be 2 E I into 1 by 4 which is 0.5 E I for the member C D 4 E I by L could be 4 E I into 2 by 6 which is 1.333 E I, then 2 E I by L will be 2 E I into 2 by 6 which is 0.667 E I.