Computer Methods of Analysis of Offshore Structures

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Module - 01
Lecture – 08
Example Problem: Continuous Beam (Part – 1)

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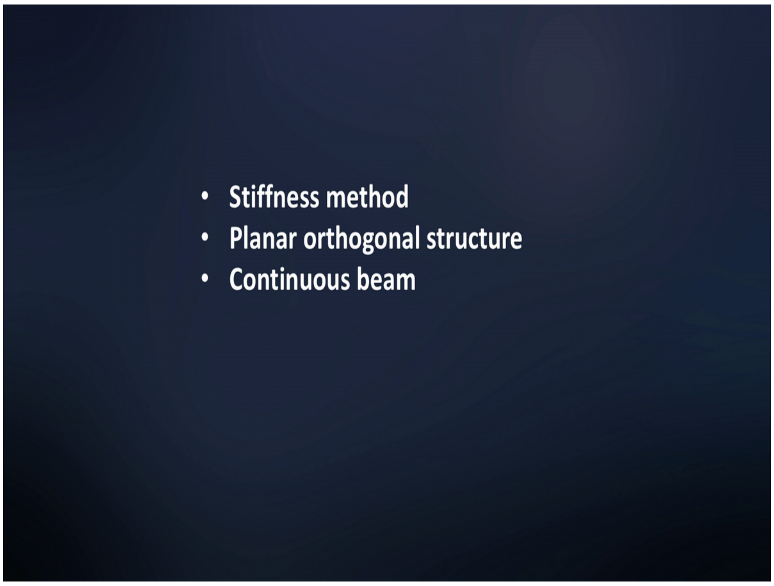

So, friends; let us continue with the example problem which is lecture 8 and module 1; we will pick up a very simple example to demonstrate this we will take a 2 span continuous beam as shown in the figure now.

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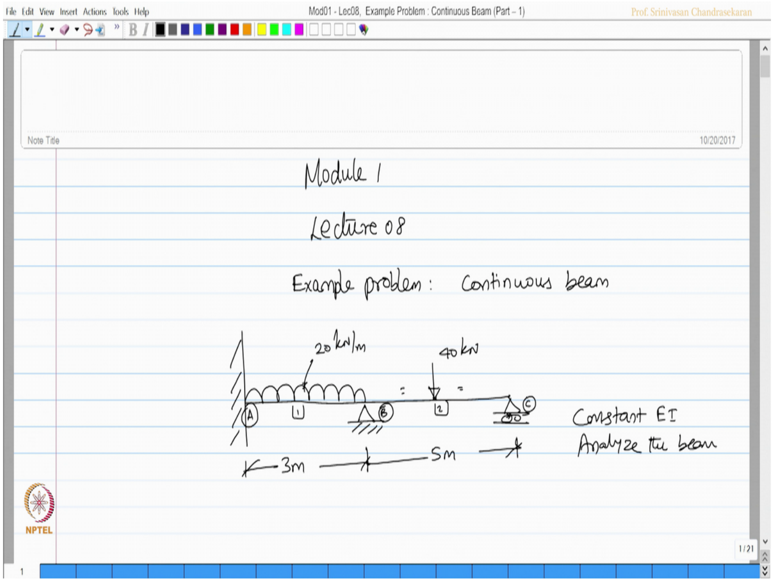

Let us say this span is 3 meters and this span is 5 meters; I call this as A, B and C; you can also name them as 1, 2, 1; 3.

So, this is my member one, member 2 subjected to some loading let us say the external loading in A B is an uniform distributed load of intensity 20 kilo Newton per meter and B C has got the concentrated load of 40 kilo Newton which is at the centre the beam has got constant E I and we need to find the reactions and end moments of this beam. So, analyze this beam that is the problem.

So, closely look at this problem, first support A is fixed support B and C or simply supported; there may be some unrestrained displacements and restrained displacements. So, we need to mark them first let us do that.

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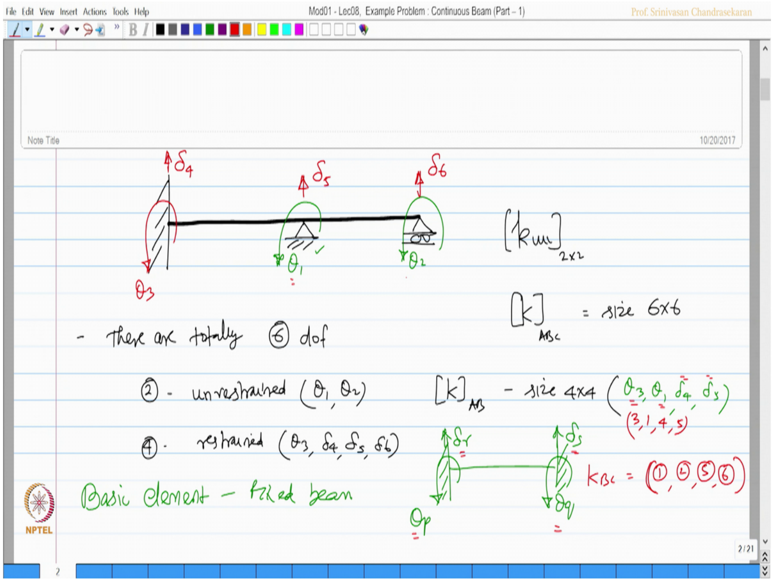

So, take the beam let us marked unrestrained displacements in green. So, fixed end cannot have a rotation. So, this can have a rotation, I call this has theta 1, I call this has theta 2, I need to continuously label the unrestrained displacements first, then the restrained displacements.

So, theta 1 theta 2 are 2 rotations which are free at ends B and C; then the restrained displacements rotation displacement and then displacements at B and C. So, this gives me the following hints, the following hints are there are totally 6 degrees of freedom out of which 2 are unrestrained which are theta 1 and theta 2 remaining 4 are restrained which are theta 3, delta 4, delta 5 and delta 6.

So, this also gives me a very clear idea that my unrestrained sub matrix K u u will be of size 2 by 2 because there are 2 unrestrained degrees of freedom and the total stiffness matrix of the member A, B C; the complete beam will be of size 6 by 6 the element stiffness matrix of A B will be of size 4 by 4 and what will be the labels of that to obtain the label; let us first compare this with the standard fixed beam. So, this is standard fixed beam which we said theta p theta q delta r and delta S.

So, the standard beam let us compare this beam with A B, I already said even though the support B is simply supported for us the basic element A is a fixed beam. So, therefore, the labels are for the K A B will be theta 3, theta 1, delta 4 and delta 5 that is the labels are 3 1 4 and 5; similarly for K B C the labels will be 1 2 5 and 6 please see the order rotations and translations rotations and translations.

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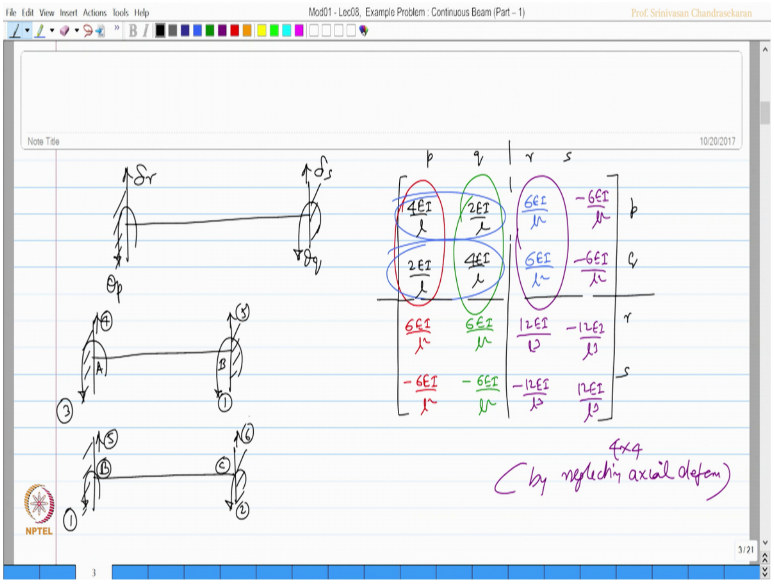

Rotations and translations that is how we have done; so, now, having said this, we already know for a standard fixed beam with theta p theta q anticlockwise delta r up and delta S up the standard stiffness matrix will have labels p q r s; p q r s we know this is actually 4 E I by l this is 2 E I by l these are the 2 rotation coefficients similarly this will be 4 E I by l and this will be 2 E I by l we can partition this matrix this way.

Remaining all can be derived we already said that adding these 2 divided by l that is 6 E I by l square and put a negative sign to this 6 E I by l square similarly adding these 2 divided by l 6 E I by l square put a negative sign 6 E I by l square then adding these 2 divide by l 6 E I by l square adding these 2 divide by l 6 E I by l square then adding these 2 divide by l 12 E I by l cube minus 12 E I by l cube and the last column is negative of the third column.

So, minus 6 E I by l square minus 6 E I by l square minus 12 E I by l cube and plus 12 E I by l cube that is a standard stiffness matrix of size 4 by 4 by neglecting axial deformation will use this for our problem let us see; now a case I want to draw 2 separate beams and mark the labels for me basic element is fixed beam; let us say A B will have labels 3 1 4 5. So, I should say A and B say it should be 3 1 4 and 5.

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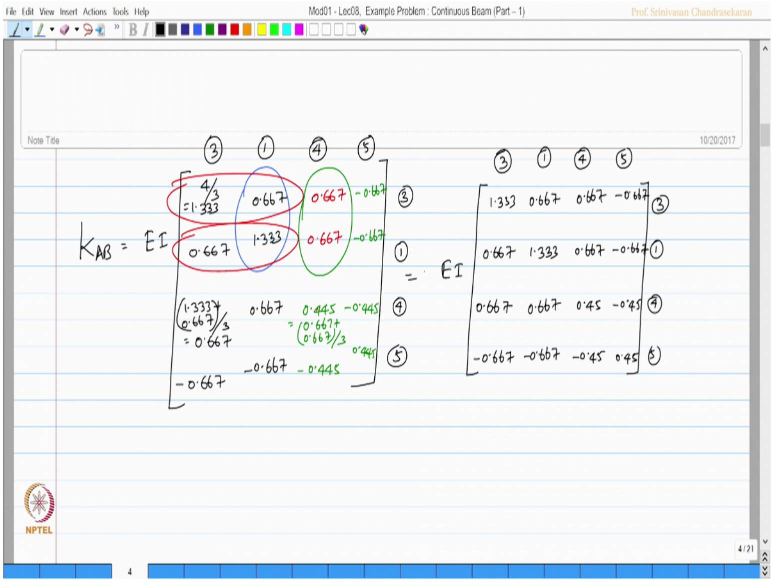

Similarly, for B C 1 2 and 5 and 6 1 2 5 and 6; let us now derive the matrix for K A B which is E I common the labels are going to be 3 1 4 and 5 3 1 4. So, this is going to be 4 E I by l this value is actually 4 E I by l which I should say is E I is constant. So, 4 by 3 because l is 3 in this case, you can see here span a b l is 3 meters.

So, 4 by 3 which is 1.333 and this will be half of that. So, 0.667; this will be some of these 2 by 3 again which is 1.333 plus 0.667 by l that is 3 meters which will be again 0.667 this will be minus of that value 0.667 and this will be 1.333 this is 0.667. So, this value will be add of these 2 addition of these 2 divide by l which is again going to be 0.667 and this is going to be minus 0.667.

As for as this value is concerned this will be addition of these 2 by l again which is the again going to be 0.667; similarly, this value will be addition of these 2 by l again. So, 0.667 and this values going to be addition of these 2 by l again which is 0.445 that is this will be actually equal to 0.667 plus 0.667 by 3 which will be 0.445 and this value will be minus 0.445 and the forth column is just reverse of this. So, it is going to be minus 0.667 minus 0.667 minus 0.445 and plus 0.445.

Let us have this matrix clearly written here K A B will have labels 3 1 4 5; 3 1 4 5; 1.333 0.667, 0.667 minus 0.667, 0.667, 1.333, 0.667 minus 0.667; this will be 0.667 again 0.667, 0.45 minus 0.45, this will be minus 0.667 minus 0.667 minus 0.45 and 0.45.

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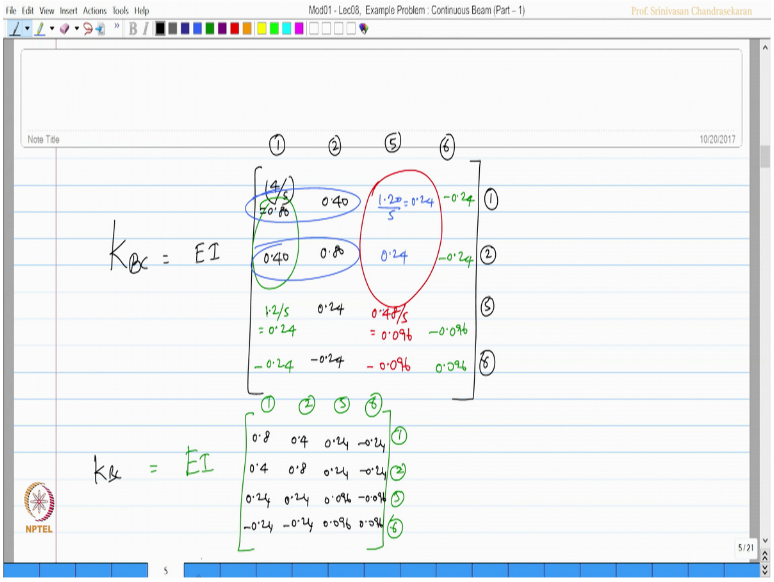

So, I have K A B as this. We can also do this for K B C; the same logic E I constant; let us look at label of K B C; look at K B C, B C will have labels 1 2 5 and 6 1 2 5 and 6 similarly 1 2 5 and 6. So, this is going to be 4 E I by l, 1 1 is going to be 4 E I by l, I write this value let us say is going to be 4 E I by l, E I is constant I will remove this; simply, it is going to be 4 by l; l is 5 meters which is 0.8 and this is going to be half of that which is 0.40 and this value will be some of these 2 by l.

That is 1.2 by 5 which is 0.24 this is minus 0.24 and this value is going to be 0.8, 0.4, again 0.24 minus 0.24 and this value will be sum of these 2 by l which is 1.20 by 5 which is 0.24, again this is again sum of these 2 which is again 0.24 and this value will be sum of these 2 by l which is 0.48 by 5 which is 0.096; this is minus 0.096.

The last column is simply the negative of the previous column. So, minus 0.24 minus 0.24 minus 0.096; 0.096, let us write down this matrix in the clearer way E I the labels are 1 2 5 and 6 1 2 5 and 6 the values are 0.8, 0.4, 0.24 minus 0.24 0.4, 0.8, 0.24 minus 0.24, 0.24, 0.24, 0.196096 minus 0.24 minus 0.24 minus 0.96096.

So, we have now K B C and K A B; let us write down both of them here, but straightly in a different manner.

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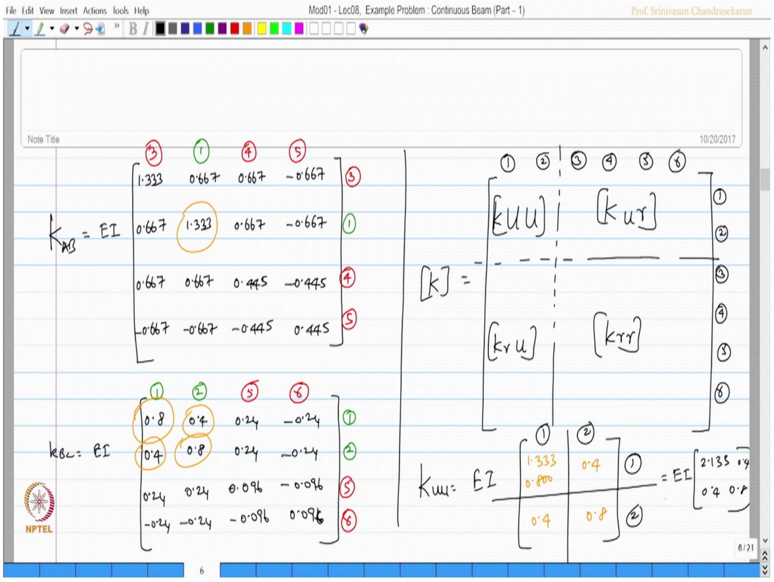

So, I am going to do this for both them the cases; this is K A B which is E I times of and this is K B C which is E I times of if you look at K A B the labels are 3, I am marking them in red because 3 is a restrained degree, then the next degree is 1 which is a unrestrained degree and then the restrained degree 4, then 5.

Similarly, 3 1 4 5 if you look at the labels of K B C; K B C labels are 1 2 5 1 6; you can see here you can see here 1 2 5 1 6. So, out of which 1 and 2 are unrestrained and 5 and 6 are restrained; so, restrained in red unrestrained in green. So, let us write down the values; we already have the matrix here and this coping the matrix again for our convenience. So, that is for K A B, similarly we can do for K B C copying the same matrix again 0.8, 0.4, 0.24 minus 0.24 done.

If you look at the K matrix and total; there will be 6 degrees of freedom. So, 1, 2, 3, 4, 5 and 6 out of which there is going to be a cross partition at 2 by 2. So, this is unrestrained; this is unrestrained row unrestrained column. So, I call this as K u u, this is unrestrained row restrained column this is restrained row unrestrained column; this is restrained row restrained column. So, I want to pick up and only write K u u value here is which is E I times of. So, what you 2 by 2 matrix.

Let us write down that which will be 1 2 1 1 2. So, from these 2 matrices let us pick up the values of 1 1; I am such circling them for our convenience 1 1 is this value. So, I write here 1.333, then I also have this value which is 0.800, then I want to pick up 1 2; 1

2 is this value which is 0.4 I want to pick up 2 1; 2 1 is this value which is again 0.4; I want to pick up 2 2 which is this value which is 0.8.

So, friends my K u u matrix is actually E I times of 2.133 0.4, 0.4 and 0.8.

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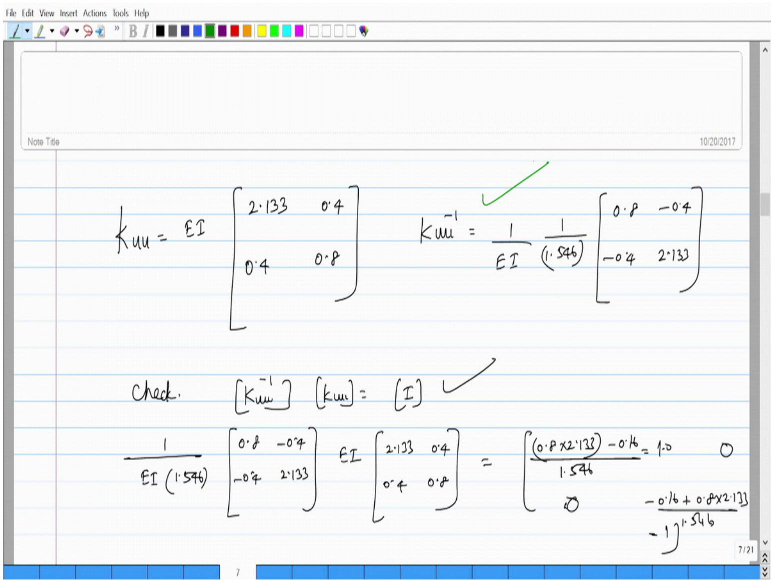

So, K u u is E times of 2.133, 0.4; 0.4 and 0.8; let us find K u u inverse which will be one by E I of determinant 5 4 6; I want you to compute this and check which I will get the values as 0.8; 2.133 minus 0.4 minus 0.4; I can also check that K u u inverse multiplied by K u u should give me identity matrix let us do that.

So, 1 by E I times of 1.546 multiplied by 0.8 minus 0.4, 0.4, 2.133 multiplied by E I times of 2.133, 0.4, 0.4 and 0.8; if you multiply this lets say; for example, this value is going to be 0.8 of 2.133 minus 0.16 divided by 1.546 will give you 1.0; this value if you see 0.8 into 0.4 minus 0.8 into 0.4 this become 0; if you look at this value minus 0.4 in 2.133 plus 0.4, 2.133, this will become 0 and if you look at this value minus 0.4 that is minus 0.16 plus 0.8 into 2.133 by 1.546 which will also become one. So, it is an identity matrix. So, this statement is proved and therefore, this inverse is correct.