Computer Methods of Analysis of Offshore Structures
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Module – 01
Lecture – 08
Example Problem: Continuous Beam (Part – 2)

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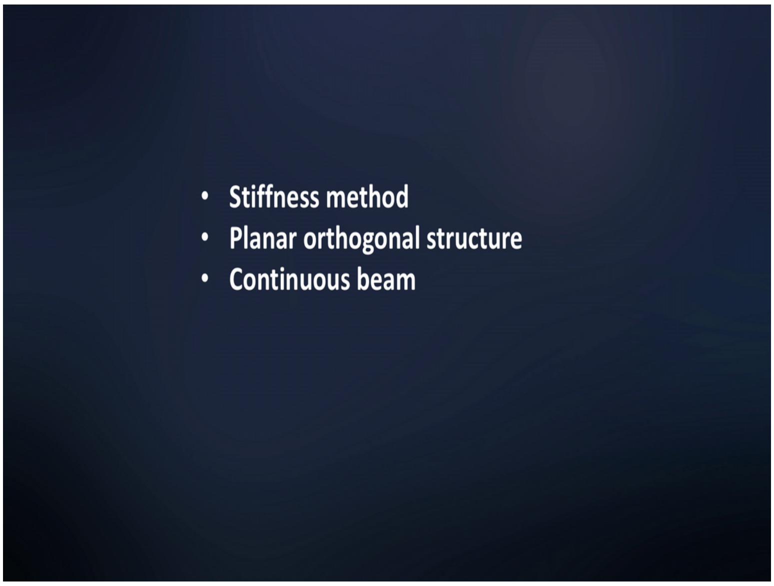

Once I have the inverse with me, I can now compute the unrestrained displacement. To do that, we need to estimate the fixed end moments.

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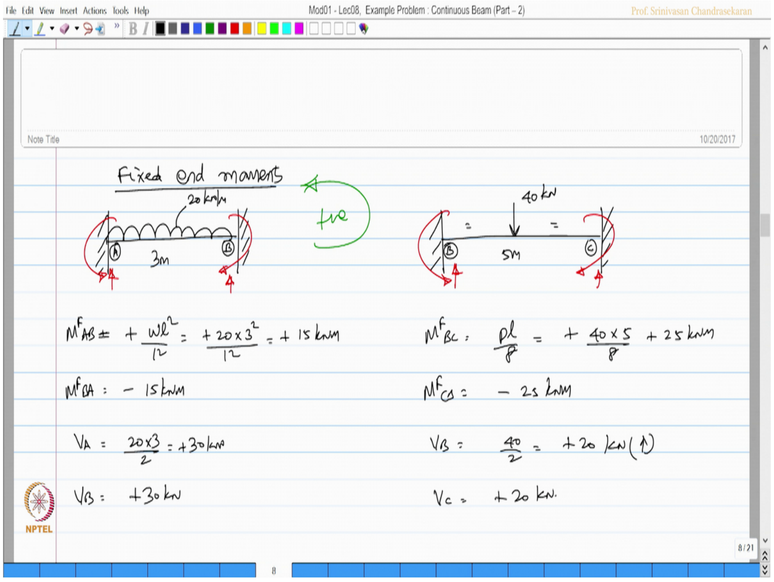

Let us take 2 beams parallel. So, beam A, B of span 3 meters subjected to uniform distributed load which is 20 kilo Newton per meter this is A and this is B the other beam is B, C which is also fixed at both the ends, do not bother about the original condition and the problem for as the basic element is a fixed beam.

So, B, C central concentrated load of intensity 40 kilo Newton equally spaced and this is 5 meters and based upon the equations we gave in the last lecture, I can compute M F A B, M F B A and the reactions at A and B can compute also M F B C and M F C B and the reactions at B and C respectively. So, let us do that M F A B will be plus w l square by 12 which is plus 20 into 3 square by 12 which will be plus 15 kilo Newton meter.

M F B A will be of the same value, but minus 15 kilo Newton meter because of the simple reason anticlockwise moments are positive, I want to find the reactions. So, for the given problem; it is easy to know that V A will be 20 into 3 by 2 that is 30 kilo Newton which is upward positive. Similarly, V B is also positive; let us go to span B C; M F B C will be p l by 8 which will be plus 40 into 5 by 8 which becomes plus 25 kilo Newton meter M F C B will be minus 25 kilo Newton meter because this is clockwise and anticlockwise is positive.

And V B is 40 by 2 which is plus 20 kilo Newton because it is upward; upward is positive V C is also upward of 20 kilo Newton these are the fixed end moments. So, the joint clause will be reversed of this.

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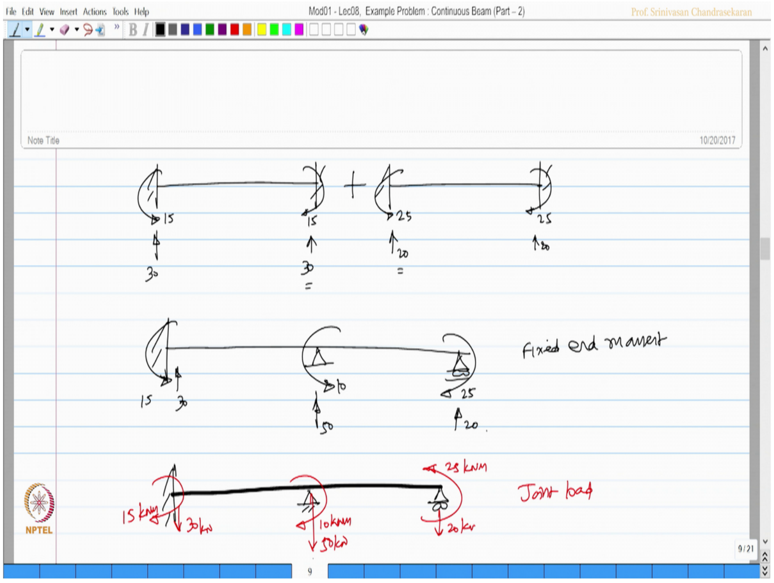

That has now summarize this I have 2 beams A B and B C; the values are this is 15, this is 15, this value is 30, this value is 30; that what we got from their last slide 15, 15, 30, 30.

Similarly, this value is 25, this is again 25 and this is 20, this is 20; I need to add this sum this make a single being. So, let us say this is as usual 15, but this one to be clockwise 15 anticlockwise 25; anticlockwise 10 and this is 25 and this upward is 30 and this upward is 50 that is 30 plus 20 and this upward is 20; this is my fixed end moments; it is not the joint load please understand

Now, I want to create the joint load I draw the beam. Now let us create the joint loads by reversing it. So, anticlockwise; so, make it clockwise 15 kilo Newton meter anticlockwise. So, make it clockwise which is 10 kilo Newton meter clockwise; make it anticlockwise 25 kilo Newton meter upward. So, make it downward 30 upward, make it downward 50 kilo Newton and upward make it downward 20 kilo Newton. So, this is my joint load.

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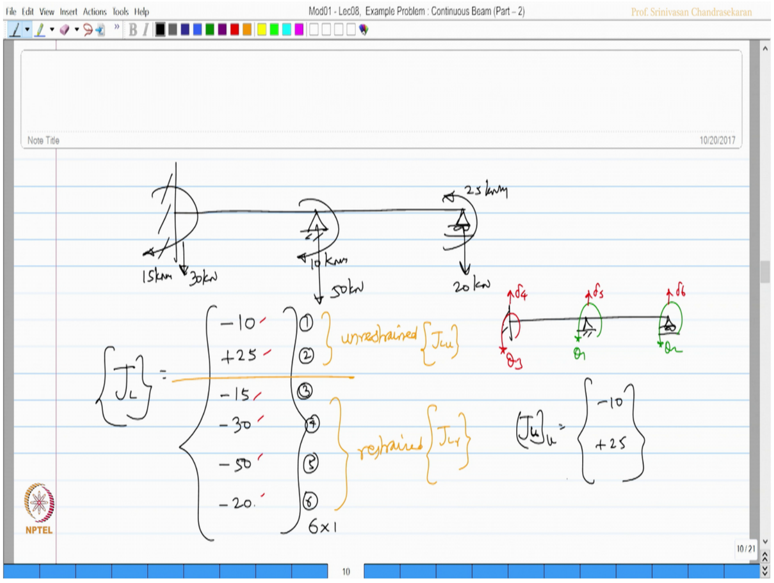

Now, let us compare this with my standard convention what I had. So, the joint loads are clockwise 15 kilo Newton meter clockwise 10 kilo Newton meter anticlockwise 25 kilo Newton meter; this is downward 30 kilo Newton. This also downward 50 kilo Newton, this is downward 20 kilo Newton, let us compare this with our standard and enter the joint load vector.

The joint load vector will have 6 rows and one column the degrees of freedom are 1, 2, 3, 4, 5 and 6 out of which there is a partition at because these are unrestrained degrees of freedom and these are restrained degrees of freedom. So, I call this vector as joint load as joint load unrestrained; I call this vector as joint load restrained. So, if you look at the degree of freedom one from this problem one corresponds to the rotation here 2 corresponds; so, 3, 4, 5 and 6; so, same logic here.

One corresponds to this anticlockwise I should say minus 10 the minus is because if you compare this is clockwise joint load, but original convention is anticlockwise. So, minus 10; similarly, this is anticlockwise; the original is also anticlockwise; therefore, we can say it is plus 25; 3 by that logic will be minus 15, 4 will be minus 30, 5 will be minus 50 and 6 will be minus 20.

You can compare you want; we can also draw that particular beam here for our understanding and mark the degrees of freedom; this was theta 1, this was theta 2 unrestrained, then this is theta 3 this is 4, this is 5 and this is 6 because these 2 are unrestrained and these 4 are restrained. So, this is clockwise, but unrestrained anticlockwise. So, minus 10 anticlockwise; anticlockwise plus 25, then clockwise, but this is anticlockwise. So, minus 50 upward, but this is downward minus 30 upward, this is downward. So, minus 50 upward this downward minus 20; so, I have this.

So, now I can write the J l u vector that is unrestrained vector is minus 10 and plus 25 correct; once I have this, I can now say the joint load displacement of unrestrained will be k u u inverse of that of J l u.

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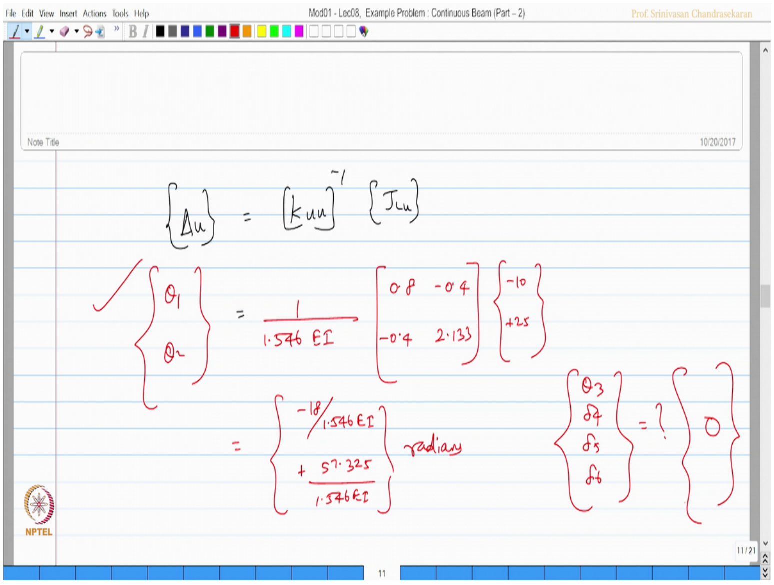

So, a k u u inverse if you see we already worked out that it is with us here this is my k u u inverse this is my k u u inverse, I will just rewrite it there which will be 1 by 1.546 E I multiplied by point 8.4 minus 0.4 and 2.133 and this should be multiplied by the unrestrained joint load vector which is minus 10 plus 25.

So, minus 10 plus 25; so, I get the unrestrained which is the theta 1 and theta 2 which will be equal to minus 18 by 1.546 E I and this theta 2 is going to be plus 57.325 by 1.546 E I in radians. So, we have solved this for theta 1, but we have to find the end moments that is important; we have only solved the unrestrained displacements may ask you a question; what will be the vector of restrained displacements which are; what are the restrained displacements in this problem theta 3, delta 4, delta 5, delta 6; what will be this theta 3, delta 4, delta 5, delta 6; what will be this vector because this are restrained degrees.

So, as you correctly guessed, it will be 0; there is no need to work out that.

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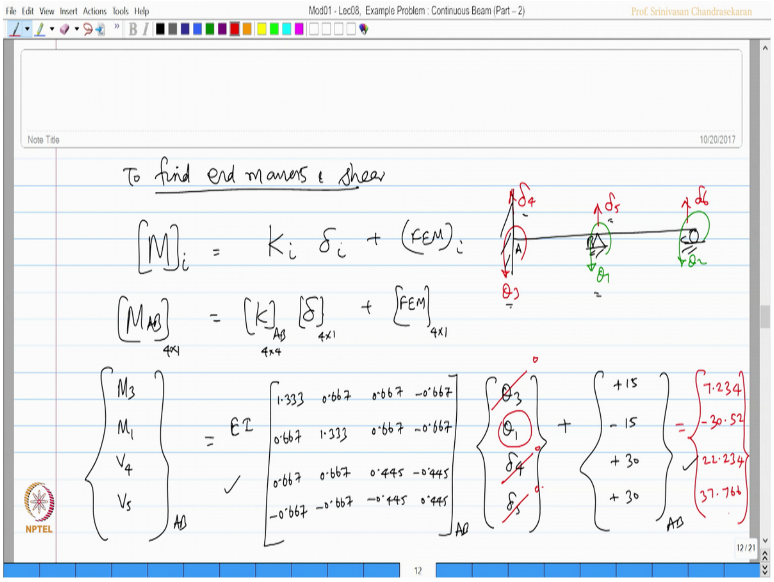

Now, our next job is to find due to the loads end moments and shear as a last step the general equations is for finding M moment of ith beam we have to say K i and displacement i plus fixed end moment of i. So, let us expand this for A B. So, M A B which will be 4 by 4 which will be K of A B which will be again 4 by 4 and displacement vector of A B which is 4 by one plus fixed end moments of A B which is 4 by 1.

So, this will be 4 by 1. So, what will this value for A B? So, let us again draw the problem this was the original problem. So, this was the degrees of freedom theta 1 theta 2, theta 3, delta 4, delta 5 and delta 6. So, for M A B, the values will be this is A; this is B and this is C M A B; the values will be as we correctly guessed is going to be M 3 that is corresponding to this M 1 corresponding to this shear 4 corresponding to this shear 5 corresponding to this; this is for A B; I have to multiple this with K matrix of A B which already we had; we can see here K matrix of A B and K matrix of B C separately.

I am just rewriting that the K matrix of A B which will be 1.333, 0.667, 0.667 minus 0.667; 0.667; 1.333; 0.667 minus 0.667, 0.667; 0.667, 0.445 minus 0.445 minus 0.667 minus 0.667 minus 0.445 and 0.445 this is for the beam A B.

And I want to now multiple this with the displacements of A B which will be theta 3 theta 1, delta 4 and delta 5 plus the fixed end moments of the member A B, we also already have this here fixed end moments of the member A B is available here. So, we can directly write those values there, I am just doing it here is going to be plus 15 minus 15 plus 30 plus 30 this fixed end moments of A B. So, simplify this, but the question is this matrix is known this vector is known; I have to estimate this vector; what happens to this vector very interesting.

Theta 3 is 0, delta 4 is 0 and delta 5 is 0 and theta 1 is known we can see here theta 1 is known. So, substitute that value here the value here and multiple process it and get that vector A B which will be actually equal to I write the values here 7.234 minus 30.52, 22.234; 37.766 that is for A B. Similarly I can do for B C. So, I can do for B C, I again draw the problem.

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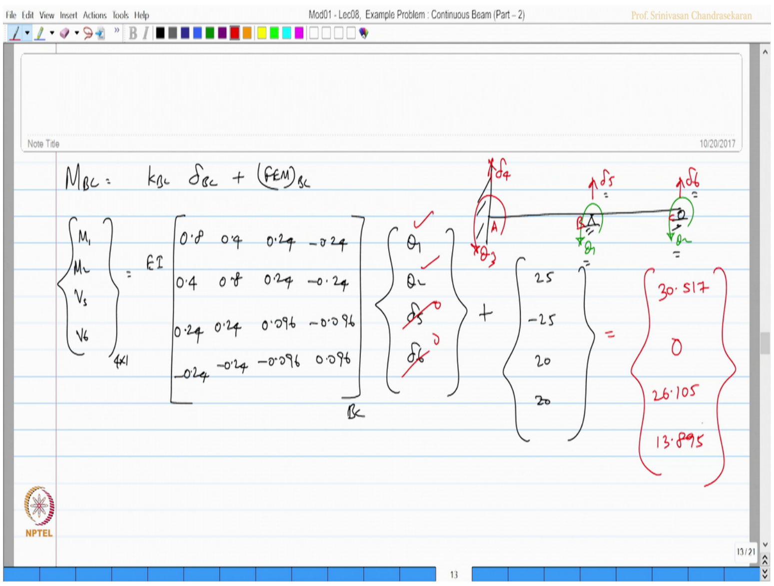

This is theta 1, this is theta 2, this is theta 3, this is delta 4, delta 5 and delta 6. This is A, this is B and this is C this is B C. So, let us do it for B C, M B C will be actually K B C delta B C plus fixed end moments of B C; M B C will have a vector which is 4 by one what will be the values M 1 corresponding to this M 2 corresponding to this; this should become 0 because this simple supported end and V 5 correspond to this and V 6 correspond to this.

Which will be equal to k B C we have the stiffness matrix of B C, I am writing it here we already derive that. So, I am just writing it here this is for the span B C and this will multiplied by delta B C which will be equal to theta 1, theta 2, delta 5 and delta 6; add this to the fixed end moments which we already have for the span B C which will be 25 minus 25 plus 20 and plus 20. So, very interesting delta 5 and delta 6 are 0 because they are restrain marked in red also theta 1 theta 2 already known to us.

So, substitute here get the value and you will see that that value will be actually equal to 30.517; this become 0, 26.105, 13.895. So, let us plot these values.

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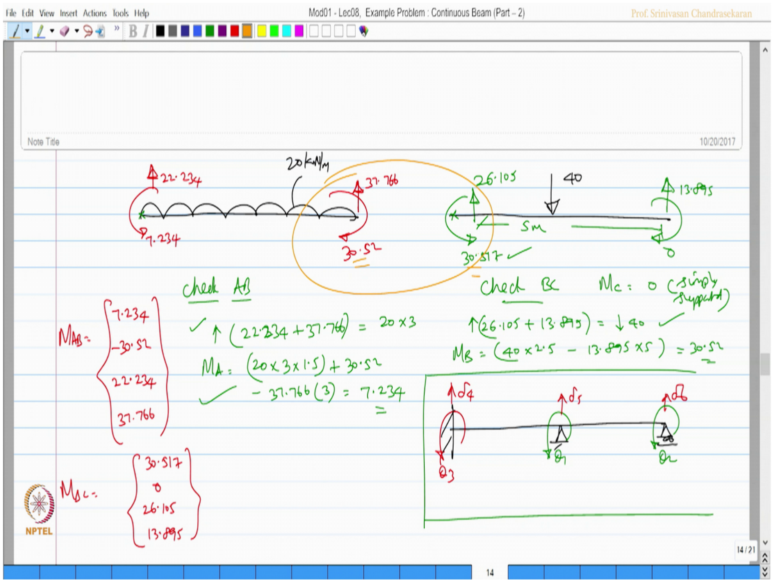

So, span A B subjected to a load of 20 kilo Newton per meter, span B C subject to a central load of 40. So, we say let us write down the values of the vector the original beam problem was like this; this was theta 1, theta 2 and this is theta 3, delta 4, delta 5, delta 6 and this vector; we look at this vector A B this is 7.234 and so on; I am just marking them. So, I am copying this vector back again here.

So, 7.234; I am marking this vector here, this is M A B 7.234 minus 30.52, 22.234 and 37.766 that is what we have here then B C, we have this vector, I am just rewriting it, here M B C is actually equal to 30.517, 0, 26.105 and 13.895 keeping this, let us now super impose this value in the original figure.

So, now this is plus. So, this is going to be anticlockwise 7.234; this is minus. So, clockwise 30.52; this is positive. So, upward 22.234 and this is upward 37.766; similarly for this positive. So, anticlockwise 30.517 and this value is 0 and this is upward 26.105 and this is also upward 13.895.

So, let us super impose this; interestingly, you will see that let us check this problem; right, I can even remove this figure this figure is actually not required at this moment. Now, let us check the span A B. So, checking the span A B the total upward force is actually 22.234 plus 37.766 which actually equal to 20 into 360 meters. So, it is checked if you take moment about of this point moment about A. So, moment about A will be 20 into 3 into 1.5 plus 30.52 minus 37.766 into 3 which will give you 7.234 anticlockwise which is also checked.

If you looked at span B C you will see M C is 0 because it is simply supported. So, a reaction 26.105 plus 13.895 is actually equal to downward reaction of 40. So, it is checked if you take moment about this point that is M B which will be equal to 40 into 2.5 because this span is 5 meters 2.5 minus 13.895 into 5 will be exactly equal to 30.52 which is here and you will also see that these reactions are matching 30.52 clockwise 30.52 anticlockwise.

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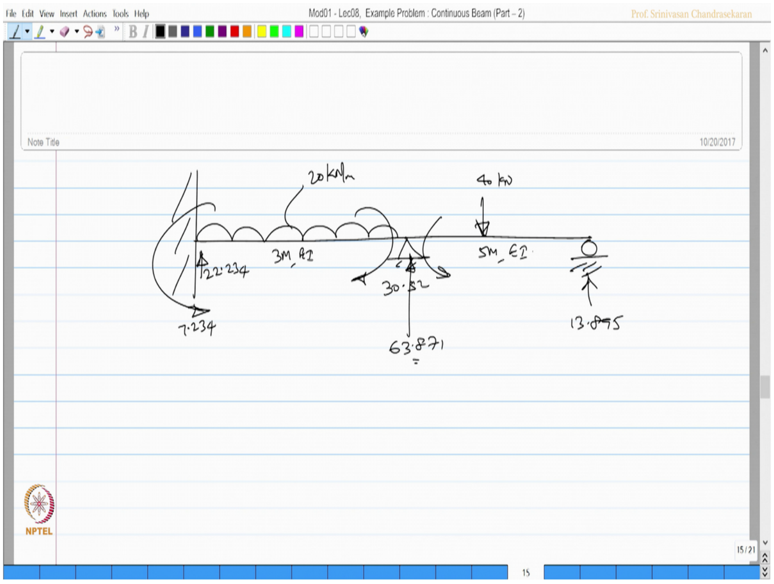

So, finally, I have a beam which is solved which is my summary for this load and for this load the reactions are 7.234, 30.52, 22.234, 63.871; 13.895 because this value will be the sum of these 2 reactions these 2 reactions. So, that is the whole problem which has been solved and this loading is 20 kilo Newton per meter and this is 40 kilo Newton; this is 3 meter E I, this is 5 meter E I. The problem is now solved; we can also check this by one more alternative which will discuss when we do the next problem.

Thank you very much.